

Cash-flow or Discount-rate Risk? Evidence from the Cross Section of Present Values

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Abstract

Realized returns comprise (ex-ante) expected returns plus (ex-post) innovations, and consequently both expected returns and returns innovations can be broken down into components reflecting fluctuations in cash flow (CF) and discount rate (DR). I use a present-value model to identify the CF and DR risk factors which are latent from the time series and cross sections of price–dividend ratios. This setup accommodates models where CF risk dominates, like Bansal and Yaron (2004), and models where DR risk dominates, like Campbell and Cochrane (1999). I estimate the model on portfolios, which capture several of the most common cross-sectional anomalies, and decompose the expected and unexpected returns into CF and DR components along both time-series and cross-sectional dimensions. I find that (1) the DR risk is more likely to explain the variations of expected returns, (2) the CF risk drives the variations of unexpected returns, and (3) together they account for over 80% of the cross-sectional variance of the average stock returns.

1 Introduction

Prices are the sums of cash flows discounted by risk-adjusted discount rates. Thus, variations in price–dividend ratio and returns are due to change in cash flow (CF), or discount rate (DR), or both. I propose a present-value model to investigate how CF and DR risk drive the stock return variations from (ex-ante) expected and (ex-post) unexpected perspectives. This model treats the time-varying CF and DR risk factors as latent, following an exogenous time-series dynamics. Upon observations of the log of price–dividend ratio ($\ln pd$), I identify the latent factors and parameters in the model on portfolios capturing several of the most common cross-sectional anomalies. Then I decompose the realized returns into expected and unexpected CF and DR components based on the model estimates. I find the DR risk is more likely to explain the variations in expected returns, while CF risk drives the unexpected returns innovations. Together, they can account for the common anomalies of cross-sectional stock returns.

In the model, the present-value and the latent-factor approaches function respectively in the analysis of expected and unexpected returns. On the expected return side, using $\ln pd$, which reflects the ex-ante expectations in both of the future CF and DR, the present-value approach facilitates to bring the CF and DR fluctuations together and evaluate their likelihood of being priced as risks. Testing whether CF or DR risk dominates the expected return has a fundamental bearing on the theoretical modeling of asset prices. Theories characterized by either CF or DR risk quantitatively explain a wide range of asset-pricing phenomena. For example, the long-run risk literature argues that the CF risk explains stock returns in time series and cross sections,¹ while the models with changing risk aversion or sentiment emphasize DR risk as the main factor

¹ See Bansal and Yaron (2004), Bansal, Dittmar and Lundblad (2005), Bansal, Dittmar and Kiku (2009), Bansal, Kiku, Shaliastovich and Yaron (2013), Drechsler and Yaron (2011), Hansen, Heaton and Li (2008), Lettau and Ludvigson (2009), Koijen, Lustig, Van Nieuwerburgh and Verdelhan (2010), Parker and Julliard (2005).

in pricing the stocks.²

In this paper, the present-value framework is general enough to accommodate both types of theories, featuring CF risk as in Bansal and Yaron (2004) and DR risk as in Campbell and Cochrane (1999). The model's building blocks include factors for the following states. Market-level dividend growth and expected return represent marketwise CF and DR. The stochastic volatilities of dividend growth and return characterize the marketwise prices of CF and DR risk. Moreover, for a piece of individual asset, the exposures of its CF and DR to the marketwise CF and DR factors capture its CF and DR risk. Either CF or DR risk channel imposes constraints on the pricing kernels of the corresponding nested model, and these constraints can be translated into hypothetical conditions on latent factors and parameters. This hierarchy of models enable the Bayes factor test to examine whether the proposed risk channel is likely to generate the cross-sectional data of *lnpd*. The results from this test emphasize the DR risk as the more likely risk channel.

Results beyond the Bayes factor test highlight the DR risk as having the main role in expected return. First, return and growth predictability is consistent with previous empirical findings under the DR risk model, but not under the CF risk model. When the DR risk dominates, the expected returns and growth constructed from DR and CF factors are significant predictors of realized returns and growth at both market and individual stock levels, with R^2 ranging from 1.9% to 8.5% for returns and 1.2% to 14% for dividend growth. The predictability of both return and dividend growth is much poorer under the model with CF risks. Second, the DR news leads CF news in contributing to the variations of *lnpd*. If the DR risk is more influential than the CF risk in pricing, the movement of prices should be driven more by DR changing, and vice versa. In

² Campbell and Cochrane (1999), Menzly, Santos and Veronesi (2004), Santos and Veronesi (2005, 2010), Wachter (2006)

the time-series variances of $lnpd$ of all test portfolios, the innovation in market DR accounts for a fraction of 91.1% on average, compared with a fraction of 26.9% from the innovation in market CF. In explaining the cross-sectional variation of level of $lnpd$, the R^2 value attributed to the level of DR exposure is 81% and the R^2 value attributed to the level of CF exposure is -18% under the unconstrained model.

On the unexpected return side, the latent-factor approach highlights the importance of the difference between the ex-ante expected return and the ex-post realized return. In my model, the CF and DR state factors describe the ex-ante expected dividend growth and expected return. They are not observable directly and subject to exogenous dynamics. Discounting the future CF with time-varying DR, I develop an exact form of $lnpd$ as a function of these latent factors. This observation equation together with the dynamics of the latent factors constitute a state-space model, and the latent CF and DR factors are readily estimated using Bayesian Gibbs sampling method. I therefore separate the ex-ante expected CF and DR from their ex-post unexpected shocks with the estimated latent factors. Under this approach, one considers $lnpd$ as a proxy for both DR and CF and is able to estimate the latent factors without bias. Without the latent-factor approach, one considers $lnpd$ as a proxy only for DR and may result in bias in DR and CF. As a consequence of this approach, anomalies in cross-sectional returns may result from ex-ante conditional DR risk or ex-post unexpected realizations in CF and DR that are not priced in the expected returns. Given that the DR risk model is a version of the conditional CAPM, I find that the conditional CAPM explains well the cross section of the ex-ante expected return. The average of ex-ante expected return is totally explained by its level and stability of the time-varying market beta. To explain the ex-post realized return, the ex-ante expected return is in line with but not sufficient. Decomposing the time-series and cross-sectional variance of realized returns, one can

see that the unexpected CF shocks mainly move the return. Intuitively, the value stocks, past winner stocks and etc, have more positive surprises than negative surprises in their dividends in the sample, which results in their high average realized returns. Together, the ex-ante DR risk and the ex-post CF shock can account for 81.2% of the cross-sectional variance of average returns.

This paper adds to the recent literature employing present-value structure to identify the DR and expected CF by using information of dividend yield and dividend growth. Ang and Liu (2004), among others,³ provide expression for the price–dividend ratio as an infinite sum of exponentially quadratic forms of expected CF and DR. The information about DR of a portfolio is characterized by a one-factor model, and it is decomposed into information from marketwise DR and time-varying DR exposure (*beta*). To extend this framework, I integrate a one-factor structure on the CF side by bringing in marketwise CF and time-varying CF exposure to explain the CF of a portfolio. Adding this new ingredient, I endow the reduced-form model with the ability to embed CF risk in cross sections and conform to the motivations in long-run risk theories.

This paper is related to the literature focused on determining the main driving force between CF news and DR news in movement of prices and returns, including Vuolteenaho (2002), Campbell and Vuolteenaho (2004), Chen and Zhao (2009), and Chen, Da, and Zhao (2013).⁴ These studies evaluate the importance of DR news and CF news in the movement of stock prices or returns. This paper focus on the validity of risk channels related to fluctuations in DR and CF, specifically, test the likelihood of two models emphasizing different risks. This is the first paper jointly test the DR risk model and CF risk model in time series and cross sections under the present-value framework. From the aspect of results, this paper provides answers unifying

³ See Kelly and Pruitt (2012), Lettau and van Nieuwerburgh (2008), Cochrane (2008), van Binsbergen and Koijen (2010), Pastor and Veronesi (2003, 2006), Pastor, Sinha and Swaminathan (2008), Ang and Bakaert (2007), and Goyal and Welch (2003).

⁴ Also see Cohen, Polk and Vuolteenaho (2003) and Vuolteenaho (2002), among others.

the seemingly conflicting results in previous literature. Emphasizing the DR risk, I argue that the DR news causes larger movement in prices or expected returns, consistent with Campbell and Vuolteenaho (2004). On the other hand, unexpected CF news, although not priced in expected returns, strongly moves realized returns. This is on the same wavelength as Chen, Da, and Zhao (2013).

2 Model

In this section, I construct a present-value model to analyze variations in expected and realized return. The cornerstone of the framework is the assumption that the price of an asset is equal to the present value of all the dividends discounted by the expected returns. I derive the closed-form expressions of the prices and returns in a general setup M_0 in subsection 2.1. Afterward, I show that both types of models characterized with CF and DR risks (M_{CF} and M_{DR}) can nest into the general framework in subsections 2.2. This approach facilitates a head-to-head comparison of the models featuring the two risks. In subsection 2.3, I briefly discuss the strategies of estimating the model.

2.1 General Present-Value Model: M_0

Let P_t be the price at time t ; it is the discount value of all the future cash flows.

$$P_t = E_t \left[\sum_{s=1}^{\infty} \left(\prod_{k=0}^{s-1} \exp(-\mu_{t+k}) \right) D_{t+s} \right], \quad (1)$$

where D_t is the dividend distributed in the period $[t-1, t)$. In this paper, all the measures of price, return and dividend are in real terms, adjusted for inflation. The DR over the period $[t, t+1)$ is

μ_t , which is defined as the log of expected return,

$$\exp(\mu_t) \equiv E_t\left[\frac{P_{t+1} + D_{t+1}}{P_t}\right] \quad (2)$$

Let Δd_{t+1} denote the log of dividend growth,

$$\Delta d_{t+1} \equiv \log\left(\frac{D_{t+1}}{D_t}\right) \quad (3)$$

I can rewrite Equation (1) as

$$\frac{P_t}{D_t} = E_t\left[\sum_{s=1}^{\infty} \left(\prod_{k=0}^{s-1} \exp(-\mu_{t+k} + \Delta d_{t+k+1})\right)\right] \quad (4)$$

As a result, the price–dividend ratio reflects variations in both CF (Δd_t) and DR (μ_t). I use conditional factor models to characterize CF and DR. This creates a hierarchy in the pricing of assets: the pricing of individual stocks depends on the pricing of the market portfolio.

2.1.1 Market Portfolio Price

I first focus on the market portfolio. I label the market-level variables with superscript M from here on. There are five factors in the market-level state vector to describe the information of aggregate CF and DR: $X_t^M = [r_t^f \ g_t^M \ (\sigma_{g,t}^M)^2 \ z_t \ (\sigma_{z,t}^M)^2]'$. The first is the risk-free rate. The next four are conditional expectations and variances of the log of market dividend growth and excess return, as defined in the following equations:

$$\Delta d_{t+1}^M = g_t^M + \sigma_{g,t}^M u_{t+1}^d \quad (5)$$

where $g_t^M = E_t(\Delta d_{t+1}^M)$ and $(\sigma_{g,t}^M)^2 = \text{var}_t(\Delta d_{t+1}^M)$; and

$$r_{t+1}^M \equiv \log \frac{P_{t+1} + D_{t+1}}{P_{t+1}} - r_t^f = z_t + \sigma_{z,t}^M u_{t+1}^r \quad (6)$$

where $z_t = E_t(r_{t+1}^M)$ and $(\sigma_{z,t}^M)^2 = \text{var}_t(r_{t+1}^M)$. These definitions are based on the assumption that the realized log of dividend growth and excess return are their conditional expectations with heteroskedastic shocks.

I include the state variables of volatility to address Jensen's terms, which are potentially important components in the risk premium. Combining equation (3) and equation (5), the log of expected dividend growth is written as

$$\log E_t(\exp(\Delta d_{t+1}^M)) = g_t^M + \frac{1}{2}(\sigma_{g,t}^M)^2 \quad (7)$$

Combining equation (2) and equation (6), the DR μ_t^M , which is the log of expected return, is of the form as

$$\mu_t^M = r_t^f + z_t + \frac{1}{2}(\sigma_{z,t}^M)^2 \quad (8)$$

Among the state factors, the risk-free rate is exogenous, and the other four factors are treated as latent and endogenous. For simplicity of estimation, I use the state factors as the demeaned ones, and their unconditional means are treated as model parameters. The state vector evolves as a VAR(1) process.⁵

$$X_{t+1}^M = \Phi_M X_t^M + \Sigma_M^{\frac{1}{2}} \epsilon_{t+1} \quad (9)$$

I specify Φ_M as diagonal for parsimonious reasons, but the variance is a set of full entries, which allows all possible correlations in the shocks to the state vector.

Under the VAR dynamic, the state vector of current CF and DR is able to capture the information of all future CF and DR. Therefore, I can link the latent state vector with the observable

⁵ It has been argued that expected dividend-price-dividend growth rates have a persistent component; see, for example, Fama and French (1988), Campbell and Cochrane (1999), Ferson, Sarkissian, and Simin (2003), and Pastor and Stambaugh (2009). Further, many authors argue that the expected dividend growth is persistent, for instance, Bansal and Yaron (2004) and Lettau and Ludvigson (2005).

market price–dividend ratio in the following form.

$$\frac{P_t^M}{D_t^M} = \sum_{n=1}^{\infty} \exp(a_n^M + b_n^{M'} X_t^M) \quad (10)$$

The detail of the derivation is documented in Internet Appendix.

