Estimating alpha in private equity

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Private equity is often considered to be an appealing asset class for investors who can afford its inherent illiquidity. The main reason for this appeal is the perception that private equity offers an opportunity for substantial excess returns, due possibly to informational advantages.

However, a problem for investors in private equity is that, because of its illiquidity, it can be relatively difficult for investors to assess the performance of a private equity portfolio on a similar basis to other asset classes. For most asset classes it has become routine to measure portfolio statistics such as mean return, volatility, and correlation with the market. Another key statistic is manager-related outperformance, or alpha, which is useful both in selecting portfolio managers and in assessing the overall performance of a portfolio. However, due to the illiquidity of private equity it has been difficult to estimate these parameters. Instead, most private equity funds report their internal rate of return (IRR), which is not comparable to any of the measures used for tradeable asset classes (Reference 1).

More recently it has been pointed out that some of these performance statistics for private equity can be estimated by some of the same techniques that are used to value private equity investments. It is very common to value private equity investments by reference to a basket of comparable public equities. It has been shown that by extending the same comparables analysis to the implied rates of return, it is possible to estimate the beta to the market and the specific volatility of a portfolio of private equity investments. While these estimates may not be as accurate as estimates for tradeable asset classes, simply because less data is available, they are as good as the corresponding private equity valuations (Ref. 2).

However, until now one key statistic has been missing. There has been no good way to estimate the alpha of a private equity portfolio. Without some estimate of alpha, it is difficult to make a case for investing in private equity. Alpha cannot be estimated from a basket of comparables, precisely because it is a measure of management-related outperformance. It must therefore be estimated from performance data, but (as discussed in Ref. 1) the usual return-regression techniques are unreliable for private equity data, due to difficulties in portfolio valuation for nontradeable assets.

Instead, an effective method of estimating alpha for private equity must use only data that are both available and reliable. This is the traditional motivation for using IRR in measuring private equity performance – it requires only actual cashflows and a current portfolio value. The current value is somewhat unreliable, but it eventually goes to zero for most private equity partnerships, which have limited investment periods. It would be desirable to find a method for estimating the alpha of private equity portfolios that used no more data than needed to calculate IRR.

It turns out that it is possible to estimate alpha for a private equity portfolio. Besides values of the chosen market index and risk-free asset, the method requires only cashflows and current value for the private equity portfolio, in addition to the market beta and specific volatility as estimated in Ref. 2. In form the calculations are identical to the standard IRR calculation, and in fact reduce to the standard IRR calculation as a special case.

The key to this method is that it's possible to calculate what final value would be expected for any one cashflow, given the modeled market beta of the portfolio and the actual market return over time. In effect, this is just an extension of the well-known Long/Nickels method of computing public-market equivalent returns (Ref. 3). Having done this, calculating alpha is straightforward.

The final version of the equation is

$$V = \sum_{i=1}^{n} \phi_i * (1+a)^{k_i} \qquad \text{(Equation 1)}$$

$$\alpha = \left(\frac{1}{\Delta}\right) \ln(1+a) \qquad \text{(Equation 2)}$$

Here

- V is the current value of the portfolio
- The ϕ_i are the modified cashflows resulting from the Long/Nickels calculations
- a is the discrete-time analog of α
- Δ is the time interval for which α is computed (typically a year).

It's easy to see that Equation 1 is identical in form to the present value equation, with a taking the place of IRR. Thus, once the modified cashflows ϕ have been generated, α can be computed using any reliable routine for IRR.

The full development of these equations can be found in the Appendix. One important result is that the modified cashflows ϕ depend on random factors summarized as specific (non-market) volatility, and also on current value of the portfolio. Since we don't know these exact values, we can never know α exactly. However, we can compute its mean and confidence interval (for example, via Monte Carlo simulation). This is true for estimates of α in any setting, including public stocks, but it is often overlooked.

As a result, the estimate of α will have a narrower confidence interval for larger, more diversified portfolios with smaller specific risk. The estimate will also have a narrower confidence interval for portfolios with a smaller uncertainty for the current valuation;

however, uncertainty in valuation naturally diminishes for most private equity portfolios as they liquidate.

This method for estimating alpha in private equity portfolios avoids the problems that are well-known to arise in trying to apply regression techniques to private equity data – by regressing against uncertain valuations, we just obtain unreliable results. At the same time, this method does take advantage of what practitioners do know about their portfolio – what's in it, what the public comparables are, and the cashflows.

