# Lecture 5
# Matrix-Matrix Product

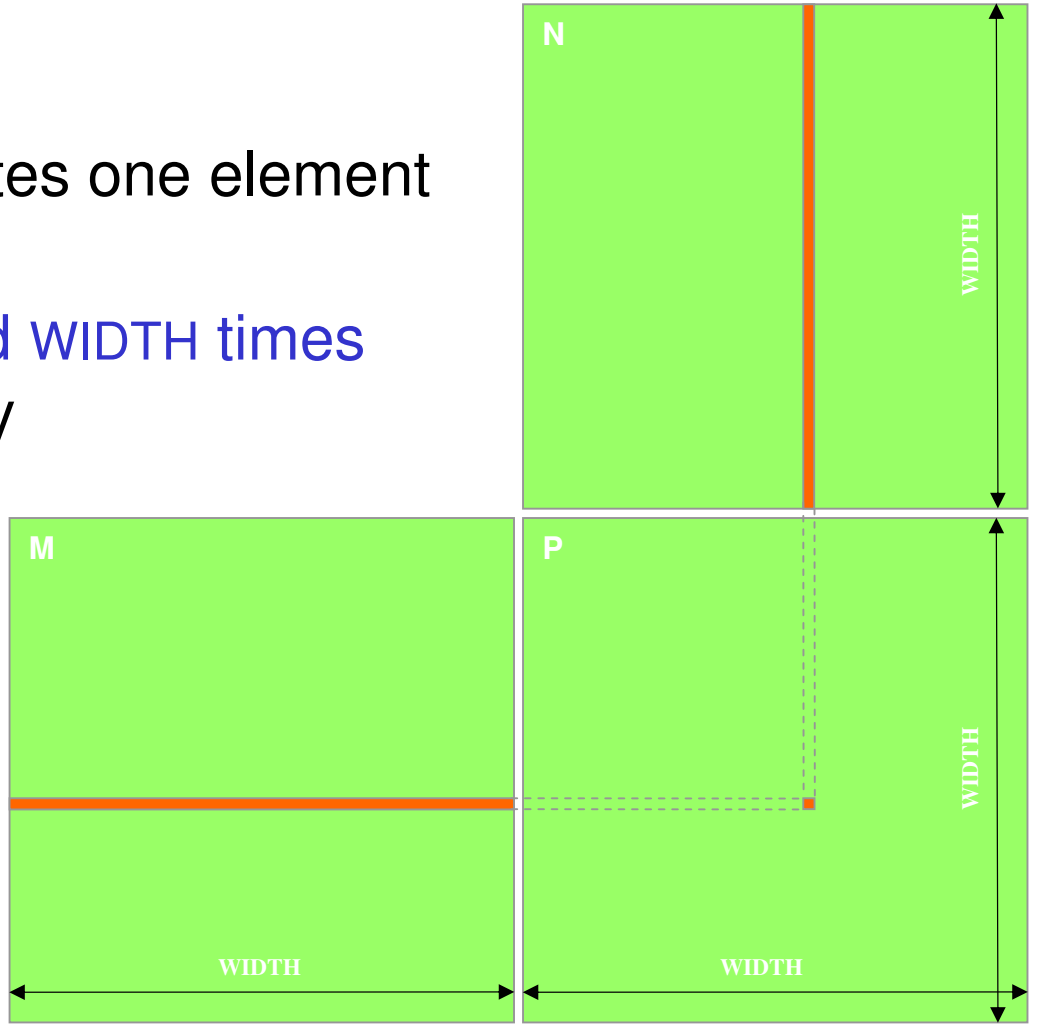
Based on the lecture materials of Hwu (UIUC) and Kirk (NVIDIA)

# Matrix Multiplication

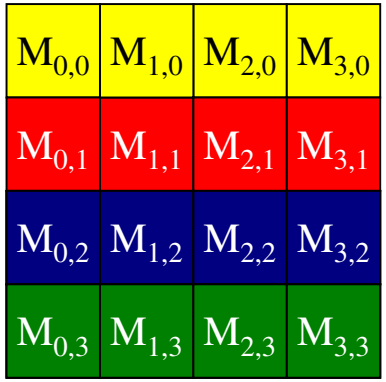
- Simple version first
  - illustrate basic features of memory and thread management in CUDA programs
  - Thread ID usage
  - Memory data transfer API between host and device
  - Analyze performance
- Extend to version which employs shared memory