Equation (10) shows the price–dividend ratio is an infinite sum of exponential functions, a nonlinear form of the state vector. To estimate the state vector more efficiently, I apply the log-linearization⁶ to equation (10). The observable market *lnpd* is then approximated by a linear function of the state vector.

$$\ln pd_t^M = A^M + B^{M'} X_t^M + \sigma_v^M v_t^M \quad (11)$$

I approve the validity of this approximation method using simulations. There is little difference between *lnpd* calculated using the exact form and the linear proxy.⁷

2.1.2 Individual Stock Price

I next develop the individual level pricing based on the market-level pricing. I label all factors related to a specific stock *P* with superscript *P*. To capture the CF and DR of individual stocks, I each employ a conditional one-factor model:

$$g_t^P + \frac{1}{2}(\sigma_{g,t}^P)^2 = c^P + \gamma_t^P (g_t^M + \frac{1}{2}(\sigma_{g,t}^M)^2) \quad (12)$$

$$\mu_t^P - r_t^f = \alpha^P + \beta_t^P (\mu_t^M - r_t^f) \quad (13)$$

In equation (12), γ_t^P stands for the exposure of stock *P*'s CF to the market CF, and c^P is the error in the level of CF under the conditional factor model. Similarly in equation (13), β_t^P is the

⁶ This methodology is proposed by King, Plosser and Rebelo (1988) and Campbell (1994)

⁷ See Internet Appendix.

sensitivity of stock P's DR to the aggregate DR and α^P is the conditional *alpha* (pricing error) in the level of DR.⁸

As a result, the state vector for individual stock P includes the time-varying CF and DR exposures in addition to the market state vector X_t^M : $X_t^P = [X_t^{M'} \gamma_t^P \beta_t^P]'$. Again, the dynamics of the state vector are subject to an exogenous VAR(1) process:

$$X_{t+1}^P = \Phi_P X_t^P + \Sigma_P^{\frac{1}{2}} \epsilon_{t+1} \quad (14)$$

Since the first five state factors of X_t^P are identical to X_t^M , the upper left 5×5 blocks of Φ_P and Σ_P are the same as Φ_M and Σ_M .

Following a similar procedure as for the market, one can represent the observable price–dividend ratio of an individual stock as a function of current state vector. Notice that the CF and DR of stock P are quadratic forms of the state vector X_t^P . The price–dividend ratio is an infinite sum of the exponential quadratic forms of X_t^P .

$$\frac{P_t^P}{D_t^P} = \sum_{n=1}^{\infty} \exp(a_n^P + b_n^{P'} X_t^P + X_t^{P'} H_n^P X_t^P) \quad (15)$$

Applying log-linearization in equation (15), the linear observation equation for stock P is

$$\ln p d_t^P = A_t^P + B_t^{P'} X_t^P + \sigma_v^P v_t^P \quad (16)$$

The details for deriving these equations can be found in Internet Appendix.

In summary, the present-value model is a state-space model. The state vector X_t^P is latent and subject to a VAR(1) process. The observation equations link the latent vector to the observable

⁸ Ample evidence support time-varying expected market returns (Ferson and Harvey, 1991,1993; Lettau and Ludvigson, 2001; among others) and time-varying market beta (Ferson and Harvey, 1999; Ang and Chen, 2007; Ang and Kistensen, 2012; among others). The structure of the model is therefore originally motivated by these empirical findings.

realized returns, dividend growth, and especially price–dividend ratios via the discounted cash flow formula for both market and individual stocks. This model structure enables me to estimate the latent factors given the observable information.

2.1.3 Returns and Expected Returns

In addition to the variation in $lnpd$ revealed by equation (15), this paper also focuses on the variation in expected and realized returns. As shown by Campbell and Shiller (1988), the log of return relates to $lnpd$ by

$$r_{t+1}^P = \kappa_0 + \kappa_1 lnpd_{t+1}^P - lnpd_t^P + \Delta d_{t+1}^P \quad (17)$$

Taking conditional expectations on both sides of equation (17), one can obtain the expected return as

$$E_t(r_{t+1}^P) = \kappa_0 + \kappa_1 E_t(lnpd_{t+1}^P) - lnpd_t^P + E_t(\Delta d_{t+1}^P) \quad (18)$$

According to this equation, the expected return reflects the expected updates in price level, as well as the expected CF. Since the expected updates in price level include updates of information about DR and CF, the expected return is supposed to be a function of both DR and CF factors. Even though the DR factors in the state vector are designed to capture the expected returns, the expected returns may still have bearings on the CF factors because (1) the DR and CF factors are potentially correlated, and (2) the expected return is related to all future DR and CF.

In addition to the variations in expected returns, the variations in realized returns also consist of variations in unexpected components. Subtracting equation (18) from equation (17), one can divide the realized returns into expected and unexpected returns. The expected returns are driven by the state variables of expected CF and DR. The unexpected returns can be separate into

DR innovation (I_{DR}), as the unexpected update in future DR, and CF innovation (I_{CF}), as the unexpected update in future CF and realization in distribution.

$$r_{t+1}^P = E_t(r_{t+1}^P) + I_{DR,t+1}^P + I_{CF,t+1}^P \quad (19)$$

For a detailed explanation of the separation, I redirect the reader to Appendix D.

Equation (18) and (19) not only describe the time-series variations in expected and realized returns, but also decide the cross-sectional variations of their average. The average of the expected return is determined by the averages of DR and CF. The conditional one-factor model depicting DR with time-varying exposure is a version of conditional CAPM. As in Jagannathan and Wang (1996) and Lewellen and Nagel (2006), the average DR of stock P features an unconditional two-factor model.

$$\overline{\mu_t^P - r_t^f} = \alpha^P + \overline{\beta^P}(\bar{z} + \frac{1}{2}\overline{(\sigma_z^M)^2}) + \text{cov}(\beta_t^P, z_t + \frac{1}{2}(\sigma_{z,t}^M)^2) \quad (20)$$

The level of expected return results from conditional *alpha* (α^P), the level of DR exposure ($\overline{\beta^P}$), as well as the covariance terms describing the stability of DR exposure. Furthermore, the present-value model also allows influence on the mean of expected returns from the CF side. By the same token as DR, the average of CF is

$$\overline{g_t^P} + \frac{1}{2}\overline{(\sigma_g^P)^2} = c^P + \overline{\gamma^P}(\bar{g}^M + \frac{1}{2}\overline{(\sigma_g^M)^2}) + \text{cov}(\gamma_t^P, g_t^M + \frac{1}{2}(\sigma_{g,t}^M)^2) \quad (21)$$

The average of expected return may also depend on the level of CF exposure and covariance terms relevant to CF.

For unexpected return, I relax the rational expectation assumptions on CF and DR and summarize the average of realized returns of stock P as the sum of the mean of expected returns, DR

innovations, and CF innovations.

$$\overline{r_{t+1}^P} = \overline{E_t(r_{t+1}^P)} + \overline{I_{DR}} + \overline{I_{CF}} \quad (22)$$

A high level of realized returns may result from a high average of expected return, as well as positive realizations of CF and DR innovations.

2.2 Models with CF and DR Risk

Notice that the model I present in the previous subsection is quite general, and the source of risks driving the expected return is not yet specified. I next show how the models featuring CF and DR risk can be nested in as special cases of the general model M_0 . This hierarchy of model facilitates my choice of the more likely risk to explain expected return as choice of the more likely model to fit the data.

2.2.1 Constrained Model with CF risk: M_{CF}

Long-run risk in cash flows is a potential way of explaining the risk premium puzzle and cross-sectional anomalies.⁹ In this subsection, I also present a long-run risk model following the models of Bansal and Yaron (2004), Bansal, Kiku, Shaliastovich and Yaron(2013) rather closely, as a special case of the present-value model featuring cash-flow risks. A crucial assumption in the long-run risk literature is the existence of a persistent long-run risk component in the dividend growth. The specification equation (5) of cash flow is in accordance with this assumption. The utility function the long-run risk models try to maximize is the Epstein and Zin (1989) utility

⁹ see Bansal and Yaron (2004), Bansal Dittmar and Lundblad (2005), Bansal Kiku and Yaron (2009), Bansal, Kiku, Shaliastovich and Yaron (2013).

function, and the log pricing kernel is therefore

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta d_{t+1}^M + (\theta - 1) r_{a,t+1} \quad (23)$$

Here δ is the discount factor, $\theta \equiv \frac{1-\nu}{1-\psi-1}$, with ν as risk aversion and ψ as the intertemporal elasticity of substitution, and $r_{a,t}$ is the log return of the claim to aggregate consumption.

The following proposition provides necessary conditions on the market state variables in the specification of long-run risk.

Proposition 1 *Given the present-value model setup on market level as in equation (5), (6), (9), and (11), if the pricing kernel takes the form in equation (23), the conditional expectation and variance in excess return, z_t and $\sigma_{z,t}^M$, are linear in conditional variance in dividend growth. There exists parameters of χ_0 , χ_1 and v_0 , v_1 , such that*

$$z_t = \chi_0 + \chi_1 (\sigma_{g,t})^2 \quad (24)$$

$$(\sigma_{z,t})^2 = v_0 + v_1 (\sigma_{g,t})^2 \quad (25)$$

As a result, the VAR coefficients of $(\sigma_{g,t})^2$, z_t and $(\sigma_{z,t})^2$ are the same, and the correlation between the shocks on the three state variables are ± 1 .

$$\Phi_{\sigma_g} = \Phi_z = \Phi_{\sigma_z} \quad (26)$$

$$\rho_{\sigma_g z} = \rho_{\sigma_g \sigma_z} = \rho_{z \sigma_z} = \pm 1 \quad (27)$$

For the proof, I redirect the reader to the Appendix A.

At the market level, this proposition emphasizes the key implications from long-run risk models, that the market CF (g_t^M) and the CF volatility ($(\sigma_{g,t}^M)^2$) are the only state factors. $(\sigma_{g,t}^M)^2$ determines the market DR (z_t) and the DR volatility ($(\sigma_{z,t}^M)^2$). This requires the persistence in the dynamics of $(\sigma_{g,t}^M)^2$, z_t and $(\sigma_{z,t}^M)^2$ are all the same, and the shocks to these state variables are perfectly correlated.

Regarding the portfolio-level implications, I state in the next proposition.

Proposition 2 *With the present-value model setup for Stock P as in equation (14), (16), and the pricing kernel of long-run risk model in equation (23), the risk exposure β_t^P linearly depends on*

the dividend growth's loading in cash-flow γ_t^P . There exists parameters of η_0 and η_1 , such that

$$\beta_t^P = \eta_0 + \eta_1 \gamma_t^P \quad (28)$$

As a result, the VAR coefficients of β_t^P and γ_t^P are the same, and the correlation between the shocks on these two state variables is 1.

$$\Phi_\beta = \Phi_\gamma \quad (29)$$

$$\rho_{\beta\gamma} = \rho_{\gamma\beta} = \pm 1 \quad (30)$$

The proof is also shown in Appendix A.

At an individual stock level, the constraints characterizing M_{CF} , require the DR exposure β_t^P to reflect the sensitivity of the individual's CF to aggregate CF. This proposition shows that the market risk in this setting is identical to the risks from CF exposure γ_t^P , and this risk is related to the fluctuation in CF. As a result, the dynamics of β_t^P and γ_t^P are supposed to have the same persistence and common shocks.

2.2.2 Constrained Model with DR Risk: M_{DR}

In this subsection, I embed the market risk of DR in the framework, specifically patterning it after the habit formation models as in Campbell and Cochrane (1999) and Santos and Veronesi (2004,2010).

I follow the notation in Campbell and Cochrane (1999), and review the model in Appendix A. The habit formation model can be captured by two key features, the habit state variable and its dynamics. Here s_t is the log of consumption ratio surplus, reflecting the external habits, and it is subject to a heteroskedastic AR(1)

$$s_{t+1} = (1 - \phi)\bar{s} + \phi s_t + \Lambda(s_t)\sigma_{c,t}v_{t+1}$$

where $\Lambda(s_t)$ is the sensitivity function characterizing the heteroskedastic innovation in s_t .

The following proposition shows how Campbell and Cochrane (1999) is linked to the present-

value model as a special case.

Proposition 3 *Given the above notation of the habit formation model, there exists a specification of $\Lambda(s_t)$, such that the expected market excess returns is a function of s_t , $z_t = f(s_t)$; and the expected excess return of asset P is given as*

$$E_t(r_{t+1}^P) + \frac{1}{2}\text{var}_t(r_{t+1}^P) - (r_t^f + \pi_t) = \beta_t^P(z_t + \frac{1}{2}(\sigma_{z,t}^M)^2) \quad (31)$$

where

$$\beta_t^P = \text{cov}_t(r_{t+1}^P, r_{t+1}^M) / \text{var}_t(r_{t+1}^M) \quad (32)$$

For the details of the proof, I redirect the reader to Appendix A.

At market level, this proposition does not impose any constraints to accommodate M_{DR} . Comparing the structures of model M_{DR} and M_0 , the state variables s_t in M_{DR} can be mapped to the state variable z_t in M_0 , showing that the DR factors may reflect the states of external habit. Because one can arbitrarily select $\Lambda(s_t)$, there must exist a choice of $\Lambda(s_t)$ to ensure that the state variable z_t , which s_t is mapped to, characterizes the DR. Since the empirical analysis of this paper focuses on the pricing in cross sections, the exact form of $\Lambda(s_t)$ is not of interest for studying. Given this choice of $\Lambda(s_t)$, one can see M_{DR} and M_0 as equivalent to price the market.

At an individual stock level, the special case of M_{DR} requires the time-varying β to purely reflect conditional sensitivity of individual stock returns to market returns. In the general model M_0 , the time-varying β may reflect other information. This restriction, however, specially clarifies that β is a risk channel associated with fluctuations in DR. Also it constitutes a crucial observation equation for the time-varying β .

2.3 Estimation Strategy

In this section, I sketch the Gibbs sampling algorithm for estimating the present-value model, which is applicable to both general model M_0 and the two constrained models M_{DR} and M_{CF} .

