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Active Risk Budgeting in Action: Assessing Risk and Return in Private Equity

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Executive Summary

Since private equity investments are not regularly traded and are limited by poor data quality, it is difficult to estimate risk and return statistics for this asset class. In addition, valuations for private equity partnerships are almost always out of date, due to the timing of underlying transactions. One might argue that the Capital Asset Pricing Model (CAPM), a basic investing framework that puts portfolio returns in the context of equilibrium, can not be applied to private equity. However, we believe private equity can be naturally decomposed in the same way as other investments: risk-free rate, exposure to a suitably chosen public index, and alpha. By comparing some elements of private equity investing to the public markets, we believe a basic investing framework can be used. The real issue does not lie in the framework, but rather, how we handle the issues that are specific to private equity when applying the framework.

We use results from modern finance theory to infer an approximate model for private equity, specific to four distinct styles: venture capital, large US buyouts, small US buyouts and European buyouts. For each style, we identify a proper public market index (or indices), then assess exposure to this index, level of residual risk and financial leverage. This allows us to estimate the beta and residual volatility for each sector. Central to this analysis is the assumption that many of the risk characteristics for private equity can be approximated by using data on publicly-traded companies.

In establishing alpha expectations, perhaps the most difficult task, we set a target information ratio for private equity against which we evaluate all other active strategies. Using this approach, we arrive at a set of expected risk and return assumptions for each private equity sector. Our data shows that increasing the number of private equity funds within each sector decreases the risk and increases the corresponding information ratio. As the data of our blended portfolio suggests, this benefit of diversification also applies across private equity sectors.

Our approach can be used to size private equity allocations relative to other investments in a portfolio. By using our assumptions on residual volatility and information ratios, our analysis shows that investors can potentially achieve higher expected returns by including private equity in a traditional portfolio of global equity and global fixed income.

Introduction

Institutional investors are thinking about their portfolios in new ways. Increasingly, they are considering exposures to passive risk on the one hand, and skill-based strategies on the other. In this context, private equity should play an important role in every investor's portfolio. The fundamental issue that investors face, however, is determining the size of the private equity allocation relative to all other assets and strategies in their portfolios.

One of the basic principles of finance is that investors should hold diversified portfolios. Thus, it is natural that investors should want to understand the risk and return characteristics of private equity, compare these figures with the risk and return of other investment opportunities, and size their allocations to private equity accordingly.

Four issues are central to this analysis:

- First, the private equity universe encompasses a variety of investment types and styles. Thus, before we can approach the issue of sizing private equity allocations, we need to develop a clear understanding of the types of private equity investments.
- Second, data on private equity investments are inconsistent. For many of the sectors that are of interest to investors, detailed and reliable information does not exist. Consequently, we need to develop a framework for portfolio analysis that lets us realistically frame the analytic issues.
- Third, this framework should be consistent with well-established principles of portfolio theory. However difficult risk and return estimation is for private equity, the determination of a private equity allocation should rest on sound theoretical principles that allow private equity to be compared with other investments.
- Finally, the framework should be flexible enough to help us understand other issues that are relevant to managing private equity allocations. Issues such as investment of committed (but not invested) capital and re-investment of private equity returns are all important components of understanding the role of private equity in a portfolio.

In this paper, we first review types of private equity and discuss the limitations imposed by data availability. Next, we introduce a basic framework and show how it can be applied to determine risk and return estimates for private equity. Finally, we discuss how the framework can be used to determine a private equity allocation.

Our conclusion is that basic principles of portfolio theory and reasonable risk and return assumptions suggest that portfolio efficiency can be improved with allocations to private equity.

I. Fundamentals of Private Equity

It is important to understand three features about private equity before we begin to apply a standard framework:

1. Differences between the types of private equity,
2. Mechanics of private equity investing, and
3. Issues associated with private equity valuation.

Each of these influences how we assess risk and return on private equity through their potential impact on parameter values.

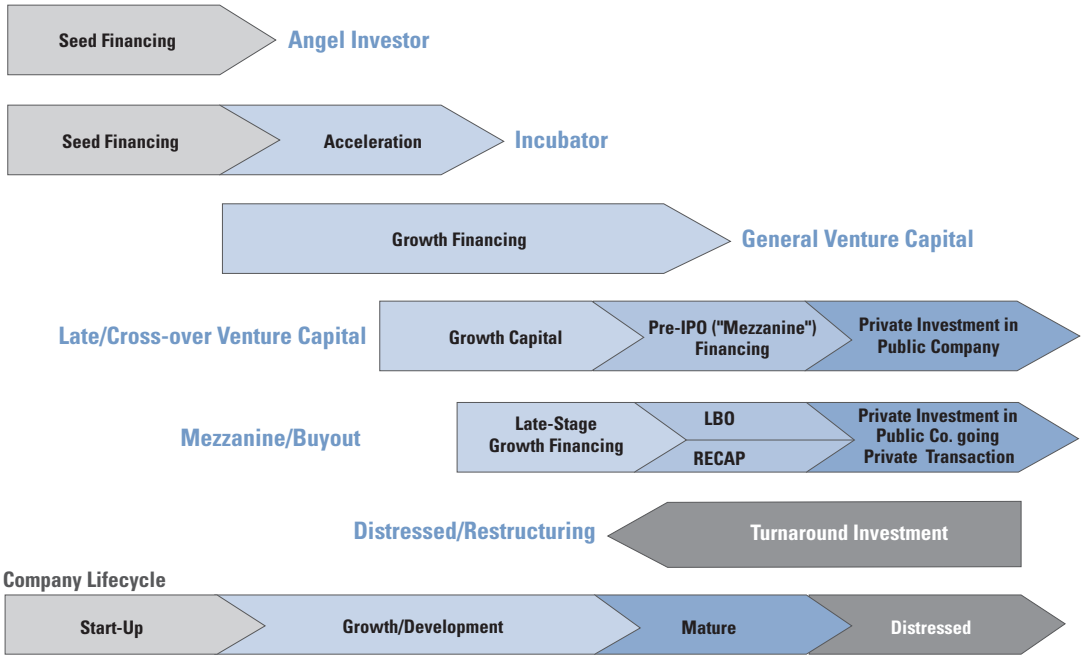
Types of Private Equity Investing

Private equity investing spans a significant number of strategies, beyond new-company start-ups or transactions that take existing public companies private. This broad range of private equity investing allows for participation in the entire life cycle of an enterprise, from a seed-stage start-up through development and expansion to possible turnaround and distressed situations. Exhibit 1 outlines the available alternatives.

