

Illiquid Alternative Asset Fund Modeling

A flexible management tool.

Dean Takahashi and Seth Alexander

Illiquid alternative assets have become an increasingly significant portion of institutional portfolios as endowments and foundations seek high returns and diversification relative to traditional stock and bond investments. Allocations to non-publicly traded investments, including venture capital, leveraged buyouts, real estate, and natural resource investments have grown from an average of 10.9% of university endowments in June 1991 to 27.0% as of June 2000.¹ Investment returns from venture capital, in particular, were spectacular in the late 1990s, often differentiating top-performing institutional funds from the rest of the pack.

Yale University was an early and leading investor in non-publicly traded alternative asset classes, with initial commitments to natural resources in 1950, leveraged buyouts in 1973, venture capital in 1976, and real estate in 1978. As of June 30, 2001, Yale's allocation to these investments totaled more than 35% of its \$10.7 billion endowment.

ALTERNATIVE ASSET FUND MODELING

Funds that invest in illiquid assets present particular challenges for portfolio managers. Asset class allocations depend not only on the level of commitment to individual funds, but also on the rate and timing of managers' drawdowns and distributions. Effective management of alternative assets requires reasonably sophisticated modeling of projected manager behavior.

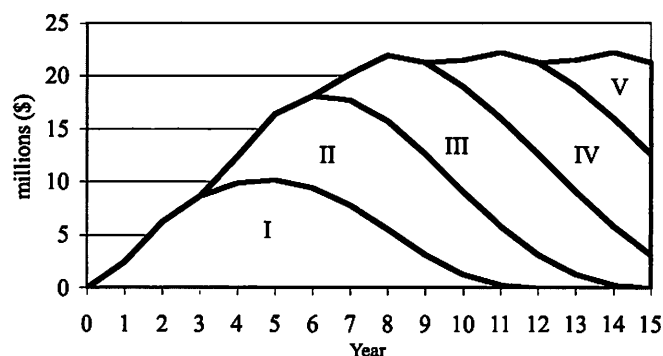
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EXHIBIT 1 NET ASSET VALUE



Commitments

Fund	Year	\$
I	0	10 million
II	3	10 million
III	6	10 million
IV	9	10 million
V	12	10 million

Institutions invest in alternative assets predominantly through commingled limited partnership funds. The funds are raised every few years on a blind pool basis by general partners who actively invest, manage, and harvest portfolio investments. At the onset of the partnership, investors commit capital that gets drawn down over several years by the general partner.

The uncertain schedule of drawdowns, unknowable changes in the valuation of the partnership's investments, and unpredictable distributions of cash or securities to the limited partners combine to make it difficult to accurately predict the future value of partnership interests. Yet because investors target a percentage allocation for illiquid alternatives that translates into an asset value, some form of projection must be employed. In addition, it is often helpful for the investor to estimate expected future cash needed to meet capital commitments, as well as projected distributions that would generate liquidity.

Prior to the mid-1990s, most institutional investors were trying to increase their relatively small and growing allocations to alternative assets. The U.S. stock market was exceptionally strong, and investors were focused on reaching their target allocations to alternative assets. At that time, simple rules of thumb relating commitment levels to expected asset values of funds were good enough.

One common method for venture capital was to invest roughly fifty cents in each of a series of funds to achieve a dollar of ongoing value aggregated across funds. This method works in certain model circumstances, shown in Exhibit 1, in which a series of \$10 million commitments every three years results in an approximate aggregate exposure of \$20 million.

As allocations to alternative assets grow, however, these models become increasingly unable to make reasonable future projections. Seeking more accurate projections, we created a model in the mid-1990s that assumed future funds would have the same pattern of capital contributions, distributions, and asset values as historical averages. This type of model, while superior to simple rules of thumb, still had a number of problems.

For example, the "average" fund experience varied greatly, depending on the time sample, raising the question of the historical period upon which to base the model. Many funds raised in the early and mid-1980s had dismal returns with little growth in asset values and delayed distributions. On the other hand, venture capital funds in the late 1980s and early 1990s exhibited strong performance and asset values significantly above capital contributed. Yale's venture capital funds exhibited marked differences in returns between time periods, as shown in Exhibit 2.

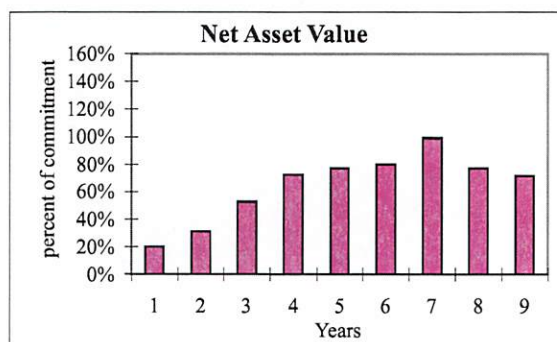
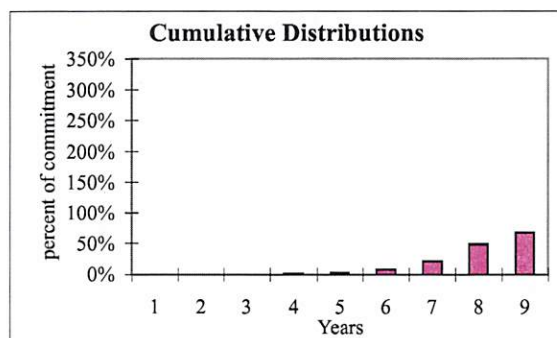
Prospective investment environments do not necessarily resemble any historical period. For example, in the late 1990s, venture capitalists drew down capital at unprecedented rates with unprecedented returns. Unlike earlier funds, which drew down capital over four to five years, funds raised in the late 1990s frequently were fully invested within two years. Performance soared as well, with many funds earning triple-digit rates of return. Historical data from any time period provided a poor template for modeling these funds.

