Unsmoothing Returns of Illiquid Funds

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Abstract

Funds that invest in illiquid assets report returns with spurious autocorrelation. Consequently, investors need to unsmooth returns when evaluating the risk exposures of these funds. We show that funds investing in similar assets have a common source of spurious autocorrelation, which is not addressed by commonly-used unsmoothing methods, leading to underestimation of systematic risk. To address this issue, we propose a generalization of these unsmoothing techniques and apply it to hedge funds and commercial real estate funds. Our empirical results indicate our method significantly improves the measurement of risk exposures and risk-adjusted performance, with stronger results for more illiquid funds.

JEL Classification: G11, G12, G23.

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Introduction

The market size of intermediaries investing in illiquid assets has grown dramatically over the last two decades.[1] However, there is a lot we do not know about their risks and performance due to the difficulty in measuring these quantities with standard techniques. Specifically, the reported returns of a fund reflect valuation changes with a partial lag when the fund's assets trade infrequently or are sporadically valued. This smoothing effect creates spurious return autocorrelation and invalidates traditional risk and performance measures (betas and alpha). The crux of the problem is that we only observe reported (or smoothed) returns, while we need economic (or unsmoothed) returns to evaluate risk exposures and risk-adjusted performance.

In some influential papers, Geltner (1991, 1993) and Getmansky, Lo, and Makarov (2004) provide different ways to recover economic return estimates by unsmoothing observed returns.[2] In this paper, we argue that while these previous techniques represent an important first step in measuring the risks of illiquid assets, they do not fully unsmooth the common component of returns, and thus understate the importance of risk factors in explaining illiquid asset returns. We then provide a novel return unsmoothing technique to address this issue and apply our methodology to hedge funds and private commercial real estate (CRE) funds, demonstrating its usefulness in measuring the risk exposures and risk-adjusted performance of illiquid assets.[3] Our main finding is that systematic risk (and risk-adjusted performance)

[1]For instance, a report by the PWC Asset and Wealth Management Research Center shows a growth of roughly 400% (from \$2.5 trillion to \$10.1 trillion) in assets under management for alternative investments (which are typically illiquid) from 2004 to 2016 (see pwc (2018)). The same report indicates that the hedge fund industry (the focus of a substantial part of our empirical analysis) has grown from \$1.0 trillion to \$3.3 trillion over the same period.

[2]Some papers follow Dimson (1979) and use the alternative approach of including lags of risk factors in the factor regressions (e.g., Bali, Brown, and Caglayan (2012), Cao et al. (2013), and Chen (2011)). While useful as a quick solution to the return autocorrelation problem, this approach does not provide a way to recover economic returns and it dramatically increases the number of parameters to be estimated in the factor regressions, which is an important limitation as investors often face the problem of measuring multiple risk exposures based on a relatively short time-series. For instance, with a 2-month autocorrelation (common in hedge funds), this method would require 25 regression parameters to estimate the 8-factor model in Fung and Hsieh (2001).

[3]We focus on hedge funds and CRE funds because smoothed returns is a common problem with these

is better measured when returns are unsmoothed using our new method.

The basic idea behind return unsmoothing methods is simple. These techniques assume observed returns are weighted averages of current and past economic returns, and thus they estimate these weights and use them to recover economic return estimates, which are otherwise unobservable.

Traditional return unsmoothing techniques (Geltner (1991, 1993) for CRE funds and Getmansky, Lo, and Makarov (2004) for Hedge funds), which we refer to as 1-step unsmoothing, seem to perform well when we only analyze unsmoothing at the fund-level. Figure 1(a) shows that fund-level autocorrelations are high in CRE and hedge funds, but effectively disappear after 1-step unsmoothing. These results suggest 1-step unsmoothed returns better reflect price movements in the funds' underlying assets than reported returns.

Despite this apparent success, we find that averaging 1-step unsmoothed returns produces aggregate returns that display significant autocorrelation, as demonstrated in Figure 1(b). This result indicates that the common component of fund-level returns is not fully unsmoothed based on 1-step unsmoothing, potentially biasing fund risk exposure estimates.

To deal with this issue, we propose a simple adjustment to traditional return unsmoothing methods. We assume the observed returns of each fund are weighted averages of current and past economic returns on both the fund and the aggregate of similar funds (i.e., an equal-weighted portfolio of funds in the same asset class). We then show how to use a 3-step procedure to estimate the weights and obtain more accurate economic return estimates. This 3-step process relies on the same information as 1-step unsmoothing, but uses the it more efficiently by incorporating information from similar funds into the unsmoothing process. As Figure 1 shows, our 3-step unsmoothing method is better able to unsmooth the common component of fund returns than 1-step unsmoothing since both the fund and aggregate autocorrelations effectively disappear after 3-step unsmoothing.

asset classes. For instance, Getmansky, Lo, and Makarov (2004) and Geltner (1991, 1993) introduced their influential return unsmoothing methods in the context of hedge funds and real estate funds, respectively.

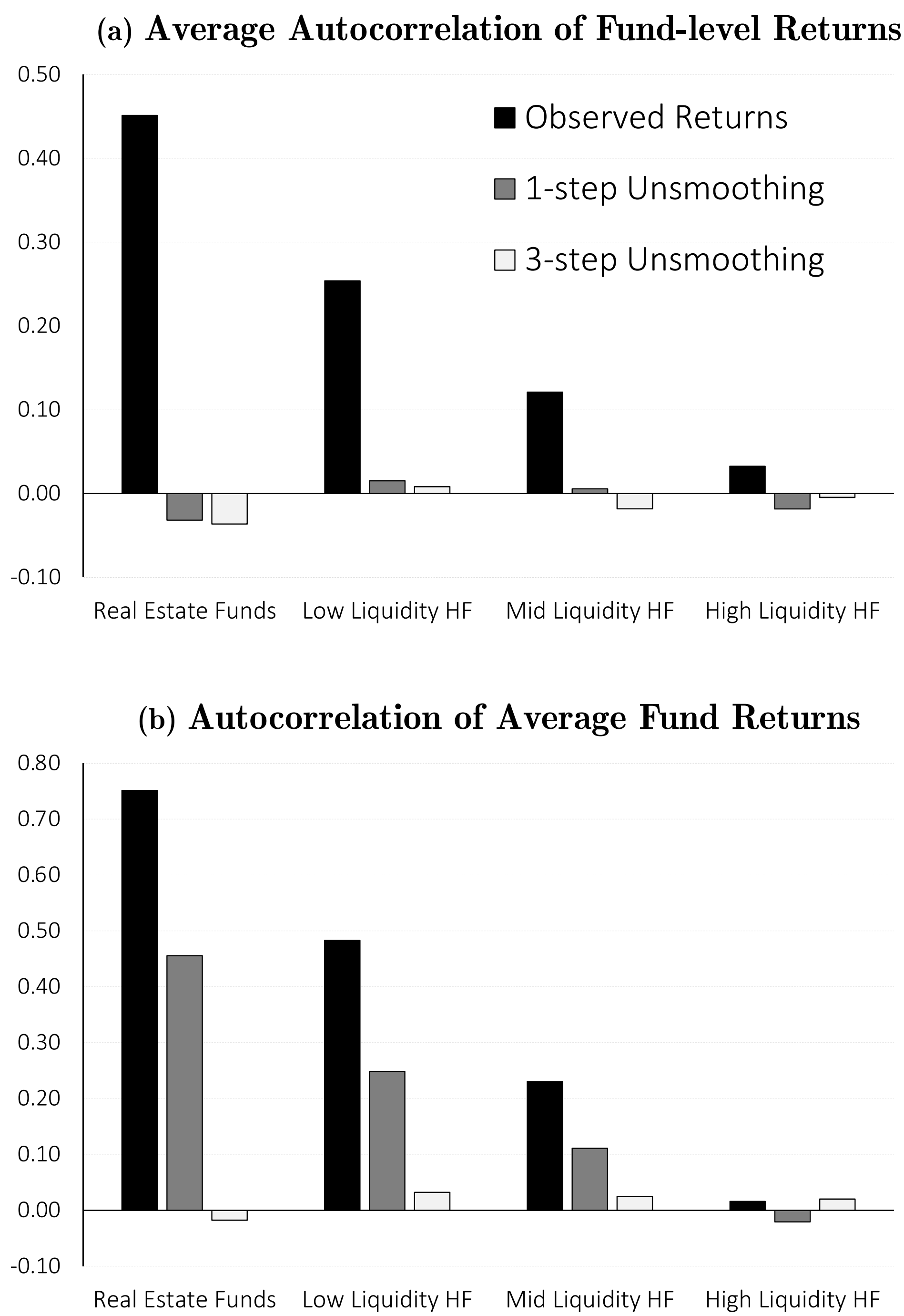

Figure 1
Real Estate and Hedge Fund Return Autocorrelation: Fund-level vs Aggregate

Panel (a) plots the average 1st order autocorrelation coefficient for returns of Commercial Real Estate funds (quarterly returns from 1994 to 2017) and Hedge Funds (monthly returns from 1995 to 2017), with the latter sorted on strategy liquidity (see Subsection 2.2 for the details on the strategy liquidity sort). Panel (b) plots the analogous measure, but for average returns (i.e., first taking the equal-weighted average of fund-level returns and then calculating the autocorrelations). We consider three definition of returns: observed returns, 1-step unsmoothed returns (Geltner (1991, 1993) for real estate funds and Getmansky, Lo, and Makarov (2004) for Hedge funds), and 3-step unsmoothed returns. See Subsections 2.1 and 3.2 for further empirical details.

The 3-step unsmoothing method we propose reduces to 1-step unsmoothing if (i) observed fund returns do not directly reflect lagged aggregate returns of similar funds or (ii) fund returns in a given category are perfectly correlated with aggregate returns in that category. However, the data do not support either of these assumptions. Specifically, we show that regressions of fund returns on lagged fund and aggregate returns strongly load on lagged aggregate returns, which invalidates condition (i), and also that the R^2 from regressing fund returns on (contemporaneous) aggregate returns tends to be below 50%, which invalidates condition (ii). We also perform a Monte Carlo simulation exercise that indicates that the autocorrelation patterns in observed returns are highly consistent with the underlying smoothing process of our 3-step unsmoothing method whereas they are inconsistent with the underlying smoothing process of the 1-step unsmoothing approach.

To understand why the smoothing process of our 3-step unsmoothing method is more consistent with the autocorrelation in observed returns, we provide a simple economic model in which funds do not observe the current value of the assets they hold, but instead receive different signals about the shocks common to all assets in their asset class and the shocks affecting the relative value of their assets. If funds report the Bayesian estimate of their asset value each period, then observed returns are based on current and past economic returns, with the smoothing process reflecting past fund and aggregate returns with different intensities, in line with the underlying smoothing process of our 3-step unsmoothing method. The intuition is that the signal-to-noise ratios of the two different signals drive the smoothing parameters and they differ across signals that are fund-specific or common to an entire asset class.

The aforementioned results point to a misspecification in unsmoothed returns obtained using 1-step unsmoothing methods. Motivated by this finding, we explore the implications of our 3-step unsmoothing method to the measurement of risk exposures and risk-adjusted performance of hedge funds and CRE funds.

In the case of hedge funds, we perform two main exercises. In the first exercise, we sort funds into three groups based on the liquidity of their underlying strategy, and apply 1-step and 3-step unsmoothing to funds in each of these groups. We then measure, for each group, average fund volatility as well as risk and risk-adjusted performance based on a standard factor model used in the hedge fund literature (the FH 8-Factor model that builds on Fung and Hsieh (2001)). We find that volatility substantially increases after unsmoothing returns, but the increase is roughly the same whether we use 1-step or 3-step unsmoothing. In contrast, 3-step unsmoothing produces economic returns that comove more strongly with FH risk factors (relative to 1-step unsmoothing) and display lower alphas as a consequence. Moreover, we also show that past alphas estimated using 3-step unsmoothed returns provide a better signal for future risk-adjusted performance than alphas obtained from observed returns or 1-step unsmoothed returns. All the aforementioned results hold only in the low and mid liquidity groups, with the performance of funds in the high liquidity group being largely unaffected by unsmoothing. This last finding indicates that unsmoothing techniques do not produce unintended distortions in the estimates of economic returns for liquid assets.

In the second exercise, we separately study funds in each major hedge fund strategy category and find results consistent with the previous paragraph. However, grouping funds based on their underlying strategies allows us to study funds exposed to similar risks, and thus to explore how our 3-step unsmoothing technique improves the measurement of systematic risk exposures. We find that our 3-step unsmoothing method tends to change risk exposure estimates in ways consistent with economic intuition despite no risk factor information being used during the unsmoothing process. For example, after unsmoothing returns using our 3-step method, the exposure of emerging market funds to the emerging market risk factor strongly increases, while other risk exposures of emerging market funds display little change.

Turning to CRE funds, the overall results are similar to what we obtain with hedge funds. However, the degree to which the 3-step unsmoothing process improves upon 1-step unsmoothing is much higher given the extreme illiquidity of real estate assets. For instance, the average beta of private CRE funds to the public real estate market increases from 0.07 to 0.34, driving the 4.3% annual alpha of CRE funds (measured with observed returns) to 1.6% after 3-step unsmoothing.

In summary, we develop a 3-step process which improves upon traditional return unsmoothing techniques for illiquid assets in order to better estimate their systematic risk exposures. We then apply our new return unsmoothing method to hedge funds, finding that the measurement of risk exposures and risk-adjusted performance substantially improves relative to what is obtained from returns unsmoothed using traditional methods. Finally, we perform a similar analysis based on CRE funds and find that results are even more pronounced for these funds given the high degree of illiquidity in their underlying assets.

Our paper provides a general contribution to the literature on illiquid assets as it develops a simple way to recover economic return estimates from observed returns to measure their risk exposures and risk-adjusted performance. Several papers in this body of literature attempt to measure the illiquidity premium (e.g., Aragon (2007), Barth and Monin (2018), and Khandani and Lo (2011)). Our contribution is particularly important in this area because unsmoothing returns is an essential part of measuring the illiquidity premium. Without properly unsmoothing the common component of returns, any attempt to measure this premium would not correctly control for exposure to other sources of risk and, as a consequence, could attribute the premium associated with other risk factors to illiquidity.

Our 3-step process improving upon Getmansky, Lo, and Makarov (2004) is a significant contribution to the hedge fund literature as it is standard practice to apply 1-step unsmoothing before studying hedge fund returns (e.g., Agarwal, Daniel, and Naik (2009), Agarwal, Green, and Ren (2018), Agarwal, Ruenzi, and Weigert (2017), Aragon and Nanda (2011), Avramov et al. (2011), Berzins, Liu, and Trzcinka (2013), Billio et al. (2012), Bollen (2013), Bollen, Joenväärä, and Kauppila (2020), Fung et al. (2008), Gao, Gao, and Song (2018), Jagannathan, Malakhov, and Novikov (2010), Kang et al. (2010), Kosowski, Naik, and Teo (2007), Li, Xu, and Zhang (2016), Patton (2008), Patton and Ramadorai (2013), Teo (2009, 2011), and Titman and Tiu (2011)).[4] Our empirical analysis further adds to previous papers

[4]While we interpret our results from the lens of illiquidity, misreporting can also induce smoothed returns (e.g., Aragon and Nanda (2017) and Bollen and Pool (2008, 2009)). We do not attempt to disentangle the two sources of return smoothness because, when considering the perspective of an investor or econometrician attempting to estimate economic returns, the degree of return smoothness is the relevant variable rather than the mechanism through which smoothness arises. Moreover, Cassar and Gerakos (2011) and Cao et al. (2017) find that asset illiquidity is the major driver of autocorrelation in hedge fund returns.

in the hedge fund literature by demonstrating that hedge fund alphas are lower than previously recognized once systematic risk is properly measured using our 3-step unsmoothing method.

Some papers in the hedge fund literature explore what drives illiquidity by studying the determinants of the unsmoothing parameters in Getmansky, Lo, and Makarov (2004) (e.g., Cao et al. (2017) and Cassar and Gerakos (2011)). We further add to this subset of the literature by demonstrating that the Getmansky, Lo, and Makarov (2004) unsmoothing parameters do not distinguish between fund-specific and systematic components of observed autocorrelation, which can directly affect inference associated with the drivers of illiquidity.

We also contribute to the real estate literature by showing that our unsmoothing technique can be used to improve upon the autoregressive unsmoothing method introduced in Geltner (1991, 1993), which is the basis for many papers unsmoothing returns in the real estate literature (e.g., Barkham and Geltner (1995), Corgel et al. (1999), Fisher et al. (2003), Fisher and Geltner (2000), Fisher, Geltner, and Webb (1994), Pagliari Jr, Scherer, and Monopoli (2005), and Rehring (2012)).

Our improvement upon traditional unsmoothing techniques also reaches beyond the hedge fund and real estate literatures. For instance, unsmoothing methods have been applied to other types of illiquid funds such as private equity, venture capital, and bond mutual funds (Ang et al. (2018) and Chen, Ferson, and Peters (2010)), to highly illiquid assets such as collectible stamps and art investments (Campbell (2008) and Dimson and Spaenjers (2011)), and even to unsmooth other economic series such as aggregate consumption (Kroencke (2017)).

The rest of this paper is organized as follows. Section 1 introduces the traditional, 1-step, return unsmoothing framework and develops our 3-step unsmoothing process to improve upon it; Section 2 applies 1-step and 3-step unsmoothing to hedge fund returns and demonstrates that the latter improves upon the former in measuring risk exposures and risk-adjusted performance; Section 3 extends the analysis to commercial real estate funds; and Section 4 concludes. The Internet Appendix provides supplementary results.

1 A New Return Unsmoothing Method

Academics and practitioners primarily rely on two methods to obtain economic (or unsmoothed) return estimates, R_t, from observed (or smoothed) returns, R_t^o. These techniques are used to unsmooth the returns of both illiquid assets and funds that invest in illiquid assets. As such, while our discussion and analysis focus on unsmoothing fund returns, our findings have important implications for directly unsmoothing asset returns as well.

Both unsmoothing methods assume R_t^o is a weighted average of current and past R_t, but the two techniques differ in how the weights are specified. The first method, developed by Getmansky, Lo, and Makarov (2004), leaves weights unconstrained, but requires a finite number of smoothing lags. We refer to this framework as MA unsmoothing as it implies a moving average time-series process for R_t^o. The second method, developed by Geltner (1991, 1993), imposes an infinite number of smoothing lags, but constrains weights to decay exponentially. We refer to this framework as AR unsmoothing as it implies an autoregressive time-series process for R_t^o. The former method is mainly used in the hedge fund literature while the latter is most commonly applied in the real estate literature.

We refer to both the MA and AR methods generally as "1-step unsmoothing" and develop a 3-step generalization for both methods, which better unsmoothes the common component of returns. Subsection 1.1 details the 1-step MA unsmoothing method; Subsection 1.2 demonstrates that it does not fully unsmooth the systematic component of returns; Subsection 1.3 develops our 3-step MA unsmoothing technique to address this issue; Subsection 1.4 explains why the 1-step method is unable to unsmooth the systematic component of returns, and Subsection 1.5 provides an economic framework that can be used to motivate our 3-step unsmoothing extension. The description of the AR unsmoothing framework is provided in Section 3, where we also apply AR unsmoothing to study the risk and performance of CRE funds.

1.1 The 1-step MA Unsmoothing Method

Table 1 provides the basic characteristics of the hedge funds we study (the data sources and sample construction are detailed in Subsection 2.1). There are 10 different hedge fund strategies (sorted by the average fund-level 1st order return autocorrelation coefficient) with a total of 5,069 funds and an average of 92 monthly returns per fund. Hedge funds display (annualized) average excess returns varying from 1.5% to 5.1% and (annualized) Sharpe ratios varying from 0.15 to 0.51, with more illiquid funds displaying higher Sharpe ratios.

It is well known that some hedge fund strategies rely on illiquid assets, and thus the observed returns of hedge funds may be smoothed. To deal with this issue, Getmansky, Lo, and Makarov (2004) (henceforth GLM) propose a method to unsmooth hedge fund returns. GLM assume the observed return of fund j at time t is given by (see GLM for the economic motivation):[5]

$$R^o_{j,t} = \theta_j^{(0)} \cdot R_{j,t} \ + \ \theta_j^{(1)} \cdot R_{j,t-1} \ + ... + \ \theta_j^{(H)} \cdot R_{j,t-H_j} \tag{1}$$

$$= \mu_j \ + \Sigma_{h=0}^{H} \, \theta_j^{(h)} \cdot \eta_{j,t-h} \tag{2}$$

where θs represent the smoothing weights with $\Sigma_{h=0}^{H}\theta_j^{(h)} = 1$ and the second equality follows from GLM's assumption that $R_{j,t} = \mu_j + \eta_{t,j}$ with $\eta_{j,t} \sim IID$.

The first equality represents the economic assumption that the observed fund return, $R^o_{j,t}$, is a weighted average of the fund's economic returns, $R_{j,t}$, over the most recent $H+1$ periods, including the current period. The second equality is the econometric implication that, under the given assumption, the observed fund returns follow a Moving Average process of order H, MA(H).

Given Equation 2, we can recover economic returns by estimating an MA(H) process for observed returns, extracting the estimated residuals, $\eta_{j,t}$, and adding the estimated expected return, μ_j, such that $R_{j,t} = \mu_j + \eta_{j,t}$. GLM also provide the basic steps to estimate θs by

[5] The fact that H does not depend on j simplifies the notation but does not imply the number of MA lags is not fund dependent. Specifically, letting H_j represent the number of MA lags with non-zero weight for fund j, we define $H = max(H_j)$ and set $\theta_j^{(h)} = 0$ for any $h > H_j$.

maximum likelihood under the added parametric assumption that $\eta_{j,t} \overset{iid}{\sim} N(0, \sigma^2_{\eta,j})$.[6] This procedure is used by several papers in the hedge fund literature to unsmooth returns (e.g., Agarwal, Daniel, and Naik (2009), Agarwal, Green, and Ren (2018), Agarwal, Ruenzi, and Weigert (2017), Aragon and Nanda (2011), Avramov et al. (2011), Berzins, Liu, and Trzcinka (2013), Billio et al. (2012), Bollen (2013), Bollen, Joenväärä, and Kauppila (2020), Fung et al. (2008), Gao, Gao, and Song (2018), Jagannathan, Malakhov, and Novikov (2010), Kang et al. (2010), Kosowski, Naik, and Teo (2007), Li, Xu, and Zhang (2016), Patton (2008), Patton and Ramadorai (2013), Teo (2009, 2011), and Titman and Tiu (2011)).