# Square Matrix Multiplication

- P = M * N of size WIDTH x WIDTH
- Without tiling:
  - One thread calculates one element of P
  - M and N are loaded WIDTH times from global memory

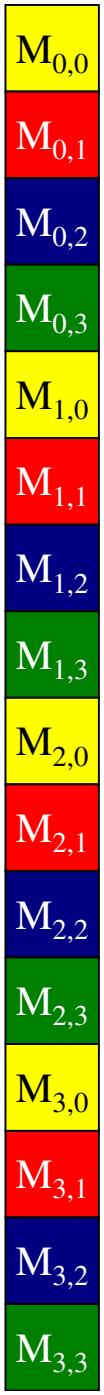


# Memory Layout of a Matrix




M


C order



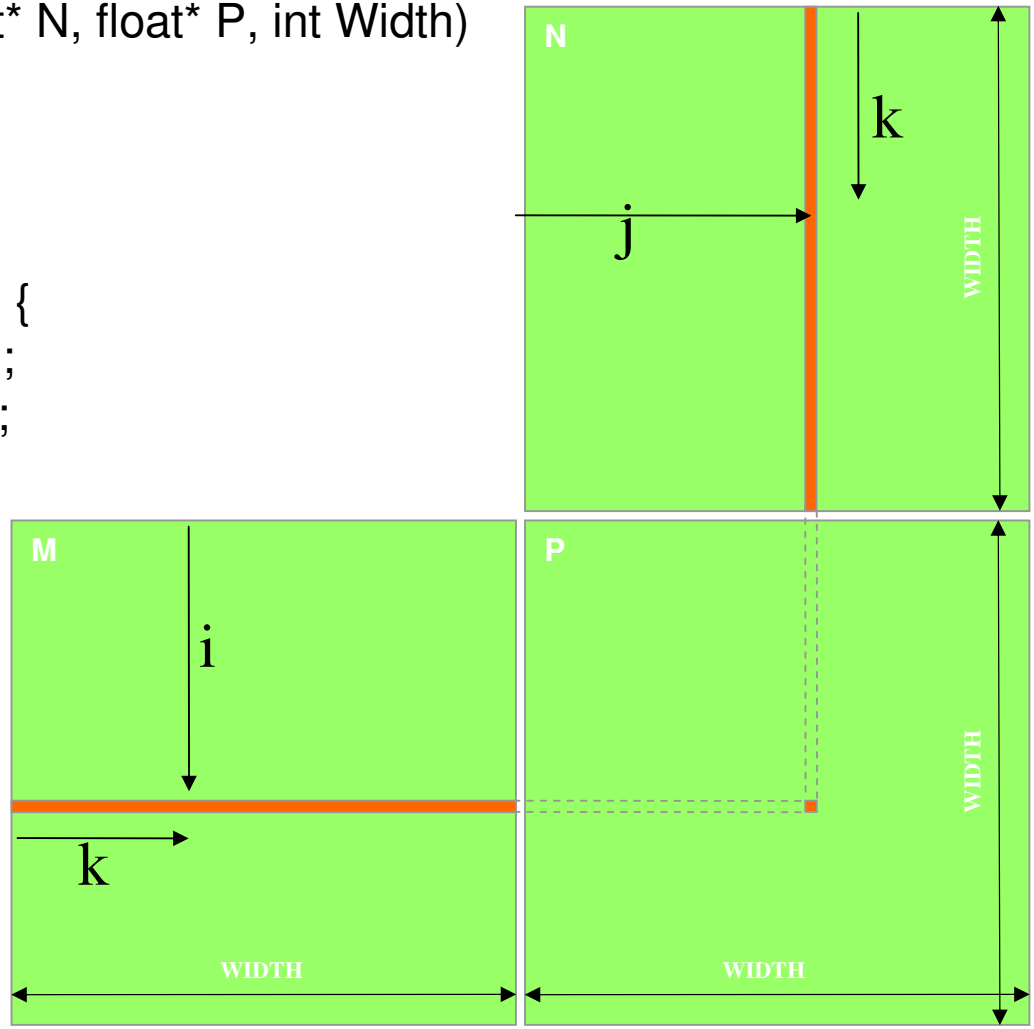

Fortran/Matlab

order

This order will be important to compute the location of the element in the matrix according to thread and block indices