NEW MODEL

In 1998, seeing a need for better estimates of future values in making private equity and real asset capital commitments, we set out to develop an alternative asset fund projection model that meets several criteria. First, the model should be simple and sensible on a theoretical basis. Second, the model should be able to incorporate and respond to actual capital commitment experience and current partnership asset values. Third, the model should be able to analyze the portfolio impact of varying return scenarios and changing rates of investment and distribu-

EXHIBIT 2 VENTURE CAPITAL

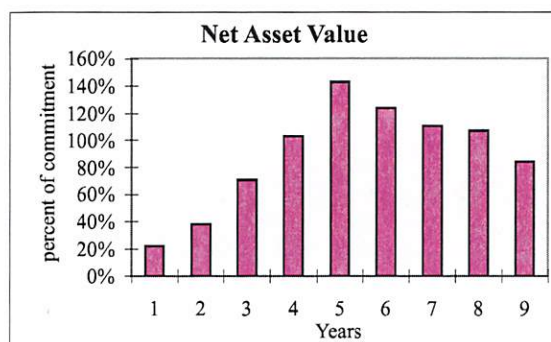
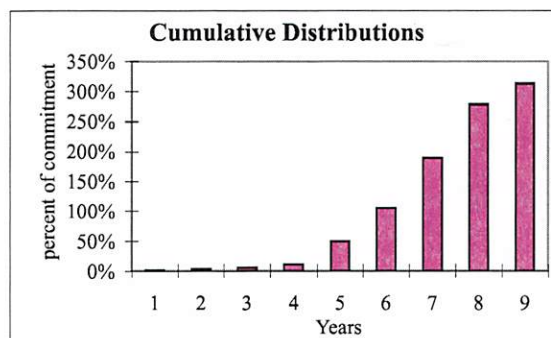
Vintage Years 1984–1986*



* Sample size: 6 funds.

** Sample size: 8 funds.

Vintage Years 1990–1992**



tion. Fourth, the model should be useful for a variety of asset types.

Recognizing that fund experience varies greatly depending on the time sample and the investment environment, we initially fit the sample model to historical data, but then allowed it to be adjusted as needed. As shown in Exhibit 3, the model uses six inputs to project capital contributions, distributions, and net asset value.

Capital Contributions

In most cases, capital contributions are heavily concentrated in the first few years of a fund's life. After the initial burst of investment activity, contributions to a fund are made at a declining rate, as fewer new investments are made, and follow-on investments and fees are spread out over a number of years.

Capital contributions are calculated by multiplying the rate of contribution (RC) by the remaining capital

commitment, or the initial capital commitment (CC), minus the sum of the paid-in capital (PIC):²

$$C_{(t)} = RC_{(t)} (CC - PIC_{(t)}) \quad (1)$$

Rather than specify a different rate of contribution every year, we simplify the model by separating the first two years of contributions from subsequent years. Typically, we would assume two similarly sized large contributions in years one and two, and geometrically declining contributions in subsequent years.

Exhibit 4 displays the resulting capital contributions with inputs as follows: 25% contribution rate in year one, 33.3% contribution rate in year two, and 50% contribution rate in subsequent years. In the model, the sum of the capital contributions never quite equals the capital commitment, a fair forecast since many funds never completely draw down all capital.³

EXHIBIT 3 MODEL INPUT AND OUTPUT

Input	Description	Output	Description
<i>RC</i>	Rate of contribution	<i>C</i>	Capital contributions
<i>CC</i>	Capital commitment (\$)	<i>D</i>	Distributions (\$)
<i>L</i>	Life of the fund (years)	<i>NAV</i>	Net asset value (\$)
<i>B</i>	Factor describing changes in the rate of distribution over time		
<i>G</i>	Annual growth rate (%)		
<i>Y</i>	Yield (%)		

Distributions

Distributions vary with the different stages of a fund's life. In the early years of a fund, distributions tend to be minimal as investments have not had time to be harvested. The middle years of a fund tend to have the most distributions as investments come to fruition. Later years are marked by a steady decline in distributions as fewer investments are left to be harvested.

Recognizing that the size and the timing of distributions (*D*) are based on the performance of the fund, we establish a distribution formula based on a rate of distribution (*RD*) and the net asset value (*NAV*):

$$D_{(t)} = RD[NAV_{(t-1)}(1 + G)] \quad (2)$$

The distribution rate (*RD*) has two components: yield (*Y*) and realizations $[(t/L)^B]$, which occur as assets are sold or investments are harvested:

$$RD = \text{Min}[\text{Max}[Y, (t/L)^B], 1] \quad (3)$$

For income-generating assets such as real estate, the yield sets a minimum distribution level. For other asset types such as venture capital, the yield term often is irrelevant and is set at zero.

As time (*t*) passes, realizations will increase and overshadow the yield term (*Y*). In the last year of the fund, when *t* equals life (*L*), the distribution rate reaches a peak as 100% of the remaining asset value gets distributed. Despite the high distribution rate, the dollar amount of the final distribution typically will be fairly small as the net asset value has declined significantly in the later years of a fund.

