**The Scientific Basis Behind G3 (Generalized Multi Asset Model v3.0)**

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Abstract

We develop a returns-based model to estimate the performance of alternative investments such as private equity and hedge funds. The model uses hierarchical Bayesian techniques to address challenges posed by limited disclosure, illiquidity, and appraisal-based valuations that are common in alternative asset classes.

By jointly estimating latent economic returns and systematic factor exposures, we construct back cast and forecast returns to assess an alternative assets’ fit within a broader portfolio. Key contributions include a unique moving average process to mimic reporting lags, use of time-weighted returns within this context, and stochastic search variable selection (SSVS) to identify significant factor exposures. The model is evaluated through explanatory power and exposure significance tests. This paper aims to improve industry analytics as applied to alternative investments and support the integration of alternatives into multi-asset portfolios.

# Introduction and Motivation

Preqin estimates that the weighted average asset allocation in 2023 of 1,785 private pension plans with $3.8tn AUM was 19.1% to alternatives, of which real estate (7.8%), private equity (5.6%), and hedge funds (4.2%) are the largest parts[[1]](#footnote-2). In Preqin’s [study of fundraising from US pensions](https://www.preqin.com/insights/research/reports/fundraising-from-us-pensions-a-guide-to-raising-capital" \t "_blank)[[2]](#footnote-3), 708 public pension funds with a total AUM of $5.9tn had an average target allocation of just under 26% to alternatives. This indicates that alternative investments as an asset class are increasing in size and importance for investors.

Portfolio allocation requires estimates of risk, return, and covariance of the focal assets. Data for alternative asset classes are not as freely available as it is for publicly available securities. Since alternative asset classes are largely exempt from public disclosure requirements, performance metrics based on actual transactions are scarce. This limited data impedes the investment process for portfolios containing such assets.

We address this problem and other issues associated with returns on illiquid and alternative investments by modeling the funds’ returns-generating process. We devise a returns-based model that uses hierarchical Bayesian modeling techniques to simulate the underlying economic returns for funds on iCapital’s Marketplace. The model helps investors research and educate themselves on the compatibility of a particular fund with their investment needs and risk preferences.

## Why Bayesian?

Bayesian techniques offer a more robust and flexible approach to capture the complex dynamics of alternative investment returns due to their ability to:

* incorporate information beyond a fund’s track record,
* handle limited data,
* account for illiquidity-induced smoothing, and
* update estimates as new information becomes available.

Bayesian techniques provide a principled and effective framework for handling sparse data in the context of alternative investment returns. By incorporating prior knowledge, Bayesian methods allow researchers to leverage existing information about market dynamics, factor relationships, and investor behavior to inform the estimation process. This is particularly valuable when dealing with limited track records, as the prior information can help to fill in gaps and provide a more complete picture of the underlying return generating process.

Another key advantage of Bayesian methods is their ability to update estimates as new data becomes available. Unlike traditional frequentist approaches, which often rely on fixed sample sizes and static model specifications, Bayesian techniques allow for the continuous updating of parameter estimates and return predictions as new information arrives. This dynamic updating process is especially useful in the rapidly evolving landscape of alternative investments, where market conditions, investor preferences, and regulatory environments can change quickly.

By leveraging prior knowledge, handling limited and illiquid data, and adapting to new information, Bayesian techniques offer a powerful and flexible toolkit for modeling the complex dynamics of alternative investment returns. As such, they have become increasingly popular among researchers and practitioners seeking to unlock the potential of these asset classes while managing their unique challenges.

### Short Track Records

Short track records in alternative assets returns are a problem that is well-known in the alternative assets’ literature. See, for example, Kaplan and Schoar [2005], who show that there is a dearth of performance metrics based on actual transactions. The available time series data often relies on non-market valuations or multiyear internal rates of return (IRR), often segmented by the vintage years of the funds. For investors seeking to optimally allocate their wealth across public securities and alternative assets, the lack of data is a significant barrier.

Bayesian methodologies allow for the integration of prior beliefs or information about specific funds into the estimation of returns. The prior information effectually supplements limited data, such as short track records. The framework aligns well with the unique characteristics of alternative investments, allowing for more accurate and informed returns estimation.

A robust corpus of academic research exists to support the use of these techniques in alternative asset analysis. For instance, Korteweg and Sorensen [2017] develop a Bayesian approach to estimate the risk and return of venture capital investments, addressing the challenges of limited data and selection bias. Ang, et al [2013] use Bayesian techniques to estimate the returns of private equity investments using cash flow data from limited partners. The authors address the challenges of limited disclosure and the irregular timing of cash flows, which are common in private equity investments. Dixon and Chong [2014] show that in the scarcity of information, the complexity of selecting successful portfolio companies, together with the absence of an empirically substantiated general formulaic approach to venture capital and private equity investments, Bayesian methodologies can be used successfully. Avramov and Zhou [2010] illustrate the efficacy of using Bayesian portfolio analysis in the study of hedge fund investing, among other more public asset classes.

One of the strengths of the model’s Bayesian estimation process provides measures of uncertainty around the estimated returns, and this allows the user to decide for themselves the degree to which they can rely on these estimated, time-weighed returns.

