First draft - comments welcome

Benchmarking Private Equity
The Direct Alpha Method

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Abstract

We reconcile the major approaches in the literature to benchmark cash flow-based returns of private equity investments against public markets, a.k.a. 'Public Market Equivalent' methods. We show that the existing methods to calculate annualized excess returns are heuristic in nature, and propose an advanced approach, the 'Direct Alpha' method, to derive the precise rate of excess return between the cash flows of illiquid assets and the time series of returns of a reference benchmark. Using real-world fund cash flow data, we finally compare the major PME approaches against Direct Alpha to gauge their level of noise and bias.

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I. Introduction

For several decades, Modern Portfolio Theory (MPT) has been very useful to investors. MPT provides a broad intellectual framework and a set of tools for measuring performance, managing risk, and constructing portfolios. For investments in illiquid assets such as private equity (PE), however, MPT has not been so helpful. The main problem is that some of the key statistics used in MPT are difficult to measure for PE. This is especially true of alpha, the rate of return from non-market sources.

Claims about alpha are very common in PE investing, but there is often confusion of how to quantify alpha. Institutional investors (LPs) very often use assumptions about the alpha they could achieve in their PE portfolios to make asset allocation decisions. PE fund managers (GPs) very often make claims about 'outperformance' in their marketing. But it is rather uncommon for either claim to be backed by a formal procedure estimating the actual alpha that has been obtained.

In recent years, a number of methods have been proposed to estimate alpha in PE. Broadly speaking, these methods are called Public Market Equivalent (PME), and the idea they all have in common is to infer alpha *indirectly* by a comparison with the return that could have been obtained from investing in some public market benchmark. Beyond that, all of these methods appear to be quite different.

In the first part of this paper we reconcile the major PME methods, and show that these are in fact quite closely related mathematically, yet remain approximations by definition. We then propose an advanced method for the exact derivation of alpha between a series of investment cash flows and the series of returns from a reference benchmark. This method, which we call *Direct Alpha*, can be applied to any illiquid portfolio for which only cash flows are observable. Finally, using a sample of real-world fund cash flow data, we compare empirical results of the major PME methods against Direct Alpha to gauge their level of noise and bias.

In the typical MPT methods, performance analysis often starts with a model for the time-evolution of the value of the portfolio in question. A standard approach uses a return model of the form

$$r(t) = b(t) + \alpha + \varepsilon(t) \tag{1}$$

where

- $r(t)$ is the return to the portfolio at time t
- $b(t)$ is the market contribution to return (which may be further broken down into beta and risk-free return factors)
- α and mean-zero $\varepsilon(t)$ are the non-market contribution to return[1]

For public equities, where frequent and reliable clearing prices are available for both the portfolio and the market reference benchmark, standard regression techniques can be applied to estimate alpha. For illiquid assets like private equity or private real estate, such clearing prices are not available by definition. Existing measures of the value of PE investments, such as net asset value (NAV) statements issued by GPs, are necessarily the product of some appraisal process and generally exhibit serious smoothing and lagging.[2] This is especially the case for NAVs prior to the adoption of FAS 157 by 2008, which attempts to bring some standardization to the valuation of illiquid assets. As a consequence, NAV estimates contribute to unreliable time series of returns, which in turn result in unreliable estimates of alpha when standard regression techniques are applied.

PME approaches come at the alpha estimation problem from the opposite direction. Rather than differencing unreliable NAVs down to unreliable series of returns, these approaches are based on observable cash flows to improve the reliability of the resulting estimates. This perspective was first documented by Long and Nickels (1996) in their Index Comparison Method (ICM), later recognized as the first of various PME methods. The Long-Nickels ICM/PME is a powerful heuristic approach, but it is not an exact solution for alpha as represented in the standard return model in Equation (1). In response to some perceived shortcomings of ICM/PME, certain extensions have been proposed, including PME+ by Rouvinez (2003) and Capital Dynamics, and mPME by Cambridge Associates (2013). These

[1] Note that in the context of one-period models like CAMP, continuously-compounded returns make α a slightly biased estimate of the mean abnormal return. This bias is an increasing function of the variance ε(t) as explained in Ang and Sørensen (2012). While the solution for the stochastic case is beyond the scope of this paper, it is addressed in Griffiths (2009).

[2] See Jenkinson, Sousa and Stucke (2013), and Brown, Gredil and Kaplan (2013).

extensions are based on the original ICM/PME method and, hence, represent heuristic approaches, too, rather than exact solutions for alpha.

Kaplan and Schoar (2005) make a key step by introducing their Public Market Equivalent ratio. Unlike its heuristic predecessors, KS-PME can actually be derived from the return model in Equation (1). However, it is limited to measuring the overall wealth generated by the illiquid asset compared to a benchmark, without regards to the rate at which this excess wealth has accrued. Therefore, investors cannot use it in their MPT tools without making additional assumptions.

Griffiths (2009) makes a key step by showing that the return model in Equation (1) can be integrated up to a model for cash flows in case of log returns and, thus, solved *directly* for alpha. This technique, in the following referred to as Direct Alpha, estimates the per-period abnormal return of the illiquid portfolio's cash flows relative to the reference benchmark. Gredil and Stucke independently arrive at similar conclusions in (2012).

Today, many PE investors are aware of some or all of these various PME techniques. However, there is a great deal of confusion about the appropriateness of the individual approaches, how these are linked to each other, and how they have to be implemented. In the following, we aim to dispel some of that confusion by reconciling each of the different techniques in detail. In fact, we show that the two exact methods, Direct Alpha and KS-PME, are both the easiest to implement and the most closely related to traditional performance measures used in PE.

Exhibit 1 provides a first illustration of the relationship between Direct Alpha and the major PME methods. It turns out that a simple transformation of the actual PE cash flows into their future values is already sufficient to derive the exact alpha relative to the chosen reference benchmark. Instead, the heuristic PME methods start by building a hypothetical portfolio in the public market first, from whose performance they then approximate alpha as a ΔIRR. Since the heuristic PME methods effectively build on the future values of the actual PE cash flows, too, their indirect approach makes the estimation of alpha unnecessarily complicated and biased. For example, ICM/PME iteratively calculates the final NAV of the corresponding investments in the public market, which is actually the result of the difference between the future value of contributions and the future value of distributions. PME+ rescales the actual PE distributions by a fixed scaling factor such that the difference between the future value of contributions and the future value of distributions is equal to the NAV of the PE portfolio. mPME uses a time-varying scaling factor to rescale both the PE distributions and the closing NAV and, finally, KS-PME is simply the ratio of the future values of distributions and contributions.

Note that the focus of this paper is to introduce the formally correct method to extract the rate of excess return between a series of PE cash flows and the time series of returns from a reference benchmark. At this point, we do not address the question about the appropriate reference benchmark for PE investments, nor account for additional factors such as beta or the risk-free rate. These considerations, albeit important, will be part of a follow-on paper.

II. A review of the different approaches

In this section we reconcile the four most widely used approaches to compare the returns from a private equity portfolio against a reference benchmark.[3] While each approach has its individual advantages and weaknesses, they all share the same spirit and, as we show, are closer aligned than commonly assumed.

The four input variables are the same in all cases:

- A sequence of contributions into the PE portfolio: $C = \{c_0, c_1, \dots, c_n\}$
- A sequence of distributions from the PE portfolio: $D = \{d_0, d_1, \dots, d_n\}$
- A residual value of the PE portfolio at time n: NAV_{PE}
- A reference benchmark (*e.g.*, the public market): $M = \{m_0, m_1, \dots, m_n\}$

The reference benchmark serves as the opportunity costs of capital, and is used to capitalize contributions, distributions and the NAV to the same single point in time to make them comparable. This can either be a present value via discounting the PE cash flows by the benchmark returns, or a future value via investing and compounding PE cash flows with the

[3] We use the term 'private equity portfolio' interchangeably for direct PE investments, PE fund investments, or investments in a portfolio of PE funds. Similarly, the methodologies presented below are not limited to private equity only, but can be applied to any type of private capital investments.

benchmark.[4] In this section we follow the perspective of future values, as this is the one behind the original ideas underlying each approach and generally more intuitive. Based on the sequence of contributions and distributions, their future values at time n are defined as follows:

- The future value of contributions at time n is: $FV(C) = \left\{c_0 \cdot \frac{m_n}{m_0}, c_1 \cdot \frac{m_n}{m_1}, \ldots, c_n\right\}$
- The future value of distributions at time n is: $FV(D) = \left\{d_0 \cdot \frac{m_n}{m_0}, d_1 \cdot \frac{m_n}{m_1}, \ldots, d_n\right\}$

A. Heuristic approaches to measure annualized excess returns

To date, three main approaches to estimate the annualized excess return between a PE portfolio and a reference benchmark have been developed and adopted by the industry. The first one is the Index Comparison Method by Long and Nickels from the early-1990s, which is also referred to as Public Market Equivalent. In the early-2000s, Rouvinez and Capital Dynamics introduced the Public Market Equivalent Plus method, followed by the Modified Public Market Equivalent method by Cambridge Associates in the late-2000s.