The bow (*B*) controls the rate at which the distribution rate changes over time. As shown in Exhibit 5, the higher the bow, the slower the initial increase of the distribution rate and the faster the later acceleration.

Net Asset Value

Partnership values are affected by three variables: investment performance, capital inflows, and capital outflows. In the model, as shown below, the net asset value (*NAV*) is calculated by multiplying the previous year's net asset value by the projected growth rate (*G*), adding contributions (*C*), and subtracting distributions (*D*):

$$NAV_{(t)} = [NAV_{(t-1)}(1 + G)] + C_{(t)} - D_{(t)} \quad (4)$$

Exhibit 6 displays a sample model using inputs as follows: 13% growth rate, 12-year life, 25% contribution rate in year one, 33.3% contribution rate in year two, 50% contribution rate in subsequent years, a bow of 2.5, and a yield of 0%.

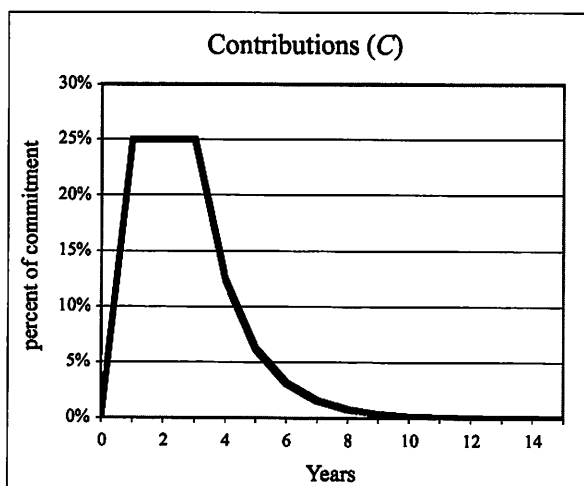
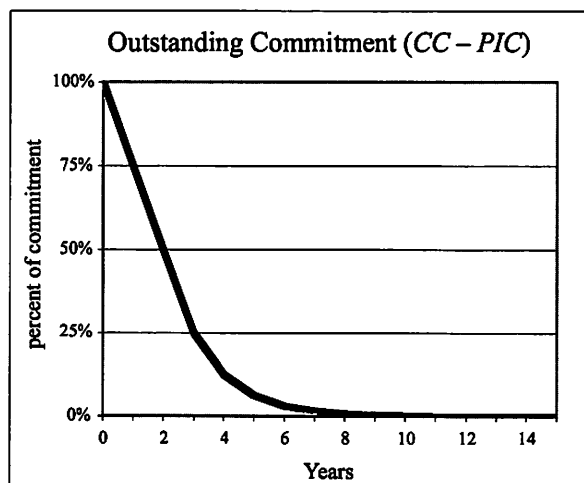
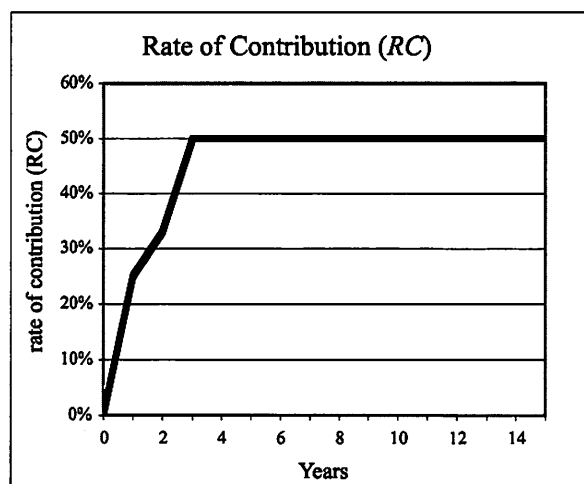
TESTING THE MODEL

To test the model, we examine our four initial criteria: 1) the model should be simple and sensible on a theoretical basis; 2) the model should be able to incorporate and respond to actual capital commitment experience and current partnership asset values; 3) the model should be able to analyze the portfolio impact of varying return scenarios and varying rates of investment and distribution; and 4) the model should be useful for multiple asset types.

The model is simple and fundamental. The net asset value is increased by contributions and a growth rate (which equals the total return net of management fees) and reduced by distributions. The basic patterns of the model graphs of net asset value, contributions, and distributions conform to reasonable expectations of fund behavior. To verify our intuition, we compare the model to historical data.

Shown in Exhibit 7 is the model (the line) graphed against a sample of Yale's historical venture capital fund data

EXHIBIT 4 CAPITAL CONTRIBUTIONS



(the bars). The inputs used for the model are as follows: 20% growth rate, 20-year life, 29% contribution rate in year one, 30% contribution rate in subsequent years, a bow of 1.2, and a yield of 0%. In this example, using the model's predicted cash flows to calculate an internal rate of return results in a 20% return, equal to the assumed growth rate of 20%.⁴

The model incorporates and responds to actual capital commitment experience and current partnership asset values. It can be updated each year with actual data, allowing it to adjust to current events. For example, the capital commitment in any given year is based on the outstanding commitment to a fund, meaning that unusually large early capital calls diminish the amount of remaining capital calls. Likewise, estimates of net asset values and distributions are based on prior years' data, allowing actual data to influence future projections.