### Smoothing

Another well-known issue for alternative asset analysis in the finance practitioner literature that returns from alternative investment such as PE, hedge funds, or real estate funds, are highly serially correlated (see, for example, Getmansky et al. [2004]). In other words, past values correlate with present values. The serial correlation occurs because of the lack of liquidity in the fund itself or some of the assets held within it. For instance, these illiquid assets may not trade frequently, leading to subjective and other- wise noisy valuations. The effect is such that when funds contain illiquid assets, their reported returns may seem steadier than their actual economic returns (returns that consider all available market information about those securities). The positive serial return correlation commonly leads to a downward bias in estimated return variance.

The effect extends to the reported returns of real estate funds (Geltner [1993]). Investors typically demand monthly or quarterly reporting. But valuations of many properties included in the funds are effectively updated only annually. Each quarter some properties have their valuations updated, and others do not. For some properties, the lack of a new valuation within a quarter might result in a carry-over of their last known value into the current quarter.

Our model deals with this issue using a moving average (MA) process with smoothing parameters estimated using a Bayesian linear regression. Note that the MA is defined in an econometric sense, and not in a literal sense, although it may be interpreted as such.

The economic assumption behind the smoothing process is simple: that the observed fund returns are a weighted average of the fund’s economic returns over the most recent periods, including the current period. This restriction is similar to the one in Getmansky et al [2004] where the observed return for some period , is a weighted average of the “true” returns over the most recent periods. In our case the observed returns are the reported returns, the “true” returns are the latent returns generated using the model.

# Conclusion

Our model represents a significant advancement in the quantitative analysis of alternative investments, such as private equity and hedge funds. By employing hierarchical Bayesian modeling techniques, we effectively address the challenges posed by limited and noisy data, illiquidity, and the need for regular updates as new information becomes available.

The model's ability to estimate unobservable true economic returns and re-smooth them to mimic reported returns provides investors with valuable insights into fund performance, enabling more informed decision-making. The use of time-weighted returns and the incorporation of stochastic search variable selection (SSVS) further enhance the model's robustness and adaptability to various market conditions and investment scenarios.

As alternative investments continue to gain prominence in the portfolios of institutional and individual investors, the need for sophisticated quantitative tools like ours will only grow. By providing a systematic and data-driven approach to evaluating alternative investment funds, we contribute to the democratization of these asset classes and support the ongoing expansion and liquidity of the alternative investment market.

In conclusion, our model represents a powerful tool for investors, fund managers, and researchers seeking to navigate the complexities of alternative investments. As the model continues to evolve and incorporate new data and insights, it has the potential to become an industry standard, driving innovation and growth in the alternative investment space.

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## Appendix

## The Choice of Prior Distributions

The influence of well-behaved priors declines as data increases. Priors are, of course, essential in Bayesian analysis. In theory, Bayesian models exhibit a property known as ‘consistency.’ This means that as the sample size grows to infinity, the posterior distribution of the parameter of interest will converge to the “true” parameter value[[3]](#footnote-4), assuming the model is correctly specified. In such a scenario, the choice of prior becomes less critical as the sample size increases.

## Stochastic Search Variable Selection (SSVS)

Integral to the returns-generating process is a type of regression known as SSVS. It is a predictor selection technique. Such techniques commonly focus on which predictors to retain, though they also aim for improved predictive performance through developing an encompassing model, or model simplification without adversely affecting predictive accuracy (Piironen and Vehtari [2017]).

SSVS is a Bayesian approach where selector variables are employed to pinpoint which subsets of predictors are worth considering. It works by identifying those predictors that have a higher chance of being relevant, based on their posterior probability. Gibbs sampling[[4]](#footnote-5) is standard when estimating these models. The MCMC approach samples from the distribution of all possible subsets of predictors. The subsets that show up more often in these samples are considered promising because they have a higher probability of being relevant.

We use the SSVS technique of George and McCulloch [1993], also known as a spike and slab regression. The term was coined by Mitchell and Beauchamp [1988] and referred to the prior for the regression coefficients used in their Bayesian hierarchy. This prior was chosen such that the regression parameters were mutually independent with a two-point mixture distribution made up of a uniform flat distribution (the slab) and a degenerate distribution at zero (the spike).

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| *Figure 4: The probability mass function of a fund’s intercept term from the SSVS (spike and slab) regression. Though we see a spike here, the bulk is in the long right tail representing substantial upside and a significantly greater weight of evidence for a positive alpha. This fund has over two years of history.* |

1. https://www.preqin.com/news/kpers-raises-alternatives-allocation-cap-as-funding-shortfalls-persist [↑](#footnote-ref-2)
2. https://www.preqin.com/insights/research/reports/fundraising-from-us-pensions-a-guide-to-raising-capital [↑](#footnote-ref-3)
3. In the frequentist sense of the word. [↑](#footnote-ref-4)
4. Gibbs sampling is a statistical technique used for generating sequences of samples from the probability distribution of multiple variables. It's a kind of Markov Chain Monte Carlo (MCMC) method. It is particularly useful in scenarios where directly sampling from the joint distribution is difficult, but sampling from the conditional distribution of each variable is feasible. [↑](#footnote-ref-5)