We apply this 1-step MA unsmoothing to each hedge fund in our dataset using the AIC criterion to select the number of MA lags (allowing for H from 0 to 3 months). Table 2 contains the (average) autocorrelations (at 1, 2, 3, and 4 monthly lags) for hedge fund returns (observed and unsmoothed) as well as the percentage of funds with a significant autocorrelation at the 10% level. Observed returns display relatively high autocorrelations. For instance, relative value funds have average 1st order autocorrelation of 0.29, with 61% of these funds displaying statistically significant autocorrelations. After 1-step MA unsmoothing, average autocorrelations are basically zero at all lags and the percentage of funds displaying statistically significant autocorrelations is in line with the statistical error of the test.

These results indicate that 1-step MA unsmoothing produces economic returns that are largely unsmoothed at the fund level. This correction is important to properly analyse hedge funds because smoothed returns understate volatilities and betas, and thus overstate Sharpe ratios and alphas, as demonstrated by GLM.

1.2 Implications to Aggregate Fund Returns

Under the assumption that economic returns are not autocorrelated, which is central to unsmoothing methods, we should also observe strategy-level returns that are not autocor-

[6]The method is almost identical to the one used in most econometric packages. The only difference is that econometric packages tend to impose the normalization $\theta_j^{(0)} = 1$ as opposed to $\Sigma_{h=0}^{H} \theta_j^{(h)} = 1$. As such, econometric packages yield $\eta^*_{j,t}$ and $\theta_j^{(h)*}$ with $\theta_j^{(h)} \cdot \eta_{j,t-h} = \theta_j^{(h)*} \cdot \eta^*_{j,t}$ and $\theta_j^{(h)*}/\theta_j^{(h)} = 1 + \theta_j^{(1)} + ... + \theta_j^{(H)}$. We can then recover $\eta_{j,t}$ and $\theta_j^{(h)}$ by dividing $\theta_j^{(h)*}$ (and multiplying $\eta^*_{j,t}$) by $1 + \theta_j^{(1)} + ... + \theta_j^{(H)}$.

related. Specifically, for any set of time-invariant weights, $\{w_j\}_{j=1}^{J}$, the assumption that $R_{j,t} = \mu_j + \eta_{t,j}$ with $\eta_{j,t} \sim IID$ implies:

$$
\begin{aligned}
\overline{R}_t &\equiv \Sigma_{j=1}^{J}\, w_j \cdot R_{j,t} \\
&= \Sigma_{j=1}^{J}\, w_j \cdot \mu_j \;+\; \Sigma_{j=1}^{J}\, w_{j,t} \cdot \eta_{j,t} \\
&= \overline{\mu} \;+\; \overline{\eta}_t
\end{aligned} \tag{3}
$$

where $\overline{\eta}_t \sim IID$, which means that $\mathbb{E}_{t-1}[\overline{R}_t] = \overline{\mu}$ (i.e., aggregate returns are unpredictable, and thus they should not be autocorrelated).

Table 3 shows monthly autocorrelations for each (equal-weighted) strategy return, with returns unsmoothed at the fund-level before aggregation. Reported returns on these strategies display quite high autocorrelations (in fact, higher than the average autocorrelations of their respective funds). Moreover, the autocorrelation coefficients remain high after aggregating 1-step unsmoothed returns. For instance, the relative value strategy has a 1st order autocorrelation of 0.51 (statistically significant at 1%) and the autocorrelation is still 0.29 (statistically significant at 1%) after 1-step MA unsmoothing.

The results suggest that 1-step unsmoothing delivers strategy indexes with substantial autocorrelation, which indicates that the approach used by the previous literature to unsmooth returns does not fully unsmooth the systematic component of returns. This result is important because risk exposure estimates are understated (and alpha estimates are overstated) even after return unsmoothing if the systematic return component is not properly unsmoothed.

1.3 The 3-step MA Unsmoothing Method

We develop a 3-step unsmoothing procedure to addresses the issue raised in the previous subsection. The basic idea is to allow the aggregate and fund-specific components of returns to be smoothed with different weights. This subsection focuses on the econometrics of the 3-Step MA unsmoothing method, with Subsection 1.5 providing an economic framework that justifies the procedure.

We refer to the "aggregate" of a generic variable, $y_{j,t}$, as $\overline{y}_t = \Sigma_{j=1}^J w_j \cdot y_{j,t}$ and (in the case of returns) its "relative return" as $y_{j,t} - \overline{y}_t$, where w_j are arbitrary (but time-invariant) weights with $\Sigma_{j=1}^J w_j = 1$. Moreover, we keep the total number of funds, J, constant over time while developing our aggregation results.

This subsection relies on the fact that, for arbitrary variables x_j and $y_{j,t}$, we have:[7]

$$\begin{aligned}\Sigma_{j=1}^J w_j \cdot x_j \cdot y_{j,t} &= \overline{x} \cdot \overline{y}_t \ + \ \Sigma_{j=1}^J w_j \cdot (x_j - \overline{x}) \cdot (y_{j,t} - \overline{y}_t) \\ &= \overline{x} \cdot \overline{y}_t \ + \ \widehat{Cov}(x_j, y_{j,t}) \end{aligned} \tag{4}$$

We generalize the underlying assumption in Equation 1 so that the aggregate and relative returns can be smoothed with different weights:[8]

$$R^o_{j,t} = \Sigma_{h=0}^H \ \phi_j^{(h)} \cdot \widetilde{R}_{j,t-h} \ + \ \Sigma_{h=0}^L \ \pi_j^{(h)} \cdot \overline{R}_{t-h} \tag{5}$$

$$= \mu_j \ + \ \Sigma_{h=0}^H \ \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} \ + \ \Sigma_{h=0}^L \ \pi_j^{(h)} \cdot \overline{\eta}_{t-h} \tag{6}$$

where $\overline{R}_t = \Sigma_{j=1}^J w_j \cdot R_{j,t}$ are aggregate returns, $\widetilde{R}_j = R_{j,t} - \overline{R}_t$ are relative returns, $\overline{\eta}$ and $\widetilde{\eta}$ are the respective shocks, and the weights add to one, $\Sigma_{h=0}^H \phi_j^{(h)} = \Sigma_{h=0}^L \pi_j^{(h)} = 1$.

In GLM, the weights on past economic returns, θs, add to one to assure that information is eventually incorporated into observed prices. Our restriction, $\Sigma_{h=0}^H \phi_j^{(h)} = \Sigma_{h=0}^L \pi_j^{(h)} = 1$, has the same effect.

[7]This result is a generalization of the typical covariance decomposition for the case of non-equal weights:

$$\begin{aligned}\Sigma_{j=1}^J w_j \cdot (x_j - \overline{x}) \cdot (y_{j,t} - \overline{y}_t) &= \Sigma_{j=1}^J w_j \cdot x_j \cdot y_{t,j} - \overline{y}_t \cdot \Sigma_{j=1}^J w_j \cdot x_j - \overline{x} \cdot \Sigma_{j=1}^J w_j \cdot y_{t,j} + \overline{x} \cdot \overline{y}_t \cdot \Sigma_{j=1}^J w_j \\ &= \Sigma_{j=1}^J w_j \cdot x_j \cdot y_{t,j} - \overline{x} \cdot \overline{y}_t\end{aligned}$$

[8]This smoothing process reduces to Equation 1 (the 1-step unsmoothing process in GLM) if we set $\pi_j^{(h)} = \phi_j^{(h)} = \theta_j^{(h)}$. Moreover, as in the 1-step method, the fact that H and L do not depend on j simplifies the notation but does not imply the number of MA lags is not fund dependent. Specifically, letting H_j and L_j represent the number of MA lags with non-zero weight for fund j, we have $H = max(H_j)$ and $L = max(L_j)$ with $\phi_j^{(h)} = 0$ for any $h > H_j$ and $\pi_j^{(h)} = 0$ for any $h > L_j$.

Aggregating Equation 6 yields:

$$\begin{aligned}
\overline{R}_t^o &= \overline{\mu} \ + \ \overline{\pi}^{(0)} \cdot \overline{\eta}_t \ + \ \overline{\pi}^{(1)} \cdot \overline{\eta}_{t-1} \ + ... + \ \overline{\pi}^{(L)} \cdot \overline{\eta}_{t-\overline{H}} \\
&\quad + \widehat{Cov}(\phi_j^{(0)}, \widetilde{\eta}_{j,t}) \ + \ \widehat{Cov}(\phi_j^{(1)}, \widetilde{\eta}_{j,t-1}) \ + ... + \ \widehat{Cov}(\phi_j^{(H)}, \widetilde{\eta}_{j,t-H}) \\
&\approx \overline{\mu} \ + \ \Sigma_{h=0}^{L} \, \overline{\pi}^{(h)} \cdot \overline{\eta}_{t-h} \qquad (7)
\end{aligned}$$

where the first equality relies on Equation 4 and the second equality is based on a large sample approximation that uses $\underset{J\to\infty}{Plim}\, \widehat{Cov}(\phi_j^{(h)}, \widetilde{\eta}_{j,t-h}) = Cov(\phi_j^{(h)}, \widetilde{\eta}_{j,t-h}) = 0$. The restriction $\Sigma_{h=0}^{L}\pi_j^{(h)} = 1$ assures the aggregate moving average parameters satisfy $\Sigma_{h=0}^{L}\overline{\pi}^{(h)} = 1$ so that aggregate information is eventually incorporated into aggregate prices.[9]

Subtracting Equation 7 from Equation 6, we have observed relative returns:

$$\begin{aligned}
\widetilde{R}_{j,t}^o &= \Sigma_{h=0}^{H} \, \phi_j^{(h)} \cdot \widetilde{R}_{j,t-h} \ + \ \Sigma_{h=0}^{L} \, \psi_j^{(h)} \cdot \overline{R}_{t-h} \\
&= \widetilde{\mu}_j \ + \ \Sigma_{h=0}^{H} \, \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} \ + \ \Sigma_{h=0}^{L} \, \psi_j^{(h)} \cdot \overline{\eta}_{t-h} \qquad (8)
\end{aligned}$$

where $\psi_j^{(h)} = \pi_j^{(h)} - \overline{\pi}_j^{(h)}$.

Equations 7 and 8 provide a simple way to recover aggregate and fund-level economic returns in an internally consistent way. First, we estimate aggregate economic returns from $\overline{R}_t = \overline{\mu} + \overline{\eta}_t$, where $\overline{\eta}_t$ are residuals of a MA(L) fit to $\overline{R}_t^o$. Second, we obtain fund-level economic relative returns from $\widetilde{R}_{j,t} = \widetilde{\mu}_j + \widetilde{\eta}_{j,t}$, where $\widetilde{\eta}_{j,t}$ are residuals from a MA(H) fit (with $\overline{\eta}_t$, $\overline{\eta}_{t-1}$, ..., $\overline{\eta}_{t-L}$ as covariates) to $\widetilde{R}_{j,t}^o$.[10] Third, we recover fund-level economic returns from $R_{t,j} = \overline{R}_t + \widetilde{R}_{j,t} = \mu_j + \overline{\eta}_t + \widetilde{\eta}_{j,t}$. This procedure summarizes our 3-step unsmoothing process.

The last four columns of Tables 2 and 3 show return autocorrelations after our 3-step unsmoothing process. From Table 2, unsmoothed fund-level returns display autocorrelations comparable to the ones obtained from 1-step unsmoothing (effectively no autocorrelation).

[9] Since $\Sigma_{h=0}^{L}\overline{\pi}^{(h)} = \Sigma_{h=0}^{L}\Sigma_{j=1}^{J} w_j \cdot \pi_j^{(h)} = \Sigma_{j=1}^{J} w_j \cdot (\Sigma_{h=0}^{L}\pi_j^{(h)}) = 1$

[10] As in the 1-step method, if the aggregate and fund-level MA processes are estimated by standard statistical packages (which would normalize $\overline{\pi}^{(0)} = 1$ in step 1 and $\theta_j^{(0)} = 1$ in step 2), then we need to divide the coefficients estimated by the package (and multiple the estimated residuals) by $1 + \overline{\pi}^{(1)} + ... + \overline{\pi}^{(L)}$ in Step 1 and by $1 + \phi_j^{(1)} + ... + \phi_j^{(H)}$ in Step 2. The $\psi_j^{(h)}$ coefficients do not need to be adjusted in step 2.

However, Table 3 shows that, in contrast to 1-step unsmoothing, our 3-step unsmoothing method drives strategy-level autocorrelations to virtually zero.[11] This evidence suggests that our 3-step MA unsmoothing method properly unsmoothes the systematic component of returns. This finding has important implications for the measurement of risk exposures and risk-adjusted performance as we demonstrate in Section 2.

1.4 Understanding Autocorrelation in 1-Step Unsmoothed Returns

The previous subsection shows that the 3-step MA unsmoothing method effectively eliminates the autocorrelation in aggregate returns while the 1-step MA unsmoothing method does not. In this subsection, we explain why this happens. Specifically, we consider the case in which the econometrician assumes Equation 2 is valid (i.e., believes the smoothing process is consistent with the 1-step method), but the true return smoothing process is given by Equation 6 (i.e., it is consistent with the 3-step method).

(a) Analytical Analysis

In this case, the econometrician's 1-step unsmoothed returns are given by:[12]

$$R_{j,t}^{1s} = \mu_j + \eta_{j,t} + \Sigma_{h=0}^{max(H,L)} \lambda_j^{(h)} \cdot \overline{\epsilon}_{j,t-h} \tag{9}$$

and

$$\overline{R}_t^{1s} \approx \overline{\mu} + \overline{\eta}_t + \Sigma_{h=0}^{max(H,L)} \overline{\lambda}^{(h)} \cdot (1-\overline{b}) \cdot \overline{\eta}_{t-h} \tag{10}$$

---

[11] Strategy-level returns from the 3-step unsmoothing method are obtained by aggregating $R_{t,j}$, not by directly using $\overline{\mu}+\overline{\eta}_t$. As such, the results reported in Table 3 are not mechanical and instead reflect the fact that the 3-step unsmoothing method is able to unsmooth the systematic portion of fund-level returns.

[12] To derive Equations 9 and 10, substitute $\overline{\eta}_t = b_j \cdot \eta_{j,t} + \overline{\epsilon}_{j,t}$ into the true smoothing process (Equation 6) to get:

$$R_{j,t}^{o} = \mu_j + \Sigma_{h=1}^{max(H,L)} \theta_j^{(h)} \cdot \eta_{j,t-h} + u_{j,t}$$

where $\theta_j^{(h)} = \phi_j^{(h)} + (\pi_j^{(h)} - \phi_j^{(h)}) \cdot b_j$ and $u_{j,t} = \theta_j^{(0)} \cdot \eta_{j,t} + \Sigma_{h=0}^{max(H,L)} (\pi_j^{(h)} - \phi_j^{(h)}) \cdot \overline{\epsilon}_{j,t-h}$. Since $\theta_j^{(0)} = 1 - \Sigma_{h=1}^{max(H,L)} \theta_j^{(h)}$ and the econometrician believes $u_{j,t} = \theta_j^{(0)} \cdot \eta_{j,t}$, s/he recovers economic returns as $R_{j,t}^{1s} = \mu_j + u_{j,t}^{1s}/\theta_j^{(0)}$, which yields Equation 9. Then, since $\Sigma_j\, w_j \cdot \overline{\epsilon}_{j,t} \approx (1-\overline{b}) \cdot \overline{\eta}_t$, we have that aggregating Equation 9 yields Equation 10.

where $\lambda_j^{(h)} = (\pi_j^{(h)} - \phi_j^{(h)})/(\phi_j^{(0)} + (\pi_j^{(0)} - \phi_j^{(0)}) \cdot b_j)$ and $\bar{\epsilon}_{j,t}$ represents the error process of the projection $\overline{\eta}_t = b_j \cdot \eta_{j,t} + \bar{\epsilon}_{j,t}$.

If $\pi_j^{(h)} = \phi_j^{(h)}$ or $\bar{\epsilon}_{j,t} = 0$, the 1-step method properly recovers the true economic returns.[13] As such, the 3-step method can be seen as a generalization of the 1-step method that allows aggregate returns and returns relative to the aggregate to have different effects on the return smoothing process ($\pi_j^{(h)} \neq \phi_j^{(h)}$). Both methods produce the same economic return estimates (as $T \to \infty$) if the underlying assumption in the 1-step method ($\pi_j^{(h)} = \phi_j^{(h)}$) is valid or if fund economic returns are perfectly correlated (in which case $\bar{\epsilon}_{j,t} = 0$ and the aggregate provides no extra information).

As discussed in the introduction, Figure 2 suggests that $\pi_j^{(h)} = \phi_j^{(h)}$ and $\bar{\epsilon}_{j,t} = 0$ are not empirically valid conditions.[14] Consequently, Equation 9 shows that $R_{j,t}^{1s}$ reflects true economic returns, $R_{j,t} = \mu_j + \eta_{j,t}$, but also a moving average component related to the portion of aggregate returns "unexplained" by fund returns, $\Sigma_{h=0}^{max(H,L)} \lambda_j^{(h)} \cdot \bar{\epsilon}_{j,t-h}$. Since fund-returns tend to be much more volatile than aggregate returns, the first term tends to dominate the autocorrelation structure so that fund-level 1-step unsmoothed returns have almost no autocorrelation. In contrast, Equation 10 shows that aggregate 1-step unsmoothed returns follow a MA(H) process so that autocorrelation is easy to detect, which explains why the autocorrelations remain high at the aggregate after unsmoothing returns with the 1-step method.[15]

Intuitively, the fund-level autocorrelation in 1-step unsmoothed returns is small because the 1-step method misspecification is related to the systematic component of returns, which

[13]Note that $b_j = Cor(\overline{\eta}_t, \eta_{j,t}) \cdot \sigma[\overline{\eta}_t]/\sigma[\eta_{j,t}]$. Since $Cor(\overline{\eta}_t, \eta_{j,t}) = 1$ implies $\sigma[\overline{\eta}_t] = \sigma[\eta_{j,t}]$, we have $b_j = \bar{b} = 1$ if $\epsilon_{j,t} = 0$ for all funds. As such, the condition $\epsilon_{j,t} = 0$ leads the 1-step method to properly recover the true aggregate economic returns even though $\epsilon_{j,t} = 0$ is not explicitly present in Equation 10.

[14]Figure 2 is only suggestive as it is based on regressions that use observed returns, not economic returns (since economic returns are unobservable without further assumptions). However, the overall results in this paper show that the economic returns recovered from the 1- and 3-step methods differ drastically, which represents a more formal way to demonstrate that the conditions $\pi_j^{(h)} = \phi_j^{(h)}$ and $\epsilon_t = 0$ do not hold. The recovered economic returns would be identical (up to sampling error) if either of these conditions were valid.

[15]Since $b_j = Cor(\overline{\eta}_t, \eta_{j,t}) \cdot \sigma[\overline{\eta}_t]/\sigma[\eta_{j,t}]$, $Cor(\overline{\eta}_t, \eta_{j,t}) < 1$ and $\sigma[\overline{\eta}_t] < \sigma[\eta_{j,t}]$ (conditions that are empirically valid) imply $\bar{b} < 1$. Moreover, we have $\overline{\pi}^{(h)} > \overline{\phi}^{(h)}$ in the data, which predicts a positive autocorrelation for aggregate 1-step unsmoothed returns, exactly what we observe empirically.

is small relative to the idiosyncratic component of returns. Yet, this misspecification has important implications for the measurement of risk exposures (and risk-adjusted performance) because these quantities heavily depend on the systematic component of returns.

(b) Simulation Analysis

To better understand the autocorrelation in 1-step unsmoothed returns from a quantitative perspective, we simulate returns on a panel of 300 funds over a 90 month period. The monthly economic returns of each fund j satisfy:

$$R_{j,t} = \alpha_j + \beta_j \cdot f_t + \varepsilon_{j,t} \tag{11}$$

where $\alpha_j \overset{iid}{\sim} N(\mu_\alpha, \sigma_\alpha^2)$, $\beta_j \overset{iid}{\sim} N(1, \sigma_\beta^2)$, $f_t \overset{iid}{\sim} N(\mu_f, \sigma_f^2)$, and $\varepsilon_t \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$.[16]

We then smooth these returns according to the smoothing process outlined in our 3-step method (i.e., Equation 5). Specifically, we consider $H = L = 1$ (i.e., MA(1) smoothing) and set $\phi_j^{(1)} = \phi^{(1)}$ and $\pi_j^{(1)} = \pi^{(1)}$ for simplicity. Finally, we estimate economic returns for each fund in the panel using the 1- and 3-step unsmoothing methods and study the properties of observed returns, 1-step unsmoothed returns, and 3-step unsmoothed returns. The average results obtained from 1,000 simulations of this panel of funds are provided in Table 4.

The first column shows results for a specification in which the aggregate and fund-specific components of returns are smoothed with the same intensity ($\phi^{(1)} = \pi^{(1)} = 0.3$). In this case, fund- and aggregate-level R_t^o display autocorrelation, but R_t^{1s} and R_t^{3s} do not, indicating that the 1- and 3-step methods both work well in unsmoothing returns when $\phi^{(1)} = \pi^{(1)}$. However, we also observe that $\overline{R}_t^{1s}$ display no autocorrelation and the amount of autocorrelation in R_t^o is roughly the same at the fund and aggregate levels (0.35 with R_t^o and 0.36 with $\overline{R}_t^o$), with both of these results being inconsistent with what we observe in our hedge fund analysis.