An example of this adjustment process is shown in Exhibit 8, which graphs the base model (unadjusted for actual data) against a 1993 vintage year venture capital fund in which we participated.⁵ The 1993 fund data through the year 2000 are actual data (shaded bars); future data (unshaded bars) are projected using the model. Although the venture capital fund performed well beyond the expectations of the base model, the use of actual data allows the model to continue to make reasonable future projections for the 1993 fund.

The model can be modified as investment environments change. For example, recent assumptions in Yale's model include a decelerated pace of capital contributions to venture capital funds, reflecting the slower pace at which funds have been calling money. This flexibility is helpful in analyzing the impact of varying market conditions. We can reflect a forecast of an unfavorable exit environment for venture capital, for example, by extending the life of funds or by slowing down the distribution rate.

To demonstrate the model's ability to depict below-average return environments, we examine Yale's venture capital funds with vintage years between 1984 and 1986. The inputs for the model are as follows: 7% growth rate, 18-year life, 20% contribution rate in year one, 25% contribution rate in year two, 30% contribution rate in subsequent years, a bow of 2.2, and a yield of 0%. As shown in Exhibit 9, the model fits historical data nicely.

Although we have focused on venture capital, the model can be used for a variety of asset types. For example, by altering the inputs, the model can be used to represent assets with cash yields such as real estate, depleting assets such as oil and gas, and more traditional private equity assets such as leveraged buyouts.

EXHIBIT 5 DISTRIBUTION RATE CHANGE

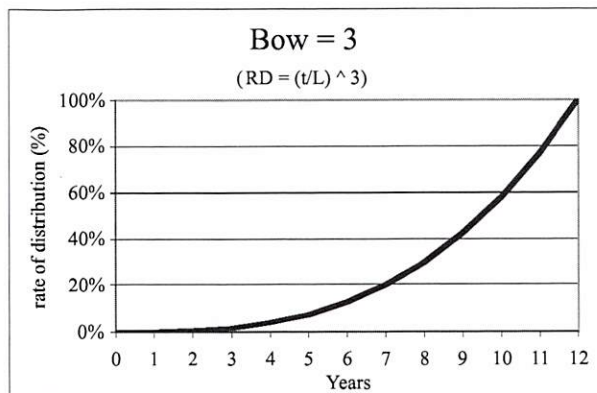
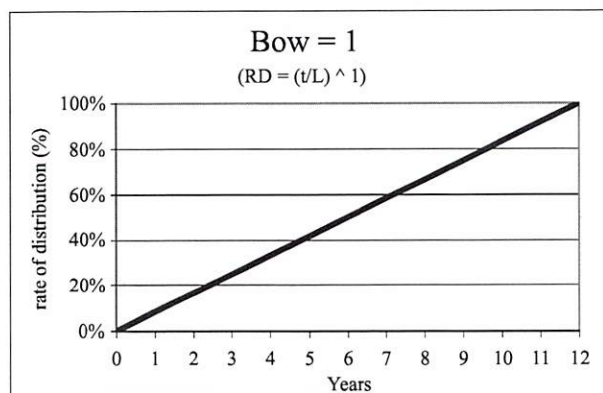


EXHIBIT 6 SAMPLE MODEL

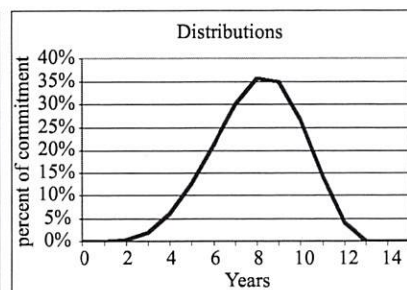
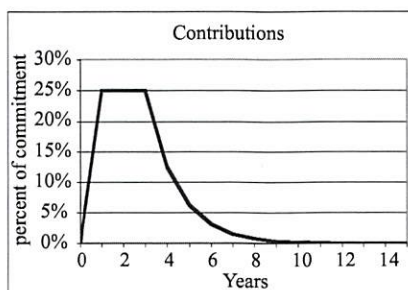
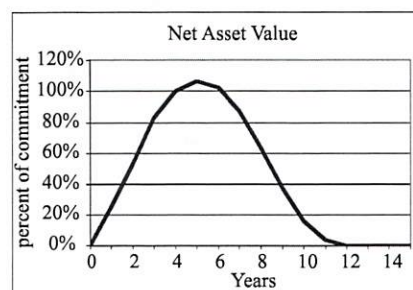
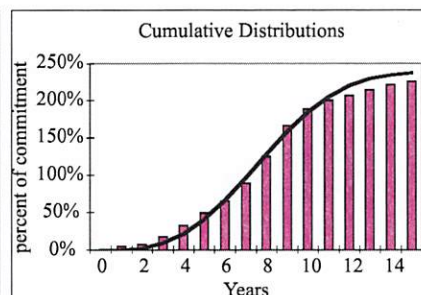
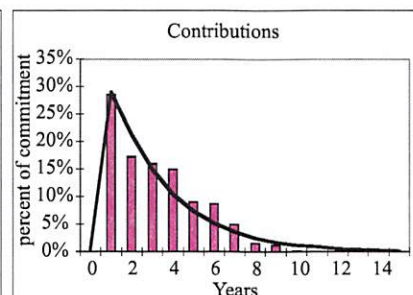
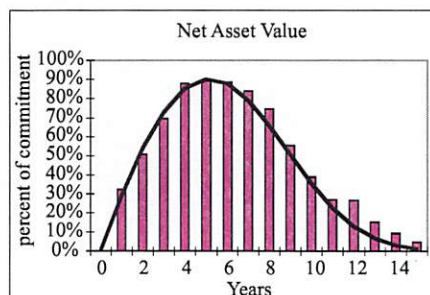
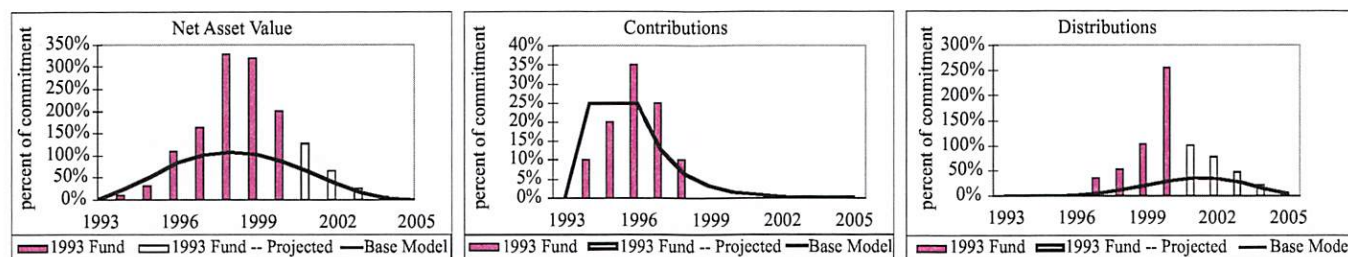