It is not surprising that these various private equity strategies have different economic sources of risk and return. For example, companies that involve innovations in technology or business models might be expected to have returns that are based substantially on the effectiveness of innovation, and could have relatively little to do with trends in the public equity market. Businesses of this kind could be classic start-up or venture investments, but could just as easily be growth or turnaround deals.

Other types of private investments might have returns that are substantially based on the public equity market. For example, some buyout investments involve nothing more than applying a higher degree of financial leverage than similar public companies. In such cases, the returns will be highly correlated with the public market. However, some start-up or venture investments are mere copies of other recent successful start-ups. To the extent that such an investment does not really have many new sources of risk and return, it might be expected to be reasonably highly correlated to some

Exhibit 1: The Private Equity Life Cycle



portion of the public equity market. This effect was clearly seen in the Internet and technology bubbles of 1998-2000, when many copycat start-ups exhibited returns that were strongly correlated to the public market.

Different private equity portfolios can have varying risk characteristics. Risk in private equity is related to several factors including:

- Strategy (discussed earlier)
- Industry or sector
- Company size
- Geography

So, for example, a portfolio of private equity investments consisting of only California-based, early-stage technology start-ups with an enterprise value under $10 million might be expected to have different risk characteristics than a portfolio of buyouts of European industrial companies in turnaround situations. Furthermore, both portfolios might be expected to differ markedly from a mixed portfolio that incorporates a blend of sectors, industries, company sizes and locations. And, none of these would be expected to closely resemble a portfolio of commitments made by the world's private equity investors in a given year.

Mechanics of Private Equity Investing

Private equity investments are not as readily accessible to investors as publicly-traded equity investments. While publicly traded securities can be bought and sold at any time, without necessarily changing control of the firm, private companies have no such open market. Most private equity transactions are between individuals or firms and typically involve a change of control, i.e., the new investors generally buy the company from the previous owners.

However, some portion of private equity investments is accessible to financial investors, most commonly through partnership interests. A partnership will be formed with a general partner, who usually has expertise in a particular type of private equity investment. The financial investors are limited partners, who don't usually provide cash when the partnership is formed, but instead, make commitments to provide cash when called upon. These commitments are for a limited size and for a limited period of time – typically five years.

When the general partner finds an attractive investment, he or she will make a capital call on the commitments of the limited partners in order to finance the purchase. For some period of time, the general partner then manages the investment, which may involve working with the investment's corporate management, installing new management, helping with operational matters, or adding value in other ways. Finally, the investment is liquidated, often by an initial public offering (IPO) or sale to another buyer, and the proceeds are distributed to the limited partners. This cycle often takes five or more years for each such investment, although shorter investments are possible.

The limited partners typically pay fees to the general partner for managing the investments. In addition, the general partner usually receives a fraction of the profits, if any, on each investment (commonly in the vicinity of 20%), which is often called a *carried interest*.

Once a commitment to a private equity partnership is made, it is not readily exited. General partners will not usually release limited partners from commitments, as this would compromise their ability to make investments on behalf of the partnership. Although there are some secondary buyers for limited partnership interests, these transactions are often time-consuming and usually require the consent of the general partner.

Valuation and Return Statistics in Private Equity

Since private equity investments do not trade regularly, there are difficulties in establishing valuations for them and resulting challenges in estimating return statistics for private equity as an asset class. Most private equity partnerships follow simple rules when reporting investment values between transactions. Typically, an investment will be held at cost, or at the value of the most recent significant transaction by an outside investor. When an investment goes public, is sold or goes out of business, the obvious valuations can be applied. In addition, general partners often have wide discretion in applying modified valuations when circumstances appear to warrant doing so.

Thus, if there are no transactions in a private business for some period of time, the valuations reported to investors can be significantly out-of-date. For example, consider the case of a business that is able to finance expansion from its own cash flow. Additional rounds of investment may not be required for some years, so the reported valuation may not change at all, while the true economic value of the business increases markedly. On the other hand, a failing business may not be able to raise additional capital, so its valuation might be held at cost until it ceases operations.

In a partnership, it is unusual for all of the investments to undergo transactions at any one time, so to some extent, the aggregate valuation of the partnership is almost always out-of-date. The valuations of a partnership can only be completely reliable at inception (when no investments have yet been made) and at termination (when all investments have been liquidated and the proceeds distributed). Since private equity valuations are always out-of-date, periodic return series are highly unreliable, as are estimates of mean return, volatility or covariances with any other assets.[1]

Overall, some elements of private equity investing can be compared with investments in public markets. Investors must determine precisely how such comparisons should be made. In the next section, we'll explore a general investing framework and how it can be modified and applied to private equity.

II. Applying a Basic Framework to Private Equity

A Basic Framework for Analyzing Portfolio Returns

Before applying an investing framework to private equity, it is important to explore a basic framework that can be applied to every asset class. The basic structure of portfolio returns is well-understood. The Capital Asset Pricing Model (CAPM) was introduced into academic literature over 40 years ago and revolutionized the way both academics and practitioners think about portfolios. Although academic literature has evolved somewhat from the CAPM, the Model still offers practitioners a useful means for sorting through investment decisions. By putting portfolio returns in the context of capital market equilibrium, the CAPM allows investors to determine the exact risks they are taking and identify the appropriate compensation for taking those risks.

Specifically, the CAPM shows that the expected return on any portfolio over a specific time horizon can be decomposed into three parts: a risk-free rate, a return due to exposure to the equity markets, and a return attributable to investor skill. In equation form, the CAPM says:

$$R_p = R_f + Beta^* (R_m - R_f) + Alpha$$

Let's take each of these terms in turn. The risk-free rate (R_f) is fairly straightforward: it represents the return that investors can receive with certainty over a specific holding period (or investment horizon). Practitioners typically use the interest rate for a zero-coupon government bond

[1] A commonly used performance measure in private equity is the internal rate of return (IRR), which can be calculated precisely once an event occurs. However, this calculation is of limited value when comparing with other investments because it is "event-dependent" and consequently cannot be easily compared to regularly reported returns generated from regular mark-to-markets.

corresponding to their investment horizon. By subtracting the risk-free return from the total portfolio return, we arrive at the portfolio's *expected excess return*.

