Internet Appendix

for

"Estimating Private Equity Returns from Limited Partner Cash Flows"

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This appendix provides further details on the estimation of the model (section I) and presents additional empirical results (section II).

I. Estimation of the Model

We restate the model here for convenience. We can merge equations (10) and (11) into one equation containing only the latent state variable, $g_t^e \equiv g_t - r_t^f$, as the excess return relative to the risk free rate. The state equation then becomes

$$g_t^e = (1-\phi)\alpha + \phi g_{t-1}^e + \beta'(F_t - \phi F_{t-1}) + \sigma_g \epsilon_t, \qquad \text{(IA.1)}$$

where the systematic factors, F_t, are observable.

We assume that the zero-NPV condition in equation (6) holds, and we specify that the log ratio of the present value of the distributions to the present value of investments of fund h is normally distributed:

$$\ln\left(\sum_{i=1}^{N^h} \frac{D_{T_i^h}^h}{g_{t_1^h} \cdots g_{T_i^h}}\right) - \ln\left(\sum_{i=1}^{N^h} \frac{I_i^h}{g_{t_1^h} \cdots g_{t_i^h}}\right) \sim N(\mu, \sigma^2), \qquad \text{(IA.2)}$$

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Equation (IA.2), which repeats equation (6), represents the likelihood function of the cash flows. To ensure that the ratio of the present value of distributions and the present value of investments are centered at one, we set $\mu = -\frac{1}{2}\sigma^2$. This is equivalent to assuming that the errors of the log ratio of the present value of distributions to the present value of investments have zero mean.

Equations (IA.1) and (IA.2) constitute a state equation and a nonlinear observation equation. The following algorithm filters the latent state variable g_t^e given the observation equations. Once g_t^e is estimated, we can infer the PE-specific return, f_t, using

$$f_t = g_t^e - (\alpha + \beta' F_t). \tag{IA.3}$$

We denote the parameters $\theta = (\alpha, \beta, \phi, \sigma_g, \sigma)$ and let θ_- denote the full set of parameters less the parameter that is being estimated in each conditional draw. We collect the exogenous PE cash flow data and the common tradable factors F_t as $Y_t = \left\{\{I_{it}^h\}, \{D_{it}^h\}, \{F_t\}\right\}$.

We estimate the model described by MCMC and Gibbs sampling. A textbook exposition of Gibbs sampling is provided by Robert and Casella (1999). Other models that involve latent state variables are estimated by Jacquier, Polson, and Rossi (2004), and Ang and Chen (2007). These papers are able to directly use observable returns. In contrast, we use nonlinear NPV equations to infer returns.

In each of our estimations, we use a burn-in period of 5,000 draws and sample for 20,000 draws to produce the posterior distributions of latent state variables and parameters. With this large number of sampling, our estimation converges in the sense of passing the Geweke (1992) convergence test. The Gibbs sampler iterates over the following sets of states and parameters conditioned on other parameters and states variables, to converge to the posterior distribution of $p(\{g_t^e\}, \theta \mid Y)$:

1. Private equity returns: $p(\{g_t^e\} \mid \theta, Y)$,
2. Parameters of the PE-specific return: $p(\beta, \phi, \alpha \mid \theta_-, \{g_t^e\}, Y)$,
3. Standard deviation of the PE return shocks: $p(\sigma_g \mid \theta_-, \{g_t^e\}, Y)$, and
4. Standard deviation of likelihood errors: $p(\sigma \mid \theta_-, \{g_t^e\}, Y)$.

We discuss the four sets of state variables and parameters in turn below.