To explore this issue further, the second column considers a specification in which the fund-level component of returns is smoothed less than the aggregate-level component ($\phi^{(1)} = 0.2$

[16]We rely on simple (but realistic) parameters given by $\mu_\alpha = 0\%$, $\sigma_\alpha = 2\%$, $\sigma_\beta = 0.25$, $\mu_f = 0.5\%$, $\sigma_f = 4\%$, $\sigma_\varepsilon = 4\%$. However, the general insights from our simulation exercise are not sensitive to these baseline parameters.

and $\pi^{(1)} = 0.4$). In this case, observed returns have an autocorrelation that is lower at the fund-level in comparison to the aggregate-level (0.32 with R_t^o and 0.46 with $\overline{R}_t^o$), which is in line with what we observed in our hedge fund analysis. Moreover, while both the 1- and 3-step methods reduce R_t^o autocorrelation to virtually zero, roughly half of the autocorrelation in $\overline{R}_t^o$ persists in $\overline{R}_t^{1s}$ (also in line with our hedge fund analysis).

Finally, the third column considers an alternative scenario in which the fund-level component of returns is smoothed more than the aggregate-level component ($\phi^{(1)} = 0.4$ and $\pi^{(1)} = 0.2$). This case results in counterfactual autocorrelation structure (relative to our hedge fund analysis) since $\overline{R}_t^o$ is more autocorrelated at the fund-level than at the aggregate level and $\overline{R}_t^{1s}$ is negatively autocorrelated.

Overall, the results suggest our hedge fund analysis is in line with a smoothing process in which the fund-level component of returns is smoothed less than the aggregate-level component, which explains why the 1-step method (which implicitly assumes the two components are smoothed with the same intensity) does not fully unsmoothes the systematic component of returns.

Given that the second column is the empirically relevant scenario, the last three rows of Table 4 suggest that a performance evaluation of hedge funds that relies on the 1-step method to unsmooth returns is likely to understate systematic risk (as $\overline{\beta} = 1$ in the simulations), and thus overstate risk-adjustment performance. We further explore performance evaluation with the 1- and 3-step unsmoothing methods in Section 2.

1.5 The Economics of 3-Step MA Unsmoothing

The previous subsections demonstrate that 1-step MA unsmoothing does not fully unsmooth the systematic component of returns and that allowing for the aggregate and fund-specific components of returns to be smoothed with different weights (i.e., $\pi_j^{(h)} \neq \phi_j^{(h)}$) through our proposed 3-step MA unsmoothing method solves the problem. This subsection provides an economic framework that clarifies why allowing for $\pi_j^{(h)} \neq \phi_j^{(h)}$ is economically more sensible than restricting the smoothing process to satisfy $\pi_j^{(h)} = \phi_j^{(h)} \equiv \theta_j^{(h)}$ as in the 1-step MA unsmoothing method. In a nutshell, $\pi_j^{(h)} \neq \phi_j^{(h)}$ arises naturally if funds do not observe the current value of the assets they hold, but instead receive different signals about the shocks common to all assets in their asset class and the shocks affecting the relative value of their assets.

There are J funds, indexed by j, operating in a common asset class. The log value of each fund in this asset class evolves as:

$$V_{j,t} = \mu_j + V_{j,t-1} + \overline{\eta}_t + \widetilde{\eta}_{j,t} \tag{12}$$

where $\overline{\eta}_t \overset{iid}{\sim} N(0, \overline{\sigma}^2)$ is an asset class shock, $\widetilde{\eta}_{j,t} \overset{iid}{\sim} N(0, \widetilde{\sigma}_j^2)$ is a shock specific to the assets of fund j, and the parameter μ_j is common knowledge.

The specification above implies log economic returns are given by (assuming no cash flows are paid from $t-1$ to t):

$$\begin{aligned} r_{j,t} &= V_{j,t} - V_{j,t-1} \\ &= \mu_j + \overline{\eta}_t + \widetilde{\eta}_{j,t} \end{aligned} \tag{13}$$

so that aggregate log economic returns are $\overline{r}_t = \sum_{j=1}^{J} w_j \cdot r_{j,t} = \overline{\mu} + \overline{\eta}_t$ and relative log economic returns are $\widetilde{r}_{j,t} = r_{j,t} - \overline{r}_t = \widetilde{\mu}_j + \widetilde{\eta}_{j,t}$.[17]

However, funds do not observe $V_{j,t}$ at time t, and thus the returns calculated from reported value (i.e., observed returns) differ from economic returns. Specifically, at each time t, each fund learns its $V_{j,t-1}$ but not its $V_{j,t}$. Instead, each fund receives a (different) signal about $\overline{\eta}_t$:

$$\widehat{\overline{\eta}}_{j,t} = \overline{\eta}_t + \overline{u}_{j,t} \tag{14}$$

as well as a separate signal about $\widetilde{\eta}_{j,t}$:

$$\widehat{\widetilde{\eta}}_{j,t} = \widetilde{\eta}_{j,t} + \widetilde{u}_{j,t} \tag{15}$$

where $\overline{u}_{j,t} \overset{iid}{\sim} N(0, \widehat{\overline{\sigma}}_j^2)$ and $\widetilde{u}_{j,t} \overset{iid}{\sim} N(0, \widehat{\widetilde{\sigma}}_j^2)$.

After receiving the signals, each fund reports the posterior mean of its log value,

[17] While $\overline{r}_t = \sum_{j=1}^{J} w_j \cdot r_{j,t} = \overline{\mu} + \overline{\eta}_t$ holds in general, the interpretation of $\overline{r}_t$ as an aggregate return relies on the approximation in Campbell, Chan, and Viceira (2003) because the aggregate of log returns is not generally equal to the log of aggregate returns.

$V^o_{j,t} = \mathbb{E}[V_{j,t}|V_{j,t-1}, \widehat{\overline{\eta}}_t, \widehat{\widetilde{\eta}}_t]$, in its books, which is given by:

$$\begin{aligned} V^o_{j,t} &= \mu_j \; + \; V_{j,t-1} \; + \; \mathbb{E}[\overline{\eta}_t|\widehat{\overline{\eta}}_t] \; + \; \mathbb{E}[\widetilde{\eta}_{j,t}|\widehat{\widetilde{\eta}}_{j,t}] \\ &= \mu_j \; + \; V_{j,t-1} \; + \; \pi_j \cdot \widehat{\overline{\eta}}_t \; + \; \phi_j \cdot \widehat{\widetilde{\eta}}_{j,t} \end{aligned} \tag{16}$$

and implies observed log returns are given by:

$$\begin{aligned} r^o_{j,t} &= V^o_{j,t} - V^o_{j,t-1} \\ &= (V_{j,t-1} - V_{j,t-2}) \; + \; \pi_j \cdot (\widehat{\overline{\eta}}_t - \widehat{\overline{\eta}}_{t-1}) \; + \; \phi_j \cdot (\widehat{\widetilde{\eta}}_{j,t} - \widehat{\widetilde{\eta}}_{j,t-1}) \\ &= r_{j,t-1} \; + \; \pi_j \cdot (\overline{\eta}_t - \overline{\eta}_{t-1}) \; + \; \phi_j \cdot (\widetilde{\eta}_{j,t} - \widetilde{\eta}_{j,t-1}) \; + \; \xi_{j,t} \\ &= \pi_j \cdot \overline{r}_t \; + \; (1 - \pi_j) \cdot \overline{r}_{t-1} \; + \; \phi_j \cdot \widetilde{r}_{j,t} \; + \; (1 - \phi_j) \cdot \widetilde{r}_{j,t-1} \; + \; \xi_{j,t} \end{aligned} \tag{17}$$

where
$\pi_j = (1/\widehat{\overline{\sigma}}^2_j)/(1/\widehat{\overline{\sigma}}^2_j + 1/\overline{\sigma}^2)$
$\phi_j = (1/\widehat{\widetilde{\sigma}}^2_j)/(1/\widehat{\widetilde{\sigma}}^2_j + 1/\widetilde{\sigma}^2_j)$
$\xi_{j,t} = \pi_j \cdot (\overline{u}_{j,t} - \overline{u}_{j,t-1}) + \phi_j \cdot (\widetilde{u}_{j,t} - \widetilde{u}_{j,t-1})$

Equation 17 is the same as the smoothing process we assume for the 3-step MA unsmoothing method (in Equation 5), except that Equation 17 applies to log returns (instead of regular returns) and the smoothing process in Equation 17 has an extra latent component, $\xi_{j,t}$, which we abstract from in our econometric framework to maintain the tractability of our unsmoothing method. Moreover, Equation 17 can be generalized to an MA(H) process by assuming that, at time t, funds only learn $V_{j,t-H}$ and have to rely on signals about the shocks that realized from $t - H$ to t in order to obtain their posterior distribution for $V^o_{j,t}$.

Given the above paragraph, it is natural to ask under which economic conditions the 1-step MA unsmoothing restriction ($\pi_j = \phi_j = \theta_j$) would hold. Inspecting Equation 17, we have that the economic condition for $\pi_j = \phi_j$ to hold is that the variance of the aggregate signal relative to the variance of aggregate returns, $\widehat{\overline{\sigma}}^2_j/\overline{\sigma}^2$, is the same as the analogous quantity for relative returns, $\widehat{\widetilde{\sigma}}^2_j/\widetilde{\sigma}^2_j$, for all funds. It seems implausible to expect such a condition to hold for hedge funds (or any set of funds), which explains why the systematic component is not fully unsmoothed when we apply 1-step MA unsmoothing to hedge fund returns, but it is when we use our proposed 3-step MA unsmoothing method.

2 Unsmoothing Hedge Fund Returns

In this section, we unsmooth hedge fund returns using the 3-step MA unsmoothing method to demonstrate that it improves upon 1-step MA unsmoothing in terms of measuring risk-adjustment performance. Subsection 2.1 explains the empirical details; Subsection 2.2 presents the main results after separating funds into liquidity groups; and Subsection 2.3 reports results by hedge fund strategy to explore the improvement in the measurement of risk exposures.

2.1 Empirical Details

(a) Hedge Fund Dataset

We combine data from two major commercial hedge fund databases to build our hedge fund dataset. Specifically, we merge the Lipper Trading Advisor Selection System database (hereafter TASS), accessed in June 2018, with the BarclayHedge database, accessed in April 2018, which produces a representative coverage of the hedge fund universe.[18] Both data providers only started keeping a so-called graveyard database of funds that stopped reporting their returns only in 1994. Hence, following the literature, we start our empirical analysis in 1995, which avoids issues associated with survivorship bias.

We apply some standard screens before including observations in the sample. We start by excluding observations with stale (for more than one quarter) Assets Under Management (AUM) or that have missing return or AUM. We then restrict the sample to US-dollar funds

[18] Joenväärä et al. (2019) combine and compare five different hedge fund databases that have been used in academic studies. Their analysis shows that the two datasets used in this study (TASS and BarclayHedge) together with Hedge Fund Research, have the most complete data in terms of the number of funds included and the lack of survivorship bias (after 1994). Joenväärä et al. (2019) also find that the average fund performance is similar across the five databases.

that report net-of-fees returns, have at least 36 uninterrupted monthly observations, and reach \$5 million in AUM at some point in the sample.

To minimize the impact of small and idiosyncratic funds and mitigate incubation bias and backfill bias, we perform two standard data screens utilized in the literature. First, funds are only included after reaching the \$5 million AUM threshold for the first time, and they are not dropped from the sample in case they fall below this threshold after reaching it. Second, after unsmoothing the returns and estimating factor regressions to obtain each fund's risk loadings, we drop all backfilled returns for each fund before calculating average excess returns, volatilities, Sharpe ratios, and alphas. We use the algorithm proposed by Jorion and Schwarz (2019) in order to identify backfilled observations.[19]

After these initial screens, we merge the data from TASS and BarclayHedge and eliminate duplicate fund observations that exist when the same fund reports to both data providers. In order to do so, we start by identifying possible duplicate funds by fuzzy-matching fund names and fund company names across the two data sources. Then, following Joenväärä et al. (2019), we calculate the correlation of returns for each potential duplicate pair and identify it as a duplicate if the correlation is 99% or higher. Finally, for each duplicate pair identified, we keep the one that has the longest series of valid return and AUM data. The final sample starts in January 1995 and ends in December 2017.

Many of our results separate hedge funds based on their strategies. We identify strategies using the "primary strategy" variable reported by TASS and BarclayHedge. We exclude funds whose strategy is classified as "other" or whose primary strategy does not fall into any of the 12 investment styles identified by Joenväärä et al. (2019). There are only a few funds

[19] Jorion and Schwarz (2019) find that dropping the first 12 monthly returns is the most common procedure used in the literature to deal with hedge fund incubation bias and backfill bias, but that this adjustment alone is not sufficient to properly measure performance and propose an algorithm that allows researchers to impute each fund's initial reporting date and thus address the entirety of backfilled returns. We follow their algorithm to input the initial reporting date and drop returns prior to it before calculating performance measures (dropping the first 12 monthly returns instead yield similar results as we demonstrate in Internet Appendix A). We still use the entire history of returns (as it is standard the literature) to estimate the unsmoothing process and factor models since the literature has found that autocorrelations and risk exposures are not affected by backfilled returns.

whose strategy is classified as short bias, hence we group them together with long/short funds. Finally, we exclude funds of funds, because these funds often invest in different fund categories and therefore they cannot be considered a homogeneous group. Table 1 (discussed earlier) provides the final list of strategies used in our analysis as well as the number of funds in each strategy.

(b) Risk Factors

Our analysis of risk exposures and risk-adjusted performance is based on the FH 8-Factor model, which augments the 7-Factor model in Fung and Hsieh (2001) with an emerging market factor. The risk-free rate and trend-following factors are obtained respectively from Kenneth French's and David A. Hsieh's online data libraries.[20] The 3 equity-oriented risk factors are calculated using equity index data from Datastream, and the 2 bond-oriented factors are calculated using data from the Federal Reserve Bank of St. Louis (both equity and bond factors follow the instructions given on David A. Hsieh's webpage).

(c) 3-step Return Unsmoothing

We perform the 1-step MA unsmoothing following a procedure similar to Getmansky, Lo, and Makarov (2004). Specifically, we use the AIC criterion to choose the number of smoothing lags (from 0 to 3) in the MA process for observed returns ($R^o_{j,t}$), extract estimated residuals ($\eta_{j,t}$), and add the average return (μ_j) back to obtain economic returns, $R_{j,t} = \mu_j + \eta_{j,t}$.[21] The MA process is estimated using maximum likelihood under $\eta_{j,t} \overset{iid}{\sim} N(0, \sigma^2_{\eta,j})$, as described in GLM.

We follow an analogous procedure for our 3-step MA unsmoothing. First, we take the average return of all funds in a given strategy each month to obtain strategy indexes and perform GLM unsmoothing (as described in the previous paragraph) for each strategy index

[20] https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
https://faculty.fuqua.duke.edu/~dah7/HFRFData.htm

[21] It is common in the literature to fix $H = 2$. We allow for H from 0 to 3 to account for heterogeneity across funds, but Internet Appendix A reports results (similar to our baseline analysis) in which we fix $H = 2$.

separately to recover unsmoothed strategy-level returns.[22] Second, we obtain unsmoothed relative returns from Equation 8 (an MA process for observed relative returns with aggregate unsmoothed returns as covariates) also relying on AIC to decide how many MA lags of relative returns to include. Third, we sum the unsmoothed strategy returns with each fund unsmoothed excess return to obtain fund-level economic returns.

For our baseline empirical analysis of hedge funds, we follow the hedge fund literature and rely on regular returns (as opposed to log returns as in our economic framework of Subsection 1.5). However, Internet Appendix A shows that our results are consistent if rely on log returns instead.

2.2 Results by Liquidity Group

This subsection demonstrates that our 3-step unsmoothing method improves the measurement of risk-adjusted performance relative to traditional (or 1-step) unsmoothing.

Since unsmoothing methods are designed to affect only the returns of illiquid funds (i.e., funds with significant return autocorrelation), our analysis classifies funds in groups based on liquidity. We sort strategies based on their first order autocorrelation coefficient to form three groups: low liquidity strategies (the three strategies with autocorrelation above 0.40), high liquidity strategies (the two strategies with autocorrelation below 0.10), and mid liquidity strategies (the other five strategies).[23] We then measure fund-level information within each group and report averages.

[22]We rely on equal-weights (instead of value-weights) to construct the strategy indexes because this approach is more consistent with our derivations of the 3-step unsmoothing method (which relies on time-invariant weights). However, Internet Appendix A provides results (similar to our baseline analysis) using value-weights to construct strategy-level returns.

[23]The liquidity ranking obtained from this approach (which is based on the first column of Table 3) is consistent with economic logic. The most illiquid strategy is the relative value strategy, which contains funds that attempt to profit from mispricing across securities. If capital markets work well, hedge funds are unlikely to find mispricing opportunities among the pool of liquid securities, and thus tend to invest in relatively illiquid assets. At the other extreme, CTAs represent the most liquid hedge funds as their underlying strategies tend to be based on trend-following and are usually executed using futures contracts, which are marked to market daily.

(a) Ex-post Performance

A fundamental question in the literature is whether hedge funds are able to generate positive risk-adjusted performance. This question naturally leads to an ex-post analysis of hedge fund performance, so we start by exploring how unsmoothing returns affects the ex-post measurement of risk-adjusted performance.

The basic problem of smoothed returns is that they understate risk (Getmansky, Lo, and Makarov (2004)), even though they do not affect average (i.e., risk unadjusted) performance. As such, unsmoothing methods are designed to increase return volatility without affecting average returns. Figure 3(a) shows, for the three liquidity groups, the average (annualized) volatility based on (i) reported returns; (ii) 1-step unsmoothed returns; and (iii) 3-step unsmoothed returns.[24] For the low and mid liquidity strategies, average volatility strongly increases after unsmoothing. For instance, the average fund volatility in the low liquidity strategies increases by 27.7% (from 9.0% to 11.5%) as we unsmooth returns. In contrast, there is almost no change in average volatility as we unsmooth returns of funds in the high liquidity strategies, which shows that unsmoothing methods work well as they should not strongly affect the returns of funds that invest in liquid assets. Comparing the 3-step method with 1-step method, we see little increase in average volatility. For instance, after the 3-step unsmoothing, the average volatility of funds in the low liquidity strategies increases by only 1.1% (from 11.5% to 11.6%). It is not surprising that the 3-step unsmoothing method has a very small effect on fund volatilities beyond 1-step unsmoothing. The 3-step approach is designed to better unsmooth the systematic portion of returns, not to increase the "unsmoothing strength." As such, the improved risk measurement provided by the 3-step method (detailed below) is not due to increased return volatility.

Figure 3(b) gauges the implications of the volatility increase after return unsmoothing to (annualized) Sharpe ratios.[25] Sharpe ratios decline for the low and mid liquidity groups

[24] We "annualize" volatilities by multiplying them by $\sqrt{12}$ (which, strictly speaking, is the correct annualization factor only for returns that are not autocorrelated). Multiplying by this fixed constant does not affect any of the relative comparisons between smoothing methods.

[25] For Sharpe ratios, we report cross-fund medians as oppose to averages since the Sharpe ratios of funds

with the effect being particularly pronounced for the low liquidity group. As with volatility, the 3-step method does not change Sharpe ratio metrics materially compared to the 1-step method.

Figure 3(c) explores systematic risk by focusing on average R^2s based on the FH 8-Factor model, which effectively captures how much of fund-level return variability is explained by the risk factors most commonly used in the hedge fund literature. 1-step MA unsmoothing has basically no effect on R^2s (if anything, R^2s decrease), which indicates that even though 1-step unsmoothing increases volatility relative to reported returns, it does not increase the fraction of volatility explained by standard risk factors. In stark contrast, 3-step MA unsmoothing substantially increases R^2s for funds in the low and mid liquidity strategies. For instance, after 3-step unsmoothing, the average R^2 of funds in the low liquidity strategies increases by 15.6% (from 34.3% to 39.6%) relative to reported returns and by 21.6% (from 32.6% to 39.6%) relative to 1-step unsmoothing.

These R^2 patterns suggest that the 3-step MA unsmoothing method allows us to better uncover the true systematic risk exposure of hedge funds, which is usually partially concealed because observed returns are smoothed. Given this R^2 result, we should be able to better measure risk-adjusted performance using our 3-step MA unsmoothing method. Figure 3(d) explores this issue by reporting average (annualized) αs for the three liquidity groups. The 3-step unsmoothing strongly decreases αs for the low and mid liquidity strategies relative to the αs obtained with observed returns or after 1-step unsmoothing. For both groups, average αs decrease by close to 1 percentage point (from 3.1% to 2.0% for the low liquidity group and from 0.9% to -0.2% for the mid liquidity group) relative to observed returns, which is about twice as large as the improvement provided by the 1-step unsmoothing method.

Figures 3(e) and 3(f) gauge how unsmoothing affects the statistical significance of fund-level αs. Overall, there is a strong decline in average t^{α}_{stat} as well as on the percentage of significant αs for the low liquidity group. The mid liquidity group still displays an effect, but

with negative average excess returns increase as volatility increases. Nevertheless, results based on cross-fund average Sharpe ratios are similar.

much weaker since αs are not (on average) very significant in the first place.

Overall, the results indicate that volatility strongly increases after unsmoothing and the fraction of volatility due to systematic risk only increases when the 3-step MA unsmoothing process is used. Moreover, unsmoothing returns decreases α, and this effect is stronger when the 3-step method is used. Finally, all of these results are only present when evaluating relatively illiquid funds.

(b) Performance Persistence

In some applications, researchers and investors are interested in identifying funds with superior performance ex-ante. To explore this issue, we follow the literature by sorting hedge funds into portfolios using past estimated αs and study the αs of such portfolios in the subsequent months. Specifically, following Bollen, Joenväärä, and Kauppila (2020), we form quintile portfolios within each liquidity group according to the t-stat of their FH 8-Factor αs calculated using a 24-month rolling window. The portfolios are formed anew at the beginning of each year and are initially equal-weighted, but not rebalanced within the year so that the portfolio weights evolve according to the realized returns of the underlying funds. We then concatenate the returns of each quintile portfolio across the sample to construct a full time-series of portfolio returns, and unsmooth these portfolio returns using the GLM method before calculating portfolio αs.