Each of the three approaches seeks to estimate the excess return in an *indirect* way, *i.e.*, by investing and divesting a PE portfolio's cash flows with the reference benchmark, and calculating the spread against the PE portfolio's IRR. Due to the non-additive nature of compound rates (see Section II.A.4), these approaches represent heuristics by definition.

A.1. The Index Comparison Method

The Index Comparison Method (ICM), first documented by Long and Nickels (1996), combines a PE portfolio's cash flows with the returns from the reference benchmark to determine the IRR (or money multiple) that would have been obtained had the PE cash flows been made instead in the benchmark. Under this approach, every capital call of the PE portfolio (*i.e.*, contribution by an LP) is matched by an equal investment in the reference benchmark at that time. Similarly, every capital distribution from the PE portfolio is matched by an equal sale from the reference portfolio. In between, actual invested amounts of capital change in value according to the change in the benchmark. The result is an identical series of contributions and distributions, but a

[4] Note that the term 'future value' refers to the point in time of the analysis, or the last occurrence of a cash flow or closing valuation. Similarly, the term 'present value' refers to the time of the first cash flow.

different residual value as derived from the reference portfolio. The IRR of this reference portfolio then serves as the basis for calculating the spread against the IRR of the PE portfolio. The residual value of the reference portfolio at time n is

$$NAV_{ICM} = \sum FV(C) - \sum FV(D) \tag{2}$$

The IRR of the reference portfolio is

$$IRR_{ICM} = IRR(C, D, NAV_{ICM}) \tag{3}$$

The IRR spread of the PE portfolio is defined as the difference between both IRRs

$$\Delta IRR = IRR_{PE} - IRR_{ICM} \tag{4}$$

Exhibit 2 presents a numerical example.

The Long-Nickels approach is deeply appealing from an intuitive point of view and has provided an excellent early guidance for institutional investors seeking to adjust annualized private equity returns for general market movements. The main issue with ICM is, however, that the hypothetical reference portfolio typically does not liquidate as the PE portfolio does. In case of a strong outperformance (underperformance) by the PE portfolio, the reference portfolio carries a large short (long) position in later years. As the PE portfolio approaches liquidation, swings in the benchmark may have essentially no impact on the value of its unrealized investments, but a big effect on the residual value of the reference portfolio. Therefore, ICM may be an unreliable measure of relative performance in those cases.

A somewhat related issue is that – following many years of matched, hence, identical cash flows – the impact of the difference between NAV_{ICM} and NAV_{PE} to ΔIRR loses significance as the reference portfolio matures. While this effect may partly mitigate the aforementioned impact of swings in the benchmark, the ΔIRR of a PE portfolio that takes a long time to eventually liquidate trends, *ceteris paribus*, towards zero.

Finally, a potential short position in the reference portfolio needs to be balanced by a closing contribution at time n. In about 5-10% of all cases the resulting stream of cash flows effectively prevents the calculation of the IRR_{ICM} and, hence, the ΔIRR.[5] By 1996, ICM has been promoted under the name Public Market Equivalent (PME) by Venture Economics.

A.2. The Public Market Equivalent Plus method

In response to the noted issues of the ICM/PME approach, Rouvinez (2003) and Capital Dynamics introduce the PME+ method.[6] PME+ is designed to generate the same residual value in the reference portfolio as of the PE portfolio at time n, and eventually to liquidate as the PE portfolio does. To arrive at identical residual values, distributions from the PE portfolio are matched against the reference portfolio after applying a fixed scaling factor.

Let s be the scaling factor for the distribution sequence. Then s is selected that

$$NAV_{PE} = \sum FV(C) - \mathrm{s} \cdot \sum FV(D) \tag{5}$$

$$\Leftrightarrow \quad s = \frac{\sum FV(C) - NAV_{PE}}{\sum FV(D)} \tag{6}$$

The IRR of the reference portfolio is

$$IRR_{PME+} = IRR(C, \mathrm{s}D, NAV_{PE}) \tag{7}$$

The IRR spread of the PE portfolio is defined as

$$\Delta IRR = IRR_{PE} - IRR_{PME+} \tag{8}$$

Exhibit 3 presents a numerical example.

While the PME+ approach effectively avoids the aforementioned issues of ICM/PME, it introduces its own difficulties. Given the sensitivity of the IRR measure to early distributions, a downscaling (upscaling) of distributions in case of an outperformance (underperformance) by the PE portfolio has an inflating effect on the positive (negative) ΔIRR. A related issue is that PME+ cannot be calculated, by definition, for younger PE portfolios, if no distributions have yet taken

[5] A modification to avoid the short position in later years in case of outperformance by the PE portfolio is to stop matching distributions from the PE portfolio against the reference portfolio when the interim NAV_{ICM} becomes zero.
[6] Note that Capital Dynamics has been granted a U.S. patent for PME+ in 2010 (#7,698,196).

place;[7] and in cases in which only a few distributions have occurred, the scaling factor s may actually be negative and turn distributions into additional contributions.

In contrast to ICM/PME, PME+ does not constitute an investable portfolio, since the distribution scaling factor s adjusts all prior distributions based on the NAV_{PE} at the time of the analysis. Therefore, PME+ is a non-causal process that cannot be followed by a real investor.

A.3. The Modified Public Market Equivalent method

The mPME method has been developed by Cambridge Associates in the later 2000s. Similar to PME+, this method aims to avoid the noted issues of ICM/PME and have the reference portfolio liquidating as the PE portfolio does. For this purpose, distributions from the PE portfolio are not matched against the reference portfolio in absolute capital terms (as for ICM/PME), but in relative terms proportionately to the succeeding interim valuations of the PE portfolio and the reference portfolio. The result are rescaled distributions from the reference portfolio such that

$$D_{mPME,i} = \left(\frac{D_i}{D_i + NAV_{PE,i}}\right) \cdot \left(NAV_{mPME,i-1} \cdot \frac{m_i}{m_{i-1}} + C_i\right) \tag{9}$$

The IRR of the reference portfolio is

$$IRR_{mPME} = IRR(C, D_{mPME}, NAV_{mPME,n}) \tag{10}$$

with

$$NAV_{mPME,n} = \left(1 - \frac{D_i}{D_i + NAV_{PE,n}}\right) \cdot \left(NAV_{mPME,n-1} \cdot \frac{m_n}{m_{n-1}} + C_i\right) \tag{11}$$

The IRR spread of the PE portfolio is defined as

$$\Delta IRR = IRR_{PE} - IRR_{mPME} \tag{12}$$

Exhibit 4 presents a numerical example.

While adjusting the distributions from the reference portfolio proportionately to the succeeding interim balances appears to be a fair treatment, the shortcomings of mPME are

[7] This is typically more often the case for venture capital funds, rather than buyout funds.

similar to those of PME+. Any rescaling of distributions, whether with a fixed or a time-varying scaling factor has an inflating effect on ΔIRR. Yet, a time-varying factor introduces a further issue. Rescaling distributions relative to interim balances of illiquid assets is likely to generate an additional bias if there are any pricing errors in the time series of the PE portfolio's interim NAVs. As a consequence, even if the PE portfolio and the reference portfolio have exactly the same true returns, mPME will return different results.

A.4. The non-additive nature of compound rates

While each of the previous three approaches is innovative in its own right, they cannot – by definition – arrive at a PE portfolio's exact rate of excess return relative to the reference benchmark. Leaving aside the individual issues mentioned the reason is the non-additive nature of compound rates such as the IRR, which follows from Cauchy's functional equation. In this context, the overall return of a PE portfolio could be expressed in the functional format

$$f(x+y)$$

with x being the 'equivalent' benchmark return generated by the reference portfolio, and y being the additional return generated by private equity, which we would like to learn about. To identify the excess return of private equity, the three approaches follow an *indirect* way by calculating the return of the reference portfolio in a first step and then subtracting them from the overall return of the PE portfolio. This could be expressed as

$$f(y) = f(x+y) - f(x) \tag{13}$$

However, this equation does not hold for compounding functions. Consequently, it is not feasible to arrive at the correct rate of excess return of the PE portfolio in this way. In Section III, we introduce our Direct Alpha method which calculates the rate of excess return *directly* based on the PE portfolio's cash flows and the time series of returns from the reference benchmark.