EXHIBIT 7 MODEL COMPARED TO HISTORICAL DATA



Bars represent historical venture capital fund data (33 funds participated in during the 1980s).

EXHIBIT 8 MODEL COMPARED TO 1993 VINTAGE YEAR VENTURE CAPITAL



Shaded bars: Actual data.

Unshaded bars: Projected data.

EXHIBIT 9 MODEL COMPARED TO 1984-1986 VINTAGE YEAR VENTURE CAPITAL

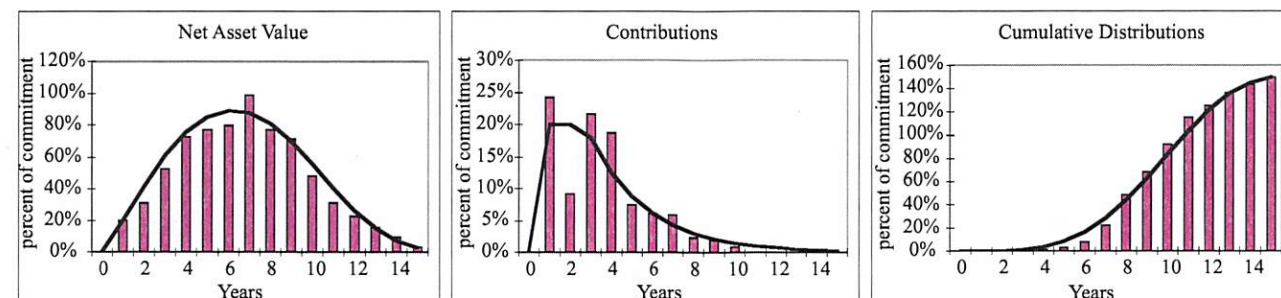


Exhibit 10 displays how the model's inputs can be modified to depict a variety of asset classes.

EMPLOYING THE MODEL

Once the model was created, we established a set of assumptions for projecting the cash flows and performance of current and future funds. Despite the recent excellent performance of many private asset classes, we used as inputs the conservative return assumptions built into our asset allocation modeling. For example, the assumptions for a new venture capital fund are as follows: 13% growth rate, 12-year life, 25% contribution rate in year one, 33.3% contribution rate in year two, 50% contribution rate in subsequent years, a bow of 2.5, and a yield of 0%. The projected 13% rate of return is significantly below the 47% annual rate of return Yale has earned on venture capital investments over the past ten years.

Once individual funds are modeled and reasonable assumptions made about commitments to new funds, the

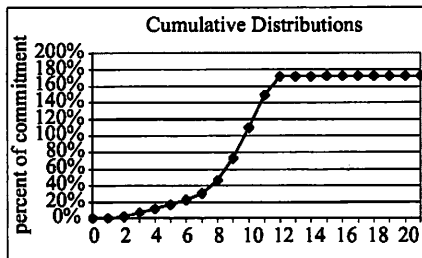
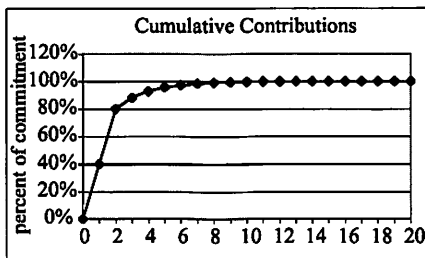
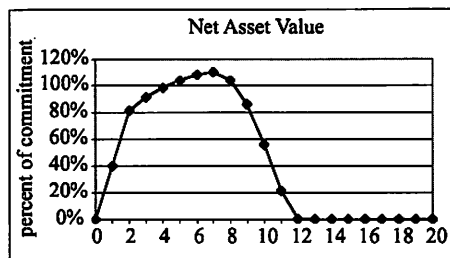
results are aggregated to examine predicted future exposures to, contributions to, and distributions from entire asset classes. Exhibit 11 displays an analysis we use to examine our asset class exposures. By varying the commitments to various managers, we can assess the effect on projected asset class exposures and cash flows. Also, we can examine the effect of different investment environments by changing the inputs for all the managers. For example, we might reduce the growth rate of each manager by several percentage points to understand the impact of lower-than-expected returns on our asset allocation.