The estimates of the latent variables and relevant parameters are used to investigate the importance of CF and DR risks in expected returns and explain the realized returns.

Notice that present-value model is a state-space model, and the latent state variables and the parameters describing their dynamics are readily estimated by using the Gibbs sampling method. The evolution dynamics are described by equation (14), for both the market level and the individual level. The observation includes equations (5), (6), and (11) for the market level, as well as equation (12), (13), and (16) for the individual stocks.

Since the pricing is ordered from the market level to the individual stock level, the estimation strategy also adheres to this sequence. I first estimate the market-level state variables of $[g_t^M, (\sigma_{g,t}^M)^2, z_t, (\sigma_{z,t}^M)^2]$ using Kalman filtering with the evolution equations and observation equations on market level.¹⁰ The set of market level parameters, $\Theta = (\bar{r}^f, \bar{g}^M, (\bar{\sigma}_g^M)^2, \bar{z}, (\bar{\sigma}_z^M)^2, \Phi_M, \Sigma_M, (\sigma_v^M)^2)$ are estimated via iteration of the Gibbs sampling method. Provided with the uniform estimates of the market-level state vector and parameters, I then draw the posterior distributions of state vectors and parameters of individual stocks using the individual-level data. The state variables $[\gamma_t^P, \beta_t^P]$ are estimated using Kalman filtering, and the parameters $\Theta = (\bar{\gamma}^P, \bar{\beta}^P, \Phi^P, \Sigma^P, (\sigma_v^P)^2)$ are obtained using iteration of the Gibbs sampler. The details of the algorithm including specification of the prior distributions are documented in Internet Appendix.

3 Data

In this section, I first describe the data and test portfolios employed by the empirical study. I then summarize the observable variables from which the latent variables can be inferred.

¹⁰ See Carter and Kohn (1994).

3.1 Data Description

In the empirical work, I use all NYSE, Amex, and NASDAQ common stocks for the period 1964 to 2012 from Center for Research in Security Price (CRSP) monthly files, merged with accounting data from Compustat. To address time-series and cross-sectional returns, I estimate the model on the market and seven groups of quintile portfolios reflecting anomalies in averages of returns. The market portfolio is formed as the value-weighted portfolio containing all common stocks in the CRSP universe. The test portfolios are sorted by size, book-to-market ratios, past returns, past idiosyncratic volatility, accrual component in earnings, capital investment, and liquidity separately. I document the labels and the details of constructing the test portfolios in Appendix B.

For each test portfolio, the observable variables are calculated as follows. Assuming the dividends are reinvested in 3-month T-bills (nominal risk-free rate),¹¹ one is able to compute the observable dividend growth and $lnpd$ from the value-weighted return with and without dividends.¹² All the dividend growth and returns are aggregated at an annual horizon to avoid seasonality, but sampled at quarterly frequency to include more observations. The data structure brings up an overlapping observation issue, as addressed by Hodrick (1992). My model can adapt to the overlapping data, and Internet Appendix address the technical details of specifications and model with overlapping data structure.

When the model M_{DR} is being estimated, there is one more observation equation about the time-varying β itself. The observable benchmark of β reflects the conditional covariance

¹¹ I follow Chen (2009) argument that when stock returns enter the calculation of dividend, the CF effect on $lnpd$ from dividend growth is contaminated.

¹² Given the surge in the share repurchase activity in this sample, I follow Bansal Dittmar and Lundblad (2005) to adjust capital gain and cash payment incorporating a candidate measure of repurchase.

between market and portfolio returns, and is directly estimated from short-window regressions, following Lewellen and Nagel (2006). At the end of month t , one can simply run OLS with all the daily returns in the following quarter after the month t to get the benchmark of β .

3.2 Data Summary

Table 1 summarizes the observable variables of the market and other test portfolios. For each sorting group, I choose the two extreme and the median portfolios for representative reporting. In columns 1-6, I report the mean and the standard error of the excess return, dividend growth, and the log price–dividend ratio. The last two columns report OLS estimates of the unconditional α ($\hat{\alpha}_{OLS}$) and the market β ($\hat{\beta}_{OLS}$) under the unconditional CAPM.

[Insert Table 1 Here]

The summary statistics reflect two features of the data. First, all three observables exhibit large time-series and cross-sectional variations. The standard deviations of the three observables are large for all portfolios, and the averages of the observables are dispersed. The valuation of CF and DR risk is based on how well the corresponding model fits these patterns. Second, the test portfolios demonstrate anomalies in the average of excess returns, which cannot be explained by unconditional CAPM.¹³ For example, the portfolios sorted by momentum display a spread of $(9.4\% - 0.8\% =) 8.6\%$ between average returns of “past winners” (MOM5) and “past losers” (MOM1). Taking CAPM market factor into account unconditionally, the spread between unconditional α ’s become even higher as 9.1%. The model in this paper breaks through with a new

¹³ An extensive literature documents the anomalies in return associated with these seven sorting: size (Fama and French 1992, 1993), book-to-market (Fama and French 1992, 1993), momentum (Jegadeesh and Titman 1993), idiosyncratic volatility (Ang, Hodrick, Xing, and Zhang 2006), accruals (Sloan 1996), capital investment (Titman, Sheridan, Wei, and Xie 2004; Liu, Whited, and Zhang 2009), illiquidity (Amihud 2002; Pastor and Stambaugh 2003).

approach from an unexpected perspective to explain these anomalies.

4 Results

In this section, I report a comprehensive analysis of the variations in expected and realized returns using the proposed present-value model. First, in examining the variation in expected return, I test whether the fluctuation of CF or DR is likely to be priced as risk, with both market-level and individual-level stock data. Second, in examining the variation of realized return, I evaluate the power of risk from expected return, as well as that of unexpected CF and DR innovations, in explaining the time-series and cross-sectional realized return.

4.1 CF vs DR: Which is the more likely risk to model expected return?

To begin with, I argue that the DR risk is the more likely risk channel than the CF risk. One can draw this conclusion by testing the hypotheses as summarized in the propositions characterizing the CF and DR risk models. I list the hypothesized constraints associated with the risks at market and individual levels in the diagram below. Proposition 1 governs the CF risk constraints at market level, and Proposition 2 characterizes the CF risk at individual stock level. For DR risk, there are no market-level constraints, because M_0 is equivalent to a version of M_{DR} at market level, as shown in section 2.2.2. Proposition 3 specifies the constraints of DR risk at the individual stock level.

	M_{CF}	M_{DR}
Market Level	Proposition 1: $\Phi_{\sigma_g} = \Phi_z = \Phi_{\sigma_z}$ $\rho_{\sigma_g z} = \rho_{\sigma_g \sigma_z} = \rho_{z \sigma_z} = \pm 1$	-
Individual Stock Level	Proposition 2: $\Phi_\beta = \Phi_\gamma$ $\rho_{\gamma\beta} = \pm 1$	Proposition 3: $\beta_t^P = \frac{cov_t(r_{t+1}^P, r_{t+1}^M)}{var_t(r_{t+1}^M)}$

One can test these propositions with classical statistical tests using the posterior mean and posterior standard error of the factors and parameters from the Gibbs estimates. However, there is one major drawback to the classical tests using Bayesian estimates: the low statistical power. In Bayesian statistics, the Bayes factor test is the formal practice for quantifying the evidence in favor of a scientific theory (Jeffreys 1935, 1960; Kass and Raftery 1995). For a model M_1 under hypothesis constraints and the general model M_0 as the alternative, the Bayes factor (LR_1) is defined as twice the difference between posterior log-likelihood of M_0 and M_1 given the observable data Y : $LR_1 \equiv 2(L(M_0 | Y) - L(M_1 | Y))$. The higher the Bayes factor, the more likely the unconstrained model is true and the greater the evidence against the hypothesis. This test is positively against the hypothesis (model M_1) if the Bayes factor is greater than 2, and strongly against it if greater than 6 (Kass and Raftery 1995). I calculate the posterior likelihood of the models with the Gibbs outputs, following Chib (1995). The details of the calculation can be found in Appendix C.

In addition, to make a choice of the more likely risk model for expected return, I examine whether the predictability of returns or dividend growth under each risk model is consistent with the existing literature. Furthermore, I evaluate the importance of CF and DR in driving $lnpd$ by decomposing the time-series and cross-sectional variance of $lnpd$. The relative importance of DR and CF variations in $lnpd$ has great relevance to modeling the expected return. For example, the crucial assumption of the long-run risk model is that $lnpd$ is purely driven by the CF factors. As a result, under the general model M_0 , the higher the portion of the variance of $lnpd$ that is attributed to a factor, the more likely fluctuation in this factor is a risk.

4.1.1 Proposition 1: CF Risk at Market Level

To begin with, I test Proposition 1 to examine CF risk at the market level. Recall this proposition hypothesizes that the state variables of $(\sigma_{g,t})^2$, z_t , and $(\sigma_{z,t})^2$ have the same VAR coefficients and perfectly correlated shocks under M_0 . In this subsection, I provide three reasons for rejecting these hypotheses.

First, I reject these hypotheses by using a classical test with the Bayes estimates. Table 2 reports the Bayes estimates of the market model parameters under M_0 and M_{CF} , separately in panel A and panel B. For each row, the posterior mean is reported as the upper number and the posterior standard error is reported as the lower one in parenthesis. The pertinent parameters in Proposition 1 are the last three estimates in the second row as VAR coefficients, and the last three in the bottom of correlation matrix of the shocks. Using these estimates, I jointly test this proposition with a χ^2_5 test and reject M_{CF} overall, since the p-value is less than 0.001.

[Insert Table 2 Here]

To further examine the difference in model structure introduced by CF risk, I compare the corresponding parameters in panel A and panel B, and perform a t-test on each parameter to check whether the estimates under the two different models are equal. As shown in the second row of both panels, the market CF and DR factors are persistent under both M_0 and M_{CF} , consistent with findings in Ferson et al. (2003) and Campbell and Cochrane (1999). However, there are significant differences between the VAR coefficients in z_t in the two panels. Under the general model, the market DR factor is more persistent than when it is restricted to reflect CF volatility only. Furthermore, the correlation between shocks of z_t , $(\sigma_{z,t})^2$, and $(\sigma_{g,t})^2$ are significantly different from 1 as proposed in the hypothesis. In summary, the hypothesis is rejected due to the

fact that variation in DR factor cannot be solely attributed to the change in CF volatility.

Second, the Bayes factor test rejects Proposition 1 and disproves CF risk as being plausible in modeling market prices. I calculate the Bayes factor statistics as 10.8, which can be interpreted as M_0 being about $(e^{10.8/2} =)$ 221 times more likely to generate the market data than M_{CF} .

The inferior performance of the CF risk model also appears in fitting $\ln pd$. Figure 1 compares the fitting of observable $\ln pd$ from estimates of M_0 and M_{CF} . The fitting R^2 values are 99% and 96%, respectively. Both models fit $\ln pd$ well overall; however, we can still observe that there is more deviation in the fitting by model M_{CF} .

[Insert Figure 1 Here]

Third, the predictabilities of market return and dividend growth under M_{CF} are less consistent with previous findings than those under M_0 . A large literature documents the existence of predictability in equity returns and dividend growth.¹⁴ As a result, a model characterized by an appropriate risk should produce expected returns and growth consistent with realizations. Since the predictability patterns for market return and dividend growth under M_{CF} contradicts former findings, I consider this as evidence for disqualifying CF risk.

Compared with the general model M_0 , M_{CF} contradicts the existence of return and dividend growth predictability. I present estimates of the latent expected dividend growth g_t^M and expected excess returns z_t at market level in Figure 2. Under M_0 , the predictive R^2 is 15% for the dividend growth, and 4.8% for the excess return.¹⁵ My results are comparable to the predictability results shown by van Binsbergen and Koijen (2010),¹⁶ and consistent with some other literature on

¹⁴ See Fama and French (1988), Campbell and Shiller (1988), Cochrane (2008), Menzly, Santos, and Veronesi (2004), Goetzmann and Jorion (1995), among others.

¹⁵ R^2 is calculated as $1 - \frac{\text{var}(\text{Realized Value} - \text{Expected Value})}{\text{var}(\text{Realized Value})}$.

¹⁶ van Binsbergen and Koijen (2010) estimate a reduced-form present value model. Using the data of dividend reinvested in cash, they show R^2 of predicting return is 8.2%, and R^2 of predicting cash flow is 13.9%.

predictability; for instance, Ang and Bakaert (2007), Lettau and van Nieuwerburgh (2008), and Cochrane (2008). All these papers report an R^2 of predictability in return is between 2% and 5%. In contrast, the expected dividend growth and excess return have negative R^2 under CF risk model M_{CF} , indicating the expectation is not a proper proxy for the realization.

[Insert Figure 2 Here]

The predictability literature better supports that $lnpd$ predicts market returns but not dividend growth (van Binsbergen and Koijen 2010). When $lnpd$ goes up, one expects a drop in return, not a rise in dividend growth. The estimates under M_0 are consistent with this argument, while the estimates under M_{CF} show the opposite. Considering the left two panels in Figure 2 for M_0 , one can see that $lnpd$ well predicts the market returns, as z_t negatively correlates with $lnpd$. When $lnpd$ was low in the late 1970s and early 1980s, z_t reached its peak, but when $lnpd$ was historically high around 2000, z_t was in the trough. According to nontabulated results, this correlation is -0.92. The CF factor g_t^M is less correlated with $lnpd$, and the correlation is about 0.44. In contrast, the two right panels tell a different story under M_{CF} . The estimates demonstrate predictability for dividend growth by $lnpd$, as the CF factor g_t^M becomes more correlated with $lnpd$. The correlation between g_t^M and $lnpd$ is 0.96. And the expected return z_t covaries less with $lnpd$, with a correlation of 0.28. However, this is inconsistent with the established empirical findings.