Private Equity Returns, $p(\{g_t^e\} \mid \theta, Y)$

We draw g_t^e using a single-state updating Metropolis-Hasting algorithm (see Jacquier, Polson, and Rossi (1994)). For a single-state update, the joint posterior is

$$
\begin{aligned}
& p(g_t^e \mid \{g_t^e\}_{i\neq t}, \theta, Y) \\
& \quad \propto p(Y \mid \{g_t^e\}_{t=1}^T, \theta) p(\{g_t^e\}_{t=1}^T, \theta, Y) \\
& \quad \propto p(Y \mid \{g_t^e\}_{t=1}^T, \theta) p(g_t^e \mid g_{t-1}^e, g_{t+1}^e, \theta, Y) \\
& \quad \propto p(Y \mid \{g_t^e\}_{t=1}^T, \theta) p(g_t^e \mid g_{t-1}^e, \theta, Y) p(g_{t+1}^e \mid g_t^e, \theta, Y) p(g_t^e)
\end{aligned}
\tag{IA.4}
$$

We can go from the second to third line in equation (IA.4) because g_t^e is Markov. In equation (IA.4), the distribution $p(Y \mid \{g_t^e\}_{t=1}^T, \theta)$ is the likelihood function in equation (IA.2). The distributions of $p(g_t^e \mid g_{t-1}^e, \theta, Y)$ and $p(g_{t+1}^e \mid g_t^e, \theta, Y)$ are implied by the dynamics of g_t^e in equation (IA.1), and they can be expressed as

$$
\begin{aligned}
p(g_t^e \mid g_{t-1}^e, \theta) &\propto \exp\left(-\frac{1}{2\sigma_g^2} (g_t^e - (1-\phi)\alpha - \phi g_{t-1}^e - \beta'(F_t - \phi F_{t-1}))^2 \right) \\
p(g_{t+1}^e \mid g_t^e, \theta) &\propto \exp\left(-\frac{1}{2\sigma_g^2} (g_{t+1}^e - (1-\phi)\alpha - \phi g_t^e - \beta'(F_{t+1} - \phi F_t))^2 \right)
\end{aligned}
\tag{IA.5}
$$

Collecting terms and completing the squares, we obtain

$$
p(g_t^e \mid \{g_t^e\}_{i\neq t}, \theta, Y) \propto p(Y \mid \{g_t^e\}_{t=1}^T, \theta) \exp\left(-\frac{(g_t^e - \mu)^2}{2\sigma_g^2}(1+\phi^2) \right) p(g_t^e), \tag{IA.6}
$$

where

$$
\mu_t = \frac{\phi(g_{t-1}^e + g_{t+1}^e + (1-\phi)\alpha + \beta'((1+\phi^2)F_t - \phi(F_{t+1} + F_{t-1}))}{1+\phi^2}. \tag{IA.7}
$$

For the prior of g_t^e, we impose an uninformative prior, $p(g_t^e) \propto 1$.

We use a Metropolis-Hasting draw with proposal density

$$
q(g_t^e) \propto \exp\left(-\frac{(g_t^e - \mu_t)^2}{2\sigma_g^2}(1+\phi^2) \right). \tag{IA.8}
$$

The acceptance probability for the (*k+1)*-th draw, $g_t^{e,(k+1)}$, is

$$
\min\left(\frac{p(Y \mid g_t^{e,(k+1)}, \{g_i^e\}_{i\neq t}, \theta)}{p(Y \mid g_t^{e,(k)}, \{g_i^e\}_{i\neq t}, \theta)}, 1 \right). \tag{IA.9}
$$

When drawing g_t^e at the beginning or the end of the sample, we integrate out the initial and end values drawing from the process in equation (IA.1).