The second term *(Beta * [$R_m - R_f$])* represents the return associated with exposure to the equity markets. To calculate this value, investors need to first calculate the expected excess return on the market, i.e., the total return on the market less the risk-free rate. Then, they must multiply the excess return times a factor called *beta*, which represents the exposure of the investment to market performance. Practitioners usually calculate market returns on the basis of capitalization-weighted indices, which represent the broad investment opportunity set of their investment. Examples of such indices include the S&P 500 Index, the Russell 2000 Index and the MSCI World Index.

The final term to consider is *alpha*. A portfolio's alpha is simply the expected return due to deviations from the market portfolio. This term plays an integral role in understanding equilibrium. Specifically, in equilibrium the expected alpha is zero, and all investors hold the market capitalization-weighted portfolio; to do otherwise is to take an uncompensated risk.

Of course, financial markets are not always in equilibrium. Deviations from equilibrium represent opportunities for investors to add return in excess of the return on the market portfolio. Thus, we should anticipate that many, if not all, investors hold portfolios that deviate from the market portfolio. After adjusting for differences in market exposure, investors' portfolios differ from one another based on views about alpha.

As a general rule, investors should allocate capital to strategies in which they are confident that skill can be used to generate alpha. In other words, they should look for skill-based deviations from the market portfolio. Because combining many independent skill-based sources of return can increase total expected return with minimal effects on portfolio risk, investors have an inherent reason to look for many such skill-based strategies. However, to meaningfully develop such a diversified portfolio, investors must be able to assess the risk and return characteristics of all assets and strategies. *This point raises the issue of estimating risk and return characteristics for private equity*.

A useful measure of alternative sources of skill is the *information ratio (IR)* – the expected alpha divided by the strategy's residual volatility.[2] The residual volatility is simply the risk associated with deviating from the market portfolio. Using this definition, the expected alpha is simply the product of the information ratio and the residual volatility, or,

*Alpha = IR * (Residual Volatility)*

Putting all of the pieces together, we can write the portfolio expected excess return as:

$(R_p - R_f) = Beta * (R_m - R_f) + IR * (Residual\ Volatility)$

This expression gives practitioners a meaningful way to decompose and estimate portfolio returns. To better understand how this framework might be used in practice, let's consider two examples:

Example 1: A traditional US active equity manager whose benchmark is the S&P 500 Index.
Objective: Find this manager's expected total return.

- Suppose the manager has a beta of 1.2 against the S&P 500 Index. This figure tells us that for every 100 basis points that US equities outperform (or underperform) cash, the manager will outperform (or underperform) cash by 120 basis points, all else equal.
- Since we are interested in finding the expected total return for the manager, we need an expected excess return for the S&P 500 Index. If the expected excess return on the market is 350 basis points, the manager is expected to outperform cash by 420 basis points, all else equal.

[2] The Information Ratio is analogous to the Sharpe Ratio. The latter shows the risk-adjusted performance of the total portfolio, while the former shows the risk-adjusted performance of the active part of the portfolio.

- Let's assume that the manager deviates from S&P 500 Index weights in their security selection and the risk (e.g., residual volatility) of doing so is 500 basis points.
- We'll assume that the manager has an expected information ratio of .5, which produces an expected alpha of 250 basis points when combined with the residual volatility of 500 basis points. Finally, suppose that the risk-free rate is 4.5%.

Combining all the assumptions, the manager's expected total return is 11.2%, with 4.5% coming from the risk-free rate, 4.2% from the market return and 2.5% from the active strategy.

Example 2: An equity long/short hedge fund.
Objective: Find this manager's expected total return.

Hedge funds are not typically measured against performance benchmarks, such as the S&P 500 Index. Nonetheless, it is useful to ask whether hedge fund manager performance comes from the application of skill-based strategies or exposure to the market (see Clark and Winkelmann, 2004).

- Suppose the hedge fund manager has a beta of 0.6 against the S&P 500 Index.
- Furthermore, suppose that the manager has an expected information ratio of .92 and a residual volatility of 500 basis points.

Under these assumptions, the expected total return for the manager is also 11.2%. That is, the total return is derived from the risk-free rate of 4.5% plus the impact of the market (.6 times 350 basis points, or 2.1%) plus the impact of skill (or .92 times 500 basis points).

These examples illustrate an important point: we arrived at the same total return, 11.2%, in both cases. However, we got there in very different ways. In the hedge fund example, more emphasis was placed on the application of skill. By contrast, in the traditional manager example, more emphasis was placed on market exposure.

This framework for finding expected total returns also has an interesting extension for thinking about risk, as measured by the volatility of excess returns. After manipulating the expression a bit, we arrive at the following expression for the volatility of excess returns:

$$\textit{Portfolio Risk} = \sqrt{(\textit{Beta} * \textit{Market Volatility})^2 + (\textit{Residual Volatility})^2}$$

Assuming that equity market volatility is 16.5%, we can apply this expression to our two examples. The risk for the traditional manager is 20.4%, while the risk for the hedge fund manager is 11.1%. Since the residual volatility and the market volatility are the same for both managers, we know that the total risk levels are different only because of differences in beta.

The examples clarify the value of the framework in analyzing different investments. A significant amount of information on any investment can be determined through three figures: beta, information ratio, and residual volatility. Thus, a good starting point for analyzing any investment is to determine the proper measure of the market for the investment, and then assess (by whatever means) these three figures.

Some would argue that the nature of private equity investing makes this framework irrelevant since the long-term horizon and poor data quality imply that any attempt at portfolio analysis is useless. However, we would argue the opposite. In fact, it is exactly these attributes that make this framework so important. How else can investors determine the exact size of an allocation to private equity, or assess allocations within private equity sectors? The real issue in analyzing private equity risk and return is not the framework, but rather, how we handle the issues that are specific to private equity when applying the framework. In the next two sections, we will address data issues within this context.