# Step 1: Simple Host Version

```
// Matrix multiplication on the (CPU) host
void MatrixMulOnHost(float* M, float* N, float* P, int Width)
{
    for (int i = 0; i < Width; ++i)
        for (int j = 0; j < Width; ++j) {
            double sum = 0;
            for (int k = 0; k < Width; ++k) {
                double a = M[i * width + k];
                double b = N[k * width + j];
                sum += a * b;
            }
            P[i * Width + j] = sum;
        }
}
```



# Step 2: Transfer Data to Device from Host

```
void MatrixMulOnDevice(float* M, float* N, float* P, int Width)
{
   int size = Width * Width * sizeof(float);
   float* Md, Nd, Pd;
   …
  // 1. Allocate and Load M, N to device memory
    cudaMalloc(&Md, size);
    cudaMemcpy(Md, M, size, cudaMemcpyHostToDevice);
     cudaMalloc(&Nd, size);
     cudaMemcpy(Nd, N, size, cudaMemcpyHostToDevice);
  // Allocate P on the device
    cudaMalloc(&Pd, size);
```

# Step 3: Output Matrix Data Transfer (Host-side Code)

2. // Kernel invocation code – to be shown later

…

3. // Read P from the device

**cudaMemcpy(P, Pd, size, cudaMemcpyDeviceToHost);**

// Free device matrices

cudaFree(Md); cudaFree(Nd); cudaFree (Pd);