Parameters of the Private Equity Specific Return $p(\beta, \phi, \alpha \mid \theta_{-}, \{g_t^e\}, Y)$

Consider the factor loadings, β. We can write the posterior

$$
\begin{aligned}
&p(\beta \mid \theta_{-}, \{g_t^e\}, Y) \\
&\quad \propto p(Y \mid \beta, \theta_{-}, \{g_t^e\}) p(\{g_t^e\} \mid \beta, \theta_{-}) p(\beta) \\
&\quad \propto p(\{g_t^e\} \mid \beta, \theta_{-}) p(\beta),
\end{aligned} \tag{IA.10}
$$

because β does not enter the dynamics of the private equity returns, g_t^e. We can rewrite equation (IA.1) as

$$
g_t^e - (1-\phi)\alpha - \phi g_{t-1}^e = \beta'(F_t - \phi F_{t-1}) + \sigma_g \varepsilon_t, \tag{IA.11}
$$

which implies a standard regression draw for β. We use a normal conjugate prior. The draws of φ and α are similar. Although they could be drawn directly in a multivariate conjugate regression draw, we separate them. This allows us to place separate priors on each parameter.

Standard Deviation of the Private Equity Return Shocks, $p(\sigma_g \mid \theta_{-}, \{g_t^e\}, Y)$

We draw σ_g^2 using a conjugate inverse Gamma draw. We select a truncated conjugate prior by confining the range of σ_g between 0.1% and 100% per quarter. We assume the prior

$$
p(\sigma_g^2) \sim IG\left(\frac{a_0}{2}, \frac{b_0}{2}\right) 1_{[10^{-6},1]}, \tag{IA.12}
$$

Where a_0=2 and b_0=10^{-6}. The peak of this prior is far left to the lower bound of our range. Therefore, the truncated prior is approximately a uniform distribution on the range.

We draw the posterior distribution of σ_g^2 from its truncated conjugate posterior:

$$
p(\sigma_g^2 \mid \theta_{-}, Y) \sim IG\left(\frac{a_1}{2}, \frac{b_1}{2}\right) 1_{[10^{-6},1]}, \tag{IA.13}
$$

where $a_1 = a_0 + T - 1$ and $b_1 = b_0 + u$, and u is given by

$$
u = \sum (g_t^e - (1-\phi)\alpha - \phi g_t^e - \beta'(F_t - F_{t-1}))^2. \tag{IA.14}
$$

Standard Deviation of Likelihood Errors: $p(\sigma \mid \theta_{-}, \{g_t^e\}, Y)$

We draw σ^2 using a conjugate truncated inverse Gamma distribution. This follows a similar method to the draw for σ_g. We assume the prior

$$
p(\sigma^2) \sim IG\left(\frac{A_0}{2}, \frac{B_0}{2}\right) 1_{[10^{-6},1]}, \tag{IA.15}
$$

with $A_0 = 10^{-6}$ and $B_0 = 10^{-6}$. Denote $s = \sum_h \left(\ln \frac{PV(D^h)}{PV(I^h)}\right)^2$.

Then the posterior distribution is

$$
p(\sigma^2 \mid \theta_{-}, Y) \sim IG\left(\frac{A_0 + N}{2}, \frac{B_0 + s}{2}\right) 1_{[10^{-6},1]}, \tag{IA.16}
$$

where N is the fund count.

Priors

Like any Bayesian procedure, the estimation requires assumptions on the prior distributions of the parameters. The priors on betas are taken from the current literature on PE as listed in Appendix Table IA.1 (Brav and Gompers (1997), Derwall et al. (2009), Ewens, Jones, and Rhodes-Kropf (2013), Cao and Lerner (2009), Jegadeesh, Kräussl, and Pollet (2015), Chiang, Lee, and Wisen (2005), Lin and Yung (2004), Elton et al. (2001), Korteweg and Sorensen (2010), Franzoni, Nowak, and Phalippou (2012), Driessen, Lin, and Phalippou (2012), Driessen, Lin, and Phalippou (2012)). These studies estimate a three-factor Fama-French model for venture capital, buyout, real estate, or high yield bonds.[2] Real estate estimates are derived from REITs, and credit estimates are derived from industrial BBB-rated bonds of 10-year maturities. The weighted average across subclasses takes the four subclasses averages and weights them by the number of funds in each subclass. The loadings are rounded at 0.05. The average loading in each category is used as the prior.

