The Use of MPT for Real Estate Portfolios in an Uncertain World

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Executive Summary. *This study examines the impact of uncertainty on the asset allocation algorithm. By "bootstrapping" the error terms from OLS-generated equations, multiple frontiers are estimated and "areas of indifference" are created. Since these alternative portfolios are as "efficient" as the original portfolios on the unperturbed frontier, the unique solution provided by traditional mean-variance optimization techniques is called into question. The analysis concludes that ranges, rather than specific investment targets, should be the preferred method of mean-variance optimization. Ranges are not only statistically robust, but also circumvent many of the practical problems associated with owning an illiquid asset, such as valuation issues and lumpiness.*

Second, the study also examines the role of multifamily housing in a real estate portfolio in the presence of uncertainty. Using econometrically determined ex ante property-type returns, the model's results suggest that most institutional portfolios are likely to be significantly underweighted in apartments using traditional asset allocation techniques.

by Richard B. Gold

> *Uncertainty is a salient feature of security investment. Economic forces are not understood well enough for predictions to be beyond doubt.*
>
> Harry M. Markowitz[1]

INTRODUCTION

In recent years asset allocation modeling has become an increasingly popular tool for real estate investors. Not only do these models help investors understand the relationship between risk and return, but they also provide the practical benefit of determining property-type allocations. Using asset allocation software, investors are asked to choose portfolios that lie along the "efficient frontier" in order to maximize their return for a given level of risk.[2]

This study addresses two different issues regarding portfolio construction. It examines the role of multifamily investments (apartments) in a mixed-asset real estate portfolio. Using econometrically derived return forecasts for four property types (apartment, retail, office, and industrial), this allows investors to build a "core" real estate portfolio that they can compare to a "naive" market portfolio. Then, uncertainty is introduced into the mean-variance model by bootstrapping the error terms from these property-specific return equations. Bootstrapping produces alternative return paths from which expected returns, risk and correlations can be calculated. They are then used as inputs in a classic portfolio optimization model. The resulting "fuzzy" frontiers give some definition to the shape and range of risk as expected by

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investors. The resulting collection of frontiers determine specific "areas of indifference" in which investors are free to choose any portfolio, or create allocation ranges that are statistically indistinguishable from the original unperturbed frontier.

MULTIFAMILY'S ROLE IN A REAL ESTATE PORTFOLIO

What is the proper role of multifamily investments in a real estate portfolio? Even today, many institutional investors are divided as to whether multifamily properties should be considered a "core" asset in their real estate portfolios. As a result, multifamily investments have generally been assigned a secondary role in institutional real estate portfolios. Often cited reasons for multifamily's limited presence in institutional portfolios are competition from taxable investors, perceived management intensiveness, and the small average-size investment, relative to office and retail properties.

Increasingly, however, evidence is accumulating that this approach is suboptimal for real estate portfolios. At a minimum, multifamily investments have been shown to be important diversifiers in any real estate portfolio. In fact, the data suggest that multifamily, like office and retail, should be considered a core property type in an institutional real estate portfolio.

Exhibit 1 shows the results of an Arthur Andersen study that estimated the size of the institutional real estate universe.[3] An investor who wishes to bring the systematic behavior of real estate to bear on the return and risk of a mixed-asset portfolio should allocate his/her real estate portfolio holdings in line with these weights. This then would constitute the naive and passive investment strategy.

Exhibit 1: Estimated Investable Universe

	Dollar Value ($ Trillions)	Percent
Multifamily	1.122	32.3
Retail	1.115	32.2
Office	1.009	29.1
Warehouse	0.223	6.4
Total	3.469	100.0

Source: Arthur Andersen Real Estate Services Group, 1993

How do the current pension fund holdings of U.S. real estate compare to the "naive" Arthur Andersen market portfolio? Two reference points with respect to the actual behavior of U.S. pension funds are available for consideration. These are the allocations by property type captured by the combined (levered and unlevered) index of the NCREIF property performance database (1994 third quarter) and the allocations documented by Institutional Property Consultants in its database on pension fund investments in real estate. The weights produced by each source are presented in Exhibit 2.

Exhibit 2: Pension Fund Real Asset, Allocations by Property Type

	NCREIF (%)	IPC (%)
Multifamily	12.2	19.0
Retail	29.0	40.0
Office	27.0	28.7
Warehouse	19.0	11.7
Other	6.0	7.4

Source: NCREIF and Institutional Property Consultants

There is little substantive difference between the two weighting schemes. Each scheme places a greater weight on office and retail, and lesser weight on industrial and multifamily property types. It is clear in both cases that institutional investors are currently underweighted in multifamily investments, relative to the market or naive benchmark.

Exhibit 3 presents an efficient frontier calculated for the period 1980 to 1994, using the historic NCREIF data on returns, risk and resulting correlations between property types. Several observations may be made. First, the mix of holdings actually held by institutional investors clearly was not on the frontier! These portfolios held an excess share of office assets during a time period when office returns were the lowest of all four property types. Second, none of the mean-variance-efficient portfolios contained any office. Rather, they contained a significant share of multifamily assets, along with a significant exposure to retail real estate. This analysis suggests that over the past fourteen years, a portfolio comprised of retail and multifamily assets would have generated the most mean-variance-efficient results for real estate investors.

