Results from Modified Insertion Sort

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Chapter 1

Introduction

These theorems have been introduced to identify the cases where modified insertion sort performs better than the orthodox insertion sort. The performance has been measuredin terms of number of comparisons involved in sorting the data set.

1.1 Theorem 1

The number of comparisons required to perform the insertion operation in the worst case permutation is $ceiling(n^2/4)$

1.1.1 PROOF

Number of comparisons in the worst case for insertion sort is

$$\frac{n(n-1)}{2} \tag{1.1}$$

Since insertion sort is performed from both the sides,then the total number of compar-
isons is

$$\frac{\frac{n}{2} * (\frac{n}{2} - 1)}{2} \tag{1.2}$$

The number of comparisons required for merging the two results is $n/2$

Summing up all the comparisons, we get

$$\frac{\frac{n}{2} * (\frac{n}{2} - 1)}{2} + \frac{\frac{n}{2} * (\frac{n}{2} - 1)}{2} + \frac{n}{2} = \frac{n^2}{4}$$

(1.3)

The number of comparisons is :

$$ceiling(n^2/4) \tag{1.4}$$

1.1.2 Inference

The difference between the number of comparisons of insertion sort and modified in-sertino sort is:

$$\frac{n(n-1)}{2} - ceiling(n^2/4) \tag{1.5}$$

Now,when n=even,equation 1.5 becomes

$$\frac{n(n-1)}{2} - \frac{n^2}{4} \tag{1.6}$$

Solving,we get:

$$\frac{n(n-2)}{4} \tag{1.7}$$

Now,when n=odd,equation 1.5 becomes,

$$\frac{n(n-1)}{2} - \frac{n^2}{4} - 1 \tag{1.8}$$

Solving,we get:

$$ceiling(\frac{n(n-2)}{4}) - 1 \tag{1.9}$$

For all n¿2,we find that modified insertion sort performs better than the normal insertionsort.

1.2 Theorem 2

The number of comparisons required performing the modified insertion sort is

$$floor(\frac{3n}{2}) - 1 \tag{1.10}$$

1.2.1 Proof

The number of comparisons for best case situation is:

$$(n-1) \tag{1.11}$$

Let us assume that n is even.

Now,the number of comparisons required to sort each half is:

$$(\frac{n}{2}-1) \tag{1.12}$$

Number of comparisons needed are

$$(\frac{n}{2}) \tag{1.13}$$

Adding all of the above, we get

$$\frac{n}{2}-1+\frac{n}{2}-1+\frac{n}{2}=(\frac{3n}{2})-1 \tag{1.14}$$

Now,when n is odd,

The number of comparisons required to sort each half is:

$$(floor(\frac{n}{2})+1)-1+(floor(\frac{n}{2})-1)-1+((floor\frac{n}{2})+1)=floor(\frac{3n}{2})-1 \tag{1.15}$$

Considering both the even and the odd cases,we find that the number of comparisons is:

$$floor(\frac{3n}{2})-1 \tag{1.16}$$

1.2.2 Inference

Subtracting the number of comparisons of the orthodox insertion sort and the modifiedinsertion sort,we find that:

$$(floor(\frac{3n}{2})-1)-(n-1)>0 \tag{1.17}$$

So, insertion sort performs better than the insertion sort when the best case situation ispresented.

1.3 Theorem 3

The number of comparisons required to sort a data set with the order

$$< a1, a2, a3, \ldots, a(n), a(n-1) > \tag{1.18}$$

in modified insertion sort is

$$floor(\frac{3n}{2}) - 2 \tag{1.19}$$

where a1,a2,a3,... are sorted. The only element out of order is a(n-1).

1.3.1 proof

Let the number of elements in the prescribed data set

be n.Then the number of swaps required in insertion

sort is n.

Therefore,the number of comparisons in the modified insertion sort is

$$(\frac{n}{2}) - 1 + (\frac{n}{2}) - 1$$

(1.20)

Then the number of comparisons for the two sorted halves will be

$$(\frac{n}{2}) \tag{1.21}$$

Therefore,the number of comparisons is:

$$(\frac{n}{2}) - 1 + (\frac{n}{2}) - 1 + (\frac{n}{2}) = floor(\frac{3n}{2}) - 2 \tag{1.22}$$

1.3.2 Inferences

The difference between the number of comparisons of insertion sort and the modifiedinsertion sort is

$$n - (\frac{3n}{2}) - 1 > 0 \tag{1.23}$$

Therefore,insertion sort performs better than the modified insertion sort in this case.

1.4 Theorem 4

The number of comparisons required to sort a data set with the order

$$< a(n), a(n-1), a(n-2), \ldots, a(0), a(1) > \tag{1.24}$$

in modified insertion sort is

$$ceiling(\frac{n^2}{4}) \tag{1.25}$$

where a(n),a(n-1),a(n-2),... are reverse sorted.

1.4.1 proof

The number of comparisons for converting the array into two sorted halves is

$$\frac{(\frac{n}{2})(\frac{n}{2} - 1)}{2} \tag{1.26}$$

The number of comparisons for merging the data is:

$$(\frac{n}{2}) \tag{1.27}$$

Adding all the terms,we get

$$\frac{(\frac{n}{2})(\frac{n}{2} - 1)}{2} + \frac{n}{2} = ceiling(\frac{n^2}{4}) \tag{1.28}$$

1.4.2 Inference

Modified insertion sort performs better than the regular insertion sort in the above case.