B. The Public Market Equivalent method by Kaplan and Schoar

Kaplan and Schoar (2005) introduce a different method to compare the returns of a PE portfolio against a reference benchmark, which they also refer to as public market equivalent (in the following, we refer to it as KS-PME). Their approach does not aim for an annualized rate of excess return, but seeks to answer the question, how much wealthier (as a multiple) an investors has become at time n by investing in the PE portfolio instead of the reference benchmark. As before, contributions are assumed to be invested in the benchmark. Similarly, distributions are reinvested in the benchmark. A residual value in the PE portfolio is taken at face value at time n. It follows

$$KS\text{-}PME = \frac{\sum FV(D) + NAV_{PE}}{\sum FV(C)} \tag{14}$$

A ratio above one indicates that the PE portfolio has generated excess returns over the reference benchmark. A ratio below one represents the opposite. Effectively, KS-PME is the money multiple, or TVPI, of the future values of the PE portfolio's cash flows.

$$KS\text{-}PME = TVPI(FV(C), FV(D), NAV_{PE}) \tag{15}$$

Exhibit 5 presents a numerical example.[8]

Consequently, KS-PME represents the returns of a strategy that finances the contributions into the PE portfolio by short-sales of the reference benchmark and reinvests all distributions back into the benchmark until time n. The clear advantage of this approach is that it always yields a valid and reliable solution. The principal drawback is that it gives no information about the (per-period) rate at which the excess wealth has accrued.

KS-PME is similar to the scaling factor s of the PME+ approach, and in case the PE portfolio is fully liquidated (NAV_{PE}=0) it is exactly the multiplicative inverse of s. Along comes a similar interpretation. KS-PME is a factor that indicates by what percentage the returns of the PE portfolio have exceeded the returns of the reference benchmark over its lifetime, *i.e.*, by what factor contributions in the benchmark would have to be increased to meet subsequent distributions from the PE portfolio.[9] The scaling factor s indicates by what percentage distributions from the PE portfolio would have to be reduced to match the value generated from the contributions into the benchmark.

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[8] Note that, instead of future values, KS-PME can be equally calculated via present values, *i.e.*, discounting all PE cash flows and the final NAV_{PE} back to the date of the very first cash flow.
[9] Note that, in line with the three heuristic approaches, KS-PME makes no assumption about investor preferences or a compensation for different levels of market risk.

III. The Direct Alpha method

This section describes the Direct Alpha method which avoids the noted issues of the heuristic approaches to measure a PE portfolio's annualized rate of return relative to a benchmark. As a result, Direct Alpha is both a robust and a reliable measure. In fact, the Direct Alpha method actually formalizes the calculation of the exact alpha (in a continuous-time log-return sense) that a PE portfolio has generated relative to the chosen reference benchmark. The underlying methodology including the stochastic generalization that is outside the scope of this paper has been independently developed by the authors in the past.[10]

A. The general form

In contrast to heuristic approaches, such as ICM/PME, PME+ and mPME, which aim to estimate a PE portfolio's relative performance *indirectly* by calculating a ΔIRR against matched investments in the reference benchmark, the Direct Alpha method represents the *direct* calculation of the PE portfolio's exact alpha

$$\alpha = \frac{\ln(1 + \mathrm{a})}{\Delta} \tag{16}$$

where a is the discrete-time analog of α

$$\mathrm{a} = IRR(FV(C), FV(D), NAV_{PE}) \tag{17}$$

and Δ is the time interval for which alpha is computed (typically one year). The underlying derivation of Direct Alpha can be found in Appendix A.

Note that by pursuing the formally correct *direct* method, Direct Alpha is not a 'public market equivalent' measure in the literal sense. That is, we do not (need to) calculate an *equivalent* public market rate of return in the first place from which to infer a ΔIRR as an approximation of alpha.

[10] See Griffiths (2009) for the first available documentation, including treatment of time-varying structure of systematic returns, multivariate reference indexes with betas other than 1.0, and construction of estimation error bounds that depend on portfolio-level specific risk.

B. Numerical examples

Exhibit 6 presents a simple numerical example for the calculation of the arithmetic alpha a. The actual contributions and distributions of the PE portfolio are compounded by the returns of the public equity index up to Dec-31, 2010, and then combined with the final NAV_{PE} to form the series of future values of net cash flows. As shown in Equation (17) and derived in Appendix A, the IRR over this series of cash flows represents the arithmetic alpha a of the PE portfolio relative to the reference benchmark. In this example, the IRR of the PE portfolio's actual net cash flows is 17.5%. The corresponding arithmetic alpha is 12.6%, representing the annualized rate of return beyond the returns of the public equity index.

The underlying rationale of compounding all PE cash flows to the same single point in time is to 'remove' or 'neutralize' the impact of any changes in the public equity index from the series of actual PE cash flows. By doing so, the resulting capitalized net cash flows do no more 'contain' any changes of the index, but reflect only the sole PE returns above or below the index returns.

As explained, it is critical to capitalize all PE cash flows by the public equity index to the same single point in time. In line with the natural process of value creation and the intuition underlying the heuristic approaches, we have followed the perspective of future values above. However, it is equally possible to capitalize all PE cash flows (and the final NAV_{PE}) by the index returns to any other point in time with the arithmetic alpha a remaining the same. For example, instead of future values one can equally follow a present value perspective.

Exhibit 7 adds the present value calculation to the current example, which discounts the PE portfolio's actual contributions, distributions and the NAV_{PE} back to Dec-31, 2001. As a result, the series of capitalized net cash flows changes in nominal terms. However, the series of present values and the series of future values differ only by a single constant factor (1.31) and, hence, the 'relationship' of the cash flows within each series remains unaffected. As a result, the arithmetic alpha (and the KS-PME) remains the same.

While it is only a matter of taste, whether to compound the actual PE cash flows to their future values, or to discount them to their present values, some people may find the present value perspective more intuitive. It can be interpreted as 'removing the contribution' of the public equity index from all (of the subsequent) PE cash flows. The future value approach, in turn, has the advantage of keeping the NAV_{PE} at face value.

C. The relationship between Direct Alpha and other methods

Direct Alpha and KS-PME are intimately related. In a sense, one can think of Direct Alpha as an annualized KS-PME taking into account both the performance of the reference benchmark and the precise times at which capital is actually employed. In Appendix B we show this link more formally through a Net Present Value perspective. Note that, by construction, Direct Alpha is zero whenever KS-PME equals one.

By this logic, combining Direct Alpha and KS-PME is a particularly convenient way to learn about the effective duration of the PE portfolio that is comparable across different market return scenarios.[11]

$$Direct\ Alpha\ Duration = \frac{ln(KS\text{-}PME)}{ln(1 + Direct\ Alpha)} \tag{18}$$

The relationship between Direct Alpha and KS-PME is also equivalent to that of the two traditional PE performance measures, IRR and TVPI. Just as the ratio of TVPI to KS-PME describes a fund's lifetime gross return due to the market-related factor, the difference between IRR and Direct Alpha describes[12] the market-related *rate of return*:

$$Market\text{-}related\ Multiple = \frac{TVPI}{KS\text{-}PME} \tag{19}$$

$$Market\text{-}related\ Rate\ of\ Return = IRR - Direct\ Alpha \tag{20}$$

Note that the heuristic methods essentially reverse the direction in Equation (20), *i.e.*, they subtract an estimate of the market-related rate of return from the IRR. As the next section demonstrates, the precision and bias of the heuristic methods depends on how closely each approach mimics the market-related return. One may then expect that mPME should be getting closer, since it adjusts both distributions and NAVs. However, this is not necessarily the case due

[11] It is defined and positive whenever KS-PME is not exactly equal to one.

[12] As explained in Section II.A.4, IRRs are non-additive by nature. One can either estimate the non-market rate of return correctly and refer to the difference against the PE IRR as "market-related rate of return", or try to estimate a hypothetical market portfolio IRR and subtract it from the PE IRR. In both cases, the residual is NOT a rate of return but an approximation thereof that depends on the cash flow schedule, etc.

to the non-additivity of compound returns as discussed in Section II.A.4. Except for very special cash flow and market return scenarios, neither approach will be exactly equal to Direct Alpha.