Exhibit 3: Optimal Historic Allocations 1980-1994
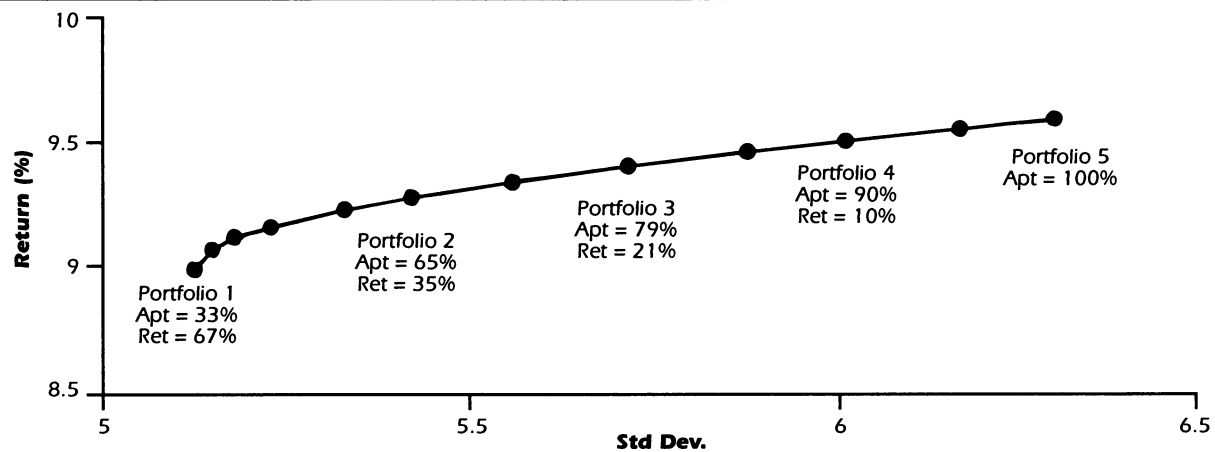
EXPECTED ASSET CLASS PERFORMANCE

Equations for both appreciation and income were estimated for each of the four major property types using quarterly NCREIF return data from 1980:1 through 1994:3. A third equation for total return was defined as the sum of its two afore-mentioned components such that:

$$IR_{it} = \alpha + \Sigma\beta_j X_{it} + \varepsilon_{it} \qquad (1)$$

$$AR_{it} = \alpha' + \Sigma\beta'_j X'_{it} + \varepsilon'_{it} \qquad (2)$$

$$TR_{it} = IR_{it} + Ar_{it} . \qquad (3)$$

Therefore by substitution:

$$TR_{it} = (\alpha + \alpha') + \Sigma\beta_j X_{it} + \Sigma\beta'_j X'_{it} + (\varepsilon_{it} + \varepsilon'_{it}), \quad (4)$$

where:

i = ith property type (apartments, office, retail, or warehouse),

t = 1980:1 to 1994:3,

IR_{it} = Income Return for the ith property type in period t,

AR_{it} = Appreciation Return for the ith property type in period t,

TR_{it} = Total Return for the ith property type in period t,

X_i = Vector of exogenous variables for the ith property types, and

ε_i = Error term for the ith property type.

Exogenous variables in the income and appreciation equations include cycle and trend variables that are sensitive to movements in both the space

and capital markets. All the equations had acceptable Durbin-Watson statistics and the exogenous variables had the correct sign and were significant at the 99% level of confidence.

While historic analysis can offer insights into past market behavior, investors are actually rewarded for the degree to which they can predict future market activity. Exhibits 4 and 5 show an expected set of returns, standard deviations (i.e., risk), and correlations for the four major property types over a five-year forecast horizon (1995 to 1999) generated by the model. These results were produced using forecasts of the exogenous variables generated by fully specified macro and MSA-level models from a nationally recognized economic forecasting firm.[4] The base case forecast has several interesting features.

Exhibit 4: Expected Property-Type Returns 1995-1999

	Return (%)	Std Dev. (%)	Sharpe Ratio (%)
Multifamily	9.88	1.22	8.1
Industrial	8.55	4.27	2.0
Office	10.88	5.22	2.1
Retail	7.49	.83	9.0

Source: Boston Financial

Exhibit 5: Expected Correlations

|	Multifamily	Industrial	Office	Retail
Multifamily	1.00			
Industrial	-0.96	1.00		
Office	-0.96	0.97	1.00	
Retail	-0.70	0.60	0.65	1.00

Source: Boston Financial

First, apartments and retail real estate continue to show the highest Sharpe ratios.[5] Second, in absolute terms, office investors should receive the highest expected yield. However, they are also expected to bear the greatest risk. Finally, and perhaps somewhat surprising, is the large negative correlation between apartments and all other property types.

The negative correlation, however, arises from the continuation of already observed differences in recent property-type performance as evidenced in Exhibit 6. Since 1992, apartment returns, on average, have not only recovered more rapidly than the other three property types, but have also had the highest absolute returns. Between 1995 and 1999, the base case forecast projects that apartment returns will decline slightly, relative to their recent performance, as market conditions stabilize. Conversely, office, retail and warehouse returns will ratchet upwards in response to improving market conditions. The net result is the negative correlation between apartments and the remaining property types during the forecast period.