Bollen, Joenväärä, and Kauppila (2020) form their portfolios by sorting on the t-stat of αs based on 1-step unsmoothed returns. We are interested in studying the extent to which unsmoothing improves the ability of past α estimates to predict future αs. Therefore, we repeat the sorting procedure using αs estimated using three different versions of past returns: observed returns, 1-step unsmoothed returns, and 3-step unsmoothed returns. Importantly, we unsmooth returns using data available only up to the month of portfolio formation to ensure that the portfolios could have been formed in real time. In the first few years of the sample, some fund strategies include only a few funds and therefore the estimated MA parameters would be unstable. For this reason, we require at least 6 years of data to unsmooth returns before forming the portfolios. That is, the first set of portfolios is formed at the end of December 2000, using data from January 1995 to December 2000 to unsmooth returns.

Figure 4 summarizes the results from our portfolio exercise. Figure 4(a) shows that sorting on αs obtained based on 3-step unsmoothed returns (as opposed to using observed returns or 1-step unsmoothed returns) produces quintile portfolios that have a larger spread in ex-ante αs, a result that holds only for the low and mid liquidity groups. For instance, in the low liquidity group, the α spread increases from 3.4% (3.6%) to 4.5% as we move from observed returns (1-step unsmoothed returns) to 3-step unsmoothed returns. The statistical significance of the spread also becomes stronger, with $t^{\alpha}_{stat} = 2.89$ when αs based on 3-step unsmoothed returns are used in the construction of the portfolios (in contrast to $t^{\alpha}_{stat} = 2.38$ and $t^{\alpha}_{stat} = 2.24$ when αs are based on observed and 1-step unsmoothed returns, respectively).

Since we cannot easily short a hedge fund portfolio, the α spreads analysed in the previous paragraph do not reflect αs that can be achieved in the market. While our point is simply that αs estimated using 3-step unsmoothed returns better predict future αs, and thus the tradability of the strategy is not relevant, Figure 4(b) also provides the αs of the highest past α quintiles, which reflect only the long positions on the strategies analysed in the previous paragraph. The results are largely similar, with αs estimated using 3-step unsmoothed returns providing a better signal for future α, and thus allowing researchers and investors to identify hedge funds that, ex-ante, have higher αs.

Overall, the results indicate 3-step unsmoothing helps not only to measure risk-adjusted performance ex-post, but also ex-ante. Moreover, this finding helps validate our ex-post α estimates. Specifically, our ex-ante α results should mitigate concerns that our ex-post estimated αs are somehow mechanically lower whereas the true αs are the ones obtained based on 1-step unsmoothed returns.

2.3 Results by Hedge Fund Strategy

The previous results suggest that our 3-step MA unsmoothing method improves the risk-adjusted performance measurement relative to the 1-step method. To better understand what risks (betas) are better measured, it is useful to analyze groups of funds that engage in similar activity, and thus are exposed to similar risks. As such, this subsection reports results by hedge fund strategy.

(a) Volatilities, Sharpe Ratios, R^2s, and Alphas

Figure 5 shows several average statistics by hedge fund strategy. In our description, we refer to "illiquid strategies" as the strategies with autocorrelation coefficient higher than 0.20 (these include all strategies in high and mid liquidity strategies of the previous section, except for market neutral, which tends to display results more consistent with the high liquidity group).

Figure 5(a) demonstrates that, for each of the illiquid strategies, volatility increases after unsmoothing, but it also shows 3-step unsmoothing has little effect beyond 1-step unsmoothing. Figure 5(b) shows how the volatility increase affects (annualized) Sharpe Ratios, which makes it clear that the effect is much larger for the more illiquid strategies. Figure 5(c) plots R^2s relative to the FH 8-Factor model and shows that the results observed in the low and mid liquidity groups are present for each of the illiquid strategies separately (with basically no effect on liquid strategies). That is, R^2s do not increase as we 1-step unsmooth returns (if anything, they decrease), but they strongly increase after our 3-step unsmoothing. Figure 5(d) shows that the average α declines in every illiquid strategy and Figures 5(e) and 5(f) make it clear that the statistical decline (i.e., decline in average t^{α}_{stat} and in the percentage of funds with significant α) is much larger for the more illiquid strategies.

Figure 6 reports the main statistics analyzed in Figure 5, but focuses on the economic and statistical changes to these metrics as analyzed returns move from (i) observed returns to 1-step unsmoothed returns and (ii) 1-step unsmoothed returns to 3-step unsmoothed returns. The t_{stat} for each average change is provided on the top of the respective bar. Figures 5(a), 5(b), and 5(c) reinforce the inferences from Figure 5 and add that the changes tend to be statistically significant as well. Figure 5(c) also emphasizes that 1-step unsmoothing significantly changes the measurement of risk-adjusted performance (i.e., decreases αs) relative to reported returns, with the change obtained from moving from 1-step unsmoothing to

3-step unsmoothing being comparable to (sometimes even larger than) the change obtained by unsmoothing through the 1-step process. This result suggests that moving from the 1-step to 3-step unsmoothing is at least as economically important as unsmoothing returns in the first place.

Overall, the results when separating funds into three liquidity groups are largely present for individual strategies as well.

(b) Risk Exposures

The previous results indicate that the FH 8-Factor model explains a significantly higher fraction of the volatility of hedge funds than suggested by looking at observed returns (or at 1-step unsmoothed returns). Below, we ask how much each risk factor contributes to the improvement.

In a factor model with two risk factors, $R_t = \alpha + \beta_1 \cdot f_{1,t} + \beta_2 \cdot f_{2,t} + \epsilon_t$, R^2 can be decomposed as (the decomposition is analogous for an arbitrary number of risk factors):

$$\begin{aligned} R^2 &= Var(\alpha + \beta_1 \cdot f_{1,t} + \beta_2 \cdot f_{2,t})/Var(R_t) \\ &= Cov(\alpha + \beta_1 \cdot f_{1,t} + \beta_2 \cdot f_{2,t}, R_t)/Var(R_t) \\ &= \underbrace{\beta_1 \cdot \frac{Cov(f_{1,t}, R_t)}{Var(R_t)}}_{R_1^2} + \underbrace{\beta_2 \cdot \frac{Cov(f_{2,t}, R_t)}{Var(R_t)}}_{R_2^2} \end{aligned} \tag{18}$$

where the second equality follows from the projection orthogonality condition, $Cov(f_{1,t}, \epsilon_t) = Cov(f_{2,t}, \epsilon_t) = 0$, and R_i^2 represents the R^2 portion due to risk factor i.

Figure 7 reports the average R^2 due to each of the risk factors in the FH 8-Factor model for each hedge fund strategy. For all illiquid strategies except the emerging market strategy, the results indicate market risk and emerging market risk are significantly more important in explaining returns after 3-step unsmoothing. For instance, market risk accounts for about 16% of the volatility of Event Driven funds after 3-step unsmoothing (an increase of 57.5% relative to the importance of market risk when we look at observed returns). For Emerging Market funds, the only risk factor that displays a substantial increase after 3-step unsmoothing is the emerging market risk factor itself, which is consistent with economic intuition. Similarly, the exposure of Event Driven funds to the size factor increases substantially after 3-step unsmoothing, which is also in line with economic intuition as these funds are often focused on relatively small and illiquid firms. For liquid funds, there is no change in the importance of different risk factors after unsmoothing returns.

Overall, the results indicate that most of the improvement coming from the 3-step unsmoothing method stems from better measuring exposures to market risk and emerging market risk in the underlying illiquid assets held by hedge funds.

3 Unsmoothing Returns of Commercial Real Estate Funds

The previous two sections introduce our 3-step MA unsmoothing method and demonstrate that it provides a substantial improvement relative to 1-step MA unsmoothing in the context of hedge funds. While unsmoothing hedge fund returns is a natural application of our 3-step unsmoothing process, unsmoothing is even more important for CRE funds, which are highly illiquid given the appraisal nature of real estate valuation. As such, this section demonstrates how we can extract economic returns for CRE funds using a 3-step version of the AR unsmoothing framework proposed in Geltner (1991, 1993), which is more commonly applied in the real estate literature. Subsection 3.1 outlines the 1-step AR unsmoothing method and extends our 3-step process to improve upon it; and Subsection 3.2 applies our 3-step AR unsmoothing to CRE fund returns.

3.1 Autoregressive Return Unsmoothing Framework

The baseline unsmoothing framework for real estate assets comes from Geltner (1991, 1993) and is often referred to in the literature as AR unsmoothing since it implies observed returns follow an autoregressive process.

Geltner (1991, 1993) assume the observed return of fund j at time t is given by (see original paper for the economic motivation):

$$R^o_{j,t} = \theta_j^{(0)} \cdot R_{j,t} \; + \; \Sigma_{h=1}^{H} \, \theta_j^{(h)} \cdot R^o_{j,t-h} \tag{19}$$

$$= \mu_j \; + \; \Sigma_{h=1}^{H} \, \theta_j^{(h)} \cdot (R^o_{t-h} - \mu_j) \; + \; \theta_j^{(0)} \cdot \eta_{j,t} \tag{20}$$

where θs capture the level of "staleness" in observed returns with $\Sigma_{h=0}^{H} \theta_j^{(h)} = 1$, and the second equality follows from $R_{j,t} = \mu_j + \eta_{t,j}$ with $\eta_{j,t} \sim IID$.[26]

The first equality represents the economic assumption that prices are only partially updated so that observed returns partially reflect the economic returns of the reporting period as well as observed returns of the H previous periods. The second equality is the econometric implication that, under the given assumption, the observed fund returns follow an AR(H) process.

Given Equation 20, we can recover economic returns by estimating an AR(H) process for $R^o_{j,t}$, extracting the estimated residuals, $\epsilon_{j,t} = \theta_j^{(0)} \cdot \eta_{j,t}$, and using them to obtain economic returns, $R_{j,t} = \mu_j + \epsilon_{j,t}/(1 - \Sigma_{h=1}^{H} \theta_j^{(h)})$. There are many methods to estimate AR(H) processes, with Ordinary Least Squares (OLS) being the simplest consistent estimator, and thus the method we use. This procedure is used by several papers in the literature to unsmooth the returns of real estate assets and funds (e.g., Barkham and Geltner (1995), Corgel et al. (1999), Fisher et al. (2003), Fisher and Geltner (2000), Fisher, Geltner, and Webb (1994), Pagliari Jr, Scherer, and Monopoli (2005), and Rehring (2012)).

However, as we empirically demonstrate in the next subsection, this AR unsmoothing method faces the same aggregation issue observed with the MA unsmoothing. As such, analogously to the MA case, we generalize the assumption in Equation 19 so that aggregate economic returns are directly included in the return smoothing process. Specifically, we assume:[27]

[26]Note also that, under invertibility, Equation 20 implies an MA(∞) representation with coefficients (i.e., weights) summing to one.

[27]This smoothing process reduces to Equation 20 (the smoothing process in Geltner (1991, 1993)) if we set $\pi_j^{(h)} = \phi_j^{(h)} = \theta_j^{(h)}$.

$$
\begin{aligned}
R^o_{j,t} &= \phi_j^{(0)} \cdot \widetilde{R}_{j,t} \;+\; \Sigma_{h=1}^H\, \phi_j^{(h)} \cdot \widetilde{R}^o_{j,t-1} \;+\; \pi_j^{(0)} \cdot \overline{R}_t \;+\; \Sigma_{h=1}^H\, \pi_j^{(h)} \cdot \overline{R}^o_{j,t-1} && (21)\\
&= \mu_j \;+\; \Sigma_{h=1}^H\, \phi_j^{(h)} \cdot (\widetilde{R}^o_{j,t-1} - \widetilde{\mu}_j) \;+\; \Sigma_{h=1}^H\, \pi_j^{(h)} \cdot (\overline{R}^o_{t-1} - \overline{\mu}) \;+\; \phi_j^{(0)} \cdot \widetilde{\eta}_{j,t} \;+\; \pi_j^{(0)} \cdot \overline{\eta}_t\\
&= \mu_j \;+\; \Sigma_{h=1}^H\, \phi_j^{(h)} \cdot (\widetilde{R}^o_{j,t-1} - \widetilde{\mu}_j) \;+\; \Sigma_{h=1}^H\, \pi_j^{(h)} \cdot (\overline{R}^o_{t-1} - \overline{\mu}) \;+\; \epsilon_{j,t} && (22)
\end{aligned}
$$

where $\Sigma_{h=0}^H \phi_j^{(h)} = \Sigma_{h=0}^H \pi_j^{(h)} = 1$, the second equality follows from $R_{j,t} = \mu_j + \eta_{t,j}$ with $\eta_{j,t} \sim IID$, and the third equality defines $\epsilon_{j,t} = \phi_j^{(0)} \cdot \widetilde{\eta}_{j,t} \;+\; \pi_j^{(0)} \cdot \overline{\eta}_t$.

Since the covariates in Equation 22 are observable (in contrast to the MA(H) process), we can directly estimate Equation 22 (by OLS) and obtain coefficient estimates, $\phi_j^{(h)}$ and $\pi_j^{(h)}$ (including $\phi_j^{(0)} = 1 - \Sigma_{h=1}^H \phi_j^{(h)}$ and $\pi_j^{(0)} = 1 - \Sigma_{h=1}^H \pi_j^{(h)}$), as well as residual estimates, $\epsilon_{j,t}$. The challenge is that $\epsilon_{j,t}$ reflects both $\widetilde{\eta}_{j,t}$ and $\overline{\eta}_t$. We rely on an aggregation step to separate the two components. Specifically, aggregating $\epsilon_{j,t}$ yields:

$$
\begin{aligned}
\bar{\epsilon}_t &= \overline{\pi}^{(0)} \cdot \overline{\eta}_t \;+\; \widehat{Cov}(\phi_j^{(0)}, \widetilde{\eta}_{j,t})\\
&\approx \overline{\pi}^{(0)} \cdot \overline{\eta}_t && (23)
\end{aligned}
$$

Similar to the MA(H) case, this framework provides a simple way to recover aggregate and fund-level economic returns in an internally consistent way given the estimates for $\phi_j^{(h)}$, $\pi_j^{(h)}$, and $\epsilon_{j,t}$ obtained from Equation 22. First, we obtain aggregate economic returns from $\overline{R}_t = \overline{\mu} + \overline{\eta}_t$ where $\overline{\eta}_t = \bar{\epsilon}_t / \overline{\pi}^{(0)}$ with $\bar{\epsilon}_t = \Sigma_{j=1}^J w_j \cdot \epsilon_{j,t}$ and $\overline{\pi}^{(0)} = \Sigma_{j=1}^J w_j \cdot \pi_j^{(0)}$. Second, we obtain fund-level economic relative returns from $\widetilde{R}_{j,t} = \widetilde{\mu}_j + \widetilde{\eta}_{j,t}$ where $\widetilde{\eta}_{j,t} = (\epsilon_{j,t} - \overline{\pi}^{(0)} \cdot \overline{\eta}_t)/\phi_j^{(0)}$. Third, we recover fund-level economic returns from $R_{t,j} = \overline{R}_t + \widetilde{R}_{j,t} = \mu_j + \overline{\eta}_t + \widetilde{\eta}_{j,t}$. This procedure summarizes our AR 3-step unsmoothing method.

Similarly to the MA(H) case, our 3-step AR unsmoothing procedure can be seen as a generalization of Geltner (1991, 1993) that allows aggregate and excess economic returns to have different effects on observed fund-level returns ($\pi_j \neq \phi_j$), but that would be identical (up to sampling variation) to the 1-step AR unsmoothing if the underlying assumption in Geltner (1991, 1993) ($\pi_j = \phi_j$) was empirically valid.

In our empirical analysis of real estate funds, we unsmooth log observed returns instead of regular observed returns and then transform the unsmoothed log return into unsmoothed regular returns to calculate all reported statistics. This approach is consistent with the 1-step unsmoothing framework (derived from a model of appraisal valuation) in Geltner (1991, 1993). However, as demonstrated in Internet Appendix A, the overall results are similar whether we log returns or not.

3.2 Unsmoothing Returns of Commercial Real Estate Funds

(a) Dataset of CRE Funds

CRE covers all real estate product types other than single-family homes and is the primary way institutional investors invest in real estate. While CRE has historically been a significant sector in the overall economy, its importance as an investment asset class has grown dramatically over the last 35 years. The average target allocation for institutional investors has grown from around 2% in the early 1980s to between 10% and 12% in 2019 (PREA (2019)).

There are a number of ways institutional investors can invest in CRE - direct investments, separate accounts, joint ventures, club deals, commingled funds, and publicly traded REITs. Our analysis focuses on US private CRE funds, which are a subset of commingled funds. Our CRE fund dataset comes from the National Council of Real Estate Investment Fiduciaries (NCREIF), which is the leading collector of institutional real estate investment information for properties within the US. We have quarterly data and the sample period goes from Q1 1994 through Q4 2017, with the starting date selected because there are few funds available before 1994 (starting in 1994 also makes the CRE analysis period roughly consistent with the hedge fund analysis).

Our sample includes all CRE funds that report return data to NCREIF and have at least 36 quarterly observations.[28] The sample consists of 66 funds that are observed, on average,

[28] We require at least 36 observations to be consistent with the standard used in the Hedge Fund literature and to assure the smoothing process is estimated with some precision. However, including all funds available in the dataset regardless of the number of observations yields similar results to the ones we report.

for 56 quarters within the 96 quarters studied. As of Q4 2017, the sample comprises 37 funds with approximately $233 billion in assets under management.

Our final dataset is composed of 29 open-end funds and 37 closed-end funds.[29] Open-end CRE funds are similar to mutual funds and some hedge funds in the sense that they are open to issuing and redeeming shares on a regular basis (quarterly) at stated Net Asset Values (NAVs). In contrast, investors in closed-end CRE funds typically only have their positions liquidated as the fund sells its underlying assets and returns capital. Besides asset sales, investors are primarily rewarded through cash distributions. In both types of funds, NAVs are based on the cumulative appraised values of the individual assets they hold, and thus NAV-based (i.e., observed) returns reflect (highly) smoothed returns. Therefore, these CRE funds provide a natural asset class to explore the effects of our 3-step AR unsmoothing method.

There is no consensus on the appropriate factor model to measure risk exposures of real estate funds. As such, our analysis relies on two simple factor models. The first has only one risk factor: returns (in excess of the risk-free rate) on an index capturing the real estate public market. The second factor model includes the same real estate factor, but adds returns (in excess of the risk-free rate) on an index capturing the public equity market.[30] Despite this simplified approach, we show that these factor models drive the average α of CRE funds to (close to) zero after 3-step unsmoothing.[31]

[29] Our results are similar if we focus on the 29 open-end CRE funds in our dataset (out of the 66 CRE funds available), as demonstrated in Internet Appendix A.

[30] For excess returns on the equity market, we use the market risk factor in Kenneth French's data library (https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). For excess returns on the public real estate market, we use excess returns on the FTSE NAREIT All Equity REIT Index available on the website of the National Association of REITs (https://www.reit.com/). We compound the monthly returns on both indexes to obtain quarterly returns and subtract the one-month Treasury bill rate compounded over each quarter to get excess returns.

[31] Some CRE funds also invest in mortgage-related assets. We explore a third model that adds to our equity and real estate factors a mortgage factor based on public REITs. The results are very similar to the ones we report, and thus are omitted for brevity.

(b) Autocorrelations of CRE Fund Returns

Table 5 provides average autocorrelations for CRE fund returns as well as autocorrelations for the aggregate (equal-weighted average) of all CRE fund returns. Confirming the intuition that the appraisal nature of real estate valuation induces (highly) smoothed returns, we find that the average 1-quarter autocorrelation of observed returns is 0.45, with 63.6% of the funds displaying a statistically significant autocorrelation. In fact, returns are so persistent that the average autocorrelation is still 0.21 at 4-quarters (and statistically significant for 40.9% of the funds). Returns are even more autocorrelated at the aggregate level, with the aggregate CRE returns displaying a 1-quarter autocorrelation of 0.75 (with p-value=0.0%) and a 4-quarter autocorrelation of 0.31 (with p-value=0.3%).

The fund-level partial autocorrelations indicate the return autocorrelation structure of most funds is well described by an AR structure with one or two lags. As such, we apply AR unsmoothing to these CRE funds using an AR(2) model, which nests the AR(1) structure.

After 1-step AR unsmoothing, the return autocorrelations at the fund level mostly disappear. However, the 1-quarter autocorrelation for the aggregate series remains extremely strong (at 0.46 with p-value=0.0%) and even the 2-quarter autocorrelation remains high (at 0.31 with p-value=0.3%). This result indicates the 1-step AR unsmoothing does not fully unsmooth the systematic component of CRE fund-level returns.

In contrast, 3-step AR unsmoothed returns display little autocorrelation both at the fund-level and aggregate-level, with the highest autocorrelation being the aggregate 2-lags autocorrelation of 0.24 (p-value=2.3%). This result suggests the 3-step AR unsmoothing goes a long way in unsmoothing the systematic component of CRE fund-level returns.

(c) Performance of CRE Funds after 1-step and 3-step AR Unsmoothing

The upper panel of Table 6 reports CRE fund statistics based on observed return, 1-step unsmoothed returns, and 3-step unsmoothed returns. Annualized expected returns are 5.0% and, by construction, do not change as we unsmooth returns, while (annualized) volatility starts at 13.1% for observed returns, increases to 25.3% after 1-step unsmoothing, and remains stable at 24.1% after 3-step unsmoothing. Interestingly, in the case of CRE funds, R^2 increases (from 3.2% to 7.9% in the 1-factor model and from 4.6% to 9.9% in the 2-factor model) as we 1-step unsmooth returns. R^2 increases even further (from 7.9% to 13.7% in the 1-factor model and from 9.9% to 16.0% in the 2-factor model) as we move from 1-step to 3-step unsmoothing. These results indicate that unsmoothing has the potential to largely affect risk measurement and, consequently, estimated risk-adjusted performance.

Analyzing the performance relative to the 2-factor model (results for the 1-factor model are similar), CRE funds seem to provide a substantial average α of 4.0% per year with roughly half the funds displaying statistically significant α. This result is a consequence of the extremely low average exposure to the real estate ($\beta_{re} = 0.02$) and equity ($\beta_e = 0.10$) public markets. After 1-step unsmoothing returns, the average exposures to the real estate ($\beta_{re} = 0.15$) and equity ($\beta_e = 0.15$) markets increase, driving the average α down to 2.4%, with 13.6% of the funds displaying statistically significant α. Average risk exposures increase even further after 3-step unsmoothing ($\beta_{re} = 0.21$ and $\beta_e = 0.26$) so that the average α becomes 0.8% and statistically insignificant (or significantly negative) for 90.9% of the funds.