The model can be used to size commitments to individual managers, providing insight into exposure to a single organization or investment strategy. We recently considered our options in making a commitment to a new fund. The University had committed to five previous funds with excellent results, but wanted to examine the effect of an increased commitment size.

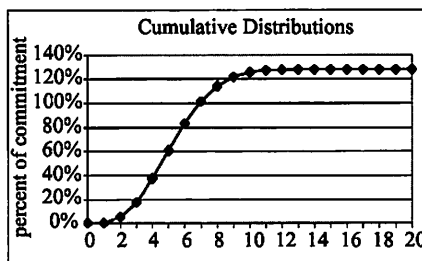
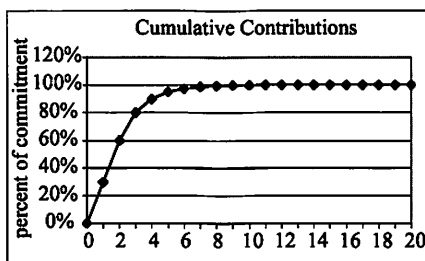
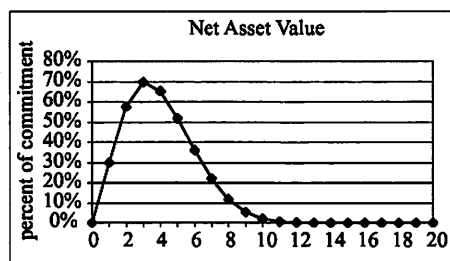
As shown in Exhibit 12, we set up an analysis that depicts the effect of making a new commitment of \$150

EXHIBIT 10 APPLICATION TO DIFFERENT ASSETS

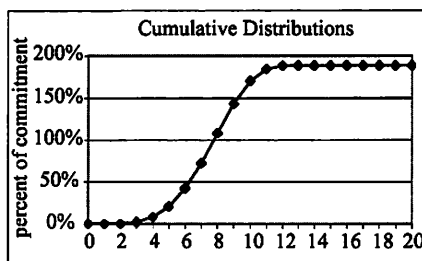
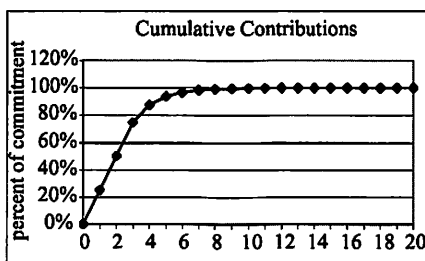
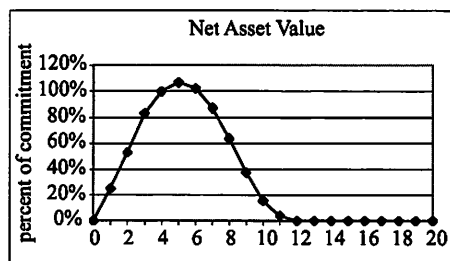
Real Estate



Oil and Gas



Leveraged Buyouts



Real Estate Assumptions

Growth	8.0 %
Life	12 years
Initial Contribution Rate	40.0 %
Contribution Rate	40.0 %
Bow	5.0
Yield	5.0 %

Oil and Gas Assumptions

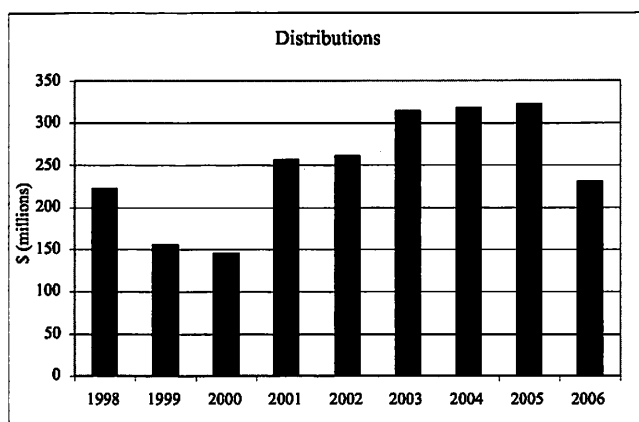
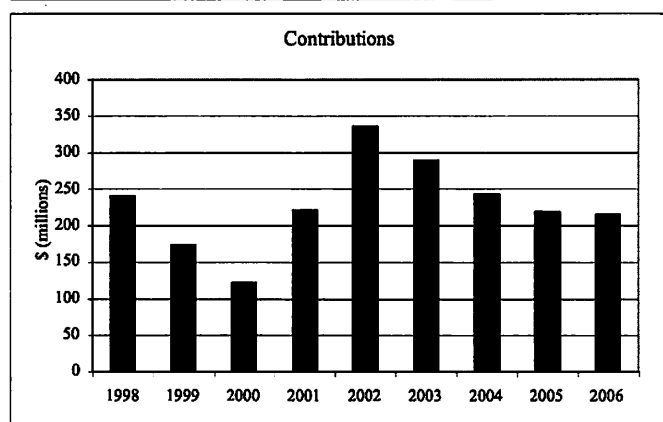
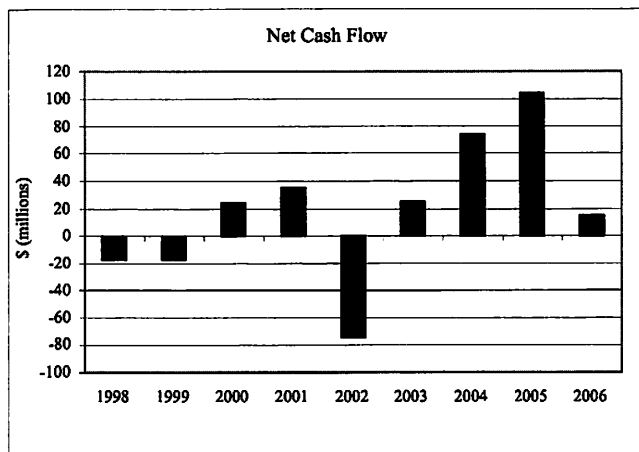
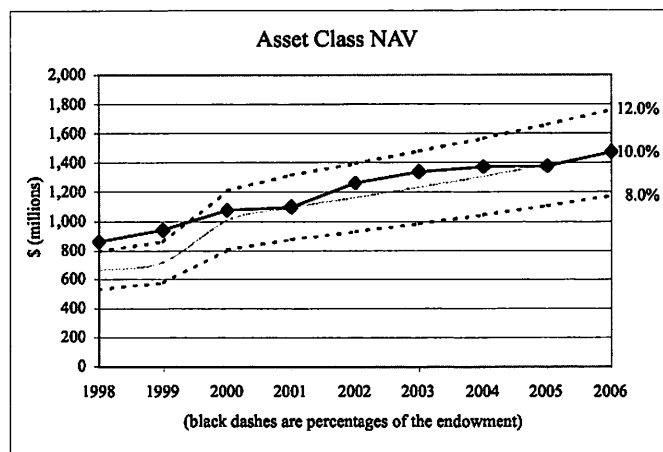
Growth	8.0 %
Life*	15 years
Initial Contribution Rate	30.0 %
Contribution Rate	50.0 %
Bow	1.0
Yield	15.0 %