The efficient frontier found in Exhibit 7 was estimated using the espected returns, expected standard deviations, and expected correlations shown in Exhibits 4 and 5. Recognizing the likelihood that investors would like to maintain a minimal presence across all property types, the frontier was estimated such that no one property type would receive less than a 10% weighting. Conversely, this meant that 70% was the upper bound of any property type's weighting at any point along the curve. The objective of this exercise is not to definitively state the optimal mix of property types within a real estate portfolio, but rather to demonstrate the role of the multifamily asset, given a set of reasonable assumptions about prospective property type and investor behavior.

Exhibit 6: NCREIF Annualized Returns by Property Type Forecast Begins 1995:1

Apartment
Industrial
Office
Retail
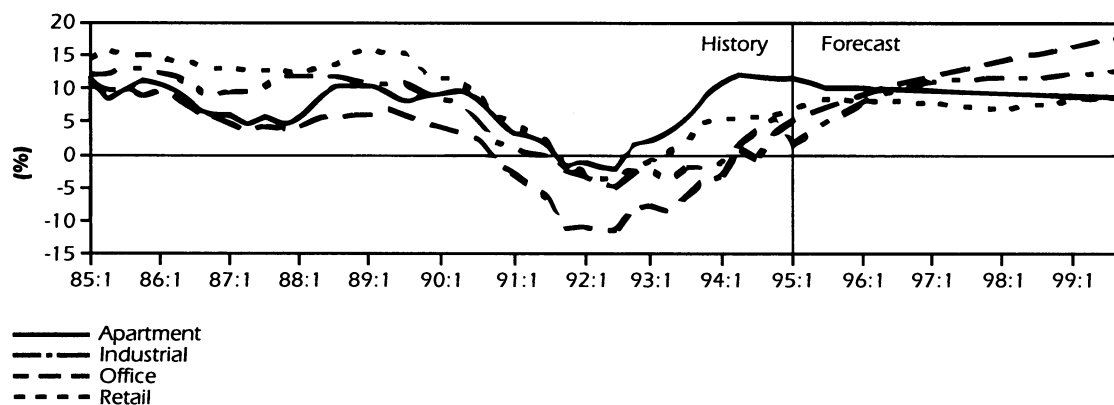
Source: NCREIF and Boston Financial

Exhibit 7: Base Case Frontier

Risk Acceptance Point	3	7	10	13	17	20	23	27	30	33	37
Expected Return (%)	9.6%	9.7%	9.7%	9.7%	9.8%	9.9%	9.9%	10%	10%	10%	10%
Risk (Std Dev.)	0.3%	0.6%	0.8%	1.0%	1.3%	1.6%	1.8%	2.1%	2.4%	2.6%	2.9%
Sharpe Ratio	32	16.2	12.1	9.7	7.5	6.1	5.5	4.7	4.2	3.8	3.4
Industrial	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%
Apartment	69	64	60	57	52	48	44	40	36	32	37
Retail	10	10	10	10	10	10	10	10	10	10	10
Office	11	16	20	23	28	32	36	40	44	48	53

The asset allocation software rounded the risk acceptance points to the nearest whole number, returns and standard deviations to the nearest one tenth of a percent.

The results strongly suggest a very important risk reduction role for multifamily properties in a real estate portfolio over the forecast period (1995-1999). As risk tolerance grows, as represented by larger Risk Acceptance Points, the portfolio allows more and more office property to enter, so that the most risk-tolerant investor will prefer a portfolio dominated by office properties (70% maximum weighting).[6] In addition, the estimated frontier suggests that investors should maintain only the forced minimum (10%) for retail and industrial holdings. Both results are consistent with the expected property-type return paths. More important, however, is the implication for existing institutional portfolios.

That is, if actually implemented, the resulting portfolio allocations would require major changes in property-type allocations and investment strategies for most institutional investors. As this frontier suggests, the core institutional portfolio should largely be comprised of multifamily and office properties with selected forays into industrial and retail property. Multifamily's stable and high absolute returns and office's higher, but more volatile returns, combined with their negative correlation, are optimal in mean-variance space.

There are two reasons why multifamily investments reduce risk. First, the standard deviation for multifamily housing is relatively low. Second, and perhaps more important, multifamily housing exhibits very little correlation with other property types. Thus, even in a part of the cycle where offices are expected to offer returns commensurate with their well-understood risks, multifamily housing and retail real estate continue to play important roles as the stable core of a real estate portfolio.

It is now possible to examine current institutional real estate allocations relative to real estate's historic performance, its expected behavior over the next five years, and the naive diversification strategy implied by the Arthur Andersen study. Regardless of which time period is chosen, current multifamily allocations, as represented by the NCREIF and IPC data, are generally well below the weight implied by both the naive diversification strategy and the implied allocations generated by either the historic or forecast efficient frontiers. This suggests that institutions may be unknowingly exposing themselves to more risk than probably desired. In the future, institutional investors may elect to reduce their office exposure in favor of multifamily properties, especially given the relative flatness of the efficient frontier and the corresponding small marginal benefits associated with a high-risk office portfolio.