The lower panel of Table 6 reports the same results as the upper panel, but focuses on how βs and α change as we unsmooth returns. The key message is that the increase in βs and decline in α obtained by 1-step unsmoothing returns is about the same as the improvement obtained when moving from 1-step to 3-step unsmoothing. The t_{stat} values in brackets also show that the average changes are highly significant from a statistical perspective.

Overall, the results indicate that the 3-step AR unsmoothing method provides a substantial improvement over 1-step AR unsmoothing in terms of measuring risk exposure and risk-adjusted performance of CRE funds. The economic gains obtained by moving from 1-step to 3-step unsmoothing are roughly similar to the gains of unsmoothing in the first place.

4 Conclusion

In this paper, we find that traditional return unsmoothing methods used to recover economic return estimates from observed returns of illiquid assets do not fully unsmooth the systematic component of the returns, and thus understate systematic risk exposures and overstate risk-adjusted performance. To address this issue, we provide a novel 3-step return unsmoothing method and apply it to hedge funds and CRE funds.

In doing so, we find that the measurement of risk exposures and risk-adjusted performance substantially improves. Overall, the improvement in risk adjusted performance is stronger for more illiquid funds and the increase in the estimated risk exposures is particularly strong when we evaluate CRE funds, which invest in highly illiquid assets.

Our results demonstrate that it is economically important to properly unsmoothing the returns of illiquid assets before measuring risk exposures. They also raise the possibility that some previously estimated alphas of funds that invest in illiquid assets are partially due to mismeasured systematic risk. We provide initial evidence consistent with this argument in the context of hedge funds and CRE funds and leave further explorations in this direction to future research.

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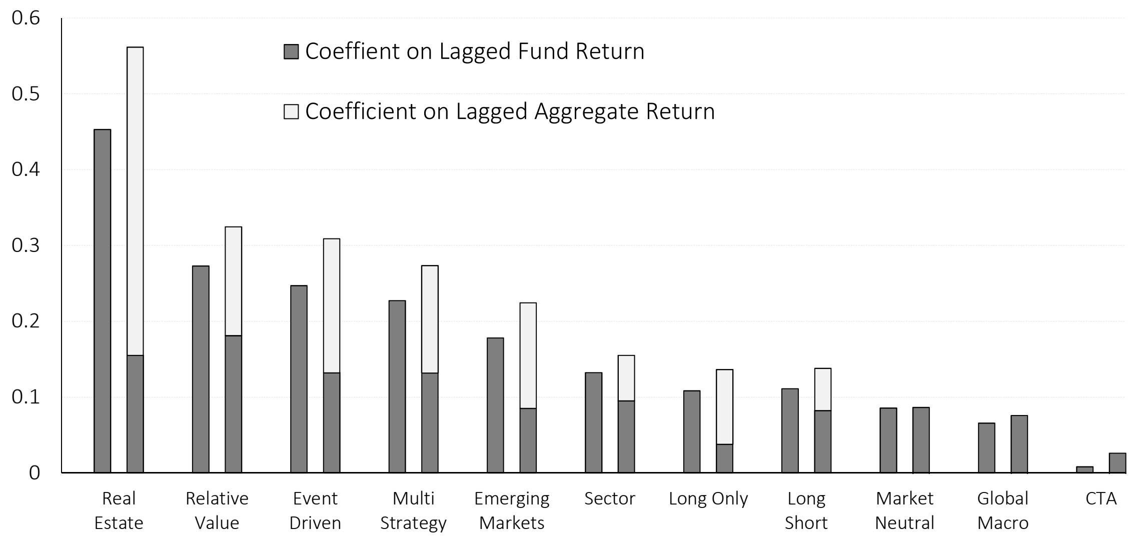
(a) **Regressing R^o_{jt} on R^o_{jt-1} and $\overline{R}^o_{t-1}$**



(b) **R^2 from Regressing R^o_{jt} on $\overline{R}^o_t$**

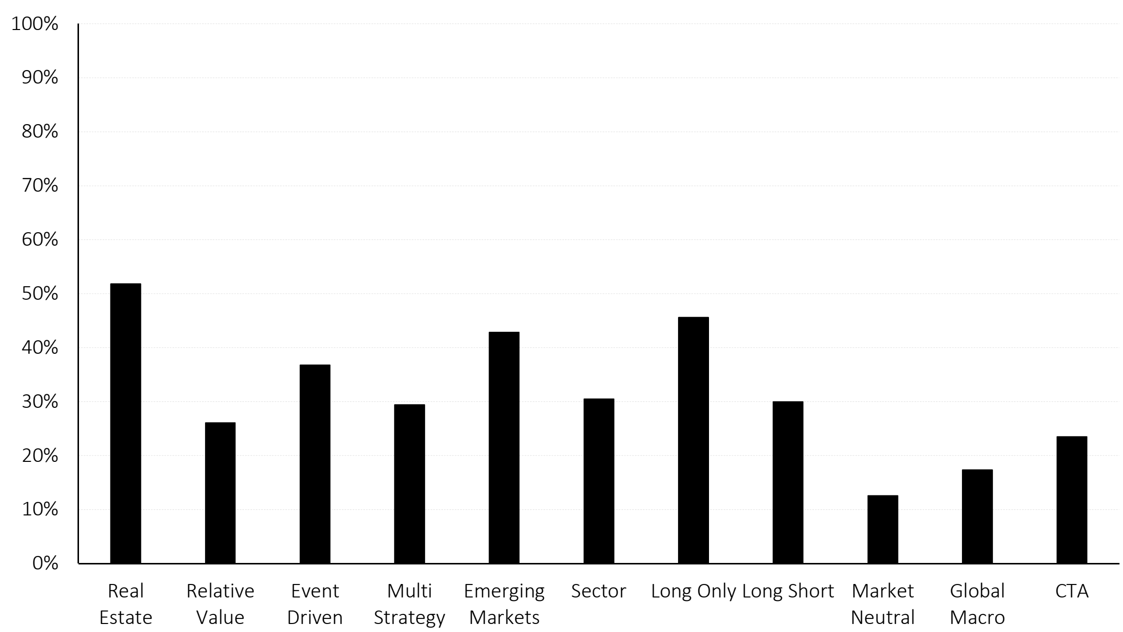


Figure 2
Checking Conditions for 1-Step Method

Panel (a) plots coefficients from regressing (observed) fund returns on lagged (observed) fund and aggregate returns. The first bar for each fund category is based on a univariate regression while the second bar relies on a bivariate regression. Observed returns are normalized to have a unit standard deviation so that the coefficients are comparable. Panel (b) reports the R^2 from regressing (observed) fund returns on (observed) aggregate fund returns. The first category in each figure is based on Commercial Real Estate funds (sample from 1994 through 2017) while the other categories reflect different hedge fund strategies (sample from January 1995 to December 2017). See Subsections 2.1 and 3.2 for further empirical details.

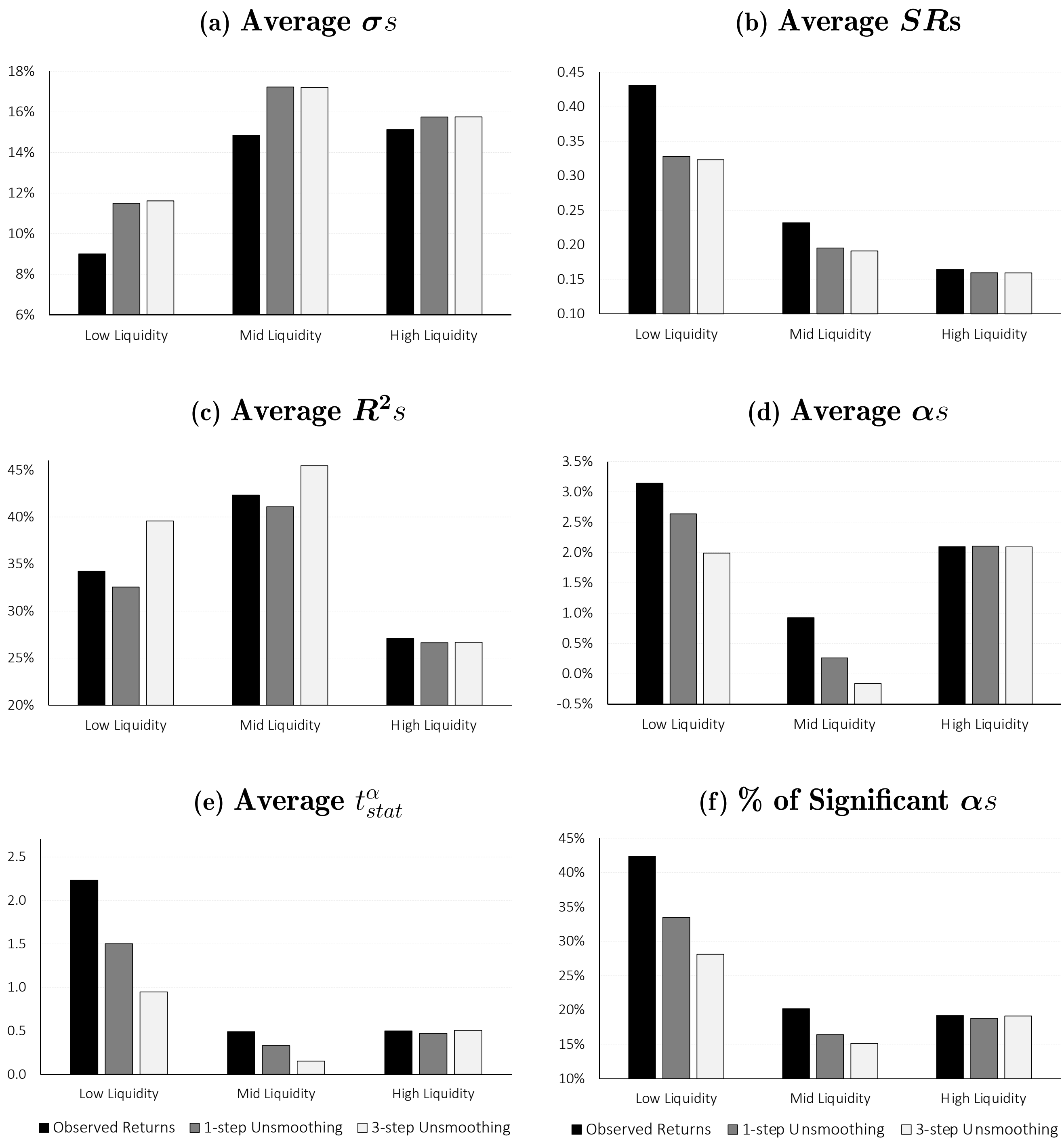


Figure 3
Hedge Fund Risk and Performance by Strategy Liquidity

The figure plots average fund-level results for three groups based on hedge fund strategy liquidity using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. We sort strategies based on their first order autocorrelation coefficient to form three groups: low liquidity strategies (the three strategies with autocorrelation above 0.40), high liquidity strategies (the two strategies with autocorrelation below 0.10), and mid liquidity strategies (the other five strategies). R^2s and αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001) and statistical significance for fund-level αs is at 10%. The sample goes from January 1995 to December 2017. See Section 1 for unsmoothing methods and Subsection 2.1 for further empirical details.

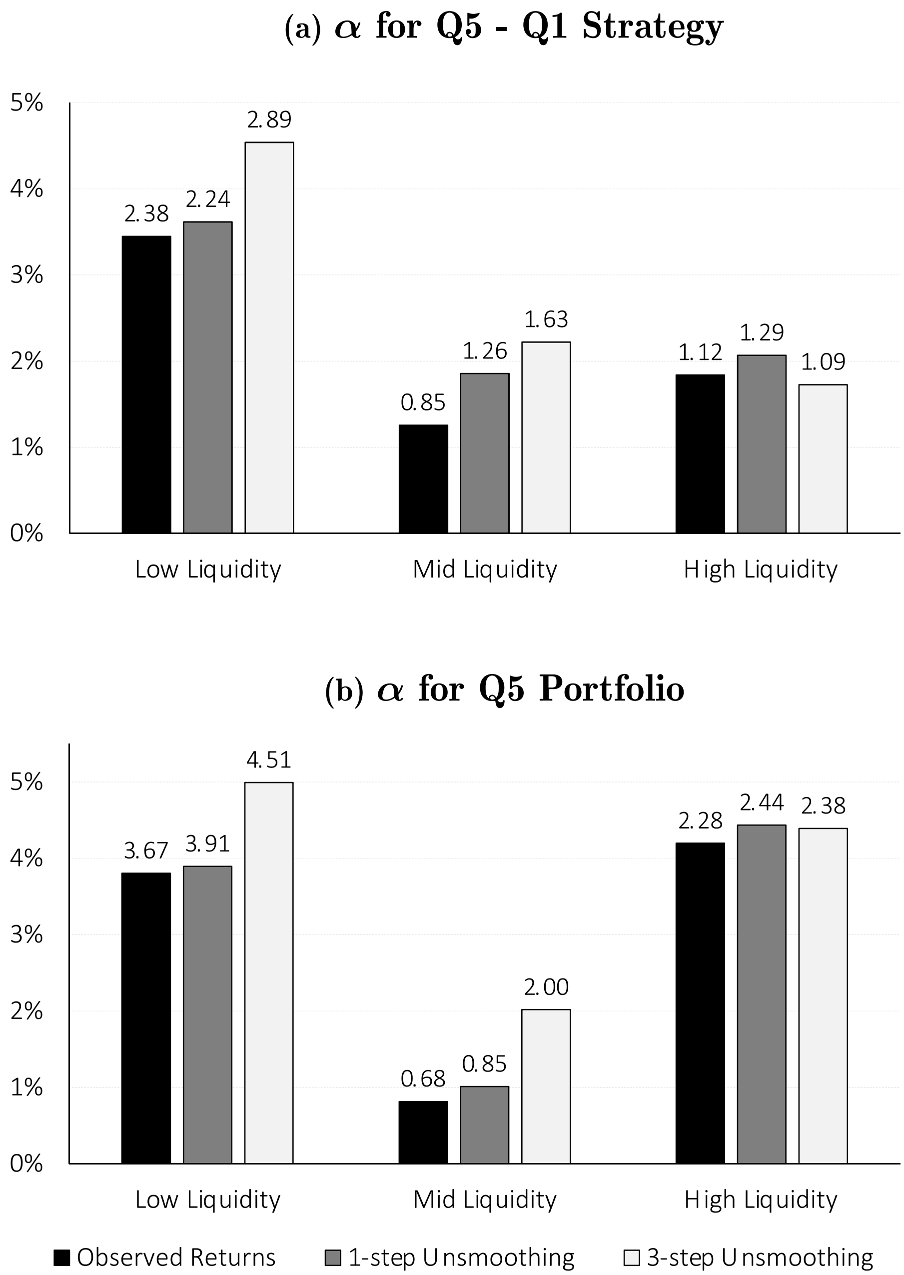


Figure 4

αs of Hedge Fund Quintiles Portfolios formed Based on the t-stat of Past αs

The figure plots αs (with their t_{stat} on the top of each bar) of quintile portfolios formed by sorting hedge funds based on the t-stat of their past αs (on a 24-month rolling window) measured using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. Panel (a) focuses on αs for a strategy that buys the highest and sells the lowest past α quintiles. Panel (b) focuses only on the highest past α quintiles. The portfolios are constructed separately using each of our three liquidity groups: low liquidity strategies (the three strategies with autocorrelation above 0.40), high liquidity strategies (the two strategies with autocorrelation below 0.10), and mid liquidity strategies (the other five strategies). αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001). The sample goes from January 1995 to December 2017, but the first portfolio formation is on December 2000 so that we have at least six years of data to unsmooth the hedge fund returns. See Section 1 for unsmoothing methods and Subsections 2.1 and 2.2 for further empirical details.

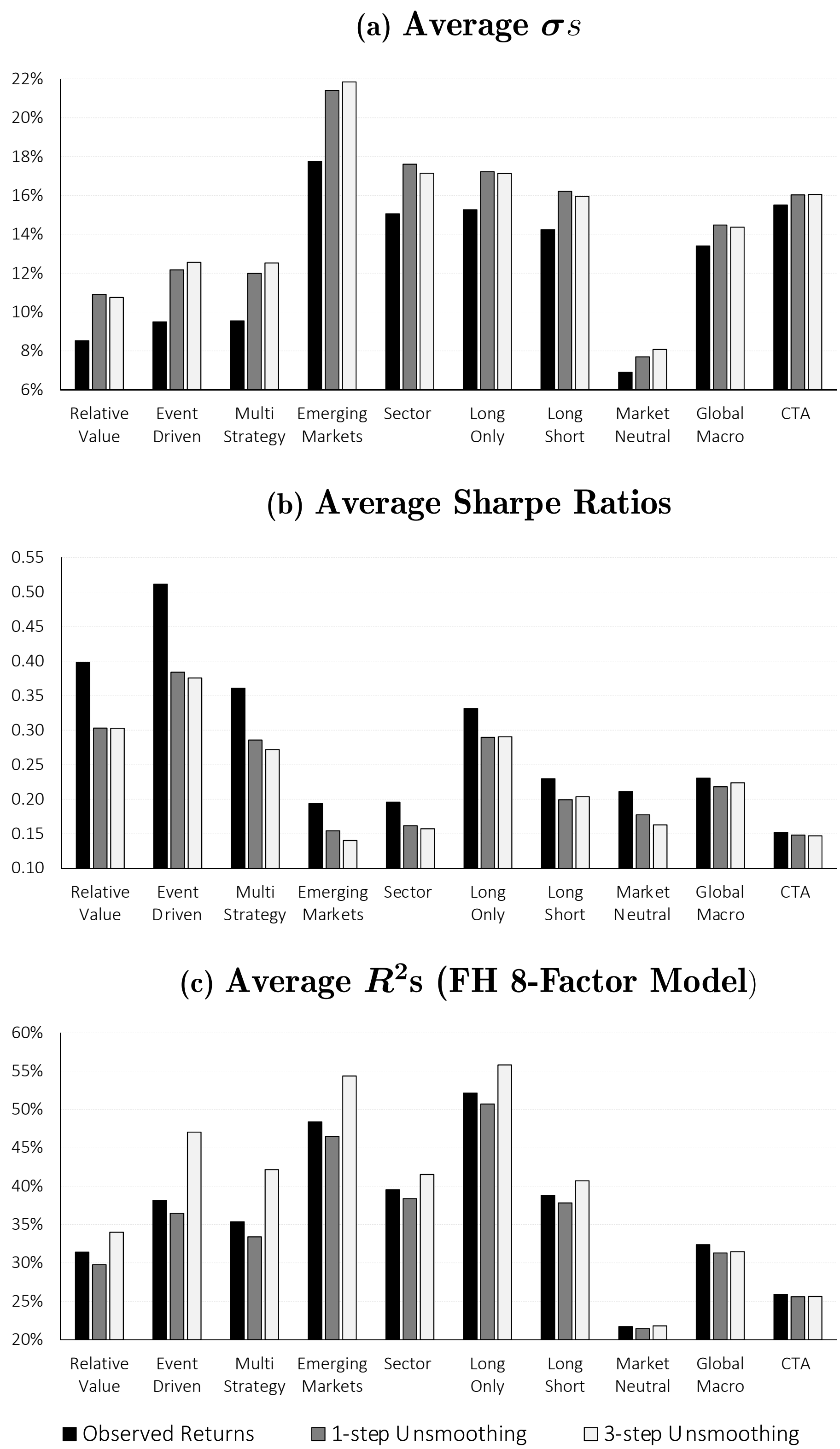
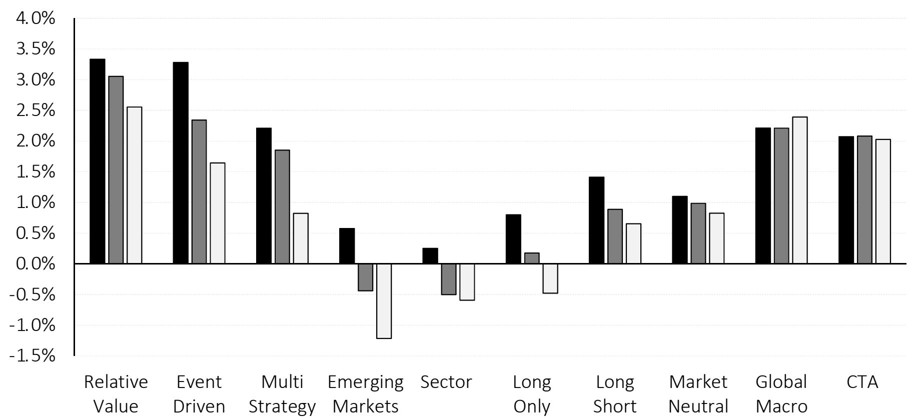


Figure 5

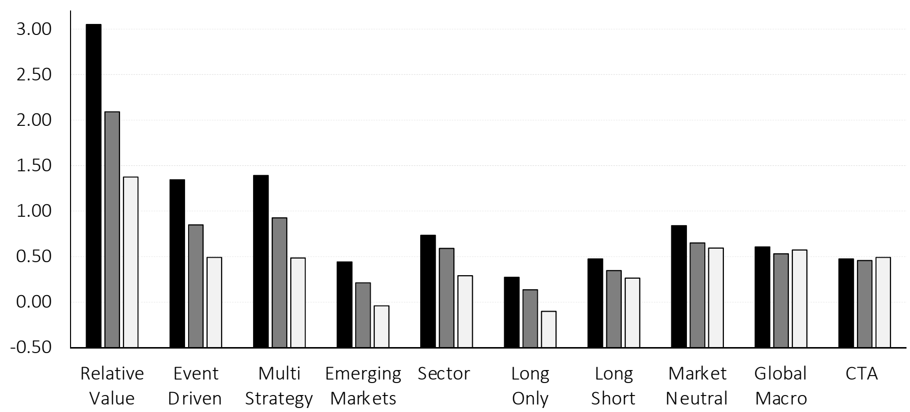
Hedge Fund Risk and Performance by Strategy

The figure plots average fund-level results by hedge fund strategy using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. R^2s and αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001). The sample goes from January 1995 to December 2017. See Section 1 for unsmoothing methods and Subsection 2.1 for further empirical details.

