Dynamics of Learning and Iterated Games
- Concise Notes -

MATH60007

Year 3

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Content from previous years to be known.

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Contents

0 Introduction

1 Replicator Dynamics for one population

Consider population, where individuals employ one of n pure strategies

1. x_i - frequency of strategy i in population
2. $(x_1, \ldots, x_n)$ a probability vector
3. $\Delta_n = \{x \in R; 0 \leq x_i \leq 1, \sum_{i=1}^{n} x_i = 1\}$

Take e_i the unit vector in the i^{th} dimension. n often fixed so we write Δ often...
Take a population, with invader who chooses strategy i against a strategy j to recieve payoff a_{ij}.
Given a population uses mixed strategy $(\ y\ )_1, \ldots, y_n$, with random matching - givin us the `linear payoff`

$$a_i(y) = \sum_{j=1} a_{ij} y_j = (Ay)_i$$

For A the matrix (a_{ij}).
Using a mixed strategy $x \in \Delta$ we have a payoff

$$Payoff(x, y) := x \cdot Ay$$

A probability vector $\hat{x} \in \Delta$ is called a **Nash Equilibrium (NE)** iff

$$x \cdot A\hat{x} \leq \hat{x}, \forall x \in \Delta$$

and a **strict Nash Equilibrium** if

$$x \cdot A\hat{x} < \hat{x} \cdot A$$