D. Empirical comparison of the different methods

In this section we provide empirical evidence on the differences in the heuristic methods vis-à-vis Direct Alpha, using fund cash flow data from Burgiss, a leading provider of portfolio management software, services, and analytics to limited partners investing in private capital. Burgiss maintains one of the largest databases of precisely timed fund level cash flows, containing over 5,300 private capital funds sourced directly from around 300 limited partners.

We compute the relative performance measures for the total of 1,044 buyout and 1,173 venture funds incepted between 1980 and 2007, using the total returns of the S&P 500 as the reference benchmark. We begin by examining magnitudes. Exhibits 8 through 10 present scatter-plots including all 2,217 funds, with Direct Alpha values plotted against the horizontal axis and the values of the respective heuristic measure against the vertical axis. In all left-hand panels, axes are limited to the -25% to +25% per annum interval; in all right-hand panels, axes zoom out to -50% to +50%. The 45-degree lines denote one-to-one relations. Thus, asymmetries of observation around the red lines indicate nonlinearities and biases.

Exhibit 8 shows that the ICM/PME ΔIRRs often deviate notably from Direct Alpha values. This is particularly the case for mature funds with higher excess returns. In an unreported analysis we confirm that most of the clustering of the ΔIRRs around zero for high values of Direct Alpha can be avoided if we constrain NAV_{ICM} in Equation (2) to be non-negative. However, such an ad-hoc adjustment does not resolve the non-monotonicity of the error in the distance of Direct Alpha from zero. In about 9% of all cases, the negative ICM_{NAV} effectively avoids the calculation of an ICM/PME IRR and the corresponding spread.

In contrast to ICM/PME, ΔIRRs of PME+ provide a better approximation of the precise excess returns for small values, as show in Exhibit 9. However, the slope of the relation appears to be biased as shown by the higher density of observations above (below) the red line for excess returns beyond +10% (-10%). As mentioned earlier, a slope above one is the result of downscaling (upscaling) distributions in earlier years. It should be noted that slightly over 2% of all funds, primarily from 2006 and 2007, have not yet distributed any capital back to LPs. Consequently, it is not feasible to apply PME+ in those cases.

As suggested in Exhibit 10, the overestimation (underestimation) pattern appears to be much less pronounced for mPME ΔIRRs compared to PME+. The difference looks more random and the variance is largely independent of the magnitude of excess returns.

We conclude the scatter-plot analysis with Exhibit 11, which compares annualized KS-PMEs against Direct Alpha values. We obtain the former by raising KS-PME to the power of 1/D and subtracting one, where D is the distance in years between the weighted-average dates of all distributions and contributions. In this case, the differences represent the different weighting schemes applied for the cash flow duration assumption. As opposed to being fixed at one, Equation (18) implies that distributions following above average market returns receive less weight.[13] Thus, the deviations from the red line are mostly a statement about market trends that prevailed during the life of those funds. The main takeaway is that the magnitude of the differences tends to be smaller than for the heuristic approaches, yet starts becoming meaningful above +/- 10%, too.

Next, we study how the identified differences affect the ranking of funds within each strategy and vintage year.[14] Exhibit 12 reports transition probabilities between the performance quartiles as measured by Direct Alpha (in rows) and each method. If the differences plotted on Exhibits 8 to 11 were inconsequential for funds comparisons (at a quartile-rank granularity), there should be 100s on the diagonals and zeroes everywhere else. Clearly, this is not what we find in the data.

Panel A1 reports that only 75.2% of the top-quartile funds, as measured by Direct Alpha, will also be classified as top quartile if the ICM/PME method is used, whereas 15.4% of these top quartile funds appear in the 3rd ICM/PME quartile. As one would expect, such transitions occur more frequently in the middle two quartiles. They also turn out to be rather asymmetric: in both cases, conditional on a discord with the Direct Alpha quartile, ICM/PME is more likely to assign a higher quartile (if an upper and lower alternative is present). In Panel A2 we constrain

[13] The positive impact of a given distribution on the PME value decreases in the market returns preceeding it. It is easier to see this when KS-PME is written in present value terms as in Appendix B. Thus, *ceteris paribus*, the numerator of Equation (18) decreases in the market returns as well, and so does the duration implied by Direct Alpha (albeit the effect is attenuated by opposite changes in the denominator).

[14] Pre-1990 we pool funds with the preceding vintage year (separately for buyout and venture), so that there are at least 20 funds to be ranked in each group to reduce the probability of outlier effects. The results are virtually insensitive to the pooling method.

ICM_{NAV} to be non-negative, *i.e.*, we avoid a short position in ICM/PME's reference portfolio. The accuracy of the approximation increases materially, especially in the middle two quartiles.

Panel B suggests that the discrepancies in ranks are smaller for the PME+ method with more than 90% of all funds having a concordant quartile ranking. However, the transitions in the middle two are again asymmetric. As opposed to ICM/PME, PME+ seems to be more likely to assign a lower quartile other than the same.

Consistent with the impression of a "homoscedastic, random but sizable difference" from Exhibit 10, there is more symmetry in transitions for the middle two quartiles in the mPME case, as shown in Panel C. However, the overall level of concordance (values along the diagonal) turns out to be lower than in the PME+ comparison.

Again, in-line with the expectations from the scatter-plots, Panel D1 shows that the concordance is highest and the off-diagonal is symmetric for annualized KS-PMEs. As a reference, we also compute quartile transition probabilities for the original KS-PME multiple in Panel D2. Both the diagonal and symmetry characteristics appear to be very close to those in the mPME case.[15]

We conclude this section with a multivariate analysis of the differences in the market-adjusted return estimates vis-à-vis Direct Alpha and the resulting ranking implications. Exhibit 13 reports estimates from a linear regression model.

The dependent variable in specifications (1), (3) and (4) is the absolute difference of the Direct Alpha values from the ΔIRRs based on the mPME, ICM/PME, and PME+ approaches, respectively. The explanatory variables are: (i) the absolute level of Direct Alpha, (ii) the Direct Alpha duration, the interaction of (i) and (ii), the mean market return (iii), and the volatility of the market return (iv) over a fund's life, as well as a dummy variable indicating venture funds (v). In specifications (2), (4) and (5), we change the dependent variable to the absolute percentile rank differences within each respective strategy and vintage group, and add the standard deviation of Direct Alpha within each strategy and vintage group as an additional explanatory variable.

Specification (1) suggests that over one-third of the variation of absolute differences between mPME and Direct Alpha can be explained by the five covariates. In contrast to the bivariate evidence from Exhibit 10, the distance between mPME and Direct Alpha increases by

[15] This does not imply that KS-PME quartiles are highly concordant with those of mPMEs, however.

about 0.3 percent for each one percent that Direct Alpha deviates from zero, yet this effect is mitigated for long-duration funds. Note that a very trendy and volatile benchmark is also associated with a large diversion of Direct Alpha and mPME values. However, it does not look as if any of this variation affects the fund ranking in a systematic way (specification 2).

In contrast to mPME, ICM/PME and PME+ level-discrepancies with Direct Alpha appear to be very fund-idiosyncratic or non-linear since specifications (3) and (5) fail to explain much of the variation. However, when it comes to rank-discrepancies (specification 4 and 6), almost each regressor becomes individually significant in both models.

In this context, note how substantial the effect of the benchmark's trend and volatility are. Adjusting for the scaling difference, it is ten times as large for ICM/PME as that of the magnitude of Direct Alpha. Note also that the coefficient of the VC-dummy is significantly positive in (3) and (6), even though we control for the dispersion of Direct Alphas among the fund being ranked.[16] This indicates that the heterogeneity in cash flow patterns plays an important role in the rankings of ICM/PME and PME+, although both approaches attempt to difference it away.

IV. Summary and outlook

Reflecting a strong call for market-adjusted performance analysis, private equity research has developed several methods over the past two decades. We provide a comprehensive review of these methods in this paper. While each approach answers a viable economic question, some also constitute an investable long private equity and short public market strategy.

However, if one's objective is to assess private equity performance based on the closest equivalent to the CAPM intercept and deploy some of the MPT tools as part of portfolio construction, we propose the adoption of our Direct Alpha method. The additional insights that the heuristic alternatives may provide are ambiguous to interpret and their implementation is often less straightforward.