THE ONLY CERTAINTY IS UNCERTAINTY

Up to this point, only asset allocation as a simple mathematical exercise, the solution of which in two dimensional mean-variance space is clear and precise, has been examined, Given assumptions about returns, risk and correlations, investors can quickly and accurately choose the combination of risk and return that most closely matches their individual expectations. While other asset combinations may approach the frontier, only those portfolios directly on the frontier are considered "efficient" since they optimize a given return/risk combination. Investors who find that their portfolios are not "efficient" must rebalance them in order to obtain the desired combination of risk and return.

While the actual calculus of mean-variance optimization cannot be questioned, its assumption regarding the certainty of its inputs and by extension, the resulting output, can.[7] Given that investors are asked to predict uncertain outcomes with certainty, the probability that they will always choose the exact outcome is effectively zero. That is, by using expected means and standard deviations, investors assume that the return path of the assets in question will not vary over time. For those using econometrically derived forecasts, it is as if the error term has an expected value of zero over the entire forecast period as well as at every discrete point along the forecast horizon. While the former assumption will hold for unbiased estimators, the very existence of an error term precludes the possibility that the latter will take place. Even for those using heuristic-based expectations, mean-variance optimization assumes a known distribution of risk, which is best represented by the standard deviation. This, however, is for the convenience of the user, not a reflection of the real world.

Another way to view the problem is to imagine drawing period-by-period expected returns from a fishbowl full of ping pong balls.[8] Inside the fishbowl there is one ball marked with the correct expected return for a given interim time period. There are also, however, additional balls that represent the entire distribution of possible future outcomes. While a large percentage of the bowl is filled with balls approaching the expected number, there is only one ball with the exact expected return for that time period printed on it. This procedure is replicated over and over again until all future time periods have been chosen. The probability of actually picking the correct ball for each time period is again zero for all practical purposes. Markowitz himself tacitly recognized this situation when he stated, "Only the clairvoyant could hope to predict with certainty. Clairvoyant analysts have no need for the techniques of this monograph."[9] Yet mean-variance optimization forces its users to become clairvoyants because it requires the collapsing of all future asset behavior into a single possibility.

For example, users of asset allocation software often create multiple scenarios altering various input assumptions. Typically, they find that small changes in any of the model's inputs can result in changes that are significant in an absolute sense. But what if there is no statistical difference between these alternative portfolios? Stated differently, what if both had an equal probability of occurring, given the same set of initial conditions?

If small changes in expectations can result in major shifts in allocation targets, then several important implications emerge for holders of illiquid assets, such as real estate. One is the potential difficulty of undertaking the costly and lengthy rebalancing process based on short-term changes in long-term expectations. This strongly suggests that in an uncertain world, mean-variance optimization should be considered suggestive, not definitive.

Given uncertainty, allocators would appear to have created their own prisoner's dilemma. On the one hand, they know that market behavior will at best approximate their current expectations. Yet, on the other hand, traditional mean-variance optimization provides no clue as to the underlying shape and/or impact of uncertain outcomes on the allocation process. Because of the nonlinearity of the mean-variance algorithm, traditional forecast error "banding" is not an accurate descriptor of risk. Fortunately, resampling techniques such as "bootstrapping" provide a solution to this problem. Specifically, by directly reintroducing uncertainty back into the allocation process, bootstrapping explicitly estimates both the shape and impact of uncertainty.

Bootstrapping is based upon the idea that resampling allows the user to recreate the population mean from the sample mean, or in the case of asset allocation, estimate the probability density function of returns for a given asset class. The power of bootstrapping was pointed out by Efron (1979), and the basic concept is not new. Monte Carlo simulations are quite similar to bootstrapping and were commonly used well before Efron's work. As shown by Hall (1992), resampling is powerful because the bootstrap is merely an integral of the sample distribution function, which in turn is representative of the population distribution function.[10]

AREAS OF INDIFFERENCE

A reexamination of equation 4 shows that the error terms from the total and income return equations are explicit parts of the expected future returns. It is this error term that will be resampled to create the probability density function of expected returns and standard deviations. (Usually forecasters ignore the error terms of stochastic equations under the false assumption that all future errors have an expected value of zero rather than being a single point in a mean-zero distribution.) The historic error terms contain valuable information that the analyst was unable to capture and that is likely to reoccur over the forecast horizon. Even if the error term was a random event, the likelihood that no random events will occur in the future can be no less than the likelihood that they would not have occurred in the past. In fact, since the future is infinitely long compared to the past, future return cycles will undoubtedly be more extreme, especially given the extremely limited time period for which real estate performance data are available.

When the error term is not systematically reintroduced into the forecast process, it is explicitly

assumed that the equation has explained all of the historic variation. Therefore, asset allocators relying on point estimates of returns, standard deviations, or correlations assume that the future will unfold exactly as expected. This is an unlikely event at best.

However, while the specific ex ante value of future errors is unknown, in any time period, $t+n$, they have an expected value of which zero is only one of an infinite number of equally probable outcomes. In order to replicate these probabilities the equation's historic error terms are reintroduced into the forecast path. This is done by restocking the fishbowl. This time the ping pong balls represent the error terms from equation 4. On each ball is the error term for each of the four property types for a specific historic observation. Once a base case forecast is available, the alternative bootstrapped forecasts are built using draws from the fishbowl, such that in period $t+n$ the total return for a given property type is the base case total return plus its respective randomly drawn error term. This is done for each of the twenty forecast quarters to create an alternative return path for each property type. Since each draw is done with replacement, it is entirely possible that the same error term is reintroduced multiple times, while others are not

drawn at all. From each new return forecast, standard deviations, as well as a correlation matrix, can be calculated in order to estimate a new efficient frontier.

