Let be the historical returns of indices corresponding to CMA asset classes for periods. Generally, . Using Singular Value Decomposition (SVD) –

where are the left and right orthonormal eigenvectors and has the singular values along its diagonal. We may reduce the dimensionality by limiting to specific eigenvalues –

where .

Given , as the historical returns of risk factors, we can propose a linear relationship between the factors and the CMAs as –

(the OLS with asset classes on RHS, factors on LHS)

(reduced set of principal components)

where . So, we can estimate .

From the above equations, we have –

Thus, given a vector of projected returns for the CMAs, , we can estimate a vector of projected returns for risk factors, .