Evaluating Risk and Return in Private Equity

Now that we have a better understanding of both the investment characteristics of private equity and a general risk/return framework, we can see how the two fit together. Because measurements of periodic returns in private equity are highly problematic, we can't use standard regression tools to estimate model parameters. Instead, we use results from modern finance theory to infer an approximate model for private equity. We take the approach of identifying several distinct styles of private equity, evaluating their relationships with publicly traded asset classes, and estimating model parameters from our knowledge of capital structures. This framework has two benefits for investors: first, the approach uses easily understood terms to help investors identify the key issues in assessing risk and return in private equity; second, the framework provides investors with a way to make meaningful comparisons with investments in other asset classes. This approach will not be as accurate as the approach for publicly traded asset classes. However, by calibrating the model in this way, we can reduce our reliance on spurious regression estimates obtained from data that inaccurately represents the performance characteristics of private equity investments.

As discussed in the preceding section, we can distinguish four broad categories of private equity investing:

- Venture capital
- Large US buyouts
- Small US buyouts
- European buyouts

Although we could distinguish several more detailed styles of private equity, many practitioners in private equity would agree that this list captures most of the essential types. Moreover, roughly equal amounts of capital have been committed to funds in each of these areas in recent years.[3] Our objective, then, is to assess the following for each of the types of private equity:

- Proper public market index or indices
- Exposure to this index (or indices)
- Level of residual risk
- Financial leverage
- Levels of alpha that might be obtained by various managers

Central to our analysis is the assumption that many of the risk characteristics for private equity can be approximated by using data on publicly traded companies. We chose this approach because framing the issue in this way helps us add precision to our estimates and direct further research.

To illustrate, let us consider the estimation of residual volatility. Our initial estimate of residual volatility for private equity will be drawn from the experience of public companies. However, we can reasonably ask whether, all else equal, a private company is inherently riskier than a public company (or for that matter, vice versa). Because a public company has more immediate access to both capital markets and credit, one might argue that a public company has an inherently lower residual risk than a private company. Alternatively, it could be argued that residual risk is lower in private companies precisely because they do not face the same short-term reporting pressures.

Our analysis does not take a particular stand on the issue of whether private companies are inherently riskier than public companies. Regardless of one's position on this issue, we believe the first step in systematically analyzing private equity investments is to use as much publicly available information as possible.

[3] Source: Venture Economics VentureXpert Database

Venture Capital

Venture capital involves new companies, largely in technology-related areas (especially information technology, communications and life sciences). As such, venture capital is considered to be most closely related to the NASDAQ. This is a natural index to use, as many companies newly taken public are initially traded on the NASDAQ.

Exhibit 2 shows the sector allocations and estimated sector betas for the NASDAQ. This analysis is important, as it helps us calibrate the beta for venture capital relative to the NASDAQ. For instance, if the venture capital allocations were significantly different from the NASDAQ, we would be inclined to assume that the beta for venture capital would be approximately one. Since we have no prior information to suggest that the sector breakdown for venture capital differs significantly from that of the NASDAQ, we assume that the beta of venture capital deals is equal to one, before the managers take a carried interest. Venture investments, like public companies traded on the NASDAQ, typically employ low levels of financial leverage.

Exhibit 2: NASDAQ Sector Allocations

Sector	Allocation (%)	Estimated Beta
Banks	6.9	0.15
Finance ex-Banks	1.5	0.63
Industrial	29.3	0.90
Insurance	2.0	0.14
Transportation	1.5	0.39
Computers	35.5	1.22
Telecommunications	15.2	1.06
Biotechnology	8.0	0.97

Source: DataStream, Bloomberg. The allocations are as of Feb 2005 and the betas were calculated using monthly data from Nov 1993 to Feb 2005.

Most, if not all, venture companies start off as ideas, with no or low revenues and earnings. Thus, some venture capitalists may have the ability to enhance investment returns through superior technical judgment or business skill, which the private equity investor is hoping to exploit, and is captured through the alpha term. Of course, this return is not guaranteed, and could well be negative. The uncertainty around the skill-based return is captured through the residual volatility (or residual risk) term.

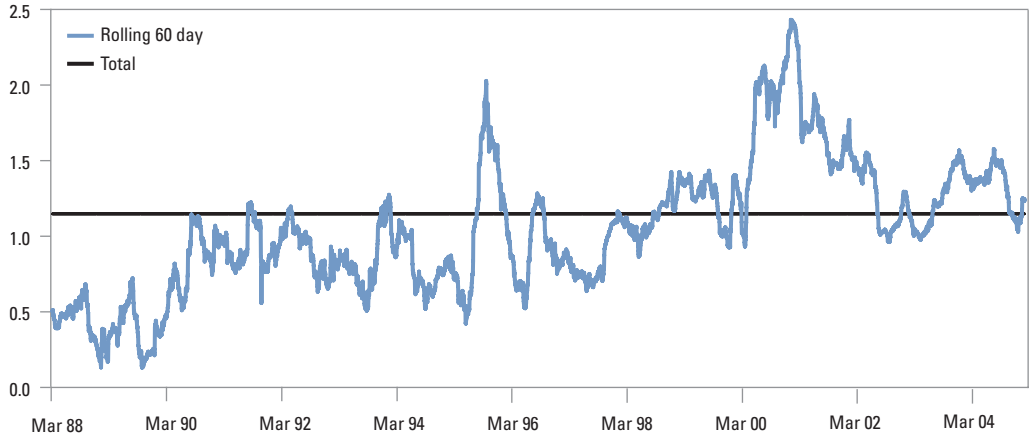
As a proxy for residual volatility, we could use the residual volatility for a company recently taken public. One of the benefits of this proxy is that it is relatively easy to calibrate. Conceptually, the data could be used as follows: select a portfolio of companies recently taken public and calculate the residual volatility of this portfolio (relative to the NASDAQ) over a suitable period. The start date for the estimation should be immediately after the IPO.

In practice, we cannot follow such a straightforward approach. Few companies are taken public on the same date. Consequently, we have chosen to start with the estimated residual volatility for individual companies in the 60 days immediately following their IPO, and then calculate the average of these estimated residual volatilities. Using this procedure, the average estimated residual volatility is around 114%.[4]

[4] The drawback of this approach is that it estimates individual residual volatilities over different time periods. As a result, the residual volatility from this method is not directly comparable with the average residual volatility of the entire NASDAQ over any particular period.