For the loading on the liquidity factor, there is only the Franzoni, Nowak, and Phalippou (2012) estimate available in the literature and it is available only for buyouts. The authors report a beta of 0.7, which we then use as a prior for buyout funds. For the other subsamples and the pooled PE fund samples, we use a prior of 0.5.

From the estimates in Table IA.1, we set prior means for the market, size, value, and liquidity factors at 1.30, 0.55, 0.05, and 0.50, respectively, for the PE sample. When we estimate factor loadings per fund category (venture capital, buyouts, etc.), we use the average estimate in the literature in the corresponding category. The prior for alpha is set at zero for simplicity. The estimation is insensitive to the alpha priors.

To compute the prior for premium persistence, we use the individual buyout investment return database of Lopez-de-Silanes, Phalippou, and Gottschalg (2015). We compute the correlation between successive investment IRRs of the same PE firm (IRR of investment *i* and IRR of investment *i+1*) and find that it is equal to 0.25. The average spread in starting dates is around six months and investments last for four years. If the process is assumed to be AR(1), this implies an autocorrelation coefficient of 0.5 at yearly frequency, which is what we use as a prior.

[2] Note that Jegadeesh, Kräussl, and Pollet (2015) use a data set that contains predominantly but not exclusively buyout-related vehicles (the rest of their sample is made up of venture capital funds).

We set bounds of ±1 around the prior mean for betas. We impose no bounds on the alpha. The autocorrelation parameter, φ, is restricted to lie between 0 and 0.9 and the latent return is restricted to lie in between -0.50 and 1.00 per quarter.

The standard deviation of the prior captures how diffuse the priors are. We choose a large standard deviation for the priors equal to 10. This would represent an extremely diffuse prior in most contexts. We find, however, that the posterior distributions depend on the volatility of the latent factors more than the priors of the parameters. The volatility of the latent factor is equivalent to determining the R^2, that is, how much of the private equity return is attributable to systematic factors. Since f_t is latent, we could exactly match any private equity return process if there were no restrictions. This can be clearly seen in a traditional linear context, but also occurs in the likelihood for our private equity cash flows. Thus, as expected, the volatility of the latent process influences the informativeness of the priors. We cap the volatility of the latent returns at a large 100% per quarter. For the prior for the volatility of the latent return, we use a mean of 20% per quarter.

Table IA.I

Prior Estimates of the Risk Exposures of Private Equity Funds

This table shows the factor loading estimates shown in the literature. Selected papers are those that estimate a three-factor model for venture capital, buyouts, real estate, or high yield bonds. Jegadeesh, Kräussl, and Pollet (2015) use a data set that contains predominantly but not exclusively buyout related vehicles (the rest of their sample is venture capital related). Real estate estimates are derived from REITs, and credit estimates are derived from industrial BBB-rated bonds of 10-year maturities. The weighted average across subclasses takes the four subclass averages and weights them by the number of funds in each subclass. The loadings are rounded to increments of 0.05. The average loading in each category is used as priors in our Bayesian estimations.

		Venture capital funds		
Authors	Year	β_{mkt}	β_{smb}	β_{hml}
Brav, and Gompers	1997	1.10	1.30	-0.70
Driessen, Lin, and Phalippou	2012	2.40	0.90	-0.25
Ewens, Jones, and Rhodes-Kropf	2013	1.05	-0.10	-0.90
Korteweg, and Sorensen	2009	2.30	1.00	-1.55
Average venture capital funds		**1.70**	**0.80**	**-0.85**
		Buyout funds		
Authors	Year	β_{mkt}	β_{smb}	β_{hml}
Cao, and Lerner	2007	1.30	0.75	0.20
Driessen, Lin, and Phalippou	2012	1.70	-0.90	1.40
Ewens, Jones, and Rhodes-Kropf	2013	0.80	0.10	0.25
Franzoni, Nowak, and Phalippou	2012	1.40	-0.10	0.70
Jegadeesh, Kräussl and Pollet	2010	1.05	0.60	0.35
Average buyout funds		**1.25**	**0.10**	**0.60**
		Real estate		
Authors	Year	β_{mkt}	β_{smb}	β_{hml}
Chiang, Lee, and Wisen	2005	0.55	0.40	0.50
Derwall, Huij, Brounen, and Marquering	2009	0.65	0.40	0.60
Lin, and Yung	2004	0.55	0.40	0.70
Average real estate		**0.60**	**0.40**	**0.60**
		Credit		
Authors	Year	β_{mkt}	β_{smb}	β_{hml}
Elton, Gruber, Agrawal, and Mann	2001	0.70	1.30	1.45
Average high yield debt		**0.70**	**1.30**	**1.45**
Weighted average across subclasses		**1.30**	**0.55**	**0.05**