An important aspect that is outside the scope of this paper involves the selection of the reference benchmark and the adjustment for additional factors such as the exposure to market risk, etc. We intend to address these and further issues in a separate work.

[16] It remains the same if we control for the within-group skewness and kurtosis as well.

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Appendix A: Derivation of Direct Alpha

In line with the heuristic approaches, the Direct Alpha method is based on the assumption that the continuous-time log rate of return to the PE portfolio follows the standard model for public equities (Merton (1971)), including both a market return and a non-market return to skill

$$r(t) = b(t) + \alpha \tag{21}$$

where r(t) is the continuous-time log return to the PE portfolio, b(t) is the continuous-time log return to the reference (public equity) benchmark, and α is the constant continuous-time non-market log return to skill. We can see that the value of the PE portfolio at final time t_n due to any single contribution c_i at t_i must be

$$v_i(t_n) = c_i \cdot exp\left\{\int_{t_i}^{t_n} [b(t) + \alpha]\, dt\right\} \tag{22}$$

But since b(t) is just the continuous-time log return to the reference benchmark, then by definition

$$\frac{m_n}{m_i} = exp\left\{\int_{t_i}^{t_n} [b(t)]\, dt\right\} \tag{23}$$

If we discretize time by some interval Δ, so that

$$t_i = i\Delta \tag{24}$$

and define the arithmetic equivalent of the log rate α by

$$1 + \mathrm{a} = exp(\alpha\Delta) \tag{25}$$

then we can see that the above equation simplifies to

$$v_i(t_n) = \left[c_i \cdot \frac{m_n}{m_i}\right] \cdot (1 + \mathrm{a})^{n-i} \tag{26}$$

When we combine the effects of all cash flows at final time, we find that

$$NAV_{PE} = \sum_{i=0}^{n} d_i \cdot \frac{m_n}{m_i} \cdot (1 + \mathrm{a})^{n-i} - \sum_{i=0}^{n} c_i \cdot \frac{m_n}{m_i} \cdot (1 + \mathrm{a})^{n-i} \tag{27}$$

Consequently, a is just the IRR of the future values of the cash flows and α is its equivalent log rate.

Appendix B: The net present value perspective

Instead of using future values, Direct Alpha and KS-PME can be similarly derived from a series of present values. For example, KS-PME can be expressed as the ratio of all present values of distributions and the residual NAV to all present values of contributions, since multiplying the numerator and denominator by the same factor m_0/m_n does not alter the ratio. Consequently, one could think of KS-PME as an "ex post" net present value (NPV) of the PE portfolio's investments, normalized by the sum of all contributions (discounted accordingly).

$$KS\text{-}PME = \frac{\sum FV(D) + NAV_{PE}}{\sum FV(C)} \cdot \frac{m_0/m_n}{m_0/m_n} = \frac{\sum PV(D) + NAV_{PE} \cdot m_0/m_n}{\sum PV(C)} \tag{28}$$

$$= \frac{[\sum PV(D) + PV(NAV_{PE})] - \sum PV(C) + \sum PV(C)}{\sum PV(C)} = \frac{NPV}{\sum PV(C)} + 1 \tag{29}$$

Intuitively, the concept of NPV is tantamount to cumulative alpha. Recall the starting point for the Direct Alpha derivation. It can be rearranged such that

$$\frac{v_i(t_n)}{c_i \cdot m_n/m_i} = (1 + \mathrm{a})^{n-i} \tag{30}$$

If there is only a single Contribution-Distribution/Value pair for the PE portfolio, the right hand side of this equation is precisely KS-PME. In general, however, $ln(KS\text{-}PME)/n$ is not equal to α because n, the number of years since inception, is not necessarily the time that the capital has been employed by the PE portfolio (*i.e.*, the duration of all investments).

However, applying the time and money-weighting of the IRR procedure to appropriately discounted cash flows, as per Direct Alpha, explicitly accounts for the effective timing of cash flows, essentially, per-period NPVs. Again, it is invariant to using future or present values as inputs variables, since re-scaling all cash flows by a constant factor does not alter the returns.

Appendix C: Robustness

While a discussion about the appropriate reference benchmark to evaluate a PE portfolio's performance is beyond the scope of this paper, we would like to consider the robustness of KS-PME and Direct Alpha to possible market beta misspecification in the context of a performance comparison across funds. Note that one can re-write the sum of the present values of cash flows as the product of the original cash flows times the *average* present value. This average present value can be interpreted as an estimate of the expectation of the discount rate. Recent work by Sørensen and Jagannathan (2013) and Korteweg and Nagel (2013) argues that since KS-PME effectively estimates the product of the expectation of the discount factor times the cash flows, it 'automatically' accounts for the beta-adjustment of factor returns. Below is a synopsis of this rationale and our take on its implications for the performance comparison across funds.

Let P be the time t price of an asset which has a payoff of X at time $t+1$, such that X/T equals the gross return R. Similarly, M and R_m are, respectively, the risk factor payoff and the return that investors require to be compensated for, whereas R_f is the gross return of the risk-free asset. Assuming that KS-PME implicitly estimates beta means that

$$P = E[MX] \tag{31}$$

implies

$$E[R] - R_f = \beta(E[R_m] - R_f) \tag{32}$$

where $E[.]$ is the expectation operator as of the information set at time t, and $\beta = cov(Rm, R)/var(Rm)$. See Sørensen and Jagannathan (2013) and Korteweg and Nagel (2013) for further details, as well as Cochrane (2005) for a text book exposition of the beta representation of (31).

While equation (32) looks like the standard CAPM equation, (31) is the cornerstone equation of asset pricing theory saying that any price today is the weighted average of all possible future payoffs. Importantly, (31) does not say what the weighting scheme is, but only that all weights (and, thus, M) must be positive if the law of one price holds. The proof of (31)→(32) makes no economic assumptions or restrictions either.

The assumptions are introduced with the choice of a particular M, the risk-factor. Discounting cash flows with the market returns means that (32) actually becomes a CAPM equation, while KS-PME and Direct Alpha become (31) if investors have a logarithmic utility function which is more restrictive than minimally required by the CAPM. Korteweg and Nagel (2013) somewhat relax this restriction at a cost of estimating additional parameters. In either case, the intuition is that, as we pool present values across more and more funds and over increasing time periods, the resulting portfolio PME approaches a ratio of $E[MD]/E[MC]$, while its distance from one expresses the abnormal return[17] of the portfolio, regardless of the betas (or even the knowledge of them).

Essentially, the pooling of the cash flows and discount factor realizations for a large sample of funds over time allows for accurate estimates of both the numerator and denominator. Nonetheless, in any real-life application the estimates will be subject to a statistical error. When it comes to comparing individual fund-level estimates, those errors might be quite substantial as the samples of cash flow and discount factor realizations are small. Moreover, the estimation errors may depend on the funds' individual betas and the mean factor return over the life of the funds. Consequently, the problem of benchmark selection for a performance comparison across funds persists, despite $P = E[MX]$ being asymptotically true for each fund. One can think of the benchmark selection exercise for KS-PME and Direct Alpha as a way to 'shrink and unbias' those estimation errors.

[17] Technically, a value statistically different from one indicates that the law of one price fails since there exist pure alpha opportunities in the economy.