Leveraged Buyout Assumptions

Growth	13.0 %
Life	12 years
Initial Contribution Rate	25.0 %
Contribution Rate	50.0 %
Bow	2.5
Yield	0.0 %

*Because of the fast distribution rate of many oil and gas investments, the stated life doesn't equal the actual life of fund. The model is representative; input factors are used as mathematical inputs and should not be taken literally.

EXHIBIT 11 COMMITMENT AND ASSET CLASS EXPOSURE ANALYSIS

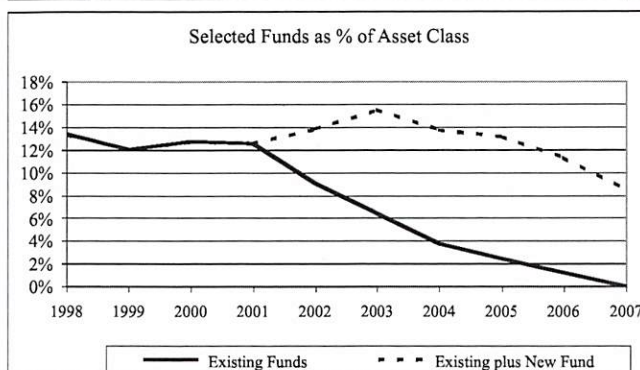
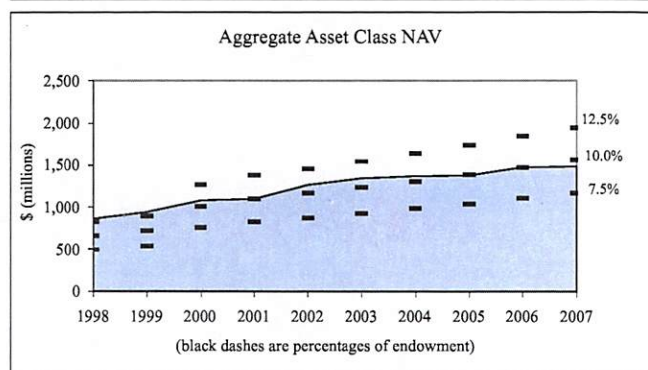
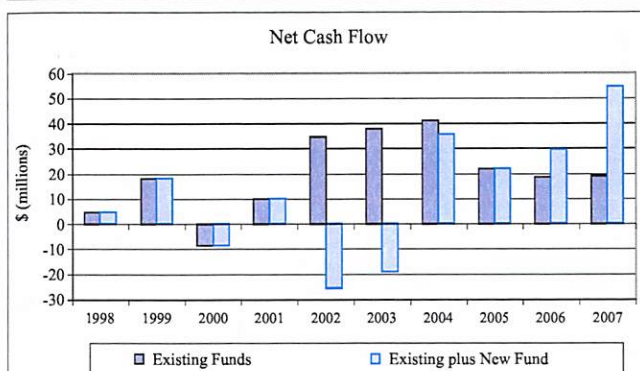
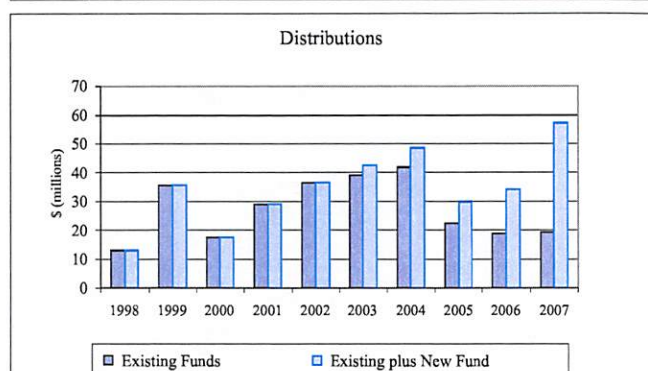
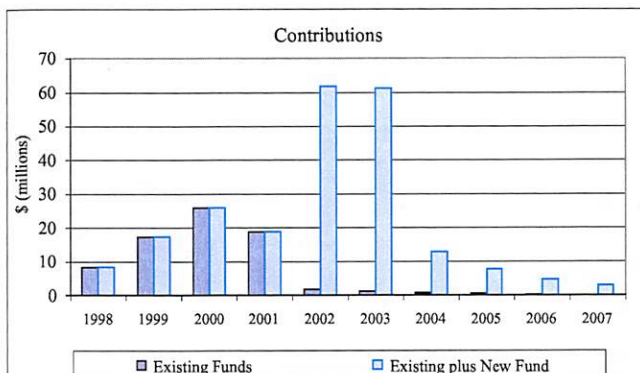
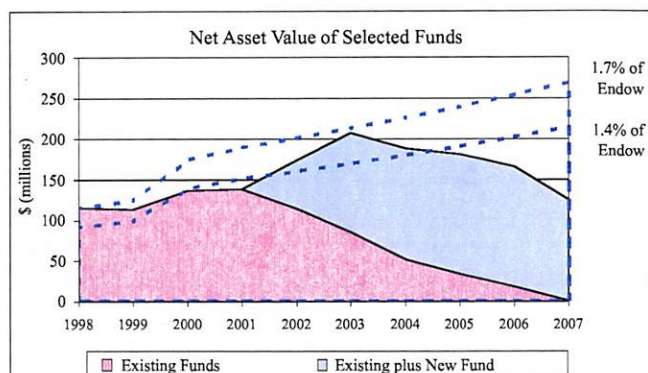