Table IA.II

Risk and Return of Other Types of Private Equity Funds

This table is the same as Table V in the main article. Statistics are shown for subsamples of private equity funds.

Panel A: Real Estate Funds (N = 161 funds)						
Model	β_{market}	β_{size}	β_{value}	$\beta_{illiquidity}$	Alpha	R^2
CAPM	0.47[a]				0.00	8.9%
	0.18				*0.01*	
3 factors (FF)	0.51[a]	0.34	0.46[b]		-0.01[c]	14.7%
	0.19	*0.26*	*0.22*		*0.01*	
4 factors (PS)	0.62[a]	0.38	0.54[b]	0.36[c]	-0.03[a]	20.3%
	0.20	*0.26*	*0.24*	*0.22*	*0.01*	
Panel B: Credit Funds (N = 144 funds)						
Model	β_{market}	β_{size}	β_{value}	$\beta_{illiquidity}$	Alpha	R^2
CAPM	0.51[a]				0.05[a]	9.0%
	0.18				*0.01*	
3 factors (FF)	0.52[a]	1.21[a]	1.25[a]		0.01	28.7%
	0.20	*0.26*	*0.21*		*0.01*	
4 factors (PS)	0.56[a]	1.20[a]	1.24[a]	0.27	-0.01	41.3%
	0.19	*0.25*	*0.22*	*0.19*	*0.01*	
Panel C: Infrastructure and Natural Resource Funds (N=81 funds)						
Model	β_{market}	β_{size}	β_{value}	$\beta_{illiquidity}$	Alpha	R^2
CAPM	0.58[a]				0.05[a]	9.9%
	0.20				*0.01*	
3 factors (FF)	0.53[b]	1.22[a]	1.34[a]		0.03[a]	34.9%
	0.22	*0.26*	*0.24*		*0.01*	
4 factors (PS)	0.56[b]	1.21[a]	1.37[a]	0.50[b]	0.01	36.4%
	0.23	*0.26*	*0.25*	*0.24*	*0.01*	
Panel D: Fund of Funds (N=189 funds)						
Model	β_{market}	β_{size}	β_{value}	$\beta_{illiquidity}$	Alpha	R^2
CAPM	0.84[a]				0.00	25.7%
	0.19				*0.01*	
3 factors (FF)	0.88[a]	1.11[a]	1.25[a]		0.00	43.8%
	0.21	*0.23*	*0.21*		*0.01*	
4 factors (PS)	0.84[a]	1.13[a]	1.28[a]	0.40[c]	0.00	47.4%
	0.19	*0.24*	*0.22*	*0.23*	*0.01*	