Exhibit 1: Illustrative relationship between Direct Alpha and the heuristic approaches

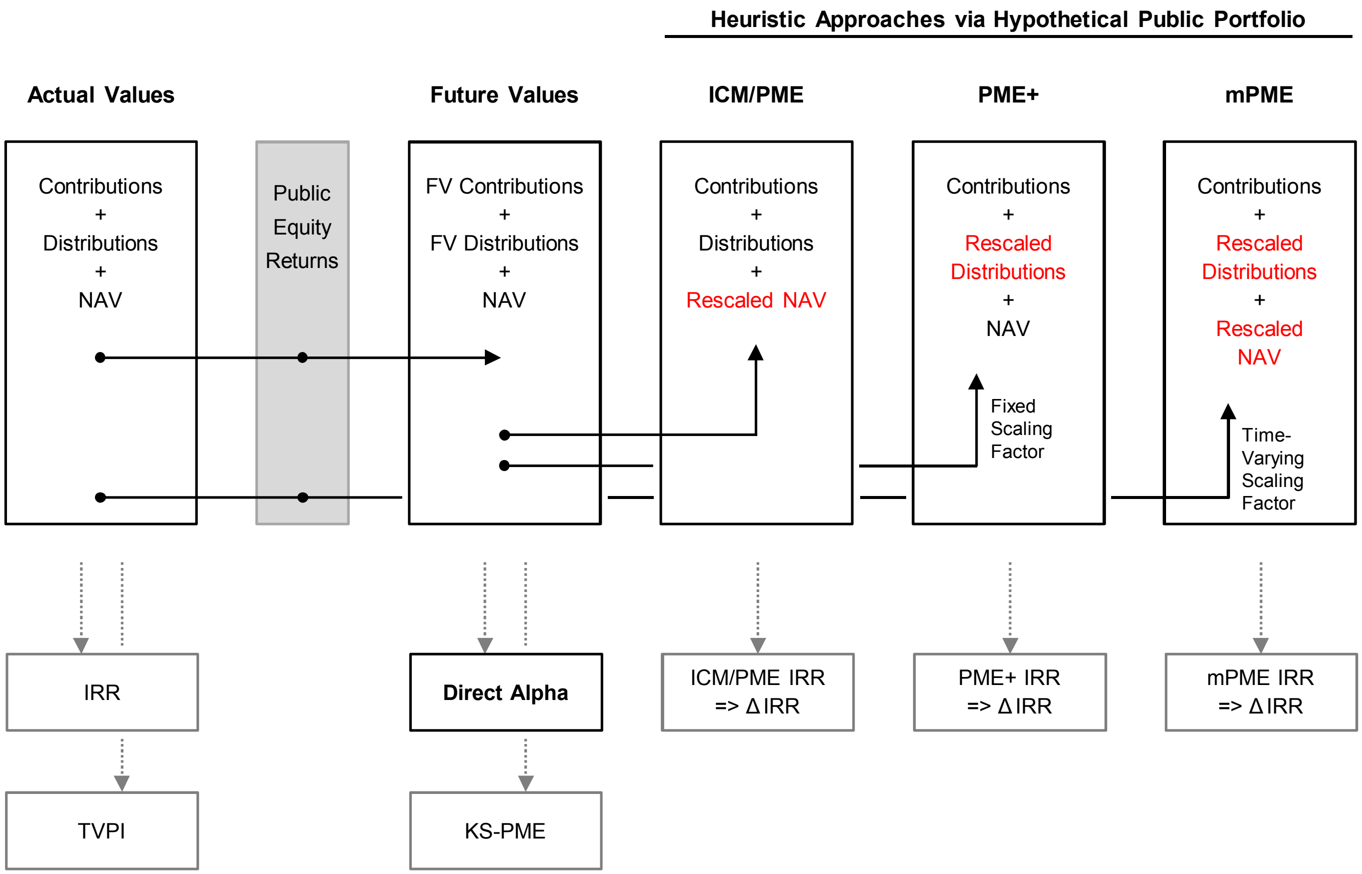

Exhibit 2: Numerical example of the ICM/PME approach

C represents contributions into, and D represents distributions from, the PE portfolio and the hypothetical public portfolio. Net CF represents the net cash flows plus the respective final net asset value (NAV_{PE} or NAV_{ICM}).

	Actual Values					ICM/PME Hypothetical Public Portfolio			
	C	D	NAV_{PE}	Net CF	Index	C	D	NAV_{ICM}	Net CF
Dec-31, 2001	100	0	...	-100	100	100	0	100	-100
Dec-31, 2002	0	0	...	0	78	0	0	78	0
Dec-31, 2003	100	25	...	-75	100	100	25	175	-75
Dec-31, 2004	0	0	...	0	111	0	0	194	0
Dec-31, 2005	50	150	...	100	117	50	150	104	100
Dec-31, 2006	0	0	...	0	135	0	0	120	0
Dec-31, 2007	0	150	...	150	142	0	150	-23	150
Dec-31, 2008	0	0	...	0	90	0	0	-15	0
Dec-31, 2009	0	100	...	100	113	0	100	-118	100
Dec-31, 2010	0	0	75	75	131	0	0	-136	-136
			IRR	17.5%				ICM IRR	6.0%
								Δ IRR	11.5%

Exhibit 3: Numerical example of the PME+ approach

C represents contributions into the PE portfolio and the hypothetical public portfolio. D represents distributions from the PE portfolio. s • D represents rescaled distributions from the hypothetical public portfolio. Net CF represents the net cash flows plus the final net asset value (NAV_{PE}).

	Actual Values					PME+ Hypothetical Public Portfolio			
	C	D	NAV_{PE}	Net CF	Index	C	s • D	NAV_{PE}	Net CF
Dec-31, 2001	100	0	...	-100	100	100	0	...	-100
Dec-31, 2002	0	0	...	0	78	0	0	...	0
Dec-31, 2003	100	25	...	-75	100	100	13	...	-87
Dec-31, 2004	0	0	...	0	111	0	0	...	0
Dec-31, 2005	50	150	...	100	117	50	80	...	30
Dec-31, 2006	0	0	...	0	135	0	0	...	0
Dec-31, 2007	0	150	...	150	142	0	80	...	80
Dec-31, 2008	0	0	...	0	90	0	0	...	0
Dec-31, 2009	0	100	...	100	113	0	53	...	53
Dec-31, 2010	0	0	75	75	131	0	0	75	75
								Scaling Factor s	0.53
			IRR	17.5%				PME+ IRR	4.0%
								Δ IRR	13.5%

Exhibit 4: Numerical example of the mPME approach

C represents contributions into, and D represents distributions from, the PE portfolio and the hypothetical public portfolio. Net CF represents the net cash flows plus the respective final net asset value (NAV_{PE} or NAV_{mPME}).

	Actual Values					mPME Hypothetical Public Portfolio			
	C	D	NAV_{PE}	Net CF	Index	C	D_{mPME}	NAV_{mPME}	Net CF
Dec-31, 2001	100	0	100	-100	100	100	0	100	-100
Dec-31, 2002	0	0	95	0	78	0	0	78	0
Dec-31, 2003	100	25	190	-75	100	100	23	177	-77
Dec-31, 2004	0	0	235	0	111	0	0	196	0
Dec-31, 2005	50	150	170	100	117	50	120	136	70
Dec-31, 2006	0	0	240	0	135	0	0	157	0
Dec-31, 2007	0	150	130	150	142	0	89	77	89
Dec-31, 2008	0	0	80	0	90	0	0	49	0
Dec-31, 2009	0	100	40	100	113	0	44	18	44
Dec-31, 2010	0	0	75	75	131	0	0	20	20
			IRR	17.5%				mPME IRR	4.6%
								Δ IRR	12.9%

Exhibit 5: Numerical example of the KS-PME approach

C represents contributions into, and D represents distributions from, the PE portfolio. Their corresponding future values FV (C) and FV (D) are as of Dec-31, 2010.

	Actual Values				Future Values		
	C	D	NAV_{PE}	Index	FV (C)	FV (D)	NAV_{PE}
Dec-31, 2001	100	0	...	100	131	0	...
Dec-31, 2002	0	0	...	78	0	0	...
Dec-31, 2003	100	25	...	100	130	33	...
Dec-31, 2004	0	0	...	111	0	0	...
Dec-31, 2005	50	150	...	117	56	168	...
Dec-31, 2006	0	0	...	135	0	0	...
Dec-31, 2007	0	150	...	142	0	138	...
Dec-31, 2008	0	0	...	90	0	0	...
Dec-31, 2009	0	100	...	113	0	115	...
Dec-31, 2010	0	0	75	131	0	0	75
Total	250	425			317	453	
		TVPI	2.00			KS-PME	1.67

Exhibit 6: Numerical example of the Direct Alpha approach I

C represents contributions into, and D represents distributions from, the PE portfolio. Their corresponding future values FV (C) and FV (D) are as of Dec-31, 2010. Net CF represents the net cash flows plus the final net asset value (NAV_{PE}).

