Couts-Goncalves-Rossi 2020

## Summary and motivation

This paper attempts to improve the de-smoothing of reutrns. I

## One-step

For a single asset, fit the following via MLE:

$$
\begin{aligned}
R_{jt} =& \mu_j + \sum_{h\in 0:H} L^h \theta_j \eta_{jt} \\
\eta_{jt} \sim& N\left(0, \sigma^2\right)
\end{aligned}
$$

Here $L$ is the lag operator, $\theta$ is the MA coefficient, $\eta$ is the fund-specific shock, and $\mu$ is the drift. For a portfolio:

$$
\begin{aligned}
\overline{R}_t =& w' R_t \\
=& w'\mu + w'\eta_t \\
=& \overline{\mu} + \overline{\eta}_t
\end{aligned}
$$

Here $w$ is the portfolio weight (summing to 1). Note that estimating the

## Three steps

Define relative returns as

$$
\tilde{R}_{jt} = R_{jt} - \overline{R}_t
$$

Then returns can be written as

$$
R_{jt} = \mu + \sum_{h\in 0:H} L^h \phi_j \tilde{\eta}_{jt} + \sum_{h\in 0:H} L^h \pi_j \overline{\eta}_{jt}
$$

Note that the above allows for the systematic returns to vary with different lags than the relative returns. Does this make sense?

Aggregating the above:

$$
\overline{R}_t = \overline{\mu} + w' \sum_{h\in 0:H} L^h \left(\phi \odot \tilde{\eta}_t\right) + \sum_{h\in 0:H} L^h \pi \overline{\eta}_t
$$

The authors note that the middle term is a covariance estimator

$$
\hat{\sigma}\left(\phi_j, \tilde{\eta}_{jt}\right) = w' \sum_{h\in 0:H} L^h \left(\phi \odot \tilde{\eta}_t\right)
$$

They assume that $\sigma\left(\phi_j, \tilde{\eta}_{jt}\right) = 0$, hence for a LARGE number of funds,

$$\overline{R}_t \approx \overline{\mu} + \sum_{h \in 0:H} L^h \pi \overline{\eta}_t$$

Subtracting this from the single asset return equation:

$$\begin{aligned}
\tilde{R}_{jt} =& \tilde{\mu}_j + \sum_{h \in 0:H} L^h \phi_j \tilde{\eta}_{jt} + \sum_{h \in 0:H} L^h \psi_j \overline{\eta}_{jt} \\
s.t.& \\
\psi_j \equiv& \pi_j - \overline{\pi}
\end{aligned}$$

(Note there is a typo here - $\overline{\pi}$ should not have a subscript in equation 8)

Thus the procedure can be summarized as:

1. Estimate an MA process for the index to get $\overline{\pi}^h$
2. Estimate a double (vector) MA process to get $\phi^h$ and $\psi^h$
3. Recover the economic returns from the shocks $R_{jt} = \mu_j + \overline{\eta}_t + \tilde{\eta}_{jt}$

From an implementation standpoint, steps 1 and 2 are econometrics (PhD) 101 application of MLE. The third step is more complex since each shock must be estimated, however, it may still be doable with standard econometrics packages.

## Thoughts

- The premise of the approach is the systematic and the idiosyncratic (deviations) have a differing temporal structure.
- The additional parameters lead to a better model fit (of course they would), but I'm not sure I believe the model is better for ex ante modeling.
- The economic story that managers observe different signals for idiosyncratic changes in value and aggregate changes in value seems questionable. It is true that a standard DCF model might vary with the exit multiple, and similarly the economic information about the fund is imperfectly observed- but the multiple is observed more or less contemporaneously based on, say, P/E ratios of public companies.
  - Note I did not work through the algebra here.