References

1. Griffiths, B., "Chapter 28: Investing in Private Equity," in Litterman et al, Modern Investment Management: An Equilibrium Approach, Wiley, 2003.
2. Winkelmann, K., S. Browne, and D. Murphy, with B. Griffiths, "Active Risk Budgeting in Action: Assessing Risk and Return in Private Equity," Goldman Sachs Asset Management, April 2005.
3. Venture Economics, 2000 Investment Benchmarks Report: Venture Capital, Summer 2000, "Bannock/Long/Nickels Coller Public Market Equivalents," p. 267.
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5. Griffiths, B., "Determination of alpha from private equity data," 10 April 2007 (private communication).

APPENDIX: Details of calculations

In this section we provide the details of the calculations to find alpha (or α) for a private equity portfolio, for the general time-varying stochastic case. Earlier versions reported a restricted result for the deterministic case (Refs. 4,5).

We assume that at discrete times $\{t_i\}_{i=0}^{n-1}$ we know the interim cashflows $\{c_i\}_{i=0}^{n-1}$, and that at current time t_n there is a good estimate for the portfolio value $V_{t_n} = V$. We also assume that we have found good estimates, possibly time-varying, for the portfolio beta vector (β_t) to some m-dimensional vector of indexes (N_t, at the value level), and the spectral density of the corresponding specific volatility (σ_t).

If the risk-free index (again, at the integrated or value level) is given by F_t, then the standard Ito differential equation for the evolution of portfolio value over time is

$$d(\ln(V_t)) - d(\ln(F_t)) = \alpha * dt + \beta_t^T * [d(\ln(N_t)) - 1 * d(\ln(F_t))] + \sigma_t * dw_t$$

(Equation A1)

Here w_t is a unit Brownian motion, and 1 is a vector with all elements exactly 1.

For the case where there is a single cashflow, we can solve Equation A1 to see that

$$\ln(V) - \ln(c_i) = \alpha * (t - t_i) + \int_{t_i}^{t_n} \beta_t^T * d\ln(N_s) + \int_{t_i}^{t_n} (1 - \beta_t^T * 1) * d\ln(F_s) + \int_{t_i}^{t_n} \sigma_s * dw_s$$

(Eq. A2)

$$V = c_i * \exp\{\alpha * (t - t_i) + \int_{t_i}^{t_n} \beta_t^T * d\ln(N_s) + \int_{t_i}^{t_n} (1 - \beta_t^T * 1) * d\ln(F_s) + \int_{t_i}^{t_n} \sigma_s * dw_s\}$$

(Eq. A3)

For the case of multiple cashflows, we can then see by superposition that

$$V = \sum_{i=0}^{n-1} \left(c_i * \exp\{\alpha * (t_n - t_i) + \int_{t_i}^{t_n} \beta_t^T * d\ln(N_s) + \int_{t_i}^{t_n} (1 - \beta_t^T * 1) * d\ln(F_s) + \int_{t_i}^{t_n} \sigma_s * dw_s \} \right) \quad \text{(Eq. A4)}$$

We assume without loss of generality that the time indexes are ordered such that

$$t_n \geq t_{n-1} \geq \ldots \geq t_0 \quad \text{(Eq. A5)}$$

We can now simplify Eq. 4. First, consider the discretization of the α term. Select some appropriate time interval Δ and define a by

$$1 + a = \exp(\alpha\Delta) \quad \text{(Eq. A6)}$$

We also define the corresponding discrete-time indexes

$$k_i = \frac{t_n - t_i}{\Delta} \quad \text{(Eq. A7)}$$

Next, consider the discretization of the N term. Define the incremental component of this term by

$$d_i = \exp\{\int_{t_i}^{t_n} \beta_t^T * d\ln(N_s)\} \quad \text{(Eq. A8)}$$

We can see that d_i follows the backward recursion relation

$$d_n = 1$$
$$d_{i-1} = d_i * \exp\{\int_{t_{i-1}}^{t_i} \beta_t^T * d\ln(N_s)\} \quad \text{(Eq. A9)}$$

Next, consider the discretization of the F term. Define the incremental component of this term by

$$f_i = \exp\{\int_{t_i}^{t_n} (1 - \beta_t^T * 1) * d\ln(F_s)\} \quad \text{(Eq. A10)}$$

We can see that f_i follows the backward recursion relation

$$f_n = 1$$

$$f_{i-1} = f_i * \exp\{\int_{t_{i-1}}^{t_i} (1 - \beta_t^T * 1) * d\ln(F_s)\} \qquad \text{(Eq. A11)}$$