(d) Average αs (FH 8-Factor Model)



(e) Average t^{α}_{stat}



(f) % of Funds with α Significant at 10%

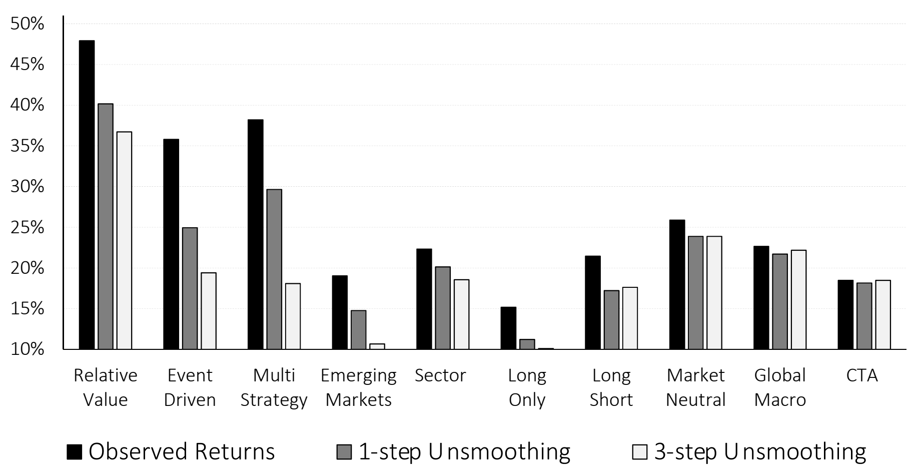


Figure 5 (Cont'd)
Hedge Fund Risk and Performance by Strategy

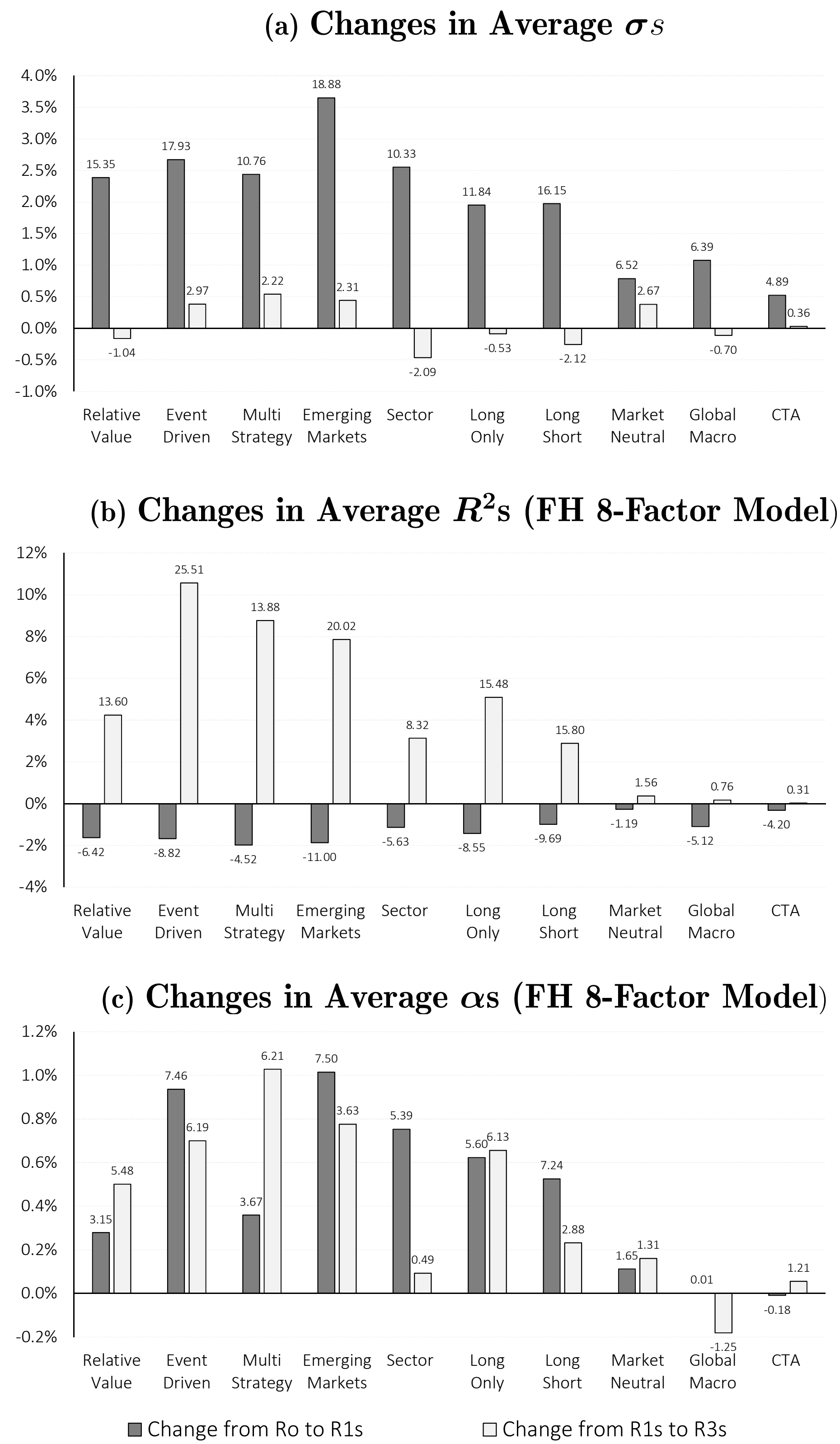


Figure 6
Changes in Hedge Fund Risk and Performance by Strategy

The figure plots increases in σ and R^2 (and declines in α) with their t_{stat} by hedge fund strategy as we move (i) from observed to 1-step unsmoothed returns and (ii) from 1-step to 3-step unsmoothed returns. R^2s and αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001). The sample goes from January 1995 to December 2017. See Section 1 for unsmoothing methods and Subsection 2.1 for further empirical details.

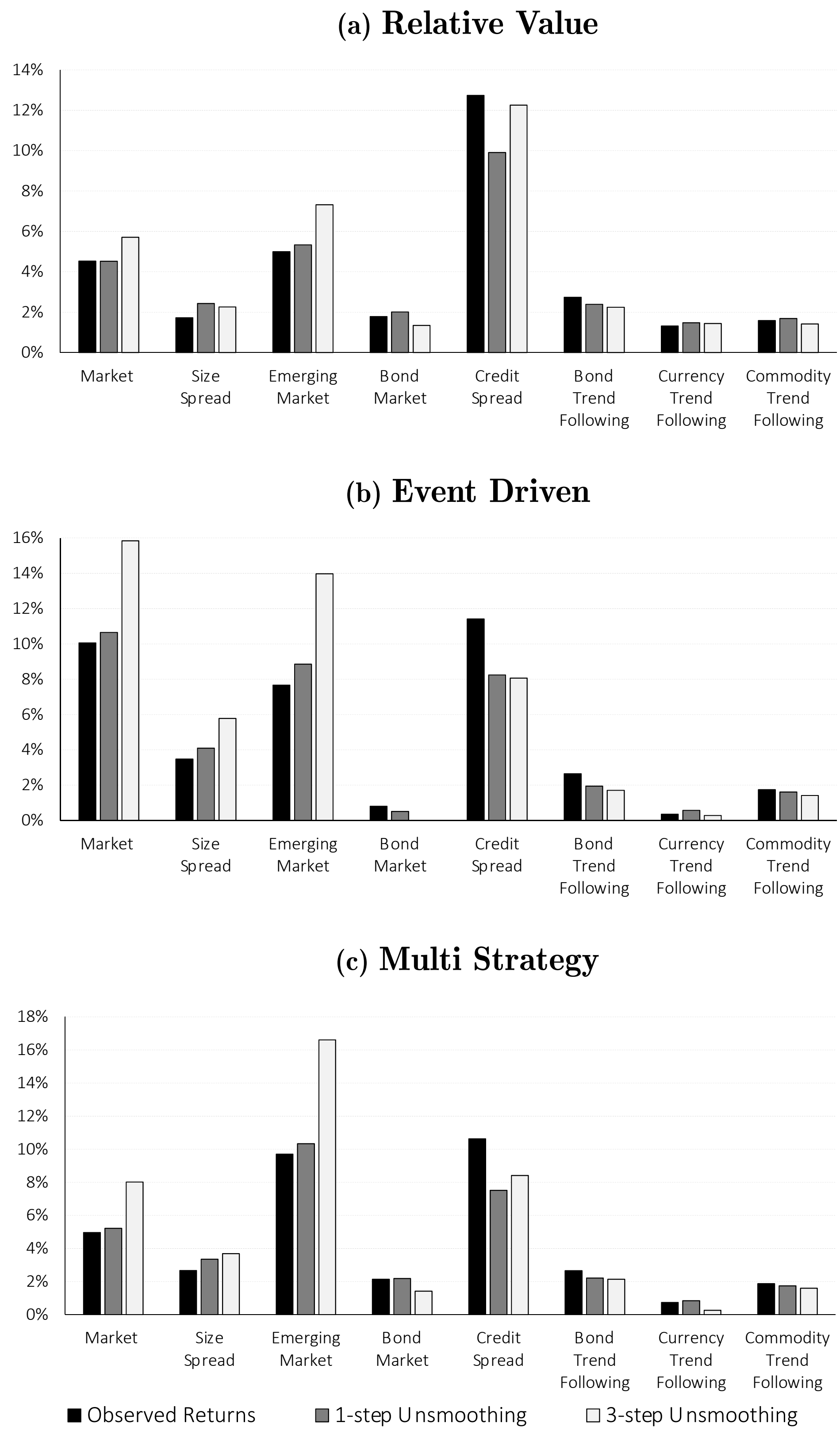
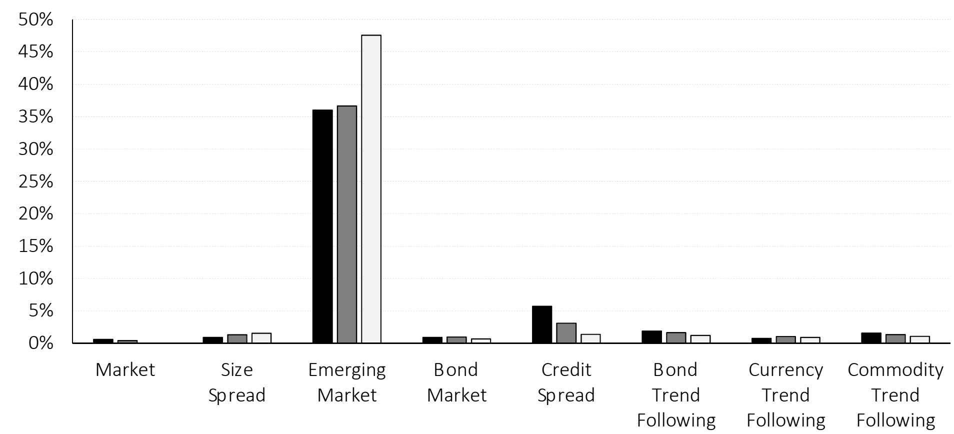


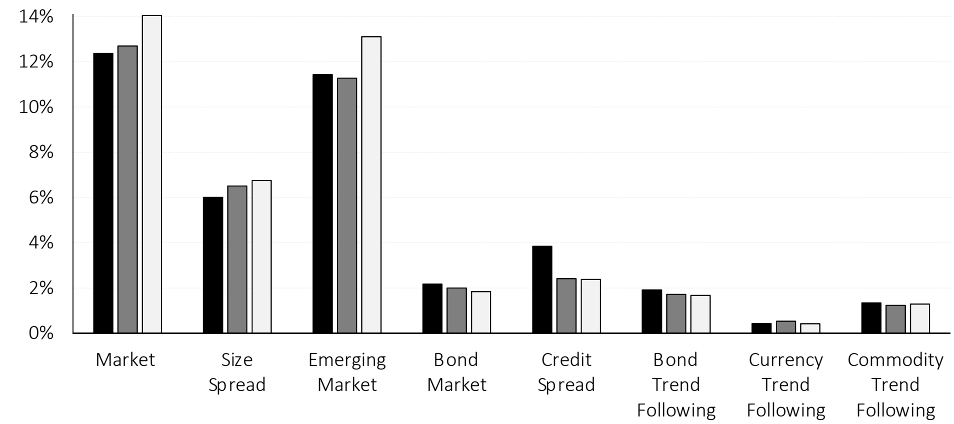
Figure 7
Decomposing Hedge Fund R^2s into the Effect of Each Risk Factor

The figure plots, for each hedge fund strategy, the average R^2 from factor regressions decomposed into the effect of each risk factor (see Equation 18). We use the risk factors in the FH 8-Factor model that builds on Fung and Hsieh (2001). The sample goes from January 1995 to December 2017. See Section 1 for unsmoothing methods and Subsection 2.1 for further empirical details.

(d) Emerging Markets



(e) Sector



(f) Long Only

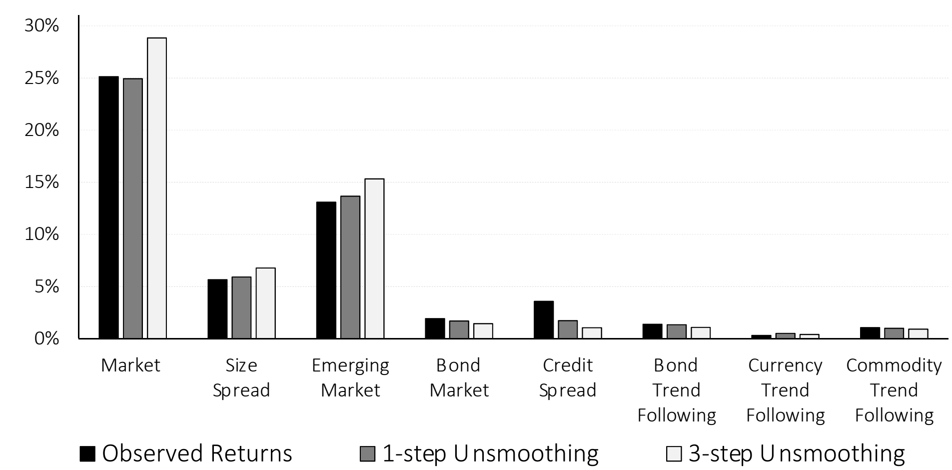
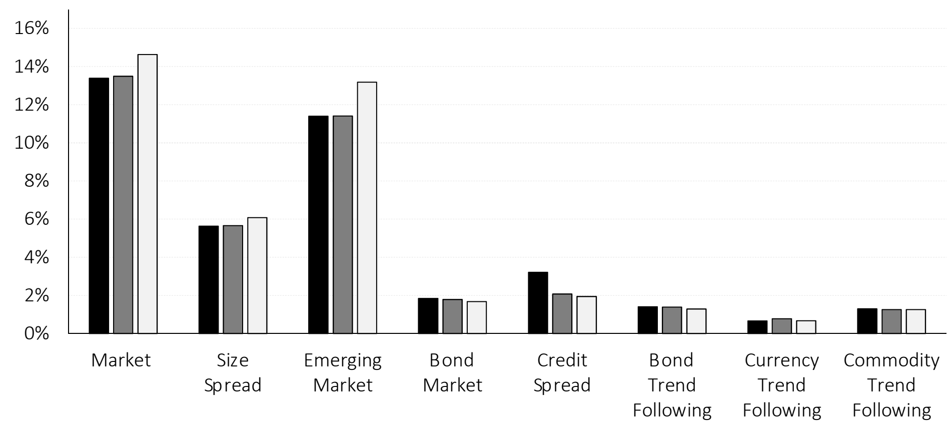
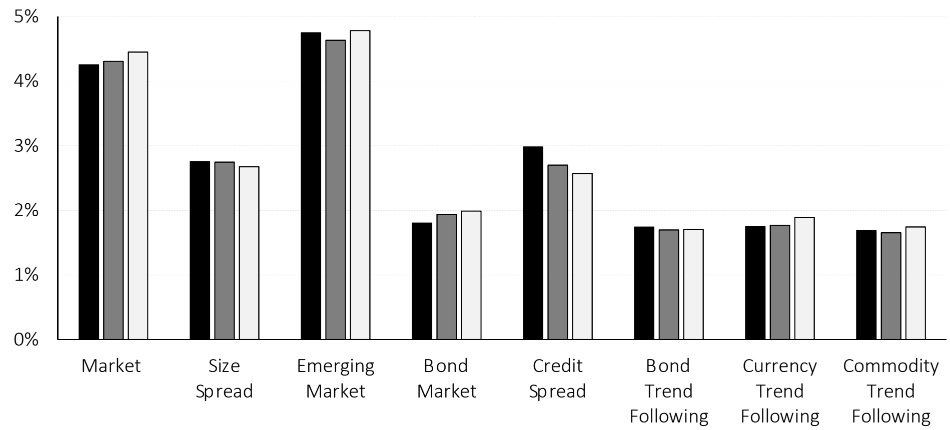


Figure 7 (Cont'd)
Decomposing Hedge Fund R^2s into the Effect of Each Risk Factor

(g) Long Short



(h) Market Neutral



(i) Global Macro

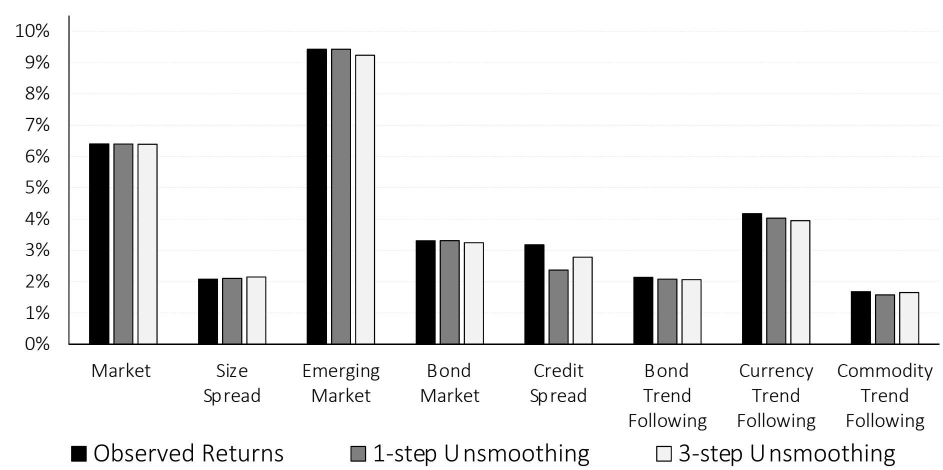


Figure 7 (Cont'd)
Decomposing Hedge Fund R^2s into the Effect of Each Risk Factor

(j) CTA

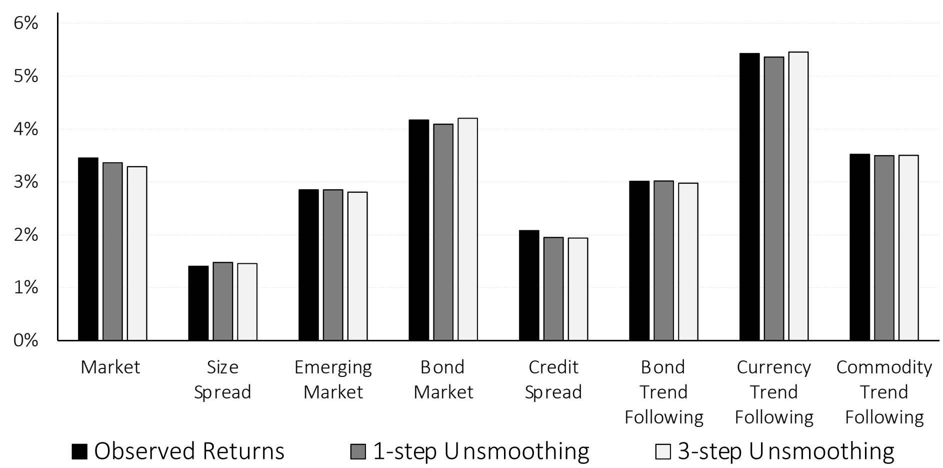


Figure 7 (Cont'd)
Decomposing Hedge Fund R^2s into the Effect of Each Risk Factor

Table 1
Hedge Fund Strategies and Summary Statistics

The table reports the total number of hedge funds (N), the average number of months per hedge fund ($\overline{T}$) and other average fund-level statistics for hedge fund returns by strategy, with strategies sorted based on the 1st order (average fund-level) autocorrelation coefficient (Cor_1). All statistics are based on observed returns. The sample goes from January 1995 to December 2017 and is restricted to US-dollar funds that report net-of-fees returns, have at least 36 uninterrupted monthly observations, and reach $5 million in AUM at some point in the sample, with fund observations included only after reaching the $5 million AUM threshold for the first time. See Section 1 for unsmoothing methods and Subsection 2.1 for further empirical details.

Hedge Fund	**Sample Size**		**Fund-level**			
Strategies	N	$\overline{T}$	Cor_1	σ	$\mathbb{E}[r]$	$\mathbb{E}[r]/\sigma$
Relative Value	670	85	0.29	8.5%	3.4%	0.40
Event Driven	433	96	0.25	9.5%	4.9%	0.51
Multi Strategy	199	97	0.22	9.5%	3.4%	0.36
Emerging Mkts	657	92	0.18	17.7%	3.4%	0.19
Sector	318	90	0.13	15.1%	2.9%	0.20
Long Only	455	102	0.11	15.3%	5.1%	0.33
Long-Short	965	92	0.10	14.2%	3.3%	0.23
Market Neutral	201	79	0.09	6.9%	1.5%	0.21
Global Macro	212	93	0.06	13.4%	3.1%	0.23
CTA	959	94	0.00	15.5%	2.4%	0.15

Table 2
Autocorrelations of Hedge Fund Returns

The table reports average fund-level autocorrelations (from 1 to 4 months) for hedge fund returns by strategy, with strategies sorted based on the 1st order (average fund-level) autocorrelation coefficient. Autocorrelations are based on observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. The numbers in parentheses reflect the fraction of funds with the respective autocorrelation being significant at 10% level. The sample goes from January 1995 to December 2017 and is restricted to US-dollar funds that report net-of-fees returns, have at least 36 uninterrupted monthly observations, and reach $5 million in AUM at some point in the sample, with fund observations included only after reaching the $5 million AUM threshold for the first time. See Section 1 for unsmoothing methods and Subsection 2.1 for further empirical details.