4.1.2 Proposition 2: CF Risk at Individual Portfolio Level

In the following steps, I test the CF risk using data at the individual portfolio level. There are two reasons for employing the cross-sectional data. First, the risk is mostly reflected by cross-section of expected returns since different levels of risks the stocks bear result in different levels

of awards in the expected returns. Second, bringing more information in relative pricing increases the sample size, therefore enhancing the power of the test.

Again, I begin with the classical statistical test. Proposition 2, associated with this scenario, hypothesizes that the DR and CF exposures have the same persistence and perfectly correlated shocks. Panel A of Table 3 reports the Gibbs estimates of Φ_β and Φ_γ and the correlation between their innovations $\rho_{\beta\gamma}$ under general model M_0 . Overall, Proposition 2 is rejected by a joint χ^2_{70} -test, with a p-value of 0.001.

[Insert Table 3 Here]

Taking a closer look at this proposition, I find the first part, $\Phi_\gamma = \Phi_\beta$, cannot be rejected using the t-test. Comparing the top left block with the middle left block in panel A of Table 3, one can see the means of Φ_γ and Φ_β for the same test portfolio are almost identical. Moreover, the standard error of both estimates in the top and middle right blocks are large enough that this part of the proposition cannot be rejected with a t-test.

The second part of Proposition 2, $\rho_{\beta\gamma} = \pm 1$ gets rejected. In the bottom left block of panel A, I show that the correlations between the shocks of β and γ are significantly different from ± 1 for all portfolios, using a t-test. All $\rho_{\gamma\beta}$'s are around zero, among which the largest deviation from zero belongs to the second largest size portfolio (SIZE2), as 0.121. The standard errors of all the estimates in the bottom right block are below 0.4. As a result, ± 1 is outside of the two standard error boundary around the posterior mean.

Furthermore, the Bayes factor test also rejects Proposition 2. In panel B of Table 3, the LR statistics is 17.58 given all the test portfolios, indicating M_0 is more than 6000 times more likely to generate the cross-section of *lnpd* as observed. I also report the Bayes factors given the

observation as portfolios within each sorting group and portfolios with different levels of returns; the model with CF risks can only weakly explain these data because the Bayes factors are larger than 6 in all cases.

At last, I show that CF risk is inconsistent with predictability of return and growth. Panel C of Table 3 compares the predictive R^2 for return and dividend growth under M_0 with R^2 under M_{CF} .¹⁷ The predictive R^2 for returns of all test portfolios are around 2% under M_0 as shown in the top left block. This result is consistent with the predictability at market level. In contrast, as shown in the top right block, 21 out of the 35 test portfolios have a negative predictive R^2 for returns under M_{CF} . For the dividend growth, the bottom left block shows predictive R^2 under M_0 ranges between 0.8% to 13.8%, while the R^2 values under M_{CF} in the bottom right block are mostly smaller and negative. The lack of predictability undermines the rationality of the expectations under the CF risks.

4.1.3 Proposition 3: DR risk at Individual Portfolio Level

Since there are no constraints associated with DR risk at market level, testing the DR risk with the data of individual portfolios is sufficient. The proposition characterizing DR risk requires the time-varying β to reflect the covariance between individual stock returns and the market returns. The results of classical test, Bayes factor test, and examination of predictability all support the DR risk.

To test this proposition in the classical way, one needs to compare β estimated under M_0 with the time-varying benchmark of β . The benchmark is calculated using short-window regression of daily data as mentioned in section 2.3.

¹⁷ R^2 is calculated in the same way as for the case of the market portfolio. The expected return and dividend growth is calculated using CF and DR exposures and the corresponding market factors, as given by equation (12) and equation (13).

[Insert Figure 3 Here]

I find that this proposition cannot be rejected for all test portfolios. As representative illustrations, Figure 3 shows the comparison of the estimated β under M_0 and the benchmark for four test portfolios. The two left panels are for the cases of two extreme portfolios sorted by book-to-market ratio, and the two right panels are for the cases of two extreme portfolios sorted by past returns. The solid lines are the benchmark of conditional β calculated by rolling regression of daily returns. The dash-dotted lines represent the posterior mean of estimated β under M_0 , and the dashed lines contour the boundary of the 95% confidence interval of the posterior distribution of β . Although the correlations between posterior mean of β and the benchmark vary for different test portfolios, the 95% confidence intervals mostly enclose the benchmark of β for all panels. In addition, the p-values of a χ^2 test of the identity in the proposition are above 0.05 for all test portfolios.

The Bayes factor test on this proposition agrees that DR risk is an eligible risk channel given individual portfolio data. The test statistics of Bayes factors are reported in panel A in Table 4. Based on the data containing all test portfolios, the Bayes factor is -0.93, which slightly favors the model with DR risks. In the test using the seven portfolios with the highest/medium/lowest return in each sorting groups, the model with DR risks fits the data well, with negative Bayes factor in all three scenarios. When testing the model by sorting groups, four out of seven sorting groups have negative Bayes factors as evidence for the DR risk model M_{DR} . Moreover, the evidence against the DR risk model from the other three sorting groups is not strong, with all three LR statistics being less than 6.

[Insert Table 4 Here]

In examining the predictability under M_{DR} , I find that the predictive R^2 for returns and dividend growth under M_{DR} is consistent with the existence of predictability. In panel B of Table 4, the R^2 values for returns range between 1.4% to 8.3%, and the R^2 values for growth range between 0.6% to 13.9% under M_{DR} across all test portfolios. They are similar to corresponding R^2 values under M_0 , as reported in panel C of Table 3. Unlike CF risk, the DR risk can result in expectations that feature in predicting their realizations.

4.1.4 Variance Decomposition in $lnpd$ under M_0

The result of this exercise demonstrates that DR dominates over CF in determining both dimensions of time-series and cross-sectional variations in $lnpd$, therefore it adds credit to the DR risk.

First, I decompose the time-series variance of $lnpd_t$ into fractions driven by CF and DR factors under the general model M_0 , following a general approach in the literature.¹⁸ I report the estimated fractions in $\text{var}(lnpd_t^P)$ of each state variable in Panel A of Table 5. The details of calculating the fractions are relegated to Appendix D.

[Insert Table 5 Here]

The results take the DR factor as the more important driving force of movement in prices, and the market-level variation as more influential than the variation in portfolio specific exposures. In the first row, I show that, on average over all test portfolios, the market DR (z) explains 91.1% of the total variance of $lnpd$, dominating over the market CF (g^M), which accounts for a fraction of 26.9%. The fractions attributed to variation in CF and DR exposure (γ and β) are 0.4% and 1.0% in the total variance of $lnpd$ on average. The exposure information is insignificant compared with market DR and CF factors, yet the DR exposure slightly outperforms the CF exposure. To show

¹⁸ see Campbell (1991), Campbell and Vuolteenaho (2004), Chen and Zhao (2009), Binsbergen and Koijen (2010), and others.

that the rank in explanatory power is not due to a special test portfolio, I report the maximum, median, and minimum of the estimated fractions over all test portfolios in row 2-4 of Panel A, as well as the decompositions using test portfolios with highest, medium, and lowest returns within each sorting group in row 5-7 in Panel A. The pattern of importance in attribution is consistent in all cases.

Second, one can cross-sectionally examine how the average of $lnpd$ is affected by the average of factors. The average of $lnpd$ depends on the average of CF and DR. Therefore, the cross-section variance is decomposed into variation in (1) the average of CF and DR exposures ($\bar{\gamma}$ and $\bar{\beta}$), (2) the conditional errors of CF and DR (c and α), and (3) the covariances describing the stability of the CF and DR exposures ($cov(\gamma, g^M)$, $cov(\gamma, \sigma_g^2)$, $cov(\beta, z)$, and $cov(\beta, \sigma_z^2)$). In Table 5, panel B reports the fraction of the cross-sectional variance of $lnpd$ attributed to each component. The details of the calculation of the fraction are also relegated to Appendix D. As the covariances of these statistics depicting cross-sectional pricing are double calculated, the sum of these fractions is not necessarily equal to one.

In explaining the average of $lnpd$, the determinants on the DR side occupy the dominant roles. $\bar{\beta}$ accounts for 81% cross-sectional variation in $lnpd$. $cov(\beta_t, z_t)$ explains a fraction of 22%, and α explains a fraction of 8%. On the other hand, the determinants on the CF side make a poor contribution to the variation in $lnpd$. The R^2 values explained by $\bar{\gamma}$ and c are negative. Yet $cov(\gamma_t, \sigma_g^2)$ is an exception, contributing 14% to the cross-sectional variation in $lnpd$. These results are based on the estimation from M_0 , but they are robust and don't rely on the model used for estimation.

4.2 CF vs DR: Which is More Important in Determining Realized Returns

Given the DR risk as the more likely risk channel being priced in the expected return, it is natural to investigate the following ideas: (1) how the expected and unexpected components together drive the realized return in time series, and (2) whether the cross-section of average expected return can fully justify the cross-section of average realized return.

To answer these two questions, I decompose the time-series and cross-sectional variance in realized returns using the same method that was applied to $lnpd$ in Section 4.1.4. Recall that the realized return is the sum of expected return and unexpected DR and CF innovations as expressed in equation (19). The variation in realized return is therefore due to the state factors affecting expected returns, as well as the DR and CF innovations.

In the time-series variance dimension, I argue that the unexpected CF plays the largest role in driving the realized returns. Panel A of Table 6 shows the fractions of time-series variance of returns attributed to the changes in expected state factors and unexpected DR and CF innovations under M_{DR} , as the DR risk is highlighted in the last section. The weights of expected DR and CF factors are dwarfed by the unexpected DR and CF innovations. According to the first row of panel A, the variation in z only accounts for 25.9% of the total variance of return on average, and g^M only accounts for 12.5%. In contrast, the CF innovation explains 59.1% on average, and the DR innovation explains 10.6% of the total variance in returns. To make sure these results are not due to only a few test portfolios, I provide a robustness check in the other rows of panel A.

[Insert Table 6 Here]

In the cross-sectional dimension, the results demonstrate that the cross-section of average expected returns, which are explained by the conditional risks, contribute to but are not sufficient

to explain the cross-section of average realized returns. When estimating the model, I pin down the cross-sectional averages of DR and CF with the information about *lnpd* only. By doing so, I relax the assumption of rational expectation for returns and growth. Therefore, the averages of unexpected DR and CF innovations may not be zero, and a large fraction of average return spread is due to the average of unexpected CF. Bringing expected and unexpected components together, I can readily explain the cross-section of average realized returns.

Before jumping to the decomposition results in detail, I first exhibit the cross-sectional patterns of the determinants affecting the level of realized returns to intuit some meanings. In Figure 4, I display the cross-sections of these determinants under the DR risk model. The patterns are robust if one chooses M_0 or even M_{CF} .

[Insert Figure 4 Here]

The upper left panel plots the cross-section of the conditional DR error α . I choose four representative sorting groups as SIZE, B/M, MOM, and LIQ to show the patterns. For all portfolios, α is very small, mostly under 0.5% annually, and the cross-sectional pattern is flat, hardly explaining any dispersions in returns.

The upper right and the lower left panels in Figure 4 exhibit $\text{cov}(\beta, z^M)$ and $\text{cov}(\beta, (\sigma_z^M)^2)$. For all four sorting groups, the cross-sections of $\text{cov}(\beta, z^M)$ and $\text{cov}(\beta, (\sigma_z^M)^2)$ are in line with the patterns of cross-sectional returns. The portfolios with higher excess returns also have higher covariance terms describing the stability of β . This relation is consistent with the conditional CAPM literature, such as Jagannathan and Wang (1996). However, quantitatively, the covariance terms are still too small to fill the gap.

Lastly, the lower right panels show the average of unexpected CF innovations ($\overline{I_{CF}}$), which

is calculated as the difference between the level of realized and expected dividend growth. One can see that the patterns of $\overline{I_{CF}}$ are in line with the realized returns for all sorting groups. For example, $\overline{I_{CF}}$ for the value portfolio (BM5) is higher than that for the growth portfolio (BM1) by 4 percentage points. For momentum sorted portfolios, the spread of $\overline{I_{CF}}$ between past winner (MOM5) and past loser (MOM1) is 8%. The quantity scale is large enough to potentially match the spreads in the unconditional *alpha* (α^u) shown in the summary Table 1.

The decomposition results substantiate the power of unexpected CF in explaining the spreads in realized returns. Panel B of Table 6 reports the fractions in the cross-sectional variation in realized returns explained by each determinant. The first column shows results under M_{DR} . The largest portion, a fraction of 64.6%, of the cross-sectional variance of returns is attributed to $\overline{I_{CF}}$. The covariance terms reflecting the stability of market beta determine the cross section of ex-ante expected return, hence realized return, as motivated by conditional CAPM: $\text{cov}(\beta, z)$ accounts for 22.2%, and $\text{cov}(\beta, \sigma_z^2)$ accounts for 14% of the cross-sectional variance of returns. The component influencing the expected CF c and $\text{cov}(\gamma, g^M)$ also contribute to the returns dispersion, but in a weaker way, accounting for 10% and 5.6% of the total cross-sectional variance of return, respectively. The levels of DR exposure $\bar{\beta}$ and CF exposure $\bar{\gamma}$, however, do not explain the return spreads. Taking all the factors into account, 81.2% of the cross-sectional average returns can be justified. These results are robust to the model I choose, and one can observe similar results in columns 2 and 3 in the table.

In summary, the realized returns reflect not only the variations in expectation, but the variations outside expectation as well. These unexpected variations are primarily attributed to the CF innovations. The unexpected CF innovations drive the realized returns' time-series variations. Furthermore, the overall level of the unexpected outcomes in dividend distribution is a key aspect

affecting the average of realized returns.