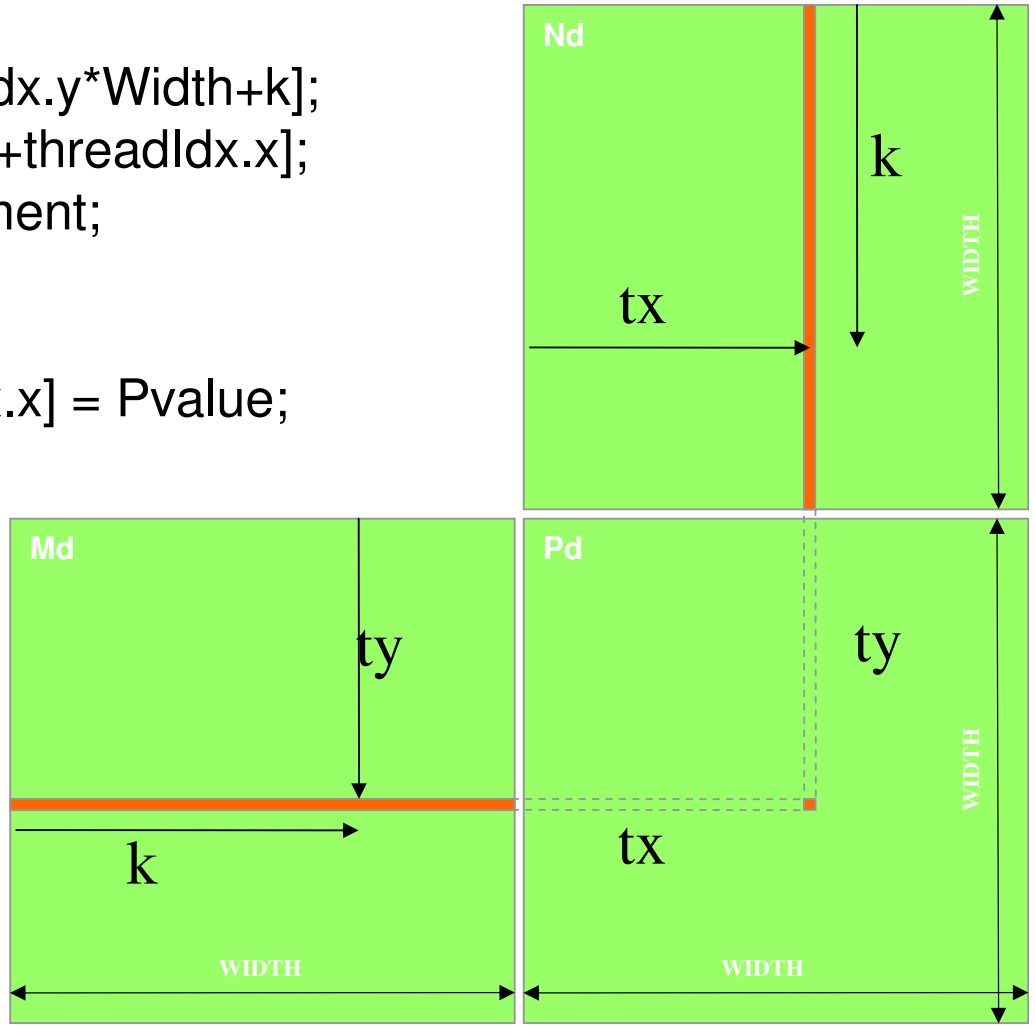
}

# Step 4: Kernel Function

```
// Matrix multiplication kernel – per thread code

__global__ void MatrixMulKernel(float* Md, float* Nd, float* Pd, int Width)
{

    // Pvalue is used to store the element of the matrix
    // that is computed by the thread
    float Pvalue = 0;
```

# Step 4: Kernel Function (cont.)

```
    for (int k = 0; k < Width; ++k) {
        float Melement = Md[threadIdx.y*Width+k];
        float Nelement = Nd[k*Width+threadIdx.x];
        Pvalue += Melement * Nelement;
    }

    Pd[threadIdx.y*Width+threadIdx.x] = Pvalue;
}
```

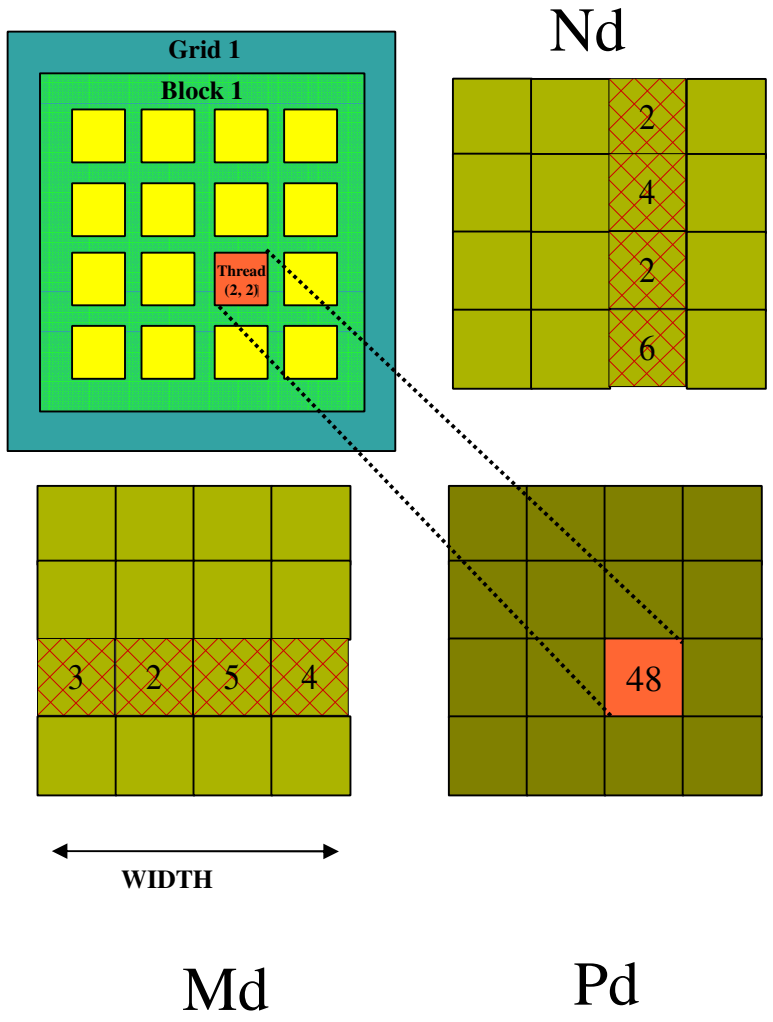


# Step 5: Kernel Invocation (Host-side Code)

```
// Setup the execution configuration
   dim3 dimGrid(1, 1);
    dim3 dimBlock(Width, Width);


// Launch the device computation threads!
MatrixMulKernel<<<dimGrid, dimBlock>>>(Md, Nd, Pd, Width);
```