Other attempts have been made to directly calibrate the risk parameters of venture capital. For example, Cochrane (2003) attempts to control for the observability problems noted earlier, and obtains beta estimates of 1.7 relative to the S&P 500 Index, with residual volatility of 86%. To compare the two approaches, we need to calculate the beta of the NASDAQ relative to the S&P 500. Exhibit 3 shows a time series of rolling beta calculations for the two indices. As the Exhibit illustrates, on average the beta of the NASDAQ relative to the S&P 500 has been about 1.2. Thus, Cochrane's calculations are roughly in line with ours.

Exhibit 3: NASDAQ Beta to the S&P 500 Index
Mar 1988 – Feb 2005



Source: DataStream

Therefore, the gross economic excess return to individual venture capital deals, i.e., the return before fees and carry, can be modeled as:

$$(R_{vc} - R_f) = Alpha_{vc} + Beta_{vc} * (R_{Nasdaq} - R_f)$$

Because the alpha for any investment can be written as the product of the expected information ratio and the residual volatility, we can easily rewrite the gross economic return to an individual venture capital deal as:

$$(R_{vc} - R_f) = IR_{vc} * Residual\ Risk_{vc} + Beta_{vc} * (R_{Nasdaq} - R_f)$$

In the remainder of this paper, we will use assume that $Beta_{vc}$ is 1.0 and 114% for residual risk of individual venture capital deals.

Buyouts

Buyouts differ from venture capital in several ways. As previously discussed, buyout investors are acquiring an existing company that generally has revenues and earnings. After re-engineering the company, the buyout firm then sells the company public again, often by an IPO. Consequently, buyouts can be viewed as being closer to existing public market equity indices. However, buyouts typically make more extensive use of financial leverage in ways that can somewhat complicate the analysis.

To analyze buyout investments, we'll make use of standard corporate finance relationships. The Modigliani-Miller theorem asserts that, absent tax effects and financial distress, the total value of an enterprise is independent of its financing. In equation form, we can write this idea as:

$$V_{firm\ total} = V_{firm\ equity} + V_{firm\ debt}$$

Expressed in terms of returns, this equation says that the return to the total value of the company is simply the weighted average of the return to equity and the return to debt, with the weights given as a percentage of total value,[5] or

$$R_{firm\ total} = W_{equity} * R_{firm\ equity} + W_{debt} * R_{firm\ debt}$$

By rearranging this somewhat, we can show that the return on equity depends on the return to the value of the firm, i.e., changes in the present value of the productive capacity of the firm, and the return to debt. As such, we obtain:

$$R_{firm\ equity} = \left(\frac{1}{W_{equity}}\right) * R_{firm\ total} - \left(\frac{W_{debt}}{W_{equity}}\right) * R_{firm\ debt}$$

Let's consider a single-factor representation for the value of the firm. In other words, suppose that the firm is 100% equity-financed. Then, we have

$$(R_{firm\ total} - R_f) = Alpha_{firm} + Beta_{firm} * (R_{equity\ index} - R_r) + Residual\ Risk_{firm}$$

In this expression, the growth rate (or excess return) to the value of the firm is represented as an exposure to the broad equity market and a company-specific term. The mean of the company-specific return represents the added value to the company, above that which an investor would receive through market exposure alone.[6] This mean is, of course, random.

Now, let's consider the case of a levered company. Combining the above equations and rearranging them somewhat, we can obtain a return model for an individual buyout deal, again, before fees and carry:

$$(R_{equity} - R_f) = Alpha_{levered} + Beta_{levered} * (R_{\substack{equity \\ index}} - R_f) - Beta_{\substack{levered \\ debt}} * (R_{\substack{debt \\ index}} - R_f) + Residual\ Risk_{levered}$$

where

$$Alpha_{levered} = \left(\frac{1}{W_{equity}}\right) * Alpha_{firm}$$

$$Beta_{levered\ equity} = \left(\frac{1}{W_{equity}}\right) * Beta_{firm}$$

$$Beta_{levered\ debt} = \left(\frac{W_{debt}}{W_{equity}}\right)$$

$$Residual\ Risk_{levered} = \left(\frac{W_{debt}}{W_{equity}}\right) * Residual\ Risk_{firm}$$

The impact of leverage can be seen quite clearly here. Buyout deals are essentially long equity and short debt compared to their less-levered counterparts. Returns of more levered companies are more sensitive to equity market moves (through the impact on beta) and more volatile (through the impact on residual volatility). The compensating impact is felt in the alpha term. If a manager is able to add alpha to the total value of the company, then a more levered company produces more alpha for the equity holders than a similar company with less leverage.

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[5] The return equation is found by differentiating with respect to time, dividing by initial value and then rearranging somewhat. Note that this expression represents the local effect of instantaneous changes in equity and debt values.

[6] The average of alpha values across all companies in an equity index is zero.

The return equation is a representation for any company, regardless of whether it is publicly traded or privately held. Thus, the real issue is how to calibrate each of the parameters. In the case of buyout, much of this is made quite straightforward, because the buyout exercise is one of buying existing companies, making appropriate operational improvements, and then reselling them.

As a consequence, many of the important analytic decisions have already been made for us. Consider, for example, equity index selection. In terms of our three broad categories, it would be reasonable to assume that the equity index for Large Buyout should be the S&P 500 Index, for Small Buyout the Russell 2000 Index and for European Buyout the MSCI Europe Index. Similarly, in the absence of any information about a specific buyout portfolio, it strikes us as reasonable to assume that in aggregate, each buyout sector is market-neutral relative to its equity index.

Switching to the debt part of the equation, buyout situations differ from public companies in terms of the structure of the debt. All companies have a mix of bank debt and bonds. Buyout differs a bit from publicly traded companies, perhaps, because of both the blend of bank debt and bonds and the appropriate sector of the bond market. In particular, many buyout situations are forced to tap into the high yield market and bank debt, rather than issue into the investment grade sector.