5 Conclusion

This paper presents a general present-value model that treats CF and DR factors as time-varying and latent. Assuming the exogenous VAR(1) dynamic for these factors, I develop closed-form formulae linking the state factors with the observable price–dividend ratios, realized returns and dividend growths. Estimating the model using Gibbs sampling method on portfolios capturing the most common anomalies, I provides a comprehensive analysis on the time-series and cross-sectional variations of stock returns.

First, the model can make a judgment of the likelihood of CF risk and DR risk. Both the model characterized by CF risk, as in Bansal and Yaron (2004), and the model characterized by DR risk, as in Campbell and Cochrane (1999), can nest as a special case of the model by imposing constraints on the pricing kernel. I argue that the DR risk dominates according to the Bayes factor test constructed by the estimates. Moreover, under the model dominated by DR risk, the expected return and dividend growth display predictability in the realizations.

Second, since the DR risk model is a version of conditional CAPM, I find the conditional CAPM can explain well the cross section of ex-ante expected return. The dispersions in the ex-ante expected return are due to the level and stability of time-varying market beta.

Third, the ex-ante return is not sufficient to justify the ex-post realized return. The difference is mainly due to unexpected CF shocks. Integrating both expected and unexpected variations, the model can explain 80% of the dispersions in realized excess returns cross-sectionally.

A future possible extension from this paper is to apply its reduced-form model to evaluate liquidity effects by including time-varying and latent liquidity factors. The liquidity effects may

be a breakout in explaining asset prices, especially those in an illiquid market.

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Appendix

A Proof of the Propositions

In this section, I present the proofs of the Propositions 3, 1, 2 in the main body of the text.

Proof of Proposition 1

In following Bansal, Kiku, Shaliastovich and Yaron (2013), I assume that the market portfolio gives the aggregate consumption.¹⁹ The total wealth portfolio's return, i.e. market return, is of the form

$$r_{M,t+1} = \kappa_0 + \kappa_1 \ln pd_{t+1}^M - \ln pd_t^M + \Delta d_{t+1}^M$$

According to the assumption of long-run risk model, the log price-dividend ratio only depend on the long-run expectation and conditional volatility of Δd_{t+1}^M .

$$\ln pd_{t+1}^M = A_0 + A_1 g_t^M + A_2 \sigma_{g,t}^2$$

In the equation above, A_0 , A_1 and A_2 are parameters, whose exact forms are ready to pinned down, but not important.

The log of pricing kernel in the Bansal Yaron (2004) model is $m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta d_{t+1}^M + (\theta - 1)r_{a,t+1}$. Without loss of generality, I identify $r_{a,t+1}$ with the market return. By log-linearization in Campbell and Shiller (1991), the return is approximated by $r_{a,t+1} = \kappa_0 + \kappa_1 \ln pd_{t+1}^M - \ln pd_t^M + \Delta d_{t+1}^M$. Here the log of price-dividend ratio depends only on the long-run component in dividend growth and its conditional volatility by the assumption of long-run risk model.

$$\ln pd_t^M = A_0 + A_1 g_t^M + A_2 (\sigma_{g,t}^M)^2$$

Plug it into the equation of aggregate returns, I can show that the innovation in the aggregate return is

$$r_{a,t+1} - E_t(r_{a,t+1}) = \kappa_1 A_1 \Sigma_{gg}^{\frac{1}{2}} \epsilon_{t+1}^g + \kappa_1 A_2 \Sigma_{\sigma_g \sigma_g}^{\frac{1}{2}} \epsilon_{t+1}^{\sigma_g} + \sigma_{g,t}^M u_{t+1}^d$$

As a result the conditional variance in the return is linear in $(\sigma_{g,t}^M)^2$.

$$(\sigma_{z,t}^M)^2 \equiv \mathbf{Var}_t(r_{a,t+1}) = (\sigma_{g,t}^M)^2 + const$$

Plug the innovation in aggregate returns in the log of pricing kernel, I get the innovation in the pricing kernel as

$$m_{t+1} - E_t(m_{t+1}) = (\theta - 1 - \frac{\theta}{\psi}) \sigma_{g,t}^M u_{t+1}^d + (\theta - 1) [\kappa_1 A_1 \Sigma_{gg}^{\frac{1}{2}} \epsilon_{t+1}^g + \kappa_1 A_2 \Sigma_{\sigma_g \sigma_g}^{\frac{1}{2}} \epsilon_{t+1}^{\sigma_g}]$$

The risk premium is therefore given by

$$\begin{aligned} z_t \equiv E_t(r_{a,t+1} - r_t^f) &= -\mathbf{Cov}_t(m_{t+1}, r_{a,t+1}) - \frac{1}{2} \mathbf{Var}_t(r_{a,t+1}) \\ &= -(\theta - 1 - \frac{\theta}{\psi}) (\sigma_{g,t}^M)^2 - (\theta - 1) \kappa^2 A_1^2 \Sigma_{gg} - (\theta - 1) \kappa_1^2 A_2^2 \Sigma_{\sigma_g \sigma_g} - \frac{1}{2} (\sigma_{z,t}^M)^2 \end{aligned}$$

I show that z_t is linear in $(\sigma_{g,t}^M)^2$ and $(\sigma_{z,t}^M)^2$, and since $(\sigma_{z,t}^M)^2$ is linear in $(\sigma_{g,t}^M)^2$, z_t is only a linear function of $(\sigma_{g,t}^M)^2$. The restrictions in the parameters are necessary given the linear relationships.

¹⁹ It is a traditional practice of substituting the return to aggregate wealth with the return on the stock marke in the static CAPM literature. It has also been practiced in empirical work based on recursive preferences (e.g., Epstein and Zin (1991) among others). The main reason of my choice of this assumption is that the empirical analysis this paper focus is the relative pricing in cross sections, this expedition in modeling has no influence on the covariances of the cash-flows and the discount rates.

Proof of Proposition 2

It suffices to show that the market risk β is linear in the cash-flow exposure γ . For a specific Portfolio P , the pricing equation is

$$\begin{aligned} E_t(r_{t+1}^P) + \frac{1}{2}\text{Var}_t(r_{t+1}^P) - r_t^f &= -\text{Cov}_t(r_{t+1}^P, m_{t+1}) \\ &= \frac{\text{Cov}_t(r_{t+1}^P, m_{t+1})}{\text{Cov}_t(r_{a,t+1}, m_{t+1})} [-\text{Cov}_t(r_{a,t+1}, m_{t+1})] \end{aligned}$$

The term in the bracket is the price of risk, and the ratio of covariances is the market risk β . Given the assumption that the only resource is the long-run risk from the cash-flow. The β is therefore linear in the cash-flow exposure γ .

Proof of Proposition 3

In reviewing Campbell and Cochrane (1999), the utility function maximized by identical agents

$$E \sum_{t=0}^{\infty} \delta^t \frac{(C_t - Hb_t)^{-\nu} - 1}{\nu - 1}$$

Here δ is the time discount factor, C_t is the level of consumption and Hb_t is the level of habit. Without loss of generality, I follow the original paper by setting the consumption growth as i.i.d.

$$\Delta c_{t+1} \equiv \log \frac{C_{t+1}}{C_t} = g_t^c + \sigma_{c,t} v_{t+1}$$

The log of pricing kernel is therefore

$$m_t = \log \delta - \nu(s_{t+1} - s_t) - \nu \Delta c_{t+1}$$

Here s_t is the log of surplus consumption ratio $S_t \equiv \frac{C_t - Hb_t}{C_t}$, and it subjects to a heteroskedastic AR(1)

$$s_{t+1} = (1 - \phi)\bar{s} + \phi s_t + \Lambda(s_t)\sigma_{c,t} v_{t+1}$$

where $\Lambda(s_t)$ is the sensitivity function characterizing the heteroskedastic innovation in s_t . Denote λ_t as $z_t + \frac{1}{2}(\sigma_{z,t}^M)^2$. If the pricing kernel can be put in the form of

$$m_{t+1} = -r_t^f - \frac{1}{2} \frac{\lambda_t^2}{(\sigma_{z,t}^M)^2} - \frac{\lambda_t}{\sigma_{z,t}^M} v_{t+1}$$

, then equations (31) and (32) are satisfied. Comparing the above form of pricing kernel with the Campbell and Cochrane (1999) pricing kernel, as long as $\Lambda(s_t)$ and g_t^M satisfying

$$\lambda_t = \nu(1 + \Lambda(s_t))\sigma_{g,t}\sigma_{z,t}$$

$$r_t^f = -\log \delta + \nu g_t^M + \nu[(\phi - 1)(s_t - \bar{s})] - \frac{1}{2}\nu^2(1 + \Lambda(s_t))^2(\sigma_{g,t}^M)^2,$$

then the Campbell and Cochrane (1999) model is embedded in the present-value model as a special case automatically.

B Detail of Portfolio Sorting

I employ 35 test portfolios, which are seven groups of quintile portfolios sorted on different stock characteristics. These portfolios are common portfolios reflecting the anomaly in the cross-sectional returns. For each sorting, I label the portfolios according to the *alpha* relative to the unconditional CAPM. The portfolios with the lowest *alpha* are labeled by number “1”, while the portfolios with highest *alpha* are labeled as number “5”.

The size portfolios (SIZE1-SIZE5) are formed at the end of each December based on the market capitalization at the end of year. The book-to-market portfolios (BM1-BM5) are rebalanced at the end of June in every year, based on the book-to-market ratio of each stock. The practice of forming both sets of portfolios follow the instruction in Fama and French (1992). We form and rebalance the momentum portfolios (MOM1-MOM5) by the rank of past returns between month $t - 12$ and month $t - 1$, at the end of each quarter of a year, following Jegadeesh and Titman (1993). The set of idiosyncratic volatility portfolios (VOL1-VOL5) are constructed as described in Ang et al (2006). In accordance with the notation in their paper, we construct a 3/0/3 portfolio, which means the portfolios are rebalanced at the end of a quarter and held for three months, according to the idiosyncratic volatility in the past quarter. We form the the quintile Accruals portfolios (ACC1-ACC5) at the end of each March in year t based on the

Accrual component in the earnings at the end of the year $t - 1$. The definition of the accrual is from Sloan (1996). We sort tertile portfolios (CI1-CI5) at the end of each June based on the capital investment of the previous year, following Titman, et al (2004) and Liu, et al (2009). The last sets of tertile portfolios (LIQ1-LIQ5) are formed by sorting on the liquidity of the stocks, using the liquidity measure constructed in Amihud (2002).

C Details of Calculating Bayes Factor

Denote M_1 as the model characterized by the hypothesis I want to test and M_0 is the general model without hypothetical constraints. The Bayes factor to test the hypothesis characterizing model M_1 is calculated as $2(L(M_0 | Y) - L(M_1 | Y))$, which is twice of the difference between marginal likelihood of the two models given observable data Y . It suffices to show how to calculate these two marginal likelihood.

The methods of calculating these two marginal likelihoods are the same, following Chibs (1995). The marginal likelihood is obtained using output from Gibbs sampling. Without loss of generality, I show this algorithm of $L(M_0 | Y)$ only.

Suppose $p(Y|\Theta, X)$ is the sampling density (likelihood function) and $\pi(\Theta)$ and $\pi(X)$ are the prior density. Then, the marginal likelihood can be written as

$$p(M_0 | Y) = \frac{p(Y|\Theta, X)\pi(X)\pi(\Theta)}{p(X, \Theta | Y)}$$

owing to normalized by posterior density of latent variables and parameters $p(X, \Theta | Y)$. This identity holds for any Θ and X_t . For any value of Θ^* and X_t^* , the proposed estimates of log of marginal density is therefore

$$L(M_0 | Y) = \log p(Y | \Theta^*, X^*) + \log \pi(\Theta^*) + \log \pi(X^*) - \log p(\Theta^*, X^* | Y)$$

The likelihood function and priors density are ready given any specification of X^* and Θ^* . I only need to estimate the posterior density. As the Gibbs sampler is defined through iterations of conditional density of X and Θ , the log of posterior density of latent variables and parameters can be written as

$$\log p(\Theta^*, X^* | Y) = \log p(\Theta^* | X^*, Y) \log p(X^* | Y)$$

The first term is the marginal coordinate, which can be estimated by initial draw of Gibbs sampler. The second term is given by

$$p(X^* | Y) = \int p(X^* | \Theta, Y)p(\Theta | Y)d\Theta$$

This can be estimated by draws from the reduced complete conditional Gibbs run.

$$\hat{p}(X^* | Y) = N^{-1} \sum_{j=1}^N p(X^* | \Theta^{(j)}, Y)$$

D Details of Calculating Fractions of Variation in Variance Decomposition

In this section, I present how to calculate the fractions accounted by state variables, in decomposition of the time-series and cross-sectional variance of *lnpd*, expected and realized returns.

In general, the state vector with I variables can be written as $X_t = (x_t^{(i)})_{i=1}^I$. Notate the state vector muting the $i - th$ variable as $X_t^{(-i)}$.

D.1 Time-series Variance Decomposition

I first address the time-series variance decomposition. As aforementioned in Equation (??), the *lnpd* is a function of these state variables.

$$lnpd_t = lnpd(X_t) \equiv A^P + B^P X_t + X_t' Q^P X_t' \quad (D.1)$$

The the fraction of time-series variance of *lnpd* explained by the $i - th$ ($i \leq I$) state variable is therefore,

$$R_i^2 = 1 - \frac{var(lnpd(X_t^{-i}))}{var(lnpd(X_t))}$$

The expected and realized return can also be written as function of state vector X_t , by virtue of *lnpd*. Using

Campbell and Shiller (1988) log-linearization, the expected return relates to the expected and current $lnpd$.

$$E_t(r_{t+1}^P) = \kappa_0 + \kappa_1 E_t(lnpd_{t+1}^P) - lnpd_t^P + E_t(\Delta d_{t+1}^P) \quad (D.2)$$

From Equation (D.1), one can derive the expected $lnpd$ as function of state vector X_t .

$$E_t(lnpd_{t+1}) = A^P + B^{P'} \Phi_P X_t + X_t' \Phi_P' Q^P \Phi_P X_t \quad (D.3)$$

Therefore one can see $E_t(r_{t+1}) = Er(X_t)$ is a function of the state vector X_t .