	Actual Values					Future Values			
	C	D	NAV_{PE}	Net CF	Index	FV (C)	FV (D)	NAV_{PE}	FV (Net CF)
Dec-31, 2001	100	0	...	-100	100	131	0	...	-131
Dec-31, 2002	0	0	...	0	78	0	0	...	0
Dec-31, 2003	100	25	...	-75	100	130	33	...	-98
Dec-31, 2004	0	0	...	0	111	0	0	...	0
Dec-31, 2005	50	150	...	100	117	56	168	...	112
Dec-31, 2006	0	0	...	0	135	0	0	...	0
Dec-31, 2007	0	150	...	150	142	0	138	...	138
Dec-31, 2008	0	0	...	0	90	0	0	...	0
Dec-31, 2009	0	100	...	100	113	0	115	...	115
Dec-31, 2010	0	0	75	75	131	0	0	75	75
			IRR	17.5%			Direct Alpha (arithmetic)		12.6%

Exhibit 7: Numerical example of the Direct Alpha approach II

C represents contributions into, and D represents distributions from, the PE portfolio. Their corresponding future values FV (C) and FV (D) are as of Dec-31, 2010. Their corresponding present values PV (C) and PV (D) as well as the PV (NAV_{PE}) are as of Dec-31, 2001. Net CF represent the net cash flows plus the respective final net asset value (NAV_{PE} or PV (NAV_{PE})).

	Actual Values					Future Values				Present Values			
	C	D	NAV_{PE}	Net CF	Index	FV (C)	FV (D)	NAV_{PE}	Net CF	PV (C)	PV (D)	PV (NAV_{PE})	Net CF
Dec-31, 2001	100	0	...	-100	100	131	0	...	-131	100	0	...	-100
Dec-31, 2002	0	0	...	0	78	0	0	...	0	0	0	...	0
Dec-31, 2003	100	25	...	-75	100	130	33	...	-98	100	25	...	-75
Dec-31, 2004	0	0	...	0	111	0	0	...	0	0	0	...	0
Dec-31, 2005	50	150	...	100	117	56	168	...	112	43	129	...	86
Dec-31, 2006	0	0	...	0	135	0	0	...	0	0	0	...	0
Dec-31, 2007	0	150	...	150	142	0	138	...	138	0	105	...	105
Dec-31, 2008	0	0	...	0	90	0	0	...	0	0	0	...	0
Dec-31, 2009	0	100	...	100	113	0	115	...	115	0	88	...	88
Dec-31, 2010	0	0	75	75	131	0	0	75	75	0	0	57	57

Future Values:

317 528

KS-PME 1.67

Direct Alpha (arithmetic) 12.6%

Present Values:

