**PAPER OUTLINE**

Tentative Title: “Bayesian Modeling of Private Equity Returns”

Abstract: We develop a methodology to estimate time series of private equity fund returns based on returns as reported by general partners. We test this methodology on several funds and find goodness-of-fit measures as high as 85\%. We use a hierarchical Bayesian model to estimate distributions of both, factor loadings and private equity returns. The model de-smooths reported returns, fills in missing data, and allows for analyzing private equity reported returns under the rubric of Modern Portfolio Theory. Using these returns we can effectively allocate weights to both public and private equity for retail investors.

1. Introduction/Motivation/Lit Review
   * There are no other papers that model PE fund returns at the asset-level.
   * We also can provide full probability distributions of the returns, and the factor loadings. We can calculate as many moments as needed.
   * Our technique can also be used to impute missing and unreliable data for fund returns.
   * PMEs are heuristic in nature, we add some mathematical and statistical rigor to the process.
   * Introduction to Bayesian techniques here. Something about the philosophy.
   * Frequentists need repeated sampling to estimate parameters accurately; Bayesians use priors formed from the data itself. This leads to Bayesian techniques being arguably better for modeling PE returns.
   * Place in the literature: PMEs 🡪 Kaplan and Schoar 🡪 Ang, et al 🡪 Us (there are more papers to be listed here, but I think this is the high-level placing).
   * We apply MPT/APT to PE and assume that fund returns can be explained using factor analysis. The factors we use are the ones we think drive PE fund returns. *This should be the economic intuition for the model.*
2. Data
   * We use returns data provided to iCapital by fund managers. Unique dataset! Already adding value.
3. Methodology
   * Some more education on Bayesian techniques here. Focus on the parts that are relevant for us, with appropriate citations (BDA, etc.).
   * A math-light version of the model. None of the algebraic derivations, but the key equations and the priors. I’m thinking some version of the first four pages of your model notes.
   * We use weakly informative priors which are better than uninformative priors (citation provided). Standard practice in social sciences is to use uninformative priors.
   * What did we do? We estimated the smoothing parameter, we estimated the factor loadings, we estimated the factor returns, and finally, we used all those to estimate the actual fund returns. We work under the assumption that fund returns can be explained by the factors that we chose. This is the economic intuition and a way by which practitioners can place us in the MPT/empirical literature.
   * A nice feature of the model is that we can also use the factor loadings as risk metrics.
4. Results and Efficacy
   * For the first draft of the paper, in-sample cross-validation using R^2 as you have already done on the slides should suffice. Your write-up on CV is enough for this, along with the Vehtari citation.
   * Maybe we might do AIC or WAIC because it’s more easily understandable, with the caveats provided in Vehtari so people take it with a grain of salt.
5. Conclusion and Further Research
   * Estimate returns for funds with pre-specified correlations?
   * Other factors to be used?
   * Etc.

**Priors**

The general philosophy followed:

* Small changes to the prior should not have a significant influence on the inferences, or if it does, then that is suggestive that the data isn’t driving the conclusions.
  + Implication: employ sensitivity analysis
* We can also plot the posterior and the prior to see that the prior is or is not driving the shape of the distribution (see Jacquier-Polson-Rossi 2004)
* The priors should be proper- see Rossi 2013 (I added it to the textbooks folder in the lit folder). This allows us to integrate and compute Bayesfactors without needing to worry if we are working on a valid probability distribution. More generally, I am worried about technical regularity conditions that we may inadvertently violate, given that neither of us consider ourselves experts on the econometric theory.
* The priors can be informed by our outside knowledge- see Avramov-Zhou 2009.

Additional Questions:

* Should we highlight the fact that we focus on PE funds and not VC and other types of “PE” funds as in most of the economic literature?