Then the fraction of time-series variance of expected return explained by the $i - th$ ($i \leq I$) state variable is,

$$R_i^2 = 1 - \frac{var(Er(X_t^{-i}))}{var(Er(X_t))}$$

For the realized return, Campbell and Shiller (1988) shows it has three components, expected return, unexpected shock in $lnpd$ and unexpected distribution in CF.

$$r_{t+1} = E_t(r_{t+1}) + \kappa_1(lnpd_{t+1} - E_t(lnpd_{t+1})) + (\Delta d_{t+1} - E_t(\Delta d_{t+1})) \quad (D.4)$$

As the shock in $lnpd$ has both DR and CF information, I further decompose it into two shocks: one is purely due to change in DR (I_{DR}) and the other is purely due to change in CF (I_{CF}).

$$\kappa_1(lnpd_{t+1} - E_t(lnpd_{t+1})) = I_{DR,t+1} + I'_{CF,t+1} \quad (D.5)$$

where

$$I_{DR,t+1} = \frac{1}{2} \kappa_1 [(lnpd(\mu_{t+1}, \omega_{t+1}) - lnpd(\mu_t, \omega_{t+1})) + (lnpd(\mu_{t+1}, \omega_t) - E_t(lnpd_{t+1}))] \quad (D.6)$$

$$I'_{CF,t+1} = \frac{1}{2} \kappa_1 [(lnpd(\mu_{t+1}, \omega_{t+1}) - lnpd(\mu_{t+1}, \omega_t)) + (lnpd(\mu_t, \omega_{t+1}) - E_t(lnpd_{t+1}))] \quad (D.7)$$

I then combine the shocks due to update in CF and unexpected realization of CF together to get $I_{CF,t+1} = I'_{CF,t+1} + (\Delta d_{t+1} - E_t(\Delta d_{t+1}))$.

As a result, besides the state vector X_t , the realized return is also function of these two unexpected components, $r_{t+1} = r(X_t, I_{DR,t+1}, I_{CF,t+1})$. For convenience, I notate $I_{DR,t+1}$ as x_t^{I+1} , and $I_{CF,t+1}$ as x_t^{I+2} .

The fraction of time-series variance of realized return explained by the $i - th$ ($i \leq I + 2$) state variable is

$$R_i^2 = 1 - \frac{var(r(X_t^{-i}))}{var(r(X_t))}$$

D.2 Cross-sectional Variance Decomposition

I first illustrate the cross-section variation decomposition in a general notation. Suppose one is interested in analyzing the cross-sectional variation in the average of f , and the average of f is of the function form of K cross-sectional statistics s_1, \dots, s_K .

$$\bar{f}^P = \bar{f}(s_1^P, \dots, s_K^P)$$

Therefore, the fraction of the cross-sectional variance attributed to the $k - th$ cross-sectional statistics is calculated as the remaining portion that cannot be explained by the other arguments.

$$R_k^2 = 1 - \frac{var(\bar{f}(s_1^P, \dots, s_{k-1}^P, 0, s_{k+1}^P, \dots, s_K^P))}{var(\bar{f}(s_1^P, \dots, s_K^P))} \quad (D.8)$$

To analyze the cross-sectional variance of $lnpd$, expected and realized returns, one need to find the function forms of their mean. Taking unconditional expectation on both sides of Equation (D.1), (D.2) and (D.4), one can see the mean of $lnpd$ and returns as function of cross-sectional statistics characterizing mean of DR and CF of individual stocks. I use over-line to notate for the average.

$$\overline{lnpd_t^P} = \overline{lnpd}(\bar{\beta}^P, \alpha^P, \text{cov}(\beta^P, z), \text{cov}(\beta^P, (\sigma_z)^2), \gamma^P, c^P, \text{cov}(\gamma^P, g), \text{cov}(\gamma^P, (\sigma_g)^2)) \quad (D.9)$$

$$\overline{E_t(r_{t+1}^P)} = \overline{Er}(\bar{\beta}^P, \alpha^P, \text{cov}(\beta^P, z), \text{cov}(\beta^P, (\sigma_z)^2), \gamma^P, c^P, \text{cov}(\gamma^P, g), \text{cov}(\gamma^P, (\sigma_g)^2)) \quad (D.10)$$

$$\overline{r_{t+1}^P} = \bar{r}(\bar{\beta}^P, \alpha^P, \text{cov}(\beta^P, z), \text{cov}(\beta^P, (\sigma_z)^2), \gamma^P, c^P, \text{cov}(\gamma^P, g), \text{cov}(\gamma^P, (\sigma_g)^2), \overline{I_{DR}}, \overline{I_{CF}}) \quad (D.11)$$

Plugging these functions forms in Equation (D.8), one can obtain the fraction of the total cross-section variance of various observables attributed to each explaining components.

Table 1: Summary Statistics

I report summary statistics of the market portfolio, as well as the other test portfolios which are sorted on size, book-to-market, momentum, idiosyncratic volatility, accrual component in earnings, capital investment and liquidity. I choose the two extreme and the median portfolio within each quintile sorting group to report. Data are from CRSP and Compustat, spanning from Jan 1964 to Dec 2012. The variables are sampled at quarterly frequency but measured at annual horizon. The mean and standard error are annualized. The excess returns are defined as returns in excess of the annualized 3-month Treasury bill rate. In the last two columns, I report OLS estimates of the alphas and betas from unconditional CAPM, by regressing the quarterly portfolio annual excess return on the market excess returns.

	$\mu(R^e)$	$\sigma(R^e)$	$\mu(g)$	$\sigma(g)$	$\mu(\ln pd)$	$\sigma(\ln pd)$	OLS Estimates	
							α^u	β^u
Market Portfolio	0.054	0.174	0.011	0.069	3.616	0.406	0.000	1.000
Size Portfolios								
SIZE1 (Large Cap)	0.051	0.169	0.006	0.069	3.575	0.442	0.000	0.948
SIZE3	0.074	0.204	0.026	0.121	3.932	0.546	0.016	1.082
SIZE5 (Small Cap)	0.078	0.259	0.036	0.176	4.372	0.516	0.014	1.186
B/M Portfolio								
BM1 (Growth)	0.046	0.188	0.010	0.125	4.083	0.387	-0.009	1.026
BM3	0.063	0.169	0.008	0.111	3.400	0.448	0.016	0.882
BM5 (Value)	0.096	0.193	0.031	0.265	3.436	0.476	0.046	0.928
Momentum Portfolios								
MOM1 (Losers)	0.008	0.249	-0.059	0.410	3.769	0.540	-0.057	1.216
MOM3	0.044	0.164	0.000	0.142	3.506	0.403	-0.004	0.886
MOM5 (Winners)	0.094	0.209	0.059	0.362	4.087	0.588	0.034	1.098
Idiosyncratic Portfolios								
VOL1 (Volatile)	-0.007	0.349	-0.073	0.712	5.428	1.070	-0.092	1.572
VOL3	0.067	0.246	0.011	0.247	4.379	0.799	-0.004	1.310
VOL5 (Stable)	0.058	0.151	0.012	0.067	3.469	0.410	0.013	0.829
Accrual Portfolios								
ACC1 (High Accrual)	0.023	0.206	-0.012	0.214	4.201	0.580	-0.037	1.110
ACC3	0.057	0.169	0.002	0.166	3.579	0.508	0.008	0.912
ACC5 (Low Accrual)	0.071	0.199	0.041	0.198	3.798	0.528	0.013	1.068
Capital Investment Portfolios								
CI1 (High Cap Invest)	0.050	0.181	0.007	0.190	3.860	0.499	-0.004	0.999
CI3	0.054	0.160	0.009	0.163	3.542	0.413	0.006	0.887
CI5 (Low Cap Invest)	0.080	0.202	0.038	0.222	3.924	0.602	0.021	1.099
Liquidity Portfolio								
LIQ1 (Liquid)	0.050	0.162	0.007	0.094	3.464	0.391	0.002	0.905
LIQ3	0.083	0.200	0.025	0.183	3.704	0.466	0.029	0.986
LIQ5 (Illiquid)	0.090	0.238	0.043	0.319	4.002	0.456	0.031	1.095

Table 2: Estimates of Market Parameters under M_0 and M_{CF}

Panel A: Estimates under General Model M_0

	r^f	g^M	$(\sigma_g^M)^2$	z	$(\sigma_z^M)^2$
Average	0.009 (0.002)	0.011 (0.005)	0.004 (0.000)	0.057 (0.013)	0.032 (0.000)
VAR coeff. Φ	0.938 (0.017)	0.958 (0.034)	0.671 (0.321)	0.984* (0.008)	0.709 (0.182)
Shock Variance Σ	0.084 (0.009)	0.041 (0.047)	0.007 (0.006)	0.011 (0.006)	0.461 (0.240)

Correlation Between Shocks: ρ				
	g_t	$(\sigma_{g,t}^M)^2$	z_t	$(\sigma_{z,t}^M)^2$
r_t^f	-0.244 (0.302)	-0.041 (0.368)	0.269 (0.260)	0.148 (0.377)
g_t		-0.053 (0.401)	-0.369 (0.277)	-0.142 (0.355)
$(\sigma_{g,t}^M)^2$			-0.027* (0.412)	-0.032* (0.395)
z_t				0.199* (0.358)

Panel B: Estimates under CF Model M_{CF} .

	r^f	g^M	$(\sigma_g^M)^2$	z	$(\sigma_z^M)^2$
Average	0.009 (0.002)	0.012 (0.005)	0.004 (0.000)	0.056 (0.012)	0.033 (0.000)
VAR coeff. Φ	0.939 (0.017)	0.975 (0.009)	0.684 (0.184)	0.684 (0.184)	0.684 (0.184)
Shock Variance Σ	0.084 (0.009)	0.017 (0.016)	0.008 (0.004)	1.017 (1.045)	0.444 (0.630)

Correlation Between Shocks: ρ				
	g_t	$(\sigma_{g,t}^M)^2$	z_t	$(\sigma_{z,t}^M)^2$
r_t^f	-0.548 (0.316)	0.551 (0.289)	0.551 (0.289)	0.551 (0.289)
g_t		-0.514 (0.299)	-0.514 (0.299)	-0.514 (0.299)
$(\sigma_{g,t}^M)^2$			1.000 (0.000)	1.000 (0.000)
z_t				1.000 (0.000)

Note to Table 2 I report the Gibbs estimates of posterior mean and standard error of the parameters related to the market. For each parameter, the upper row is the posterior mean, and the posterior standard error is reported in the lower row with parenthesis. Panel A shows estimates under M_0 or M_{DR} , as they are equivalent at market level. Panel B shows estimates under M_{CF} . For each parameter, I perform a t-test on the equality of estimates under different models, and using asterisk showing the significance (“*”: $p < 0.01$) in panel A.

Table 3: Rejecting the CF Risk Model M_{CF} at Individual Level

Panel A: Classical Test

Joint χ^2 test of Proposition 2: $p_{\chi^2} = 0.001$

	$m(\Phi_\beta)$					$s(\Phi_\beta)$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	0.845	0.780	0.886	0.904	0.908	0.469	0.464	0.469	0.469	0.469
BM	0.876	0.832	0.914	0.903	0.865	0.473	0.467	0.475	0.474	0.468
MOM	0.831	0.830	0.716	0.776	0.789	0.454	0.456	0.453	0.455	0.459
VOL	0.868	0.858	0.809	0.842	0.867	0.448	0.458	0.460	0.469	0.474
ACC	0.763	0.700	0.683	0.671	0.725	0.452	0.447	0.450	0.448	0.446
CI	0.638	0.988	0.812	0.714	0.825	0.430	0.494	0.424	0.442	0.463
LIQ	0.895	0.903	0.912	0.930	0.906	0.479	0.469	0.470	0.475	0.464
	$m(\Phi_\gamma)$					$s(\Phi_\gamma)$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	0.851	0.764	0.866	0.895	0.901	0.474	0.461	0.476	0.474	0.468
BM	0.877	0.787	0.892	0.891	0.863	0.476	0.466	0.482	0.471	0.455
MOM	0.827	0.816	0.674	0.773	0.772	0.451	0.473	0.442	0.449	0.437
VOL	0.872	0.866	0.825	0.821	0.847	0.447	0.452	0.456	0.474	0.472
ACC	0.773	0.682	0.673	0.671	0.748	0.451	0.444	0.451	0.446	0.442
CI	0.643	0.929	0.909	0.721	0.833	0.435	0.464	0.455	0.449	0.459
LIQ	0.901	0.892	0.905	0.923	0.896	0.485	0.472	0.476	0.476	0.463
	$m(\rho_{\gamma\beta})$					$s(\rho_{\gamma\beta})$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	0.042*	0.140*	-0.062*	0.071*	0.119*	0.336	0.393	0.320	0.356	0.377
BM	0.121*	-0.038*	0.076*	0.058*	0.025*	0.351	0.280	0.384	0.368	0.358
MOM	0.060*	-0.038*	0.039*	0.063*	0.128*	0.362	0.287	0.359	0.387	0.368
VOL	0.080*	0.041*	0.008*	-0.010*	0.048*	0.354	0.357	0.355	0.306	0.381
ACC	0.109*	0.004*	-0.018*	0.123*	0.036*	0.390	0.320	0.309	0.382	0.375
CI	0.088*	-0.200*	0.036*	-0.012*	-0.010*	0.377	0.260	0.236	0.341	0.329
LIQ	0.002*	0.048*	0.104*	0.115*	-0.057*	0.322	0.354	0.367	0.396	0.308