Other parameters are similarly straightforward to calibrate. Consider, for example, the debt-to-equity ratio. On average, public companies' equity is about 70% of the capital structure at most, with debt representing the remaining 30%. In buyout situations, these numbers are skewed toward debt. On average, it is common for buyout situations to have 40% of the capital structure in equity and 60% in debt.[7] (There are some distinctions between the structure of the debt component across sectors, with large US and European buyouts tapping into the high yield market more commonly than small US buyouts.) These differences in capital structure are important, as they serve to "lever up" the equity betas for buyout situations relative to comparable public companies. For example, suppose that an unlevered public company has a beta of 1.0 and a residual volatility of 25%. These figures produce a total volatility of 30% (assuming that equity market volatility is 16.5%). Levering up the company by 50% increases the total volatility to 60%.

A final parameter that is relatively straightforward to calibrate for buyout situations is the residual term, or sigma. Remember that buyout is the exercise of taking public companies private, adding value, and then selling them. Consequently, a reasonable way to calibrate the residual is to use the average residual for public companies. Of course, these figures will change over time with changes in market conditions. However, since private equity investing is meant to represent a longer-term time horizon, it is worthwhile to consider long-term average residuals.

Exhibit 4 shows the average residual volatility calculations for our three sectors. These figures represent the average residual volatility for the existing stocks in the indices, calculated over the period 3-Jan-2000 to 18-Nov-2004, using daily returns. All else equal, we would expect the average residual volatility to be about the same for the large buyout, irrespective of region, and higher for the small buyout. As Exhibit 4 indicates, that is roughly the pattern we observe.

Exhibit 4: Average Residual Volatility in Each Sector
Daily data from 3-Jan-2000 – 18-Nov-2004

Sector	Index	Average Residual Risk
Large Buyout	S&P 500	55%
European Buyout	MSCI Europe	49%
Small Buyout	Russell 2000	80%

Source: GSAM

[7] Source: Standard & Poor's

Effect of Fees and Carry

The next issue that we should consider is cost. So far, our analysis has been done purely on the economics of individual deals. However, for those investors in private equity funds, rather than individual private equity deals, fees and carry must also be taken into account.

There is a substantial degree of variation in the exact mechanics of fees and carry, but the general outlines are fairly consistent across most private equity funds. Fees do not depend on actual investment results, so they impact the alpha term, but not the market exposure. Carried interest is a fraction of profits that are paid to the fund manager; thus, for funds that ultimately achieve a negative return, there is no carry. However, for funds that achieve a positive return and pay a carry, the effects of carry are to reduce the alpha, beta, and sigma terms. If carried interest is c and the impact of fees on return is r_{fees}, then we have approximately:

$$Alpha_{net} = (1\text{-}c) * \{Alpha_{gross} - r_{fees}\}$$

$$Beta_{net} = (1\text{-}c) * Beta_{gross}$$

$$Residual\ Risk_{net} = (1\text{-}c) * Residual\ Risk_{gross}$$

Thus, we see that while one of the effects of carried interest is reducing the net alpha seen by the investor, carried interest also tends to reduce both the market exposure and the residual risk. Fees, on the other hand, are simply a negative alpha term, as they are paid regardless of the return on the investment. (For more details on the impact of fees and carry on rates of return in private equity, see Griffiths and Murphy, 2003).

Parameter Reduction

So far, we've modeled each private equity sector relative to indices specific to that sector (e.g., we've used the NASDAQ as the representative index for venture capital). The exposure to the index (as measured by beta) effectively tells an investor how much of the return is due to passive index exposure.

Each equity market index, in turn, has exposure to global equity. In other words, we can find a beta value for each of our representative indices against a global equity index such as the MSCI World. Since a global equity index is not a perfect match for each of the sector indices (e.g., the returns on the MSCI World do not perfectly match the returns on the NASDAQ), we know there will also be a residual volatility.

Because we can represent each sector index as exposures to global equity, we can also represent each private equity sector in terms of exposures to global equity. In doing so, we will have to adjust both the beta and residual volatility. Since many private equity investors typically invest in diversified portfolios, such parsimony in representing private equity risks can help develop a broader intuition for understanding private equity's role relative to other investment opportunities. Clearly, the finer detail is preferable for risk management of a specific private equity portfolio.

Exhibit 5 shows the beta and residual volatility values for each of the private equity sectors and each value relative to the sector's specific index and relative to the MSCI World Index. The values are assumed to represent the exposure of a single private equity deal in each sector. In the remainder of this paper, we will assume that each private equity sector is measured relative to the MSCI World.

Exhibit 5: Beta and Residual Volatility by Sector
Using daily data from Jan 2001 to Feb 2005

Sector	Index	Sector Beta to Index	Sector Residual Volatility	Index Beta to Global Equity	Index Residual Volatility	Sector Beta to Global Equity	Sector Residual Volatility
Venture Capital	NASDAQ	1	114	1.5	17.2	1.5	116.9
Large US Buyout	S&P 500	1	55	1.1	7.8	1.1	55.5
Small US Buyout	Russell 2000	1	80	1.1	12.5	1.1	81.0
European Buyout	MSCI Europe	1	49	1.0	12.5	1.0	50.6

Source: DataStream, GSAM

Estimating Alpha in Private Equity Portfolios

Of course, private equity investors are not simply investing in a single deal. Rather, they are purchasing a portfolio of opportunities. Such a portfolio will usually consist of a number of funds, with each fund including (on average) 10-20 individual deals. Some investors have portfolios consisting of a single fund, while other investors have portfolios with hundreds of fund commitments.

One parameter that we still need to calibrate is alpha. Unfortunately, this is also the most difficult. The alpha that we wish to estimate represents the company-specific added value, as enhanced by financial leverage, net of fees and carry, on average throughout the portfolio. This is precisely the commodity that investors are looking for when they make commitments to a private program. Unfortunately, ex-ante alpha cannot be observed directly, nor can it be calibrated from other information in the way that we have found other parameters.[8] Rather than trying to estimate private equity alpha directly from fragmentary data, we suggest that investors take a different approach.

Since private equity investing can be viewed as active management, investors could establish alpha expectations for private equity by looking at private equity relative to other active strategies. Since different types of active strategies are likely to take on different levels of active risk (in the form of residual volatility), a more convenient way to compare strategies is by looking at information ratios. As the expected alpha is simply the product of the residual volatility and the information ratio, we can always derive the expected alpha after setting the relative information ratios.