This procedure was repeated 250 times, resulting in the same number of new frontiers. Like the base case frontier, constraints were imposed such that no property type could receive more than a 70%, or less than a 10%, allocation. Exhibit 8 shows the 250 alternative frontiers generated by the bootstrapping technique. Introducing the error terms gives some form to the shape and direction of uncertain outcomes by recognizing that the error terms themselves contain valuable informtion that cannot be systematically captured in the allocation process by point estimates. More important, because the allocation algorithm is nonlinear, it is necessary to "connect the dots" along the entire breadth of the distribution, not just at its outer limits.

Each new portfolio created by the bootstrap has its own property-type mix and risk/return characteristics. More important, because each of the new frontiers is parallel to the original frontier, cross-sectional analysis can be performed across the eleven Risk Acceptance Points (RAP) along the frontier generated by the allocation paradigm. The

Exhibit 8: Bootstrapped Frontier
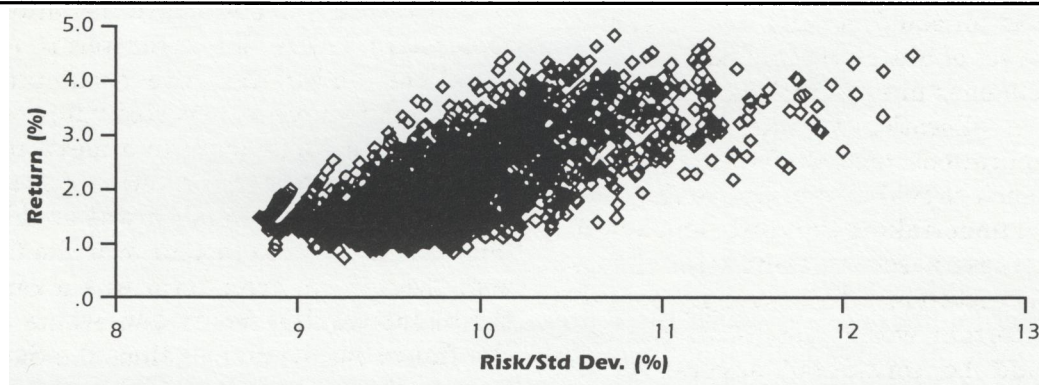
higher the RAP, all else being equal, the more systematic risk the investor is willing to bear to receive higher returns. Cross-sectional analysis is critical because it permits the user to statistically, not heuristically, determine if statistically significant differences indeed exist between the base case and bootstrapped frontiers.

The bootstrapped results are interesting for several reasons. First, they again suggest that multifamily and office property remain the core of any mixed-asset portfolio under almost any circumstances. While there are occasions when the allocations for either retail and industrial are significantly above their allocated minimum, the

bootstrapped results still largely continue to divide the allocation between multifamily and office properties. This can be seen in Exhibit 9 which shows the bootstrapped average allocations, the base case allocations and the high and low allocations for every property type at each RAP. Exhibit 10 shows, for each RAP, the expected returns, risks and Sharpe ratios produced by the bootstrapping. This suggests that the equations tend to understate the strength of the base allocation results.

Second, even though the allocation shifts are largely confined to multifamily and office, the results confirm the general impact of uncertainty on allocation targets. That is, while the results continue to generally resemble the base case, they are variable enough to clearly indicate the influence of the error terms on the allocation targets for any RAP point. Therefore, uncertainty drives asset allocations closer to a more naive position than would be indicated by point estimates. Uncertainty

Exhibit 9: Bootstrap Allocation Ranges by Property Type (250 Alternative Frontiers)

	Apartment (%)			Office (%)			Industrial (%)			Retail (%)		
RAP	High	Mean	Low	High	Mean	Low	High	Mean	Low	High	Mean	Low
2.5	70	65	36	42	14	10	31	10	10	13	10	10
5	70	60	10	70	20	10	10	10	10	10	10	10
7.5	70	55	10	70	25	10	10	10	10	10	10	10
10	70	50	10	70	30	10	10	10	10	10	10	10
12.5	70	47	10	70	33	10	10	10	10	10	10	10
15	70	43	10	70	37	10	10	10	10	10	10	10
17.5	70	41	10	70	39	10	10	10	10	10	10	10
20	70	38	10	70	42	10	10	10	10	10	10	10
22.5	70	36	10	70	44	10	10	10	10	10	10	10
25	70	35	10	70	45	10	10	10	10	10	10	10
27.5	70	33	10	70	47	10	10	10	10	10	10	10

Exhibit 10: Bootstrap Allocation Results (250 Alternative Frontiers)