Finally, consider the discretization of the w term. Define the incremental component of this term by

$$r_i = \exp\{\int_{t_i}^{t_n} \sigma_s * dw_s\} \qquad \text{(Eq. A12)}$$

We can see that r_i follows the recursion relation

$$r_n = 1$$

$$r_{i-1} = r_i * \exp\{\int_{t_{i-1}}^{t_i} \sigma_s * dw_s\} \qquad \text{(Eq. A13)}$$

Note that successive integrals on the right-hand side of Eq. A13 are independent by construction, as they are defined on non-overlapping time intervals. We can simplify Eq. A13 as follows: Define

$$y_{i-1}^2 = \int_{t_{i-1}}^{t_i} \sigma_s^2 ds \qquad \text{(Eq. A14)}$$

Also define a discrete normal random variable by

$$z_{i-1} \sim N(0,1) \qquad \text{(Eq. A15)}$$

Then Eq. A13 can be replaced by

$$r_{i-1} = r_i * \exp(y_{i-1} * z_{i-1}) \qquad \text{(Eq. A16)}$$

Now define

$$\phi_i = c_i * d_i * f_i * r_i \qquad \text{(Eq. A17)}$$

We can now see that

$$V = \sum_{i=1}^{n} \phi_i * (1+a)^{k_i} \qquad \text{(Eq. A18)}$$

It will be recognized that Eq. A18 is the standard form of a present-value relation, where ϕ takes the place of the cashflows and a takes the place of the internal rate of return. Thus, once the modified cashflows ϕ have been computed, a can be found using any reliable IRR calculator. Once a has been computed, α may be found via

$$\alpha = \left(\frac{1}{\Delta}\right) \ln(1+a) \qquad \text{(Eq. A19)}$$

Computation of sample distribution of α using Monte Carlo methods

It will be noted that the r terms are generally random variables, and also that the final value V may also be a random variable. The above may thus be simulated using Monte Carlo methods to obtain a sample distribution for α. In turn, any desired estimates and confidence intervals may be obtained from the sample distribution using standard techniques.

Piecewise-constant approximation

Two additional notes may be useful. First, if the generally time-varying vector β_t can be approximated by some constant vector b_{i-1} over the time interval $t \in [t_{i-1}, t_i)$, and since the indexes N and F are always strictly positive, then the recursion relations of Equations A9 and A11 can be reduced to

$$d_{i-1} = d_i * \prod_{j=1}^{m} \left(\frac{e_j(N_{t_i})}{e_{,}(N_{t_{i-1}})} \right)^{e_j(b_{i-1})} \qquad \text{(Eq. A20)}$$

$$f_{i-1} = f_i * \left(\frac{F_{t_i}}{F_{t_{i-1}}} \right)^{(1-b_{i-1}^T * 1)} \qquad \text{(Eq. A21)}$$

Here $e_j(x)$ is taken to represent the j-th element of vector x.

Similarly, if the generally time-varying σ_t can be approximated by some constant s_{i-1} over the time interval $t \in [t_{i-1}, t_i)$, then the integral of Eq. A14 can be reduced to the familiar

$$y_{i-1} = s_{i-1} * \sqrt{t_i - t_{i-1}} \qquad \text{(Eq. A22)}$$

Deterministic case

If the specific volatility σ_t is always zero, then the time evolution specified in Equation 1 becomes purely deterministic. In this case it can be seen that all of the $r_i(t)$ are exactly 1. If in addition the final value V has no associated uncertainty, then Eq. A18 is purely deterministic. In such a case no Monte Carlo techniques are necessary to find α.

Reduction to standard IRR

Consider the special case of a deterministic problem ($\sigma_t = 0 \forall t \in [t_0, t_n]$) where the β to the index is always zero. Suppose also that the risk-free rate is always zero – that is,

$$F_r = F_s \forall r, s \in [t_0, t_n] \qquad \text{(Eq. A23)}$$

We would expect that for this special case all of the return would be due to alpha. Solving for the modified cashflows in Eq. A17, we find that for all i,

$$d_i = 1 \qquad \text{(Eq. A24)}$$
$$f_i = 1 \qquad \text{(Eq. A25)}$$
$$r_i = 1 \qquad \text{(Eq. A26)}$$
$$\phi_i = c_i \qquad \text{(Eq. A27)}$$

Thus for this special case, Eq. A18 reduces to the standard IRR equation, as expected.