Hedge Fund	Observed Returns				1-step Unsmoothing				3-step Unsmoothing			
Strategies	Cor_1	Cor_2	Cor_3	Cor_4	Cor_1	Cor_2	Cor_3	Cor_4	Cor_1	Cor_2	Cor_3	Cor_4
Relative Value	0.29	0.18	0.19	0.10	0.03	0.05	0.11	0.07	0.04	-0.01	0.12	0.08
	(61%)	(38%)	(35%)	(26%)	(1%)	(3%)	(8%)	(18%)	(17%)	(9%)	(24%)	(20%)
Event Driven	0.25	0.13	0.10	0.05	0.01	0.03	0.04	0.03	-0.01	0.01	0.03	0.05
	(62%)	(37%)	(32%)	(20%)	(0%)	(2%)	(5%)	(11%)	(12%)	(9%)	(10%)	(14%)
Multi Strategy	0.22	0.11	0.07	0.04	0.01	0.01	0.02	0.03	0.00	-0.03	0.01	0.06
	(57%)	(37%)	(25%)	(15%)	(0%)	(2%)	(5%)	(8%)	(13%)	(3%)	(9%)	(13%)
Emerging Mkts	0.18	0.07	0.06	0.03	0.02	0.01	0.03	0.02	-0.04	0.00	0.05	0.01
	(47%)	(24%)	(16%)	(9%)	(0%)	(1%)	(5%)	(7%)	(7%)	(10%)	(13%)	(7%)
Sector	0.13	0.05	0.02	0.04	0.01	0.01	0.00	0.03	0.00	0.03	-0.02	0.04
	(36%)	(19%)	(10%)	(9%)	(2%)	(1%)	(4%)	(8%)	(8%)	(11%)	(5%)	(10%)
Long Only	0.11	0.03	0.04	0.02	0.01	0.00	0.02	0.02	-0.03	0.02	0.02	0.02
	(37%)	(10%)	(10%)	(10%)	(1%)	(2%)	(4%)	(9%)	(6%)	(8%)	(5%)	(9%)
Long-Short	0.10	0.03	0.04	0.01	0.01	0.00	0.02	0.01	-0.01	0.02	0.02	0.01
	(30%)	(14%)	(12%)	(8%)	(0%)	(2%)	(4%)	(6%)	(6%)	(8%)	(7%)	(7%)
Market Neutral	0.09	0.03	0.02	0.03	-0.01	0.00	-0.01	0.03	-0.02	-0.02	-0.03	0.04
	(27%)	(13%)	(14%)	(15%)	(1%)	(2%)	(5%)	(10%)	(1%)	(4%)	(6%)	(12%)
Global Macro	0.06	0.00	-0.02	0.01	-0.01	-0.01	-0.02	0.00	0.00	-0.01	-0.02	0.00
	(23%)	(11%)	(7%)	(9%)	(1%)	(2%)	(2%)	(6%)	(6%)	(6%)	(4%)	(8%)
CTA	0.00	0.00	-0.02	0.00	-0.02	-0.01	-0.03	0.00	-0.01	-0.01	-0.02	0.00
	(9%)	(9%)	(5%)	(8%)	(0%)	(3%)	(2%)	(7%)	(5%)	(7%)	(4%)	(8%)

Table 3
Autocorrelations of Aggregated Hedge Fund Returns

The table reports autocorrelations (from 1 to 4 months) for returns of each hedge fund strategy index (i.e., equal-weighted portfolio of all funds following the given strategy), with strategies sorted based on the 1st order (average fund-level) autocorrelation coefficient. Autocorrelations are based on observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. The numbers in parentheses reflect the p-value for the test of whether the respective autocorrelation differs from zero. The sample goes from January 1995 to December 2017 and is restricted to US-dollar funds that report net-of-fees returns, have at least 36 uninterrupted monthly observations, and reach $5 million in AUM at some point in the sample, with fund observations included only after reaching the $5 million AUM threshold for the first time. See Section 1 for unsmoothing methods and Subsection 2.1 for further empirical details.

Hedge Fund	Observed Returns				1-step Unsmoothing				3-step Unsmoothing			
Strategies	Cor_1	Cor_2	Cor_3	Cor_4	Cor_1	Cor_2	Cor_3	Cor_4	Cor_1	Cor_2	Cor_3	Cor_4
Relative Value	0.51	0.28	0.14	0.13	0.29	0.14	0.05	0.11	0.02	0.04	0.10	0.10
	(0%)	(0%)	(2%)	(4%)	(0%)	(2%)	(42%)	(7%)	(80%)	(47%)	(11%)	(9%)
Event Driven	0.46	0.25	0.18	0.10	0.23	0.12	0.10	0.08	0.03	0.03	0.05	0.08
	(0%)	(0%)	(0%)	(11%)	(0%)	(5%)	(9%)	(22%)	(64%)	(67%)	(43%)	(20%)
Multi Strategy	0.48	0.34	0.24	0.16	0.23	0.18	0.13	0.11	0.05	0.03	0.05	0.14
	(0%)	(0%)	(0%)	(1%)	(0%)	(0%)	(3%)	(7%)	(38%)	(65%)	(43%)	(2%)
Emerging Mkts	0.32	0.11	0.06	0.04	0.16	0.06	0.03	0.04	0.01	0.03	0.06	0.02
	(0%)	(7%)	(30%)	(49%)	(1%)	(34%)	(60%)	(54%)	(92%)	(64%)	(35%)	(69%)
Sector	0.21	0.05	0.04	0.01	0.08	0.01	0.04	0.01	0.02	0.04	0.03	0.01
	(0%)	(39%)	(49%)	(87%)	(21%)	(89%)	(51%)	(84%)	(70%)	(46%)	(67%)	(91%)
Long Only	0.22	0.05	0.04	0.01	0.11	0.03	0.02	0.01	0.01	0.04	0.03	0.00
	(0%)	(41%)	(56%)	(92%)	(7%)	(62%)	(70%)	(89%)	(88%)	(49%)	(67%)	(97%)
Long-Short	0.23	0.11	0.08	0.03	0.11	0.07	0.06	0.03	0.02	0.10	0.06	0.03
	(0%)	(7%)	(18%)	(59%)	(7%)	(23%)	(33%)	(65%)	(72%)	(10%)	(29%)	(61%)
Market Neutral	0.18	0.10	0.06	0.19	0.10	0.09	0.04	0.14	0.06	0.11	0.02	0.15
	(0%)	(9%)	(30%)	(0%)	(10%)	(12%)	(47%)	(2%)	(30%)	(7%)	(69%)	(1%)
Global Macro	0.05	-0.03	0.00	0.03	-0.01	-0.02	0.00	0.02	0.06	-0.04	0.00	0.03
	(37%)	(63%)	(99%)	(64%)	(91%)	(68%)	(94%)	(68%)	(32%)	(49%)	(96%)	(57%)
CTA	-0.02	-0.03	0.00	-0.02	-0.03	-0.01	0.01	-0.02	-0.02	-0.03	0.00	-0.02
	(71%)	(61%)	(98%)	(78%)	(57%)	(81%)	(90%)	(73%)	(74%)	(58%)	(94%)	(74%)

Table 4
1-Step vs 3-Step Unsmoothing in Simulations

The table reports autocorrelations and risk exposures in simulations. Specifically, we simulate returns on a panel of 300 funds over a 90 month period. The monthly economic returns of each fund j satisfy $R_{j,t} = \alpha_j + \beta_j \cdot f_t + \varepsilon_{j,t}$, where $\alpha_j \sim N(\mu_\alpha, \sigma_\alpha^2)$, $\beta_j \sim N(1, \sigma_\beta^2)$, $f_t \sim N(\mu_f, \sigma_f^2)$, and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. We then smooth these returns according to the underlying smoothing process of our 3-step method (i.e., Equation 5) with $H = L = 1$ (i.e., MA(1) smoothing), $\phi_j^{(1)} = \phi^{(1)}$, and $\pi_j^{(1)} = \pi^{(1)}$ for simplicity. Finally, we estimate economic returns for each fund in the panel using the 1- and 3-step unsmoothing methods and study the properties of observed returns, 1-step unsmoothed returns, and 3-step unsmoothed returns. The table reports the average results obtained from 1,000 simulations of this panel of funds. The first column shows results for a specification in which the aggregate and fund-specific components of returns are smoothed with the same intensity ($\phi^{(1)} = \pi^{(1)} = 0.3$), the second column considers a specification in which the fund-level component of returns is smoothed less than the aggregate-level component ($\phi^{(1)} = 0.2$ and $\pi^{(1)} = 0.4$), and the third column considers an alternative scenario in which the fund-level component of returns is smoothed more than the aggregate-level component ($\phi^{(1)} = 0.4$ and $\pi^{(1)} = 0.2$). See Section 1 for unsmoothing methods and Subsection 1.4 for further simulation details.

	Specification for Smoothing Coefficients		
	$\boldsymbol{\phi^{(1)} = \pi^{(1)}}$	$\boldsymbol{\phi^{(1)} < \pi^{(1)}}$	$\boldsymbol{\phi^{(1)} > \pi^{(1)}}$
$\boldsymbol{\phi^{(1)}}$	0.30	0.20	0.40
$\boldsymbol{\pi^{(1)}}$	0.30	0.40	0.20
$\boldsymbol{Cor_1(R_o)}$	0.35	0.32	0.32
$\boldsymbol{Cor_1(R_{1s})}$	0.00	0.00	0.00
$\boldsymbol{Cor_1(R_{3s})}$	-0.01	-0.01	-0.02
$\boldsymbol{Cor_1(\overline{R}_o)}$	0.36	0.46	0.23
$\boldsymbol{Cor_1(\overline{R}_{1s})}$	0.01	0.23	-0.14
$\boldsymbol{Cor_1(\overline{R}_{3s})}$	0.00	-0.01	0.00
$\boldsymbol{\widehat{\beta}_o}$	0.70	0.60	0.80
$\boldsymbol{\widehat{\beta}_{1s}}$	0.99	0.82	1.10
$\boldsymbol{\widehat{\beta}_{3s}}$	1.00	1.00	1.00

Table 5
Autocorrelations of CRE Fund Returns

The table reports (average fund-level and aggregate) autocorrelations (from 1 to 4 quarters) for US Commercial Real Estate (CRE) funds. Autocorrelations are based on observed returns, 1-step unsmoothed returns (as in Geltner (1991, 1993)), and 3-step unsmoothed returns. In the upper panel, the numbers in parentheses reflect the fraction of funds with the respective autocorrelation being significant at 10% level. In the lower panel, the numbers in parentheses reflect the p-value for the test of whether the respective autocorrelation differs from zero. Partial autocorrelations refer to coefficients from a multivariate regression that includes lagged returns from up to 4 quarters. The sample goes from Q1 1994 through Q4 2017 and is restricted to CRE funds that report return data to NCREIF and have at least 36 quarterly observations. See Subsection 3.1 for the AR unsmoothing methods used and Subsection 3.2 for further empirical details.

		Autocorrelations				**Partial Autocorrelations**			
	Returns	Cor_1	Cor_2	Cor_3	Cor_4	Cor_1	Cor_2	Cor_3	Cor_4
Fund Level	**Observed**	0.45	0.39	0.23	0.21	0.45	0.10	0.01	0.01
		(63.6%)	(74.2%)	(51.5%)	(40.9%)	(59.1%)	(21.2%)	(9.1%)	(12.1%)
	1-step	-0.03	0.02	0.04	0.10	-0.02	0.03	0.05	0.08
		(0.0%)	(12.1%)	(7.6%)	(21.2%)	(0.0%)	(12.1%)	(7.6%)	(21.2%)
	3-step	-0.04	0.13	-0.05	0.17	-0.02	0.11	-0.04	0.14
		(0.0%)	(27.3%)	(0.0%)	(31.8%)	(0.0%)	(12.1%)	(0.0%)	(21.2%)
Aggregate Level	**Observed**	0.75	0.67	0.42	0.31	0.65	0.42	-0.33	0.00
		(0.0%)	(0.0%)	(0.0%)	(0.3%)	(0.0%)	(0.1%)	(0.9%)	(99.7%)
	1-step	0.46	0.31	0.12	0.07	0.41	0.16	-0.09	0.02
		(0.0%)	(0.3%)	(26.6%)	(48.3%)	(0.0%)	(18.3%)	(45.0%)	(86.8%)
	3-step	-0.02	0.24	-0.09	0.19	0.02	0.20	-0.09	0.14
		(86.7%)	(2.3%)	(37.7%)	(6.9%)	(84.2%)	(6.0%)	(38.5%)	(18.5%)

Table 6
Risk and Performance of CRE Funds

The table reports (average fund-level) statistics related to the risk and performance of US Commercial Real Estate (CRE) funds. All statistics are based on observed returns, 1-step unsmoothed returns (as in Geltner (1991, 1993)), and 3-step unsmoothed returns. The upper panel reports the values of the statistics (with the % of funds with significant values at 10% in parentheses) and the lower panel reports changes in these statistics (with the t_{stat} for a test of whether the mean change differs from zero in brackets). The sample goes from Q1 1994 through Q4 2017 and is restricted to CRE funds that report return data to NCREIF and have at least 36 quarterly observations. See Subsection 3.1 for the AR unsmoothing methods used and Subsection 3.2 for further empirical details.

Statistics are	**Raw Performance**			**1-Factor Model**			**2-Factor Model**			
Related to	$\mathbb{E}[r]$	σ	$\mathbb{E}[r]/\sigma$	α	β_{re}	R^2	α	β_{re}	β_e	R^2
Observed, R_o	5.0%	13.1%	0.38	4.3%	0.07	3.2%	4.0%	0.02	0.10	4.6%
				(53.0%)	(27.3%)		(51.5%)	(9.1%)	(7.6%)	
1-step, R_{1s}	5.0%	25.3%	0.20	2.7%	0.22	7.9%	2.4%	0.15	0.15	9.9%
				(13.6%)	(47.0%)		(13.6%)	(30.3%)	(10.6%)	
3-step, R_{3s}	5.0%	24.1%	0.21	1.6%	0.34	13.7%	0.8%	0.21	0.26	16.0%
				(9.1%)	(89.4%)		(9.1%)	(30.3%)	(13.6%)	
From R_o to R_{1s}	0.0%	12.1%	-0.18	-1.6%	0.16	4.8%	-1.7%	0.13	0.05	5.3%
				[-8.71]	[8.40]		[-7.36]	[6.44]	[1.62]	
From R_{1s} to R_{3s}	0.0%	-1.1%	0.01	-1.2%	0.12	5.7%	-1.6%	0.06	0.12	6.0%
				[-3.93]	[3.70]		[-4.42]	[1.95]	[4.03]	
From R_o to R_{3s}	0.0%	11.0%	-0.17	-2.7%	0.27	10.5%	-3.2%	0.19	0.17	11.3%
				[-11.73]	[11.01]		[-11.34]	[9.62]	[8.38]	

Internet Appendix

"Unsmoothing Returns of Illiquid Funds"

By Spencer Couts, Andrei S. Gonçalves, and Andrea Rossi

This Internet Appendix contains further results that supplement the main findings in the paper by performing several robustness checks that modify some of our empirical specifications related to sample construction and econometric design.

A Robustness Analyses

This section performs a series of robustness analyses that modify different aspects of our empirical design. To conserve space, we only report the core results for each alternative specification studied, but other results are also similar to what we report in the main text.

A.1 3-step Unsmoothing with Value-weighted Aggregate Returns

While developing our 3-step unsmoothing method, we relied on time-invariant weights, w_j. For this reason, we use equal-weights in our empirical analysis (as opposed to value-weights) when unsmoothing aggregate (or strategy-level) returns. For robustness, we repeat our analysis here after replacing equal-weights with value-weights (i.e., weights based on NAV). All other aspects of the analysis are kept fixed, including the fact that we report equal-weighted averages of statistics, not value-weighted averages.

Figure IA.1 replicates Figure 3 in the main text after replacing equal-weights with value-weights to construct strategy indexes. It is clear that all results presented in the main text remain valid after relying on value-weighted strategy indexes. In particular, the 3-step method has little effect on volatility, but it increases R^2s and decreases αs.

Table IA.1 replicates Table 6 in the main text after replacing equal-weights with value-weights to construct the aggregate CRE fund return. Similar to what we report in the main text, the 3-step method continues to substantially increase risk exposures. Consequently, the 3-step method drives the average α of CRE funds to (close to) zero.

A.2 3-step Unsmoothing with Regular vs Log Returns

In the main text, we use regular returns for the hedge fund analysis and log returns for the CRE analysis to be consistent with the respective literatures. For robustness, we repeat our analysis here after replacing regular (log) returns with log (regular) returns in the case of hedge funds (CRE funds). All other aspects of the analysis are kept fixed, including the fact that the statistics reported are based on regular returns (even when we obtain regular returns from unsmoothed log returns).

Figure IA.2 replicates Figure 3 in the main text after replacing regular returns with log returns, with all results being similar to what we report in the main text. In particular, the 3-step method has little effect on volatility, but it increases R^2s and decreases αs.

Table IA.2 replicates Table 6 in the main text after replacing log returns with regular returns, with all results being similar to what we report in the main text. In particular, the 3-step method continues to substantially increase risk exposures and to drive the average α of CRE funds to (close to) zero.

A.3 MA 3-step Unsmoothing Dropping the First 12 Monthly Returns

Our baseline specification deals with backfilled returns by following Jorion and Schwarz (2019). However, their approach is relatively new. In earlier research, it was common to drop the first 12 monthly returns before calculating performance-related statistics. To provide results that are directly comparable to the previous literature, Figure IA.3 replicates Figure 3 in the main text after dropping the first 12 monthly returns when calculating performance-related statistics instead of following Jorion and Schwarz (2019)'s algorithm to identify backfilled observations. It is clear that all results presented in the main text remain valid after relying on this alternative backfilling adjustment. In particular, the 3-step method has little effect on volatility, but it increases R^2s and decreases αs.

A.4 MA 3-step Unsmoothing with $H = 2$

In the literature, it is common to rely on a MA(2) when unsmoothing hedge fund returns. In the main text, we instead use the AIC criterion to choose the number of smoothing lags (between 0, 1, 2, and 3) for the observed return process. This approach allows for heterogeneity across funds. For robustness, we repeat our analysis here after fixing $H = 2$. Figure IA.4 replicates Figure 3 in the main text after fixing $H = 2$ and demonstrates that the results are very similar to what we report in Figure 3.

A.5 AR 3-step Unsmoothing for Open-end CRE Funds

In the main text, we rely on all 66 CRE funds (29 open-end and 37 closed-end) available in our dataset to maximize our coverage. However, one could argue that open-end funds are more appropriate for the type of analysis we perform. As such, Table IA.3 replicates Table 6 in the main text after subsetting the data to the 29 open-end funds available. Results are similar, with the exposure to the public real estate market increasing strongly (and the α decreasing strongly) after 3-step unsmoothing.

References for Internet Appendix

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Geltner, D. (1991). "Smoothing in Appraisal-Based Returns". In: *Journal of Real Estate Finance and Economics* 4, pp. 327–345.

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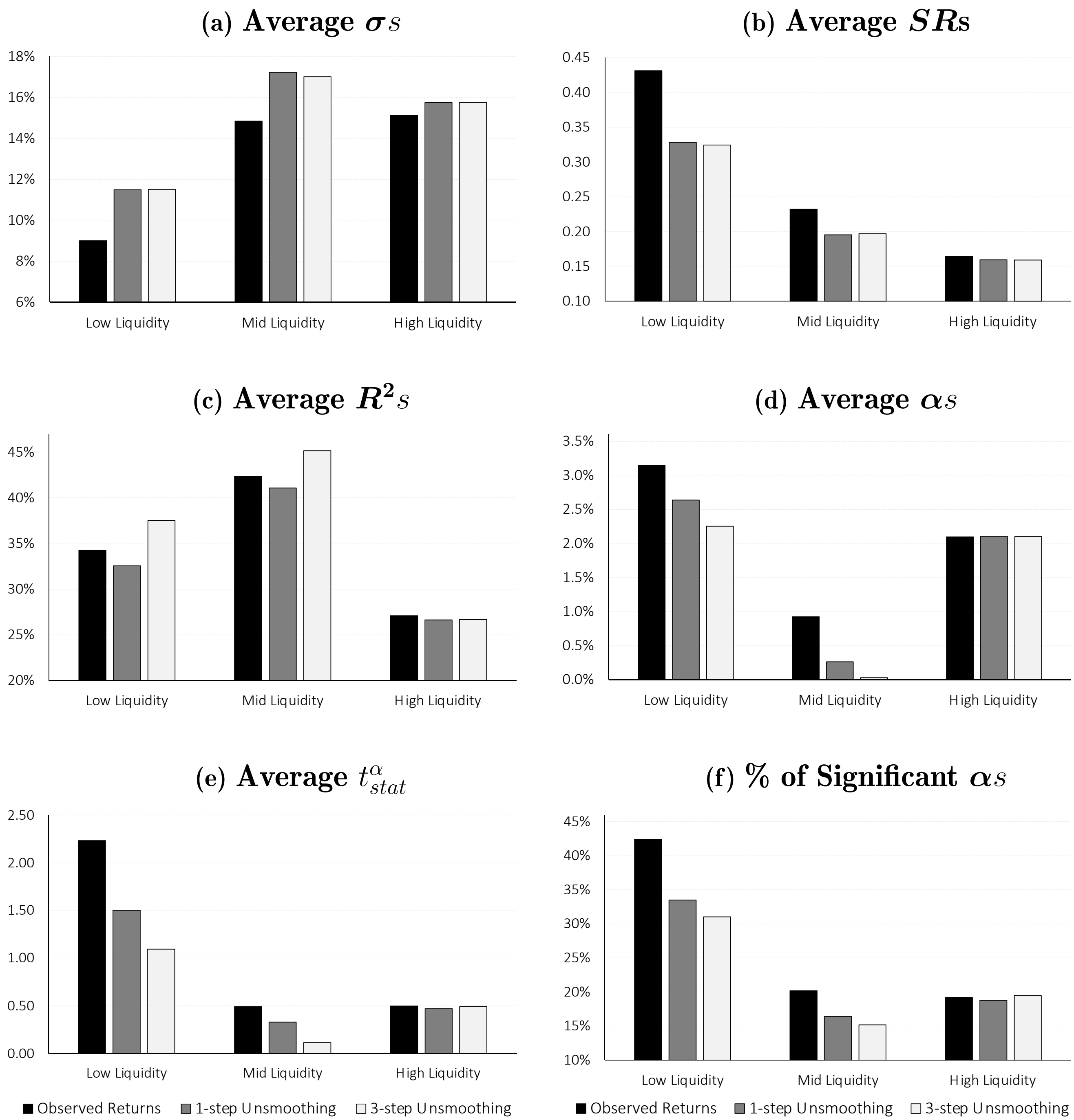

Figure IA.1

Hedge Fund Risk and Performance by Strategy Liquidity (Value-weights)

The figure plots average fund-level results for three groups based on hedge fund strategy liquidity using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. We sort strategies based on their first order autocorrelation coefficient to form three groups: low liquidity strategies (the three strategies with autocorrelation above 0.40), high liquidity strategies (the two strategies with autocorrelation below 0.10), and mid liquidity strategies (the other five strategies). R^2s and αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001) and statistical significance for fund-level αs is at 10%. The sample goes from January 1995 to December 2017. See Section 1 for the unsmoothing methods and Subsection 2.1 for further empirical details.