	Expected Return (%)			Std Dev. (%)			Sharpe Ratio		
RAP	High	Mean	Low	High	Mean	Low	High	Mean	Low
2.5	10.74	9.52	8.79	1.94	1.25	.74	5.5	7.6	11.9
5	11.86	9.66	8.79	3.12	1.42	.74	3.8	6.8	11.9
7.5	12.22	9.79	8.79	3.47	1.63	.74	3.5	6.0	11.9
10	12.22	9.90	8.79	3.95	1.84	.74	3.1	5.4	11.9
12.5	12.22	9.98	8.79	4.36	2.03	.74	2.8	4.9	11.9
15	12.22	10.04	8.79	4.36	2.20	.74	2.8	4.6	11.9
17.5	12.38	10.09	8.79	4.49	2.35	.74	2.8	4.3	11.9
20	12.38	10.13	8.79	4.49	2.47	.74	2.8	4.1	11.9
22.5	12.38	10.16	8.79	4.86	2.59	.74	2.5	3.9	11.9
25	12.38	10.19	8.79	4.86	2.68	.74	2.5	3.8	11.9
27.5	12.38	10.21	8.79	4.86	2.77	.74	2.5	3.7	11.9

creates "areas of indifference" in which there are an infinite number of statistically indistinguishable portfolios from which the investor can choose. This is in direct contrast with traditional mean-variance analysis, which posits that only one portfolio can generate a given return for any given level of risk. By bootstrapping the error terms, this is no longer the case.

The output from either 250 or 25,000 alternative frontiers has little practical meaning by itself.

Consequently, two techniques are suggested for creating statistically significant areas of indifference around each RAP. The first approach is to estimate the mean risk-adjusted return (expected return minus the expected variance) for the 250 alternative portfolios at each RAP, and eliminate all portfolios with expected risk-adjusted return greater than one standard deviation from the mean risk-adjusted return for the RAP. The remaining portfolios are then considered equally

likely to occur, creating an area of indifference in which an investor could choose any portfolio with equal confidence. Exhibit 11 shows one such area for RAP 15. Exhibit 12 compares the allocation ranges and weights for portfolios across selected RAPS. For example, because investors should be indifferent to any of these portfolios, apartment's portfolio share could range from 10% to 70% for RAP 15.

This is in sharp contrast to traditional mean-variance analysis which would calculate only one efficient portfolio for each RAP. At RAP 2.5 however, the ranges are much 'tighter. Here apartment's share ranges from 45.83% to 70% in contrast to the much wider range at RAP 15. Therefore, it would appear that uncertainty increases as one moves along the frontier, and with it the allocation ranges widen. The second technique

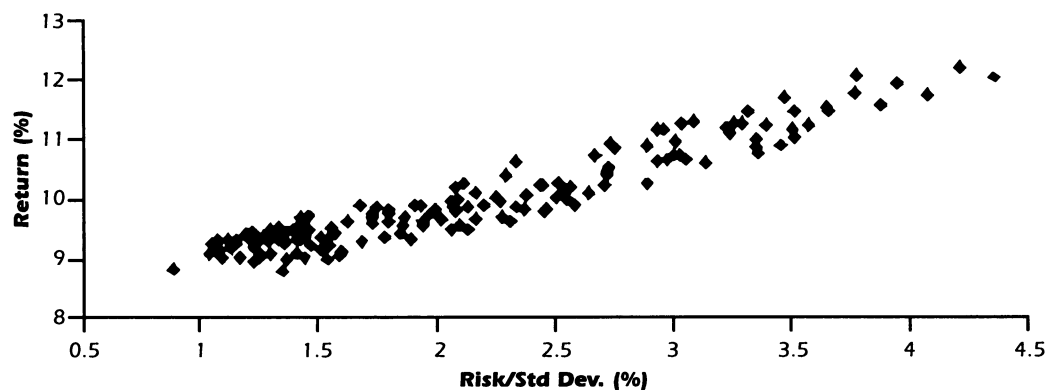## Exhibit 11: Area of Indifference for Risk Acceptance Point (RAP) 15

Exhibit 12: Bootstrapped Allocations and Returns for Selected Areas of Indifference*

	Allocation (%)				Expected		Risk-Adj**
	Industrial	Apartment	Retail	Office	Return (%)	Std Dev. (%)	Return (%)
RAP 2.5 (165 Portfolios)							
Minimum	10.00	45.83	10.00	10.00	8.87	0.88	7.35
Maximum	10.00	70.00	10.00	34.17	10.44	1.76	8.46
Average	10.00	65.81	10.00	14.19	9.49	1.25	7.92
Base Case***	10.00	69.00	10.00	11.00	9.60	0.30	9.51
RAP 15 (176 Portfolios)							
Minimum	10.00	10.00	10.00	10.00	8.79	1.04	0.73
Maximum	10.00	70.00	10.00	70.00	12.22	3.37	8.06
Average	10.00	46.46	10.00	33.54	9.86	2.04	5.30
Base Case	10.00	54.00	10.00	26.00	9.80	1.20	8.36
RAP 27.5 (132 Portfolios)							
Minimum	10.00	10.00	10.00	10.00	8.79	1.48	4.30
Maximum	10.00	70.00	10.00	70.00	12.22	3.95	6.77
Average	10.00	23.74	10.00	56.26	10.39	3.01	0.91
Base Case	10.00	40.00	10.00	40.00	9.90	2.10	5.49

* Results represent portfolios that are within one standard deviation of the mean RAP risk-adjusted return.
** Risk-adjusted Return is expected return, minus the square of the portfolio standard deviation (i.e., variance).
*** Base Case results are for the original unperturbed frontier.

is to choose portfolios located near the centroid of an indifference area. Investors can choose a particular portfolio or allocation ranges suggested by a sampling of portfolios near the geometric centroid

of the range. Selecting portfolios near the centroid affords investors some sense of security, since the location of these portfolios in mean-variance space should approximate the spacial mean of the "fuzzy"

distribution.[11] Selecting a single portfolio at the RAP's centroid, however, is similar to using traditional mean-variance results in as much as any point estimate precludes other possibilities.