Panel B: Bayes Factor Test

	ALL	Size	B/M	MOM	VOL	ACC
LR_{CF}	17.6	12.6	8.3	12.3	7.6	25.2
	CI	LIQ	High Ret	Med Ret	Low Ret	
LR_{CF}	21.7	10.5	14.0	7.3	33.2	

Panel C: Predictive R^2 in under M_0 and M_{CF}

R^2 for Return

	M_0					M_{CF}				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	0.021	0.020	0.020	0.022	0.023	0.050	-0.025	-0.029	-0.003	-0.025
BM	0.017	0.020	0.024	0.026	0.019	0.046	-0.022	-0.023	-0.005	-0.022
MOM	0.017	0.022	0.022	0.023	0.022	0.049	-0.004	-0.008	0.061	-0.003
VOL	0.016	0.014	0.020	0.022	0.025	0.022	0.052	0.031	0.027	0.018
ACC	0.021	0.020	0.020	0.021	0.020	-0.025	0.050	-0.029	-0.033	-0.016
CI	0.020	0.022	0.026	0.020	0.022	-0.008	-0.025	-0.050	-0.020	-0.023
LIQ	0.023	0.021	0.021	0.030	0.029	0.045	-0.011	-0.016	0.050	0.050

R^2 for Dividend Growth

	M_0					M_{CF}				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	0.137	0.051	0.066	0.068	0.032	-0.151	0.013	-0.044	-0.001	0.092
BM	0.040	0.041	0.039	0.062	0.048	0.010	0.074	0.048	0.022	0.060
MOM	0.018	0.015	0.055	0.031	0.015	0.058	-0.012	-0.079	-0.027	0.017
VOL	0.021	0.013	0.015	0.050	0.120	-0.009	-0.011	0.065	0.028	-0.071
ACC	0.037	0.034	0.022	0.057	0.006	0.012	-0.018	-0.095	-0.088	-0.079
CI	0.022	0.052	0.031	0.027	0.018	0.040	-0.002	-0.180	-0.059	0.049
LIQ	0.093	0.038	0.033	0.028	0.008	0.006	0.061	0.014	0.013	-0.023

Note to Table 3 I provide evidence that rejects M_{CF} using individual stock data from three aspects. The associated hypothesis is Proposition 2: the CF and DR exposure has the same VAR coefficients and their shocks are perfectly correlated.

$$\Phi_\gamma = \Phi_\beta; \quad \rho_{\beta\gamma} = \pm 1$$

Panel A reports the posterior mean ($m(\cdot)$) and standard error ($s(\cdot)$) of the pertinent parameters for the 35 test portfolios. For each parameter I also perform t-test on its specified hypothesis in the proposition and report the significance using asterisk (“*”: $p < 0.01$).

Panel B reports Bayes factor as the log of the posterior likelihood ratio of the general model M_0 over the CF risk model M_{CF} , given various test data D .

$$LR_{CF} = 2 \log \frac{Pr(M_0 | D)}{Pr(M_{CF} | D)}$$

Panel C compare the predictive R^2 under M_0 and M_{CF} at individual portfolio level, for return and growth. The R^2 for dividend growth of stock P is calculated by

$$R_g^2 = 1 - \frac{\text{var}(\Delta d_{t+1}^P - \mathbf{E}_t(\Delta d_{t+1}^P))}{\text{var}(\Delta d_{t+1}^P)}$$

The R^2 for excess return is calculated as

$$R_r^2 = 1 - \frac{\text{var}((R_t^{P,e}) - \mathbf{E}_t((R_t^{P,e})))}{\text{var}((R_t^{P,e}))}$$

Table 4: Supporting DR Risk Model M_{DR} at Individual Level

I provide evidence supporting M_{DR} with individual stock data. Panel A shows Bayes factor as the log of the posterior likelihood ratio of the general model M_0 over the DR risk model M_{DR} , given various test data D .

$$LR_{DR} = 2 \log \frac{Pr(M_0 | D)}{Pr(M_{DR} | D)}$$

Panel B reports individual portfolio level predictive R^2 for return and growth under M_{DR} . The R^2 for dividend growth of stock P is calculated by

$$R_g^2 = 1 - \frac{\text{var}(\Delta d_{t+1}^P - \mathbf{E}_t(\Delta d_{t+1}^P))}{\text{var}(\Delta d_{t+1}^P)}$$

The R^2 for excess return is calculated as

$$R_r^2 = 1 - \frac{\text{var}((R_t^{P,e}) - \mathbf{E}_t((R_t^{P,e})))}{\text{var}((R_t^{P,e}))}$$

Panel A: Bayes Factor Test

	ALL	Size	B/M	MOM	VOL	ACC
LR_{DR}	-0.93	2.0	0.3	3.0	5.6	0.1
	CI	LIQ	High Ret	Med Ret	Low Ret	
LR_{DR}	-1.2	-2.1	-2.8	-1.1	-1.2	

Panel B: Predictive R^2 under M_{DR}

	Predictive R^2 for Return					Predictive R^2 for Dividend Growth				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	0.028	0.019	0.038	0.038	0.019	0.139	0.050	0.065	0.070	0.031
BM	0.030	0.027	0.022	0.025	0.025	0.041	0.041	0.038	0.062	0.047
MOM	0.025	0.025	0.029	0.049	0.083	0.016	0.015	0.055	0.030	0.014
VOL	0.014	0.017	0.017	0.023	0.028	0.019	0.013	0.015	0.051	0.120
ACC	0.022	0.023	0.024	0.027	0.024	0.037	0.034	0.022	0.057	0.006
CI	0.026	0.032	0.032	0.025	0.018	0.022	0.051	0.033	0.027	0.017
LIQ	0.037	0.021	0.020	0.020	0.025	0.098	0.037	0.033	0.030	0.007

Table 5: Variance Decomposition of Log Price-dividend Ratio

Panel A decomposes the time-series variance of $lnpd$ and reports the percentage fractions of total variance driven by the state variables under M_0 . AVG row presents the simple average of the fraction among all test portfolios. The “Min”, “Med” and “Max” reports the minimum, maximum and median of the fractions across all the portfolios. The “High Ret”, “Med Ret” and “Low Ret” report the median of the fraction among the portfolio with high, medium and low returns, respectively. Panel B decomposes the cross-sectional variance of $lnpd$ and reports the fractions attributed to the influencing components, including average of CF and DR factors, conditional errors in CF and DR, and the covariance in CF and DR exposure with the aggregate factors. Column 1 is for the general model M_0 , Column 2 is for the DR risk model M_{DR} and Column 3 is for the CF risk model M_{CF} .

Panel A: Decompose Time-series Variance of $lnpd$

	r^f	g	σ_g^2	z	σ_z^2	γ	β
AVG	6.4%	26.9%	0.0%	91.1%	0.1%	0.4%	1.0%
Max	10.1%	56.3%	0.0%	107.0%	0.1%	2.7%	6.5%
Median	6.2%	25.3%	0.0%	98.1%	0.1%	0.1%	0.3%
Min	2.4%	7.1%	0.0%	51.7%	0.0%	0.0%	0.0%
High Ret	5.9%	23.3%	0.0%	90.4%	0.1%	0.6%	1.0%
Med Ret	5.9%	27.4%	0.0%	91.6%	0.1%	0.5%	1.2%
Low Ret	6.1%	31.0%	0.0%	85.8%	0.1%	0.4%	1.4%

Panel B: Decompose Cross-sectional Variance of $lnpd$

	M_0	M_{DR}	M_{CF}
$\bar{\beta}$	81%	53%	31%
α	8%	6%	3%
$cov(\beta, z)$	22%	11%	11%
$cov(\beta, \sigma_z^2)$	-9%	-9%	-8%
$\bar{\gamma}$	-18%	-17%	-17%
c	-5%	-1%	-3%
$cov(\gamma, g)$	14%	10%	-36%
$cov(\gamma, \sigma_g^2)$	3%	2%	27%

Table 6: Variance Decomposition of Return

Panel A reports statistics of the percentage of variance in realized return driven by the state variables (covariances between state variables) over test portfolios under M_{DR} . AVG row presents the simple average of each fraction among all test portfolios. The “Min”, “Med” and “Max” reports the minimum, maximum and median of each fraction across all the portfolios. The “High Ret”, “Med Ret” and “Low Ret” report the median of each fraction among the portfolio with high, medium and low returns, respectively. Panel B reports the fractions of the cross-sectional variance of realized return attributed to the determinants, including average of CF and DR factors, conditional intercept of CF and DR, the covariance in CF and DR exposure with the aggregate factors, and average of DR and CF shocks. Column 1 is for the DR risk model M_{DR} , Column 2 is for the general model M_0 and Column 3 is for the CF risk model M_{CF} .

Panel A: Time-series Variance Decomposition

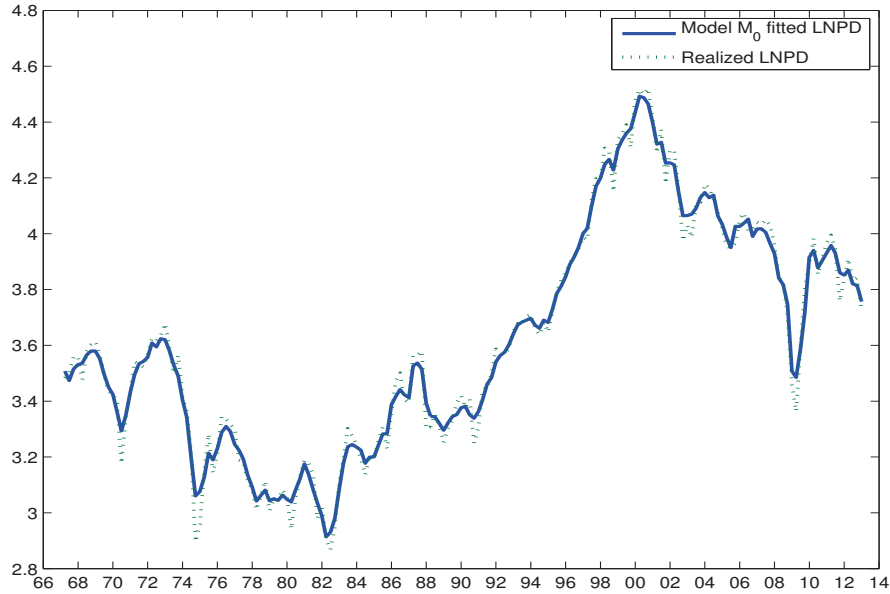
	r^f	g	σ_g^2	z	σ_z^2	γ	β	I_{DR}	I_{CF}
AVG	7.4%	12.7%	0.0%	25.7%	0.1%	0.2%	0.5%	10.6%	59.1%
Max	16.6%	41.0%	0.2%	40.3%	0.2%	1.3%	2.5%	17.1%	91.7%
Median	7.1%	11.7%	0.0%	25.8%	0.1%	0.0%	0.2%	10.3%	59.5%
Min	0.5%	3.3%	0.0%	2.5%	0.0%	0.0%	0.0%	2.3%	32.9%
High Ret	7.6%	9.8%	0.0%	23.4%	0.1%	0.3%	0.5%	9.4%	63.2%
Med Ret	9.0%	11.6%	0.0%	28.3%	0.2%	0.1%	0.2%	10.0%	59.5%
Low Ret	9.1%	18.6%	0.0%	26.4%	0.1%	0.1%	0.3%	12.6%	54.8%

Panel B: Cross-sectional Variance Decomposition

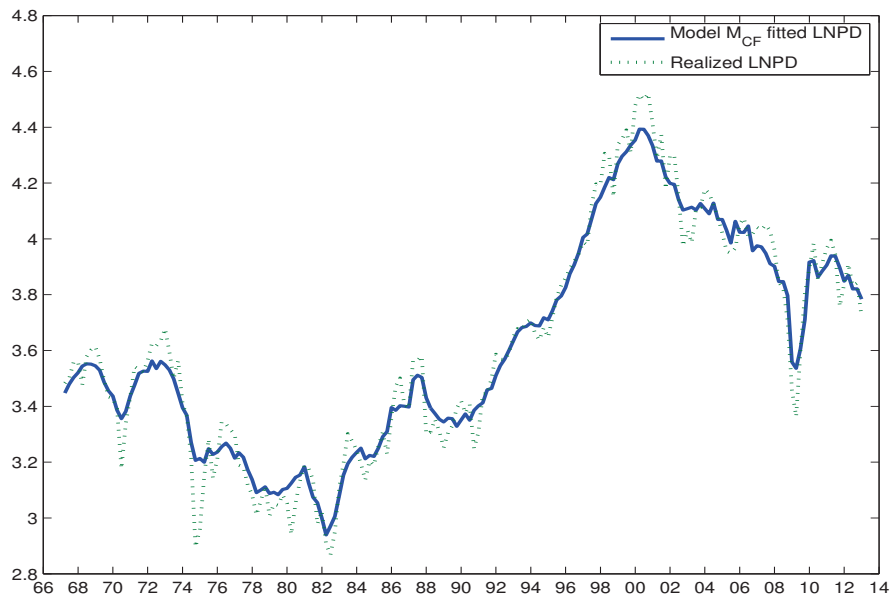
	M_{DR}	M_0	M_{CF}
β	-7.5%	-5.4%	-6.6%
α	-1.4%	-1.9%	-1.01%
$cov(\beta, z)$	22.2%	24%	19.2%
$cov(\beta, \sigma_z^2)$	14.0%	8.6%	7.7%
$\bar{\gamma}$	-2.5%	-3.1%	-2.3%
c	10.0%	13.2%	4.9%
$cov(\gamma, g)$	5.6%	6.4%	8.5%
$cov(\gamma, \sigma_g^2)$	0.3%	0.3%	0.5%
\bar{I}_{DR}	0.2%	0.5%	0.3%
\bar{I}_{CF}	64.6%	56.2%	55.2%
Total	81.2%	80.8%	78.2%