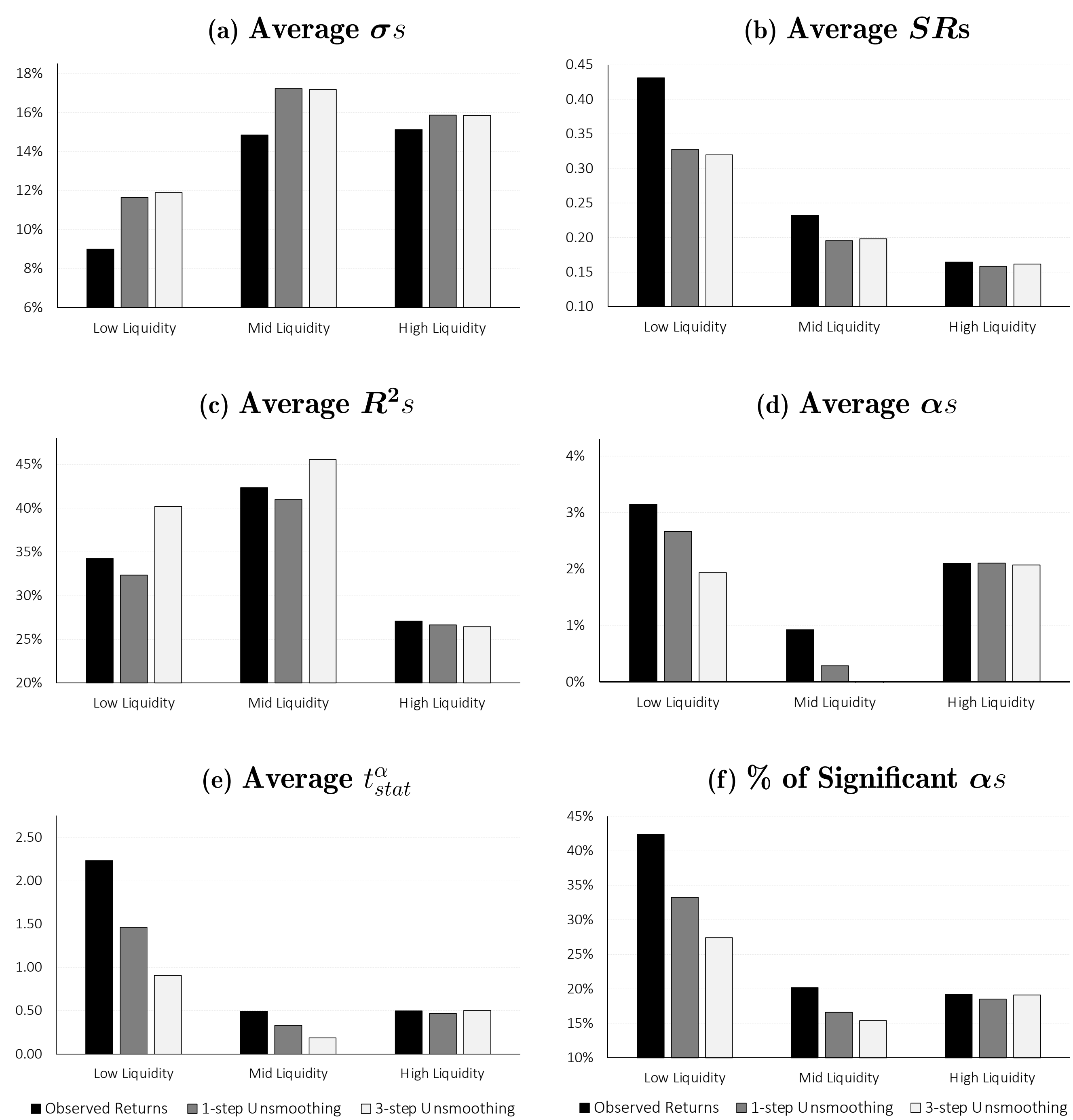

Figure IA.2

Hedge Fund Risk and Performance by Strategy Liquidity (Log Returns)

The figure plots average fund-level results for three groups based on hedge fund strategy liquidity using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. We sort strategies based on their first order autocorrelation coefficient to form three groups: low liquidity strategies (the three strategies with autocorrelation above 0.40), high liquidity strategies (the two strategies with autocorrelation below 0.10), and mid liquidity strategies (the other five strategies). R^2s and αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001) and statistical significance for fund-level αs is at 10%. The sample goes from January 1995 to December 2017. See Section 1 for the unsmoothing methods and Subsection 2.1 for further empirical details.

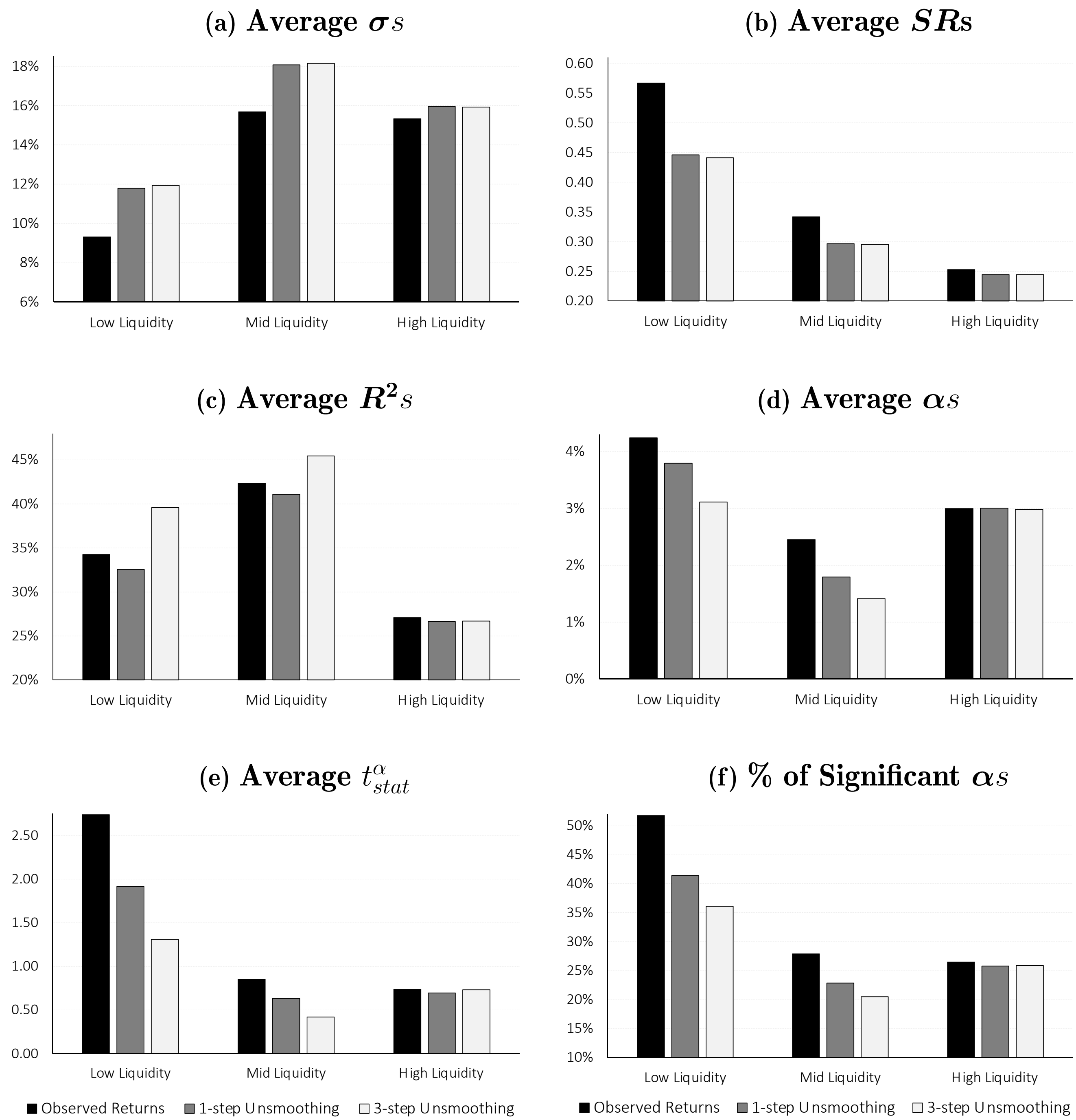

Figure IA.3

Hedge Fund Risk and Performance by Strategy Liquidity (Drop 12 Months)

The figure plots average fund-level results for three groups based on hedge fund strategy liquidity using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. We sort strategies based on their first order autocorrelation coefficient to form three groups: low liquidity strategies (the three strategies with autocorrelation above 0.40), high liquidity strategies (the two strategies with autocorrelation below 0.10), and mid liquidity strategies (the other five strategies). R^2s and αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001) and statistical significance for fund-level αs is at 10%. The sample goes from January 1995 to December 2017. See Section 1 for the unsmoothing methods and Subsection 2.1 for further empirical details.

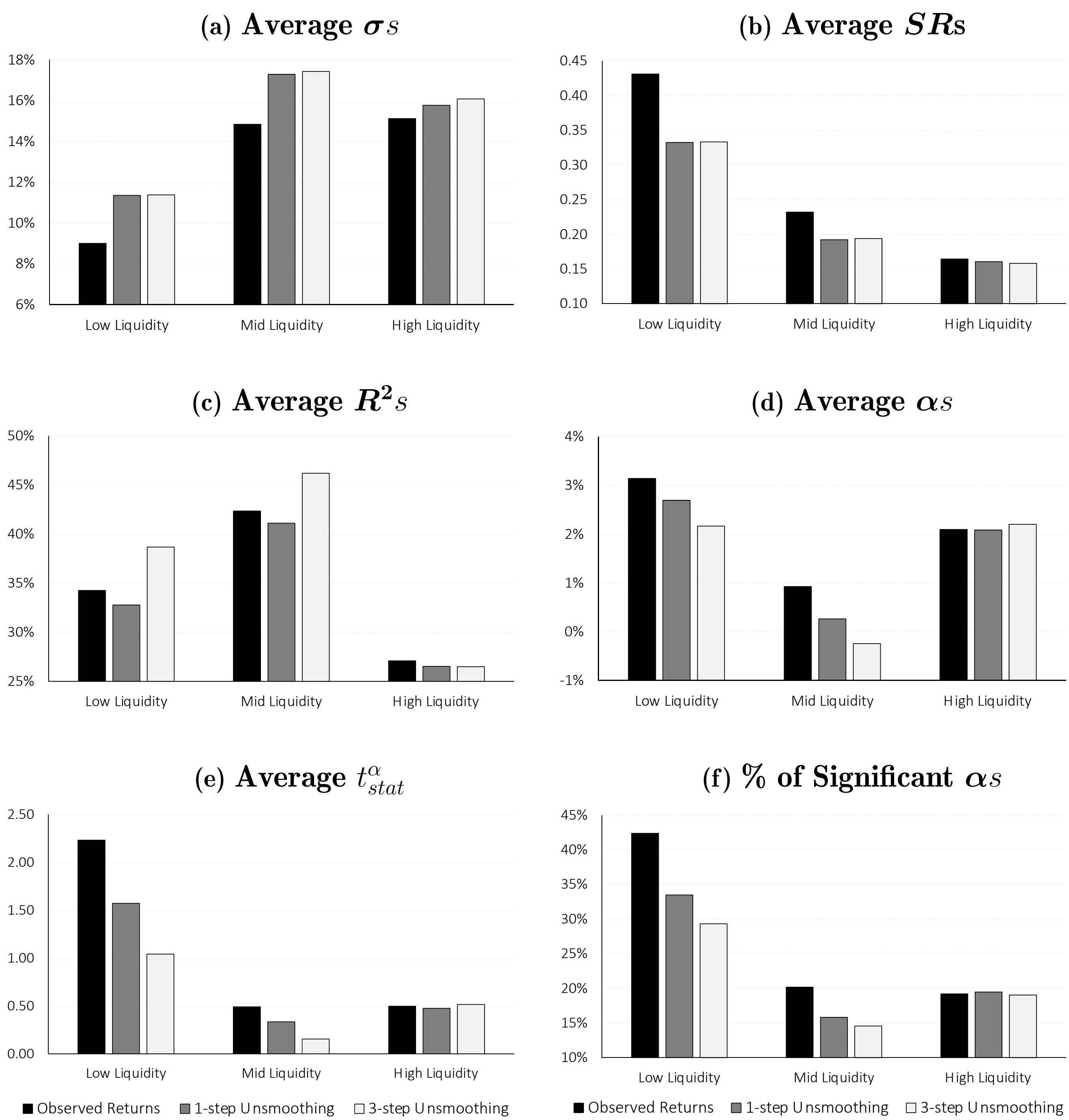

Figure IA.4
Hedge Fund Risk and Performance by Strategy Liquidity (H=2)

The figure plots average fund-level results for three groups based on hedge fund strategy liquidity using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. We sort strategies based on their first order autocorrelation coefficient to form three groups: low liquidity strategies (the three strategies with autocorrelation above 0.40), high liquidity strategies (the two strategies with autocorrelation below 0.10), and mid liquidity strategies (the other five strategies). R^2s and αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001) and statistical significance for fund-level αs is at 10%. The sample goes from January 1995 to December 2017. See Section 1 for the unsmoothing methods and Subsection 2.1 for further empirical details.

Table IA.1

Risk and Performance of CRE Funds (Value-weights)

The table reports (average fund-level) statistics related to the risk and performance of US Commercial Real Estate (CRE) funds. All statistics are based on observed returns, 1-step unsmoothed returns (as in Geltner (1991, 1993)), and 3-step unsmoothed returns. The upper panel reports the values of the statistics (with the % of funds with significant values at 10% in parentheses) and the lower panel reports changes in these statistics (with the t_{stat} for a test of whether the mean change differs from zero in brackets). The sample goes from Q1 1994 through Q4 2017 and is restricted to CRE funds that report return data to NCREIF and have at least 36 quarterly observations. See Subsection 3.1 for the AR unsmoothing methods used and Subsection 3.2 for further empirical details.

Statistics are	Raw Performance			1-Factor Model			2-Factor Model			
Related to	$\mathbb{E}[r]$	σ	$\mathbb{E}[r]/\sigma$	α	β_{re}	R^2	α	β_{re}	β_e	R^2
Observed, R_o	5.2%	12.7%	0.41	4.6%	0.06	3.2%	4.4%	0.02	0.09	4.6%
				(54.5%)	(28.8%)		(53.0%)	(9.1%)	(7.6%)	
1-step, R_{1s}	5.2%	24.1%	0.22	3.0%	0.23	8.1%	2.7%	0.16	0.14	10.1%
				(13.6%)	(48.5%)		(13.6%)	(30.3%)	(10.6%)	
3-step, R_{3s}	5.2%	26.4%	0.20	1.2%	0.40	17.0%	-0.1%	0.18	0.44	21.6%
				(7.6%)	(89.4%)		(9.1%)	(27.3%)	(57.6%)	
From R_o to R_{1s}	0.0%	11.4%	-0.20	-1.6%	0.16	5.0%	-1.7%	0.14	0.05	5.5%
				[-9.19]	[8.90]		[-7.73]	[6.73]	[1.68]	
From R_{1s} to R_{3s}	0.0%	2.3%	-0.02	-1.8%	0.17	8.9%	-2.8%	0.02	0.30	11.5%
				[-4.56]	[4.37]		[-5.23]	[0.74]	[6.43]	
From R_o to R_{3s}	0.0%	13.7%	-0.21	-3.4%	0.34	13.9%	-4.5%	0.16	0.35	17.0%
				[-9.43]	[9.23]		[-9.18]	[6.70]	[9.17]	

Table IA.2

Risk and Performance of CRE Funds (Regular Returns)

The table reports (average fund-level) statistics related to the risk and performance of US Commercial Real Estate (CRE) funds. All statistics are based on observed returns, 1-step unsmoothed returns (as in Geltner (1991, 1993)), and 3-step unsmoothed returns. The upper panel reports the values of the statistics (with the % of funds with significant values at 10% in parentheses) and the lower panel reports changes in these statistics (with the t_{stat} for a test of whether the mean change differs from zero in brackets). The sample goes from Q1 1994 through Q4 2017 and is restricted to CRE funds that report return data to NCREIF and have at least 36 quarterly observations. See Subsection 3.1 for the AR unsmoothing methods used and Subsection 3.2 for further empirical details.

Statistics are	**Raw Performance**			**1-Factor Model**			**2-Factor Model**			
Related to	$\mathbb{E}[r]$	σ	$\mathbb{E}[r]/\sigma$	α	β_{re}	R^2	α	β_{re}	β_e	R^2
Observed, R_o	5.0%	13.1%	0.38	4.3%	0.07	3.2%	4.0%	0.02	0.10	4.6%
				(53.0%)	(27.3%)		(51.5%)	(9.1%)	(7.6%)	
1-step, R_{1s}	5.0%	24.8%	0.20	2.5%	0.25	8.7%	2.1%	0.17	0.17	10.6%
				(13.6%)	(53.0%)		(13.6%)	(30.3%)	(12.1%)	
3-step, R_{3s}	5.0%	26.0%	0.19	0.8%	0.42	16.1%	-0.1%	0.26	0.32	18.5%
				(9.1%)	(90.9%)		(9.1%)	(34.8%)	(13.6%)	
From R_o to R_{1s}	0.0%	11.7%	-0.18	-1.8%	0.18	5.5%	-2.0%	0.15	0.07	5.9%
				[-9.42]	[9.01]		[-8.32]	[7.47]	[2.70]	
From R_{1s} to R_{3s}	0.0%	1.1%	-0.01	-1.7%	0.17	7.4%	-2.2%	0.09	0.15	7.9%
				[-5.16]	[4.88]		[-5.67]	[3.12]	[5.55]	
From R_o to R_{3s}	0.0%	12.9%	-0.19	-3.5%	0.35	12.9%	-4.2%	0.24	0.22	13.8%
				[-11.22]	[10.43]		[-10.78]	[9.70]	[8.43]	

Table IA.3

Risk and Performance of CRE Funds (Open-end)

The table reports (average fund-level) statistics related to the risk and performance of US Commercial Real Estate (CRE) funds. All statistics are based on observed returns, 1-step unsmoothed returns (as in Geltner (1991, 1993)), and 3-step unsmoothed returns. The upper panel reports the values of the statistics (with the % of funds with significant values at 10% in parentheses) and the lower panel reports changes in these statistics (with the t_{stat} for a test of whether the mean change differs from zero in brackets). The sample goes from Q1 1994 through Q4 2017 and is restricted to CRE funds that report return data to NCREIF and have at least 36 quarterly observations. See Subsection 3.1 for the AR unsmoothing methods used and Subsection 3.2 for further empirical details.

Statistics are	**Raw Performance**			**1-Factor Model**			**2-Factor Model**			
Related to	$\mathbb{E}[r]$	$\boldsymbol{\sigma}$	$\mathbb{E}[r]/\boldsymbol{\sigma}$	$\boldsymbol{\alpha}$	$\boldsymbol{\beta_{re}}$	$\boldsymbol{R^2}$	$\boldsymbol{\alpha}$	$\boldsymbol{\beta_{re}}$	$\boldsymbol{\beta_e}$	$\boldsymbol{R^2}$
Observed, R_o	5.5%	7.9%	0.70	5.0%	0.05	3.4%	4.9%	0.04	0.02	3.8%
				(82.1%)	(35.7%)		(82.1%)	(17.9%)	(0.0%)	
1-step, R_{1s}	5.5%	20.0%	0.28	2.7%	0.28	12.3%	2.5%	0.24	0.08	13.5%
				(10.7%)	(71.4%)		(10.7%)	(64.3%)	(0.0%)	
3-step, R_{3s}	5.5%	17.3%	0.32	2.3%	0.32	15.7%	1.7%	0.25	0.15	17.0%
				(14.3%)	(96.4%)		(10.7%)	(60.7%)	(3.6%)	
From R_o to R_{1s}	0.0%	12.1%	-0.42	-2.3%	0.23	8.9%	-2.5%	0.20	0.06	9.7%
				[-11.05]	[11.43]		[-10.68]	[6.73]	[1.44]	
From R_{1s} to R_{3s}	0.0%	-2.7%	0.04	-0.4%	0.04	3.4%	-0.7%	0.01	0.07	3.5%
				[-1.73]	[1.72]		[-2.34]	[0.22]	[1.81]	
From R_o to R_{3s}	0.0%	9.4%	-0.38	-2.8%	0.27	12.3%	-3.2%	0.21	0.13	13.2%
				[-13.45]	[15.33]		[-12.55]	[17.19]	[7.52]	