Table IA.III

Precision of Key Estimated Parameters

This table compares the estimation of beta and total volatility to the true value of these two parameters in different simulated datasets. A quarterly error term is added to investment quarterly returns (idiosyncratic volatility; I_vol), but this is not modelled by the econometrician; instead the error term in the econometrics model is on the Net Present Value of funds. By default, returns are generated with a one factor model with a true beta of 1.5, alpha of 4% p.a., plus a mean zero PE-specific return with 20% annual volatility (sigma_f), plus 20% idiosyncratic volatility. There is no autocorrelation is residuals; average holding period (duration) is 4 years, and the prior on beta is correct. The sample contains 600 funds that make 20 investments each. Panel A shows results from simulations with different fee structures. Cases with discrete fee structures are when fees are subtracted from investment value only in the quarter they are charged. Cases with continuous fee structures are when carried interest is computed each quarter as the value of a call option that reflects the current value of the carried interest that will be paid at exit. Panel B shows results from several simulations. Incorrect priors are on beta and 0.50 too low, sigma_f can be low (10%) or high (30%), true beta can be low (0.75) or high (1.50) and idiosyncratic volatility can be low (10%) or high (100%).

Panel A: Different fee structures

			True value over Estimated value			
			Beta		Total volatility	
Fee type	Mngt fees	Carry	Mean	StDev	Mean	StDev
Discrete	0.00	0.00	1.01	0.07	1.12	0.05
Discrete	0.00	0.20	1.01	0.07	1.14	0.06
Discrete	0.02	0.00	1.04	0.07	1.15	0.08
Discrete	0.02	0.20	1.11	0.10	1.22	0.08
Continuous	0.00	0.10	0.99	0.08	1.04	0.06
Continuous	0.00	0.20	0.91	0.08	1.04	0.07
Continuous	0.00	0.30	0.89	0.13	1.04	0.10

Panel B: Varying priors, beta and volatilities

				True value over Estimated value			
				Beta		Total volatility	
Priors	Sigma_f	Beta	I_vol	Mean	StDev	Mean	StDev
Correct	Low	Low	Low	1.00	0.20	1.00	0.10
Correct	Low	Low	High	1.00	0.05	1.01	0.05
Correct	Low	High	Low	1.04	0.11	1.05	0.12
Correct	Low	High	High	1.01	0.15	1.27	0.12
Correct	Mid	Low	Low	1.02	0.24	1.29	0.16
Correct	Mid	Low	High	1.01	0.07	1.12	0.05
Correct	Mid	High	Low	1.04	0.11	1.15	0.12
Correct	Mid	High	High	1.04	0.17	1.50	0.15
Correct	High	Low	Low	1.05	0.33	1.58	0.31
Correct	High	Low	High	1.01	0.09	1.23	0.08
Correct	High	High	Low	1.06	0.12	1.37	0.15
Correct	High	High	High	1.42	0.27	1.10	0.11

Incorrect	Low	Low	Low	1.51	0.62	1.13	0.19
Incorrect	Low	Low	High	1.15	0.10	1.12	0.08
Incorrect	Low	High	Low	1.29	0.21	1.26	0.17
Incorrect	Low	High	High	1.46	0.23	1.45	0.13
Incorrect	Mid	Low	Low	1.55	0.69	1.53	0.27
Incorrect	Mid	Low	High	1.18	0.11	1.24	0.08
Incorrect	Mid	High	Low	1.38	0.33	1.47	0.26
Incorrect	Mid	High	High	1.49	0.33	1.73	0.14
Incorrect	High	Low	Low	1.60	0.76	2.05	0.42
Incorrect	High	Low	High	1.18	0.17	1.36	0.10
Incorrect	High	High	Low	1.47	0.35	1.80	0.32
Incorrect	High	High	High	1.00	0.20	1.00	0.10

Table IA.IV

Monte Carlo Simulations when error structure is NOT as specified in our model

We generate a set of cash flows for 500 funds over 80 quarters. The simulation setup is described in details in the text. Each case is simulated 100 times. In the default case, each fund holds five investments, for an average of 3.5 years, factor returns and idiosyncratic volatility of investment returns is 20% p.a. Each of the following cases changes one parameter compared to the default specification. In case 2, idiosyncratic volatility is decreased from 20% to 1%. In case 3, cash flows are generated according to Metrick and Yasuda (2010) method. This means that the net of fees cash flows have lower mean (and a slightly lower volatility). In case 4, the best performing investments are those sold each quarter (funds are 'holding on to losers'). In case 5, no factor is used in the estimation. In case 6, the number of funds is reduced from 500 to 250. In case 7, the number of investments per fund increases from 5 to 20. In case 8, the average holding period is reduced from 14 quarters to 2 quarters. In case 9, investments pay an intermediate dividend of 1% of investment value each quarter. In case 10, factor volatility is decreased from 20% to 10%. In case 11, priors are incorrect (prior on alpha is zero and prior on beta is one).