Figure 1: Quality of Fitting Market Price-dividend Ratio under M_0 and M_{CF}

Model M_0 Fitted and Realized log of Price-dividend Ratio, $R^2 = 99\%$

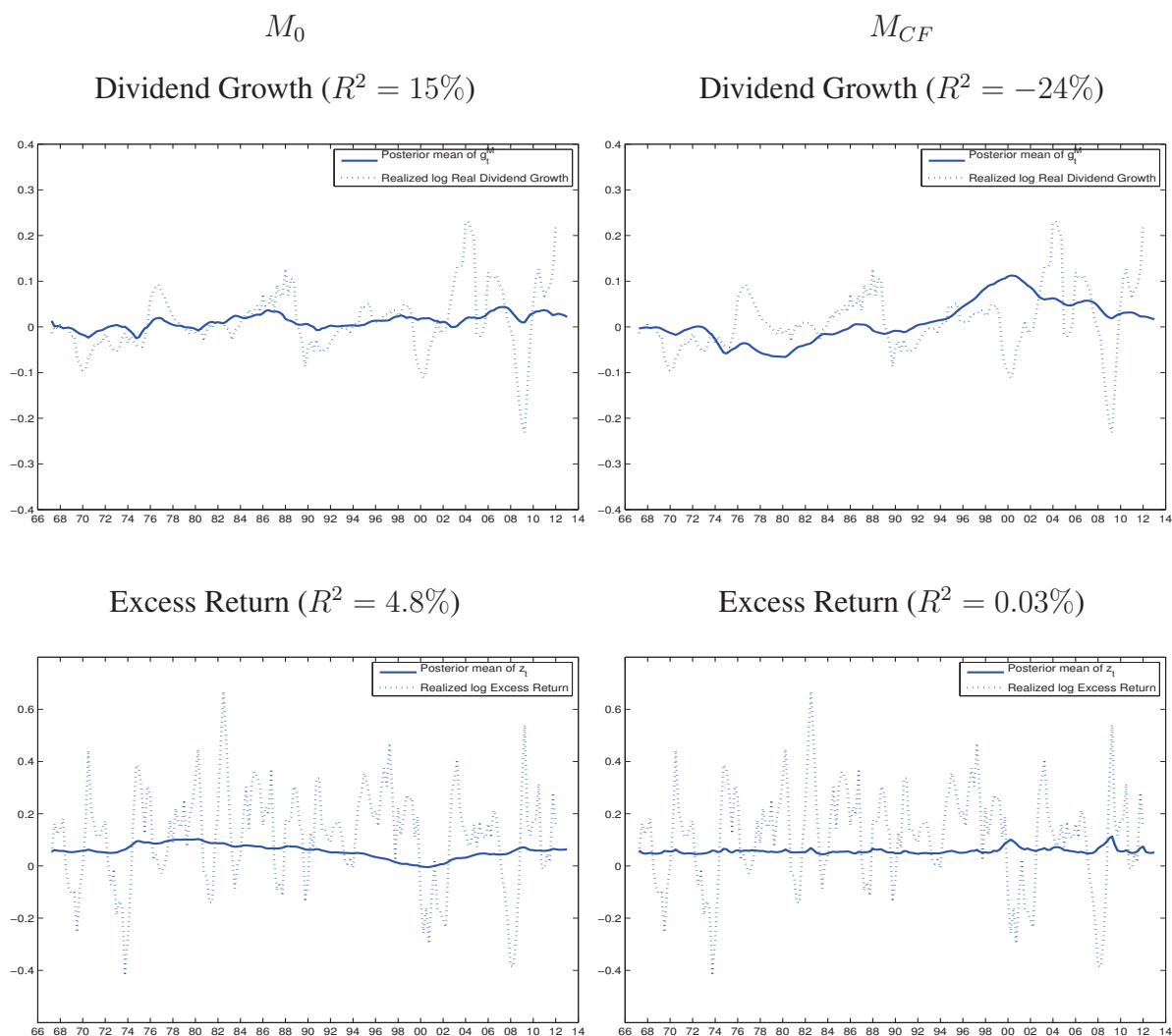


Model M_{CF} Fitted and Realized log of Price-dividend Ratio, $R^2 = 96\%$



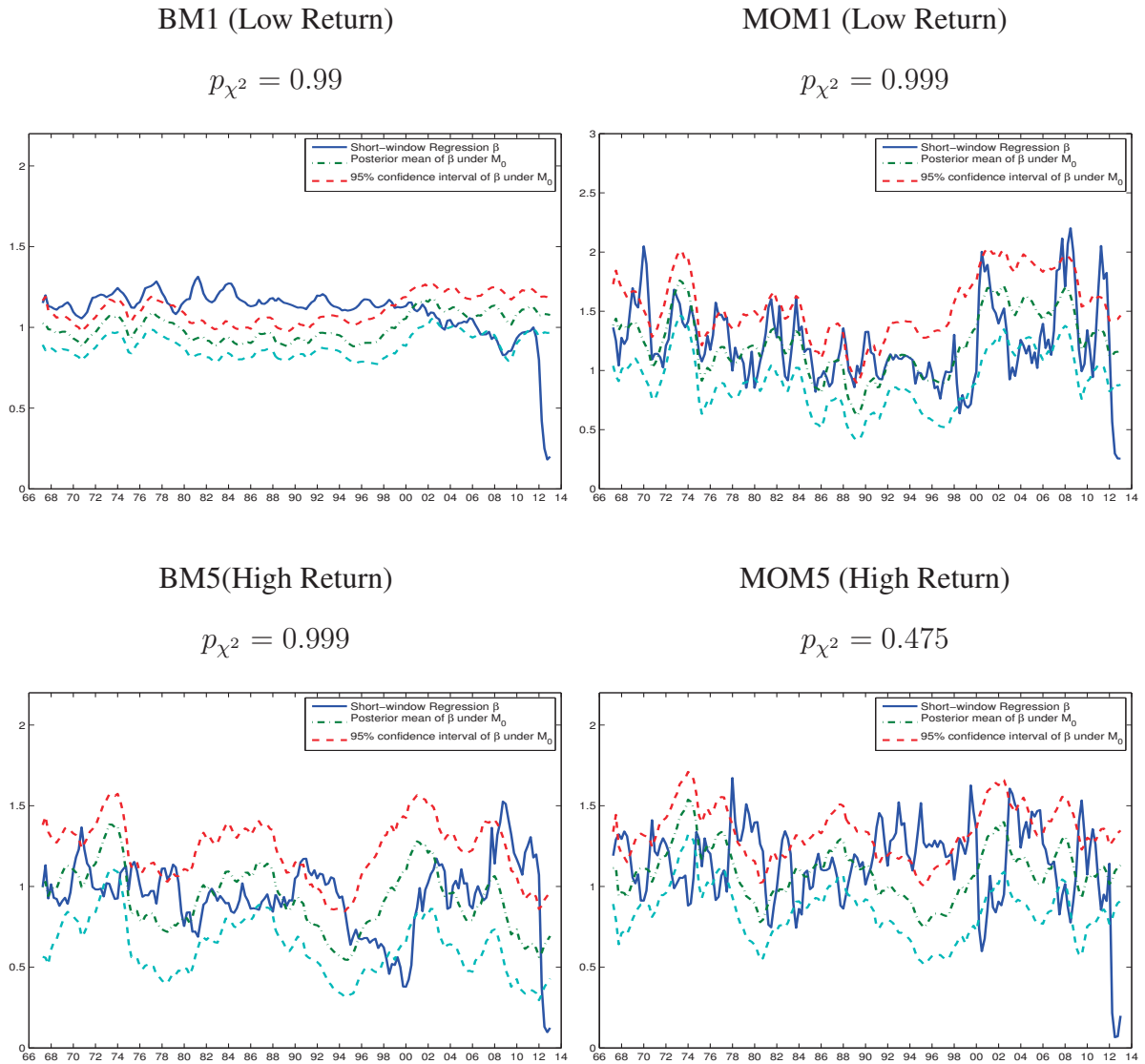
The plots compare the realized market log of price–dividend ratio and its fitted values from the models M_0 and M_{CF} . The R^2 is calculated by $1 - \text{var}(\text{Real } \ln pd - \text{Fitted } \ln pd) / \text{var}(\text{Real } \ln pd)$.

Figure 2: CF and DR Predict Dividend Growth and Return under M_0 , but not under M_{CF}



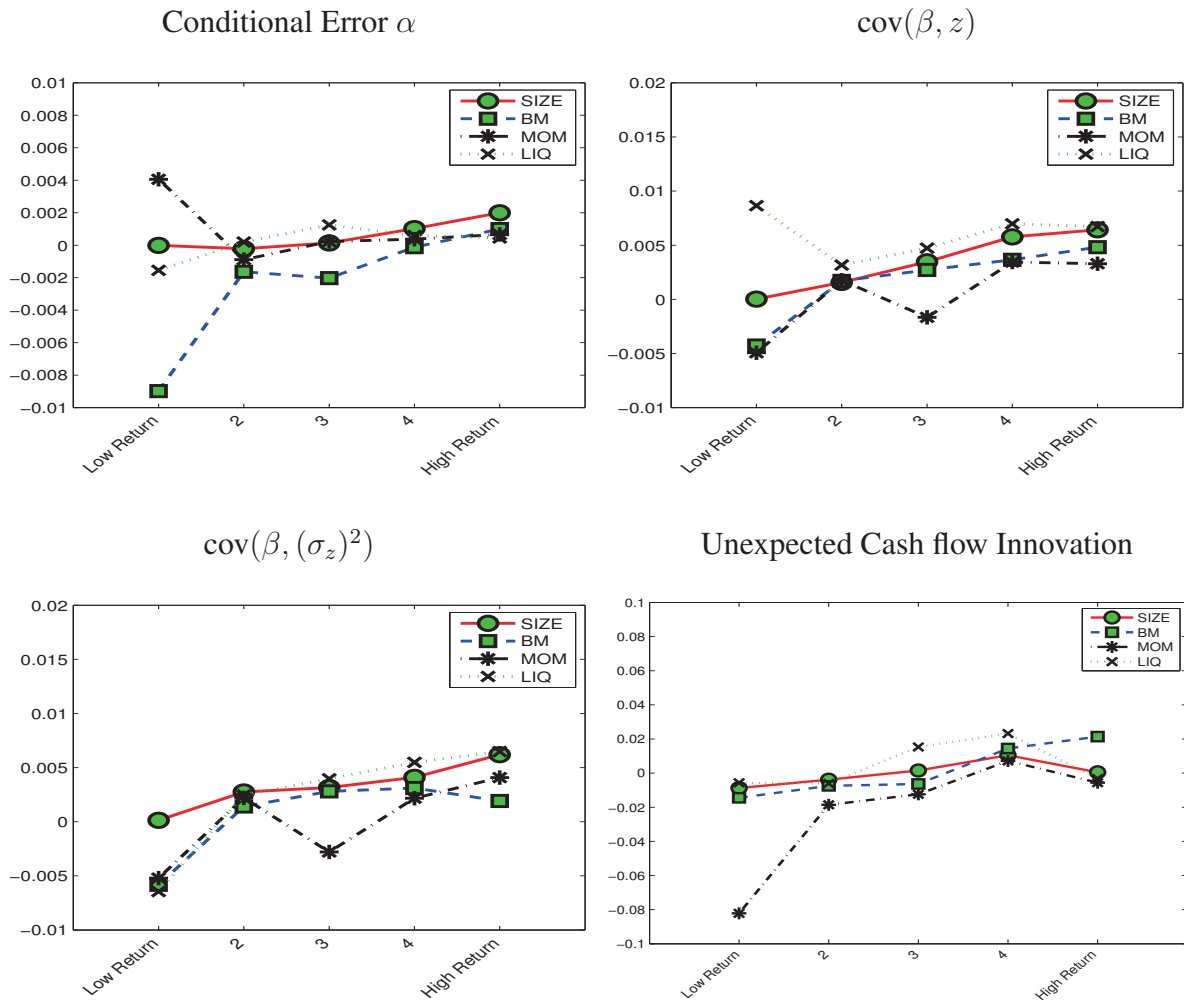
The plots compare the filtered series of expected dividend growths and excess returns (solid lines) with their realized values (dotted lines) under two specifications of risks. The upper two panels show the expected and realized log of dividend growth, while the lower two panels compare the expected and realized excess returns. The left two panels reflect estimations in the general model M_0 , and the right panels represent the CF risk model M_{CF} . The R^2 is calculated by $1 - \frac{\text{var}(\text{Realized Value} - \text{Expected Value})}{\text{var}(\text{Realized Value})}$

Figure 3: Estimates of Time-Varying β under M_0 and Short-window Regression Benchmark under M_{DR}



The plots compare the filtered series of β under the general model M_0 with the Benchmark of β under DR risk model M_{DR} . The Benchmark of M_{DR} (solid line) is calculated using short-window regression of daily individual stocks returns on daily market returns. The posterior mean of the time-varying β (dash-dotted line) and its 95% confidence interval (dashed line) is estimated using Kalman filtering iterated in the Gibbs sampling method. The p-value is associated to the χ^2 test on the equality between point estimates of β and the benchmark.

Figure 4: Determinants of Cross-sectional Returns under M_{DR}



The figure shows cross-sectional patterns of the statistics that affect the cross-sectional average realized returns. I choose four sorting groups as representatives. All the statistics are obtained under DR risk model M_{DR} . α is the conditional intercept as the pricing error in the expected return. $cov(\beta, z)$ and $cov(\beta, (\sigma_z)^2)$ characterize the stability of time-varying β . The unexpected cash flow is the average of the difference between realized CF and the expected CF.