243 404

KS-PME 1.67

Direct Alpha (arithmetic) 12.6%

Exhibit 8: Direct Alpha versus ICM/PME IRR spreads

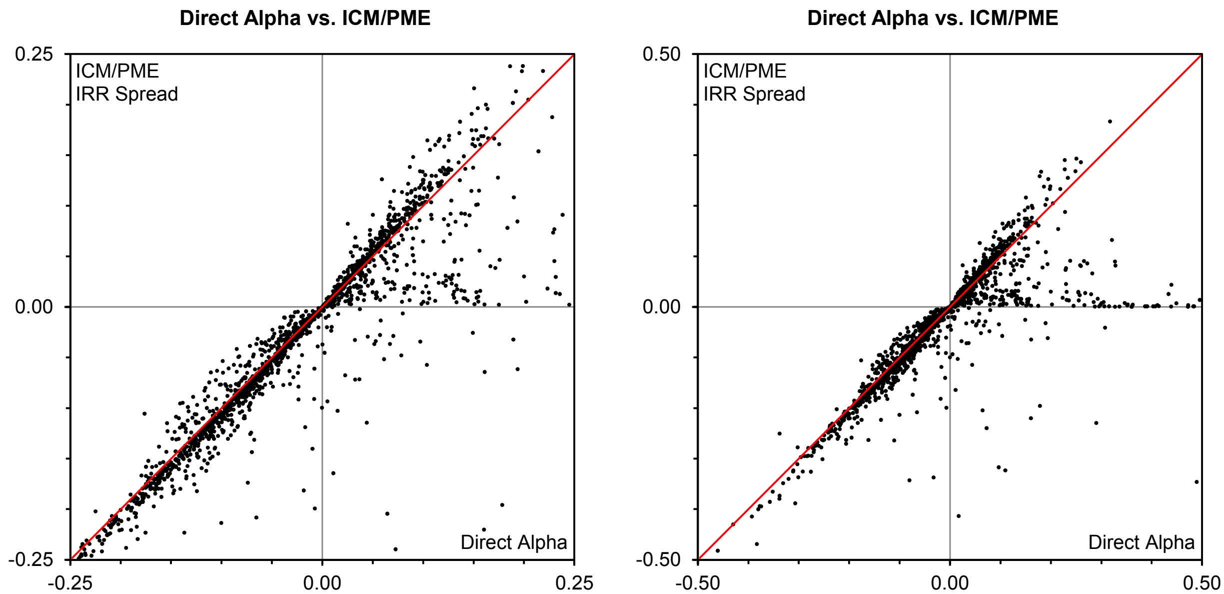

Exhibit 9: Direct Alpha versus PME+ IRR spreads

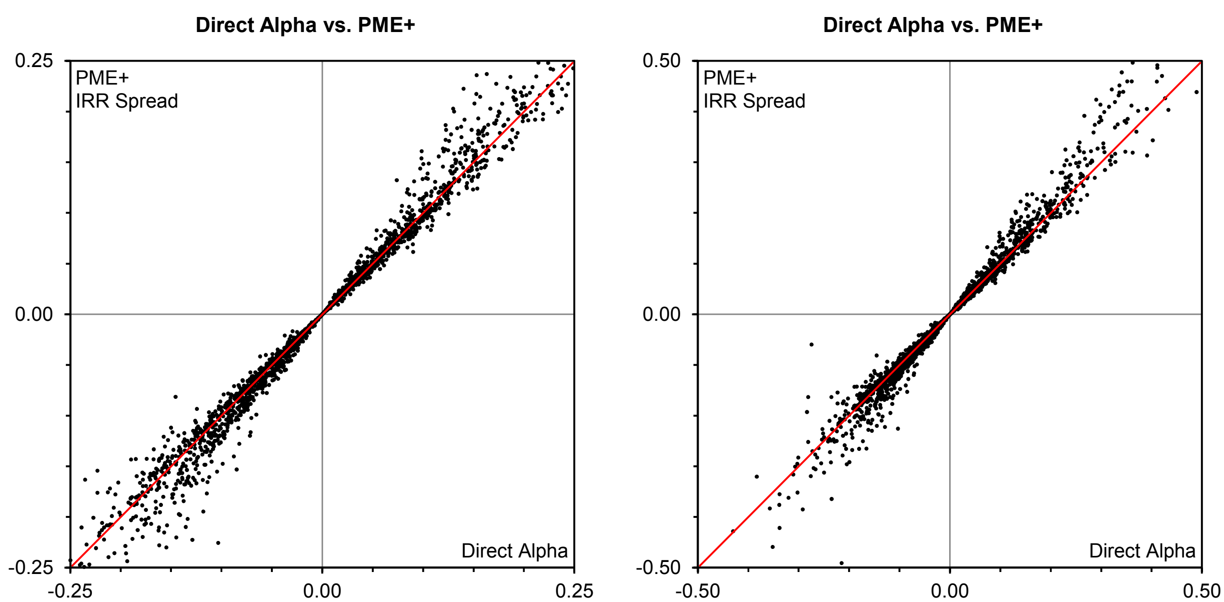

Exhibit 10: Direct Alpha versus mPME IRR spreads

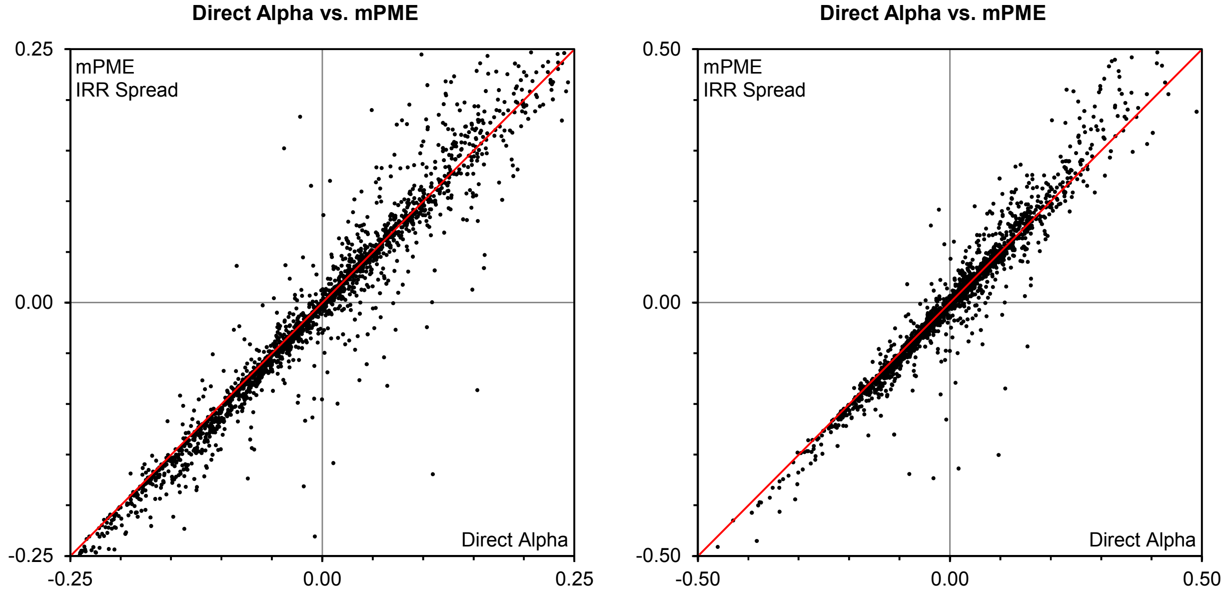

Exhibit 11: Direct Alpha versus annualized KS-PMEs

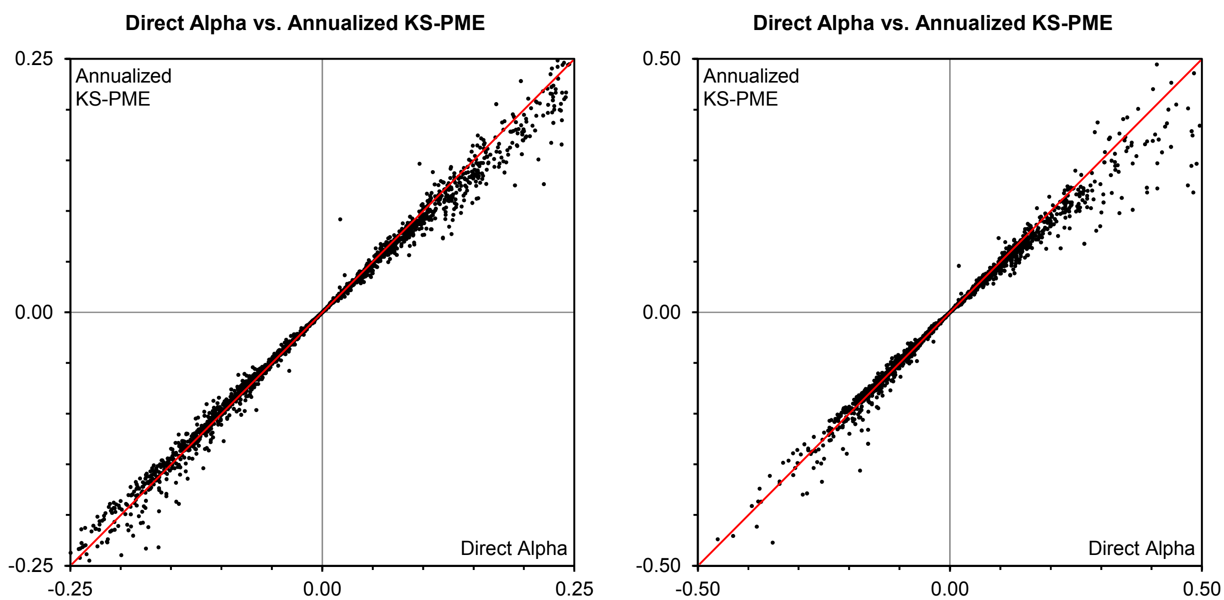

Exhibit 12: Performance quartile concordance

This table reports transition probabilities across differently-measured performance quartile-ranks. Each row corresponds to a Direct Alpha quartile. The numbers in columns represent the percentage of funds of the respective Direct Alpha quartile being in the top, third, second, and bottom quartile as measured by the approach indicated in the title of each panel.

Panel A1: ICM/PME

	Top	3rd	2nd	Bottom	Total
Top	75.23	15.37	7.57	1.83	100.00
3rd	32.92	59.67	4.73	2.67	100.00
2nd	6.79	26.23	63.02	3.96	100.00
Bottom	0.00	0.00	17.44	82.56	100.00
Total	26.02	24.63	24.23	25.12	100.00

Panel A2: ICM/PME non-short

	Top	3rd	2nd	Bottom	Total
Top	93.88	4.20	1.40	0.52	100.00
3rd	6.41	85.71	5.49	2.38	100.00
2nd	0.00	10.07	83.96	5.97	100.00
Bottom	0.00	0.00	8.54	91.46	100.00
Total	25.81	24.64	24.19	25.36	100.00

Panel B: PME+

	Top	3rd	2nd	Bottom	Total
Top	96.50	3.50	0.00	0.00	100.00
3rd	1.66	92.82	5.52	0.00	100.00
2nd	0.00	1.90	91.62	6.48	100.00
Bottom	0.00	0.00	2.66	97.34	100.00
Total	25.89	24.64	24.23	25.24	100.00

Panel C: mPME

	Top	3rd	2nd	Bottom	Total
Top	92.96	6.51	0.18	0.35	100.00
3rd	7.16	82.94	8.26	1.65	100.00
2nd	0.56	9.96	82.52	6.95	100.00
Bottom	0.00	0.00	8.60	91.40	100.00
Total	25.87	24.60	24.19	25.33	100.00

Panel D1: Annualized KS-PME

	Top	3rd	2nd	Bottom	Total
Top	97.73	2.27	0.00	0.00	100.00
3rd	2.38	94.87	2.75	0.00	100.00
2nd	0.00	2.80	94.22	2.99	100.00
Bottom	0.00	0.00	2.85	97.15	100.00
Total	25.85	24.63	24.18	25.35	100.00

Panel D2: KS-PME

	Top	3rd	2nd	Bottom	Total
Top	92.32	7.68	0.00	0.00	100.00
3rd	8.06	83.88	7.88	0.18	100.00
2nd	0.00	8.21	83.96	7.84	100.00
Bottom	0.00	0.00	7.65	92.35	100.00
Total	25.85	24.63	24.18	25.35	100.00

Exhibit 13: What drives the differences from Direct Alpha statistically?

This table reports linear regression estimates of the absolute level(rank) differences of Direct Alpha from mPME, ICM/PME and PME+. t-statistics are in parentheses and robust to error-heteroskedasticity. *, **, and *** denote statistical significance at the 10%, 5%, and 1% confidence level, respectively.

	mPME		ICM/PME		PME+	
	(1) abs(level)	(2) abs(rank)	(3) abs(level)	(4) abs(rank)	(5) abs(level)	(6) abs(rank)
Direct Alpha abs(level)	0.339***	-0.0677	2.241**	22.24**	-91.94	-0.102
	(11.58)	(-0.09)	(2.10)	(2.44)	(-1.01)	(-0.27)
Direct Alpha Duration	0.00144	0.00565	-0.0664	-1.431***	-5.250	-0.183***
	(0.77)	(0.05)	(-0.89)	(-4.24)	(-1.04)	(-4.48)
abs(level)*Duration	-0.0386***	0.159	-0.252*	6.146***	43.20	-0.329**
	(-4.23)	(0.50)	(-1.71)	(3.04)	(1.03)	(-2.19)
Benchmark Mean*100	0.0186***	0.786	0.0590	3.610***	-2.757	0.365*
	(4.59)	(1.45)	(1.49)	(3.18)	(-0.74)	(1.77)
Benchmark Volty*100	0.0165	0.796	0.281	4.631***	-6.656	2.039***
	(1.60)	(0.86)	(1.05)	(3.16)	(-0.54)	(7.82)
VC Fixed Effect	-0.000274	-0.0376	-0.303	3.221***	24.16	0.664***
	(-0.08)	(-0.11)	(-1.05)	(5.10)	(1.06)	(4.88)
Direct Alpha within-group Standard Deviation		-0.482		-22.65***		-1.984***
		(-0.60)		(-6.87)		(-5.04)
Constant	-0.0882*	-4.098	-0.763	-23.09***	46.66	-7.829***
	(-1.66)	(-0.91)	(-1.13)	(-2.89)	(0.66)	(-5.86)
R-squared	0.384	<0.001	0.0043	0.251	<0.001	0.0815
Observations	2,203	2,203	2,014	2,014	2,167	2,167