Future Commitments

Fund	Year	\$\$
Manager A	2001	25
Manager B	2001	50
Manager C	2001	75
Manager D	2001	150
Manager E	2002	25
Manager F	2002	75
Manager G	2002	75
Manager H	2002	100
Manager I	2003	25
Manager J	2003	50
Manager K	2003	50
Manager L	2003	60

Fund	Year	\$\$
Manager M	2003	100
Manager N	2003	150
Manager A	2004	25
Manager B	2004	75
Manager C	2004	100
Manager D	2005	60
Manager E	2005	150
Manager F	2005	75
Manager G	2005	50
Manager H	2006	25
Manager I	2006	75
Manager J	2006	50

EXHIBIT 12 ANALYSIS OF EFFECT OF NEW COMMITMENT



Current Funds	Inception Date	Yale Commitment	Unfunded Commitment	Approximate Size of Fund	Yale as % of Fund	NAV (Yale)	Multiple	Distributions (as a % of Contrib.)
Fund I	1993	25.0	0.0	150.0	16.7%	19.5	1.6	88.6%
Fund II	1995	47.0	0.0	200.0	23.5%	47.8	1.8	71.6%
Fund III	1996	25.0	0.0	100.0	25.0%	26.6	1.5	62.3%
Fund IV	1997	25.0	6.4	100.0	25.0%	20.3	1.5	45.8%
Fund V	1999	40.0	15.6	300.0	13.3%	23.9	1.0	0.4%
TOTAL		162.0	22.0	850.0	19.1%	138.2	1.5	57.3%

New Fund	Inception Date	Yale Commitment
Fund VI	2002	150.0

million. Despite the seemingly large size of the commitment, the model was helpful in showing that the growth in the endowment and future projected sales from older funds justified a significant step-up in the new fund.

CONCLUSION

With the growth in allocation to illiquid alternative asset classes such as venture capital, leveraged buyouts, real estate, and natural resources, institutional investors are compelled to find ways of projecting future asset values and cash flows for funds. Older tools such as simple rules of thumb or models based solely on historical data are flawed by a lack of flexibility and adaptability, and a failure to use actual data.

Our model provides a simple and sensible way to estimate future exposures and cash flows, and represents a flexible management tool to assess the impact of changing fund commitment levels and of varying assumptions regarding contributions, distributions, and underlying net returns.

ENDNOTES

¹According to the National Association of College and University Business Officers. Dollar-weighted universe.

$$^2 PIC(t) = \sum_0^{t-1} C(t)$$

³In fact, the model projects that capital contributions will persist indefinitely, an obviously incorrect expectation. Yet the dollar amounts involved are insignificant as long as *RC* is set at a reasonable level. For example, if *RC* = 25% in year one, 33.3% in year two, and 50% in subsequent years, the projected drawdown in year 15 is less than one basis point of the capital commitment. Once a fund actually draws down the total capital commitment, the model adjusts, and assumes no further drawdowns.

⁴Although we might have defined the rate of contribution to perfectly match the actual contributions each year to establish a closer fit to observed capital drawdowns, we use the standard format of the model since it is more representative of what we want to use on an ongoing basis.

⁵Vintage year refers to the year the fund was formed. The assumptions used are as follows: 13% growth rate, 12-year life, 25% contribution rate in year one, 33.3% contribution rate in year two, 50% contribution rate in subsequent years, a bow of 2.5, and a yield of 0%. These are the base model assumptions Yale uses for venture capital funds.

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