How should we assess information ratios? One way to approach this is to start with a target information ratio against which we would like to evaluate all active strategies. If we believed that one particular type of strategy was more likely to achieve the target, we would set a higher expected information ratio. By contrast, if we believed that another strategy was less likely to achieve the target, we would set a lower expected information ratio.

This approach is beneficial to investors for three reasons: first, it avoids the use of meaningless data; second, it relies on relatively straightforward comparisons across investment opportunities, and finally, it makes those comparisons on the basis of underlying financial economics.

For example, suppose that the target IR for each strategy is .25.[9] Based on arguments from financial economics, we might decide that skilled private equity managers are better able to deliver alpha than traditionally-managed, publicly traded large cap managers. To reflect these assumptions, we might assign an IR of .25 to a portfolio of private equity managers and an IR of .15 to a portfolio of large cap managers.

[8] A reasonable range of historical alpha might be established by looking at fund-level performance. Such historical performance data are available for several vendors, such as Venture Economics and Cambridge Associates. Although these data suffer from a number of problems, such as unknown selection biases, unknown industry biases, and others, it appears likely that the net alpha of the median private equity fund is very close to zero. On the other hand, a substantial number of funds have probably achieved an alpha in excess of 500 bps, and a similar number have probably achieved an alpha below -500 bps. Determining which funds are likely to achieve significant positive alpha in the future is a matter for detailed due diligence, and is beyond the scope of this article.

[9] There is nothing particularly special about .25 as a reference, with the possible exception that the Sharpe Ratio on global equity is approximately .25 with an equity premium assumption of 350 basis points. The point is not the particular target, but rather the exercise of assigning IRs around this target.

How should we put all of these pieces together to find risk and return assumptions for each of the private equity sectors? Exhibit 6 applies the framework and the corresponding calibrations to arrive at a set of expected risk and return assumptions, net of fees and carry, and accounting for diversification among deals.[10]

Exhibit 6 shows how the approach can be applied to each sector and a blended portfolio. For each sector, the exhibit shows the beta relative to global equity (as discussed, this is simply the product of the beta to the sector index and the beta of the sector equity index with respect to the global equity index.) The exhibit shows the expected excess return under the assumption of a 350 basis point global equity premium. As well, the exhibit shows the expected alpha and residual volatility for a fund of 20 deals and a portfolio of five funds for each sector. These figures are calibrated under the assumption of an IR of approximately .15 for a portfolio in each sector. This assumption corresponds to a net alpha of 250 basis points for each fund (i.e., after impact of fees and carry, as discussed above). Finally, the exhibit shows the total information ratio and the total volatility for each sector and the blended portfolio.[11]

Exhibit 6: Private Equity Risk and Return Parameters

All calculations net of fees and carry. Carry assumed to be 20%, net alpha assumed to be 250 basis points.

	Venture			Large US Buyout (S&P 500)			Small US Buyout (Russell 2000)			European Buyout (MSCI Europe)			Blended Portfolio
	1 deal	1 fund (20 deals)	Portfolio (5 funds)	1 deal	1 fund (20 deals)	Portfolio (5 funds)	1 deal	1 fund (20 deals)	Portfolio (5 funds)	1 deal	1 fund (20 deals)	Portfolio (5 funds)	5 funds each sector
Beta to Global Equity	1.32	1.32	1.32	1.30	1.30	1.30	1.43	1.43	1.43	1.28	1.28	1.28	1.33
Excess Return (%)	6.8	6.8	6.8	6.8	6.8	6.8	7.3	7.3	7.3	6.6	6.6	6.6	6.9
Residual Risk (%)	91.2	32.0	22.5	76.8	27.0	18.9	112.7	39.6	27.8	68.4	24.0	16.9	11.0
Total Risk (%)	94.6	40.7	33.7	80.0	35.0	29.3	115.8	47.7	38.4	71.9	32.7	27.9	24.1
Information Ratio	0.03	0.08	0.11	0.03	0.09	0.13	0.02	0.06	0.09	0.04	0.10	0.15	0.23

Source: GSAM Calculations

Now, after adjusting for costs, investors are in a better position to assess the aggregate expected return for their private equity portfolio. Clearly, expected returns across portfolios will vary according to allocations across styles and the corresponding risk exposures within each style. Within our framework, the return to a blended private equity portfolio will have returns associated with broad market performance and private equity-specific returns. The impact of broad market performance will depend on which public market index is most representative for the respective private equity sector and the sector's exposure to the index (or beta). The idiosyncratic component will depend on the estimated residual volatility and the expected information ratio.

An interesting point that emerges from Exhibit 6 is the benefit of diversification. We have built up to the aggregate private equity portfolio IR from assumptions about the expected IR and alpha for each deal and each fund. In the absence of other information, we've assumed that the expected IR for each deal is the same. Note from the table that this IR is substantially lower than the IR for the entire portfolio. As the table indicates, the big gains come from diversifying away some of the residual risk. Thus, to a large extent, one can conclude that investors are best off with well-diversified private equity portfolios.

For instance, we can see that a single private equity fund of any style is likely to have a fairly low information ratio. For the examples given, assuming alpha of 250 basis points, we find information ratios in the range of 0.1 to 0.2. Funds with higher alpha will have higher information ratios, but the relatively high residual risk of individual funds will limit the attainable results.

[10] Reduction of variance in a private equity portfolio does not follow the square root of the number of deals, since the deals are not equally weighted and cannot be rebalanced due to illiquidity. Instead, weights in a private equity portfolio are typically Zipf-weighted, as calculated here. See Axtell, 2003, for details.

[11] Again, it is worth noting that the assumption of a positive information ratio reflects that the investor can find skilled private equity managers.

A portfolio with about five funds in each sector will have significantly decreased residual risk compared to a single fund. However, there is a limit to how much residual risk can be reduced. Since private equity is highly illiquid, the portfolio cannot be effectively rebalanced, and the larger positions in the portfolio will tend to dominate the overall residual risk. Beyond about five funds per sector, the rate of decrease in residual risk is fairly low.

