# Summary of Factor Timing

Haddad et al, Review of Financial Studies 33(2020): pp.1980-2018

Arnav Sheth iCapital Network

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#### Introduction

- Main thesis: optimal portfolio is equivalent to the stochastic discount factor (SDF).
- Implications: an estimated SDF with properties that account for time-varying factor loadings.
  - For example, time-varying volatility ( $\sigma^2 \in [1.66, 2.96]$ ).
- Take a simple stance on the issue of factor explanatory power:
  - Use only the book-to-market ratio of each portfolio as a measure to predict its returns.

- 1. Start from a set of pricing factors  $F_{t+1}$ .
- 2. Reduce this set of factors to a few dominant components,  $Z_{t+1}$ , using principal components analysis.
- 3. Produce separate individual forecasts of each of the  $Z_{t+1}$ , that is measures of  $\mathbb{E}_t[Z_{t+1}]$ .
- To measure the conditional expected factor returns, apply these forecasts to factors using their loadings on the dominant components.
- 5. To engage in factor timing or estimate the SDF, use these forecasts to construct the portfolio given in Equation (10).

- Use fifty 'anomaly' portfolios from Kozak et al. (2020)<sup>1</sup> that effectively capture market heterogeneity.
  - These anomalies are the usual anomalies like Size, Value, ROA, SUE, etc.
- Break them into deciles, create long-short portfolios for each anomaly (Decile 10 minus Decile 1).
  - For each portfolio, they calculate the market-cap-weighted book-to-market ratio (bm) of the underlying stocks.
  - By finding the difference in log book-to-market of Portfolio 10 minus that of Portfolio 1.

<sup>&</sup>lt;sup>1</sup>Kozak, S., S. Nagel, and S. Santosh. 2020. Shrinking the cross-section. *Journal of Financial Economics* 135:271–92.

- Market-adjust and rescale the data.
  - **1** Calculate regression  $\beta$  for each anomaly.
  - ② Market-adjust returns and predictors by subtracting  $\beta \times r_{mkt}$  for returns and  $\beta \times bm_{mkt}$  for bm ratios.
  - $\odot$  Rescale to equalize the variance of market-adjiusted returns, and bm ratios for each anomaly.
- $\bullet$  So now they have 50 long-short portfolios and their corresponding bm ratios.

 They conduct a PCA to reduce the 50 long-short portfolios to five PCs that explain roughly 60% of the variance.

Table 1
Percentage of variance explained by anomaly PCs

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10
% var. explained	25.8	12.4	10.3	6.7	4.8	4.0	3.6	2.8	2.2	2.1
Cumulative	25.8	38.3	48.5	55.2	60.0	64.0	67.6	70.4	72.6	74.7

This table reports the percentage of variance explained by each PC of the fifty anomaly strategies.

- Why five?
  - **1** Campbell and Thompson  $(2007)^2$  show that the monthly  $R^2$  when predicting the market is around 75bp.
  - ② A loose upper-bound on the annual Sharpe is 1, or 8.3% monthly.
- If each included PC contributes equally to the  $R^2$ , the harmonic mean of their contribution to the total variance of returns must be  $> \frac{0.75}{8.3} \approx 9\%$ .

<sup>&</sup>lt;sup>2</sup>Campbell, J. Y., and S. B. Thompson. 2007. Predicting excess stock returns out of sample: Can anything beat the historical average? *Review of Financial Studies* 21:1509–31.

 Construct the bm portfolios for the each PC by combining anomaly-portfolio log bms according to portfolio weights:

$$bm_{i,t} = q_t' b m_t^F \tag{1}$$

- where
  - $q_i$  is the i-th column of Q, the matrix of eigenvectors, and F is the anomaly.
  - Use the difference between this quantity for the long and short leg of the PCs.
- Analyze the predictability using monthly holding periods.

#### Results

Table 2
Predicting dominant equity components with BE/ME ratios

	MKT	PC1	PC2	PC3	PC4	PC5
Own bm	0.76 (1.24)	4.32 (4.31)	1.62 (1.81)	1.80 (2.01)	4.86 (3.74)	1.56 (0.78)
bias p-value	0.68 .35	0.36 .00	0.16 .10	0.18 .07	0.10 .00	0.08 .48
$R^2$	0.29	3.96	0.74	0.56	3.59	0.50
$\cos R^2$	1.00	4.82	0.97	0.47	3.52	0.55
		oc	S R <sup>2</sup> critical val	ues		
90th	0.44	0.49	0.29	0.21	0.71	0.59
95th	0.68	0.97	0.48	0.37	1.19	0.96
99th	1.35	1.73	0.87	0.84	1.95	1.71