Another byproduct of bootstrapping is what would appear to be a collapsing of the frontier into itself. This occurs because portfolios within each indifference area overlap such that both the expected risk and return, as well as portfolio allocations, can appear in more than one indifference area. Exhibit 13 shows how similar apartment allocations exhibit considerable overlap across a wide range of the frontier created by the bootstrapping procedure. Similar results can be seen in Exhibit 14 which shows the overlap of bootstrapped portfolios from different RAPs in mean-variance space. Not only does the bootstrap create multiple efficient portfolios for any point on the curve, but the location of these portfolios in mean-variance space is so dispersed that it reduces wide portions of the efficient frontier to an area of indifference. For example, twenty-three portfolios in RAP 2.5 had risk-adjusted returns that fell within the risk-adjusted return range for RAP 17.5. Ten of these portfolios fell into the risk-adjusted return range for RAP 20. Thus, while there remains a wide variety of portfolios with a correspondingly larger range of expected outcomes, there is enough overlap across the areas of indifference to make it difficult to make statistically supportable inferences about their differences. In this fuzzy world,

investors are not constrained to modern portfolio theory's definition of efficiency, and investors are free to set ranges, rather than specific allocation targets, thus reducing many of the costs and concerns associated with a lumpy, illiquid asset such as real estate. Ranges may also serve to overcome some of the difficulties of smoothing bias associated with appraisal-based returns (see De Wit, 1993).

What are the results suggesting? First, unlike mixed asset (stocks, bonds, cash, real estate) allocation problems, the real estate-only returns, standard deviations and correlations tend to be so much alike that they generate vague results. Second, mean-variance optimization forces the parameters of the utility function to be reduced to a two-dimensional world, which is not sufficiently well-defined to be all encompassing. Third, investors live in a full equilibrium world, not a partial one as approximated by mean-variance optimization.

By forcing asset allocation decisions to be made in reduced-form space, mean-variance optimization is unable to incorporate even the most common form of uncertainty. Allocation decisions based upon data riddled with measurement error and subject to revision should be the asset allocator's first clue that, at best, mean-variance space is likely to be an approximation of the future. Fourth, it is clear that the allocator's job is more difficult than perhaps he or she would like. The presence of uncertainty implies that allocation ranges can be large enough to accommodate a statistical "Mack

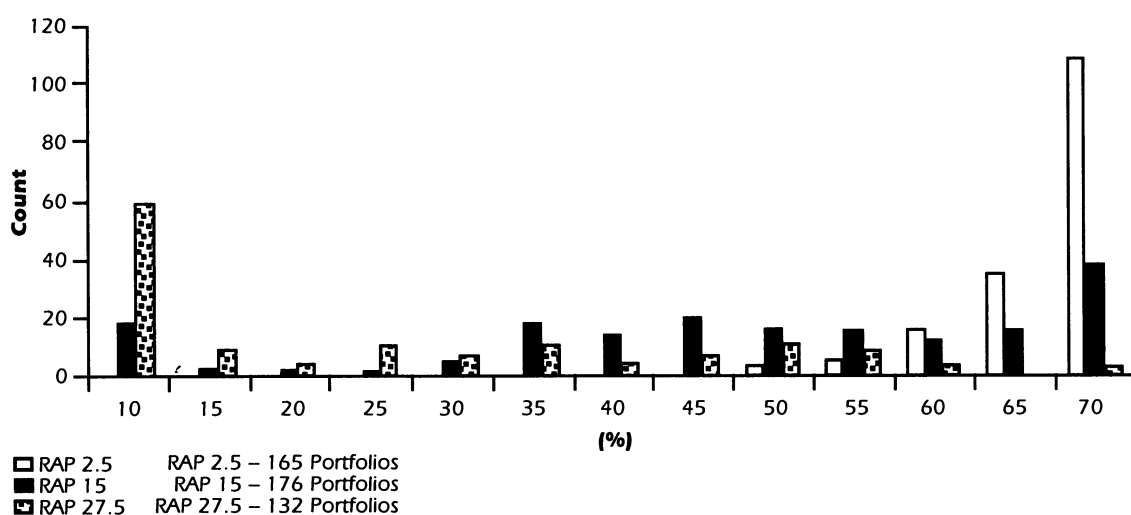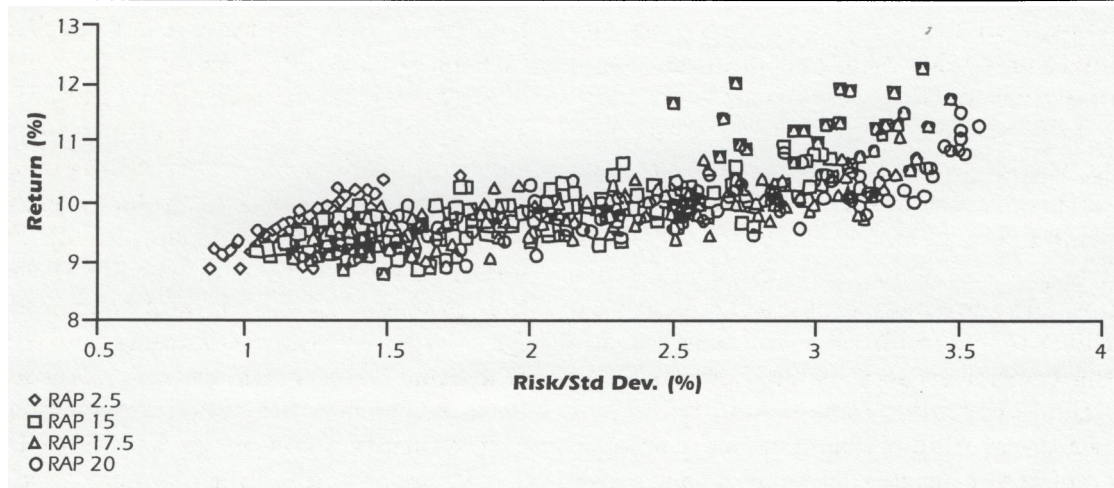**Exhibit 13:** *Areas of Indifference – Distribution of Apartment Allocations*