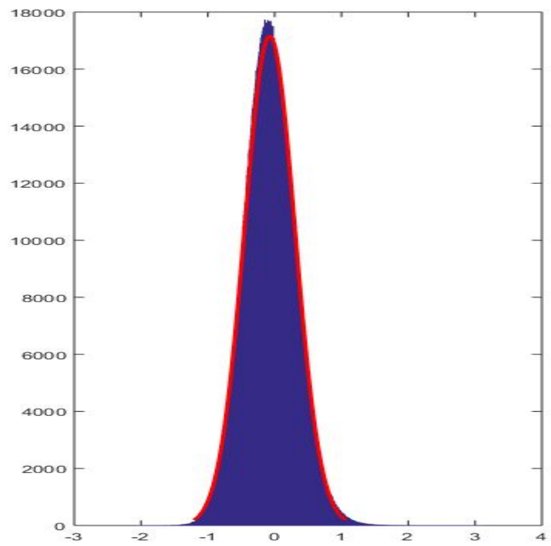
Panel A: Recovering the full time-series of returns

	True Mean	True Volatility	Estimated Mean	Estimated Volatility	Correlation	MSE
1. Default specification	0.16	0.32	0.16	0.32	96.3%	0.21
2. Decrease idiosyncratic volatility	0.16	0.32	0.16	0.32	97.8%	0.13
3. Metrick-Yasuda cash flows	0.10	0.31	0.10	0.31	95.6%	0.24
4. Hold on to losers	0.16	0.32	0.17	0.33	91.2%	0.55
5. No factor structure	0.04	0.10	0.04	0.10	74.5%	0.19
6. Decrease number of funds	0.16	0.32	0.17	0.32	92.9%	0.35
7. More investments per fund	0.16	0.32	0.17	0.32	92.1%	0.42
8. Decrease holding period	0.16	0.32	0.16	0.32	98.1%	0.12
9. Add intermediary cash flows	0.16	0.32	0.16	0.33	96.0%	0.22
10. Decrease factor volatility	0.16	0.32	0.16	0.32	96.1%	0.22
11. Wrong priors	0.16	0.32	0.16	0.30	95.9%	0.23