The additional dimension of diversification available in private equity is across sectors. Assuming that an investor can find positive alpha in each sector, additional diversification can be obtained by investing in different styles of private equity. In recent years, this was seen in practical terms by private equity investors who only committed to venture funds, while other private equity investors took a more balanced approach. While those investors who committed solely to venture funds often experienced strong results between 1994 and 1999, most experienced poor results between 2000 and 2003. The investors who made investments with positive alpha in multiple styles often performed better across this entire period.[12]

For example, consider a portfolio with 25% allocated to venture capital, and the remaining 75% split equally over the three buyout sectors. Suppose, also, that the portfolio has five active funds in each sector. Such a portfolio has about 400 underlying investments (20 companies per fund x 5 funds per sector x 4 sectors). Assuming that the investor can find an average alpha in each sector – net of fees and carry – of 250 basis points, we see from Exhibit 6 that this portfolio has an expected residual risk as low as 11% and an information ratio as high as 0.23. It also has expected excess return of 6.9% and total volatility of 24%. As expected, these statistics for such a broadly diversified private equity portfolio are all superior to those for a single fund or a collection of funds in a single style, assuming similar levels of alpha.

These results should be interpreted as a lower bound on the achievable level of residual risk, and an upper bound for achievable information ratio, given net alpha of 250 basis points. As noted above, there is no market index portfolio for private equity. A portfolio that has concentrations in particular industries, geographies, or sizes may not be as well-diversified as we have assumed here. In such cases, the residual risk would be expected to be higher and the information ratio, correspondingly lower. Of course, the information ratio may be higher if higher alpha can be achieved.

III. Finding Optimal Private Equity Allocations

One of the significant benefits of our approach is that it can be used to frame more meaningful discussions about the appropriate size of a private equity allocation. In particular, by focusing explicitly on assumptions, such as the expected information ratio and the market exposure, investors can better size the private equity allocation relative to other investments in the portfolio.

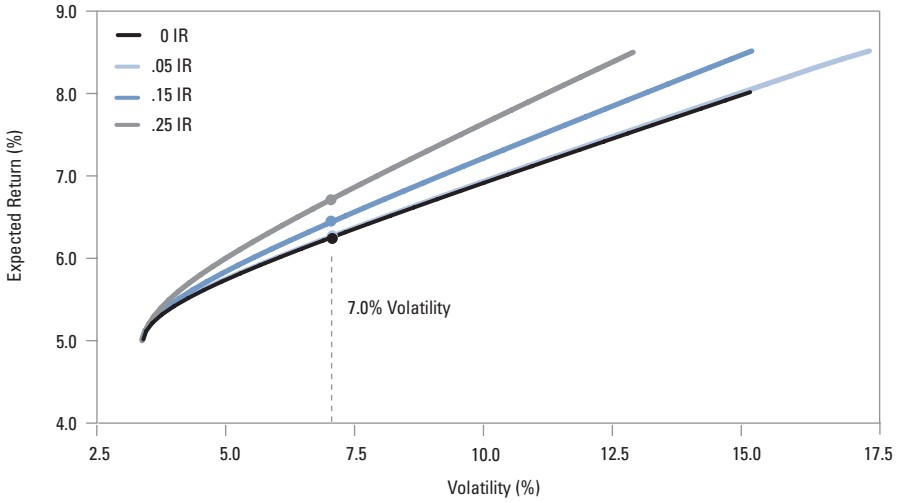
As a very simple example, suppose that an investor has three investments under consideration: global equity, private equity and global fixed income. Furthermore, suppose that the global equity and global fixed income allocations are actively managed. The challenge is to identify the optimal allocation between these three choices.

To find the optimal allocation, we need assumptions about the active risk and return for global equity and global fixed income. For illustrative purposes, suppose that the expected tracking error for a portfolio of global equity managers is 200 basis points, and the expected information ratio is .15. Similarly, suppose that the expected tracking error and information ratio for a portfolio of global fixed income managers are 75 basis points and .25 respectively. Finally, assume that the residual risk for the portfolio of private equity managers is 11%.

[12] Source: Venture Economics VentureXpert Database

Exhibit 7 shows efficient frontiers for four cases. Each efficient frontier corresponds to an assumption about the information ratio for private equity, ranging from 0 to .25. As anticipated, when the information ratio for private equity increases, the efficient frontier shifts out: at any risk level, investors can achieve a higher expected return by including private equity in their portfolios.

Exhibit 7: Efficient Frontier Analysis



Source: GSAM Calculations

The magnitude of the impact can be seen by focusing on one frontier point. To illustrate, we've chosen a volatility level of 7%. Exhibit 8 shows the allocations and returns associated with this point on the efficient frontier. By including private equity, under our assumptions, investors are able to add 20-50 basis points of extra return to the portfolio. Moreover, including private equity in the portfolio means that more capital is allocated to bonds: the inclusion of an additional active strategy means that more risk is taken in active risk and less in pure equity market risk.

Exhibit 8: Allocations and Returns at 7% Volatility

	Private Equity IR			
	0	**0.05**	**0.15**	**0.25**
Global Equity (%)	44.3	36.5	4.7	0.0
Global Fixed Income (%)	55.7	57.7	69.2	71.3
Private Equity (%)	0.0	5.8	26.1	28.7
Total Return (%)	6.2	6.3	6.4	6.7
Volatility (%)	7.0	7.0	7.0	7.0
Sharpe Ratio	0.25	0.25	0.28	0.32

Source: GSAM Calculations

IV. Conclusions and Investment Policy Implications

This paper provides a framework that investors can use to analyze their private equity investments. Our framework clearly identifies parameters of interest for private equity and shows how investors can begin to calibrate these parameters. In particular, we show that private equity returns can be naturally decomposed in the same way as other investments: risk-free rate, exposure to a suitably chosen public equity index, and alpha. Alpha, of course, is what investors are seeking when they undertake private equity investing.

Our framework puts the discussion of private equity returns and allocations on the same footing as other investments. One implication of our analysis is that investors should increase their allocations to private equity, as long as they believe the achievable information ratios are at least equal to those attainable in other investment vehicles. Our analysis also provides a foundation that investors can use to consider other important issues in private equity investing, such as benchmarking and commitment management.

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