□ RAP 2.5   RAP 2.5 – 165 Portfolios
■ RAP 15   RAP 15 – 176 Portfolios
▨ RAP 27.5   RAP 27.5 – 132 Portfolios

Exhibit 14: *Bootstrapped Areas of Indifference for Selected RAPs*

◇ RAP 2.5
□ RAP 15
△ RAP 17.5
○ RAP 20

truck." Finally, the good news. Although uncertainty may reduce the allocator's ability to precisely allocate, it also serves as a reminder that rebalancing does not have to occur every time exceptions shift. Rather, investors may find that these changes fall within the boundaries of the indifference area, thereby often eliminating the need and the expense of rebalancing.

CONCLUSION

Given a reasonable set of assumptions regarding future real estate returns, it would appear that institutional portfolios are underweighted with respect to multifamily properties. In addition, given a set of econometrically derived forecasts, it would also appear that multifamily should be considered a "core" asset, a role currently reserved for offices and retail. This conclusion is consistent with earlier studies that came to the same conclusion with respect to multifamily's role in a real estate portfolio.[12] Further reinforcement comes from the "naive" data that suggests multifamily allocations should be at least 30% of an investor's real estate portfolio.

The question of exactly how much multifamily investment property should be held, however, is complicated by the presence of uncertainty. In mean-variance space, the standard deviation of expected returns is used as a proxy for uncertain outcomes. However, given the nonlinearity of the allocation algorithm, as well as the unknown duration and

timing of the uncertain events, point estimates of returns, standard deviations and correlations are inadequate descriptors of the future. By bootstrapping the error terms from an OLS-derived return model, the resulting frontiers suggest that the two-dimensional world of mean-variance space is inadequate in an uncertain world.

The resulting fuzzy frontiers create areas of indifference from which asset allocators can freely choose statistically similar portfolios for any corresponding level of risk and return found on the traditional frontier. This contrasts sharply with the notion that each point on the frontier is unique, as is the case when only point estimates are used. Bootstrapping also has the effect of collapsing the frontier on to itself such that it is statistically impossible to differentiate between the risk/return trade-off for large segments of the curve. Like horseshoes and hand grenades, being close may be good enough, or perhaps as good as you can get!

NOTES

1. Markowitz (1991), p.4.
2. Northfield Information Services' PACO software package was used in this study.
3. Investable universe estimates were compiled by Arthur Andersen in 1993.
4. As will be shown, it is the equations' error terms that are the focus of this exercise. Goodness of fit is indeed important to the extent that it reduces or increases the impact of uncertainty. Forecasts of the exogenous inputs were provided by Regional Financial Associates (RFA).

5. The Sharpe ratio used throughout this paper is the ratio of expected return divided by its expected standard deviation. See Sharpe (1994).

6. The PACO software used in this study produces risk, return and allocations for various points along the frontier called Risk Acceptance Points (RAPs). As the name implies, a RAP is a specific point on the frontier. Each RAP represents a specific return/risk position. Associated with each RAP is a specific portfolio with its own asset mix. For this paper eleven specific RAPs were analyzed. The formula for each RAP is one divided by the slope of the frontier at a specific point. Therefore by moving to higher and higher RAPs an investor is effectively moving further out on the frontier.

7. Similar problems arise for liquid assets such as stocks and bonds and/or mixed asset portfolios containing both real estate and liquid assets, however, in the case of the former, the speed and relatively low rebalancing costs reduce the problem significantly.

8. While the mean-variance model only requires users to input expected returns and risk over the entire holding period, mean returns and standard deviations are the product of period-by-period changes in asset behavior.

9. Markowitz (1991), p.4.

10. Hall (1992).

11. For the 176 portfolios within +/- one standard deviation of the mean risk-adjusted return for RAP 15, the median portfolio had an apartment allocation of 44.5% which approximates the mean risk-adjusted allocation of 43%.

12. Gold (1995), p.41.

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