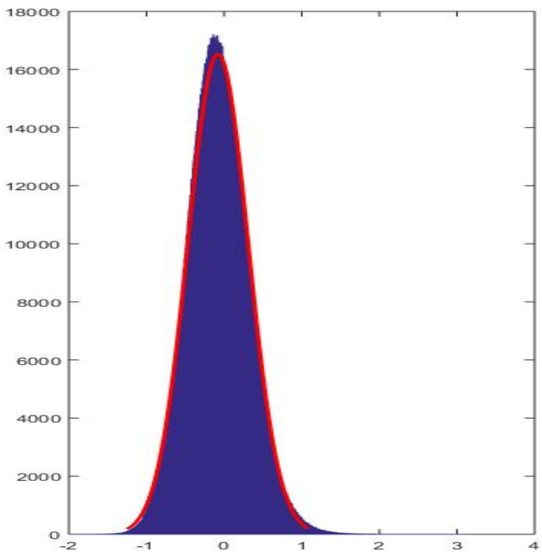
Panel B: Recovering model parameters

	Alpha			Beta		
Percentiles	25th	50th	75th	25th	50th	75th
1. Default specification	0.02	0.04	0.08	1.47	1.51	1.54
2. Decrease idiosyncratic volatility	0.02	0.04	0.07	1.49	1.52	1.54
3. Metrick-Yasuda cash flows	-0.03	0.00	0.03	1.41	1.46	1.48
4. Hold on to losers	0.03	0.05	0.09	1.43	1.51	1.59
5. No factor structure	0.02	0.04	0.08	0.00	0.00	0.00
6. Decrease number of funds	0.03	0.05	0.09	1.43	1.49	1.54
7. More investments per fund	0.03	0.05	0.09	1.42	1.48	1.54
8. Decrease holding period	0.02	0.04	0.07	1.47	1.51	1.55
9. Add intermediary cash flows	0.02	0.04	0.08	1.47	1.51	1.54
10. Decrease index volatility	0.02	0.04	0.08	1.48	1.52	1.55
11. Wrong priors	0.03	0.07	0.09	1.19	1.24	1.27

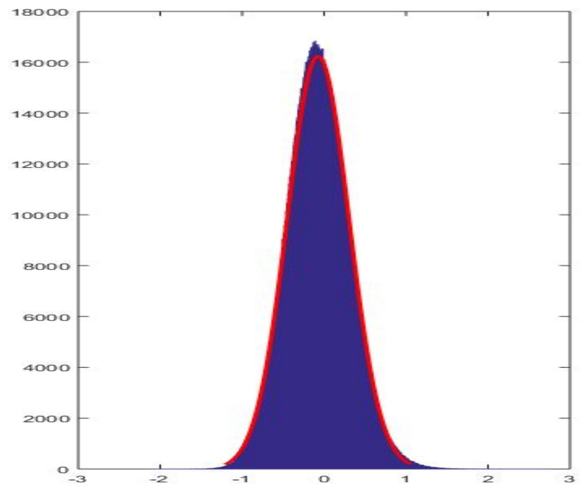
Panel A: No skew (equal size and duration)



Panel B: Skewed size (equal duration)



Panel C: Skewed duration (equal size)



Panel D: Skewed size and duration

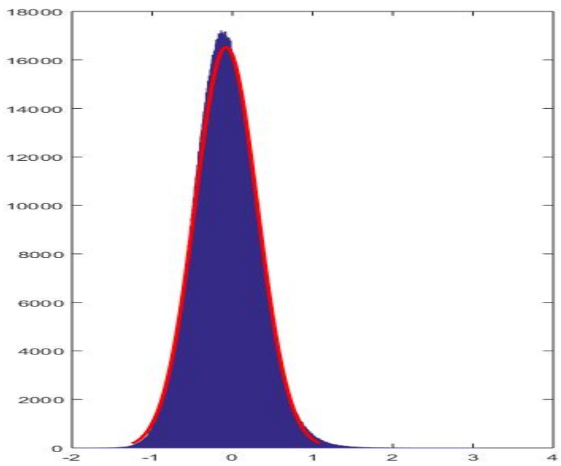


Figure A1. Validity of Assumption 2 in Small Samples and Worse Case Scenarios. There are twenty investments in a fund. Fund size is one. The size of the investments in a fund is either 0.05 each, or one investment is 0.2 and the rest of the investments are 0.8/19. Investment duration is either 16 quarters each, or one investment last for 40 quarters and the rest of the investments last for 16 quarters. We draw ϵ_t^i as i.i.d, normal distribution with a volatility of 50% per annum (because this generates a distribution of PMEs whose volatility matches the one we observe empirically). We compute U_{t_i,T_j}^i for each investment in a fund according to the definition in equation (6) and take the value weighted sum of the twenty U_{t_i,T_j}^i we have for each fund and obtain u_h for fund h as defined in equation (15). Assumption (2) holds if $\ln u_h$is normally distributed. We show below that distribution obtained by Monte-Carlo simulations (N = 1 million) and compare it to the best fitting Normal distribution.

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