# First version: One Thread Block

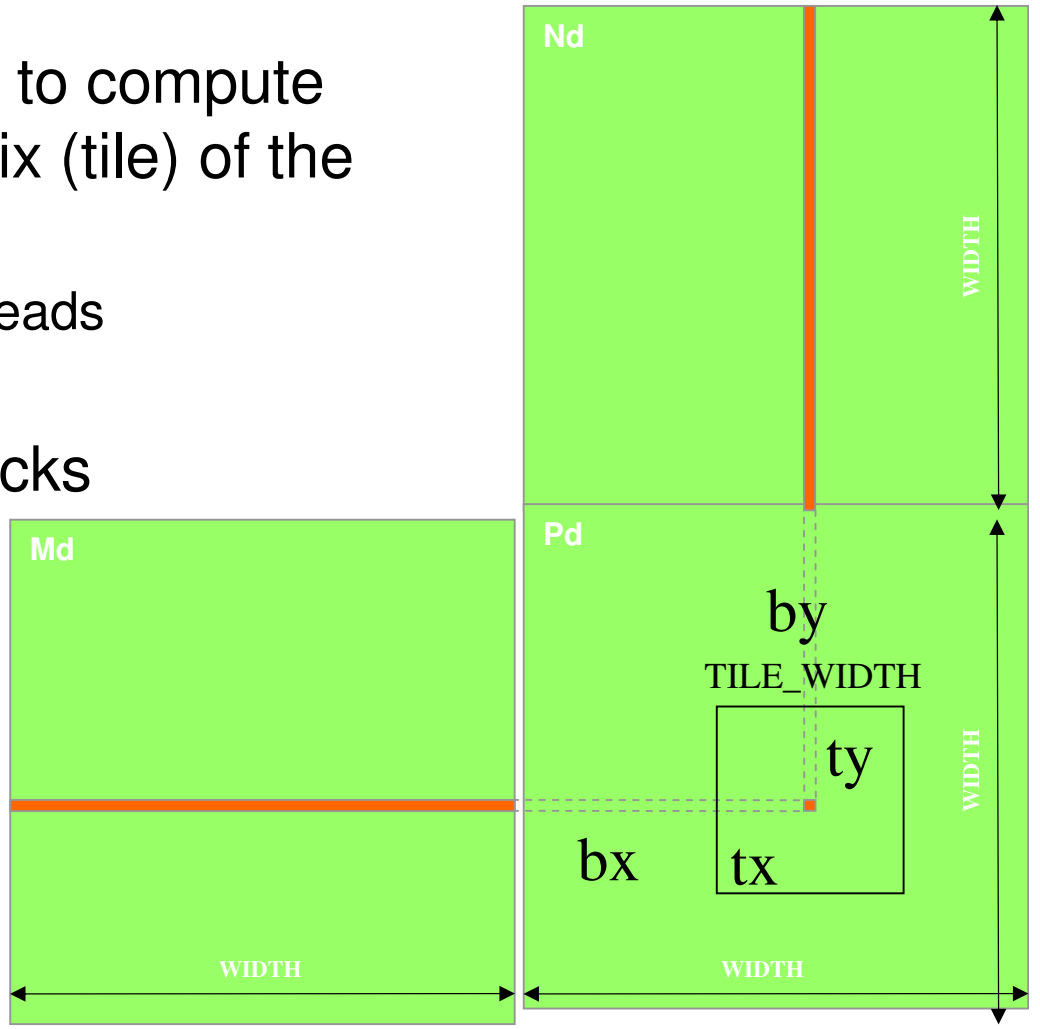
- One Block of threads compute matrix Pd
  - Each thread computes one element of Pd
- Each thread
  - Loads a row of matrix Md
  - Loads a column of matrix Nd
  - Perform one multiply and addition for each pair of Md and Nd elements
  - Compute to off-chip memory access ratio close to 1:1 (not very high)
- Size of matrix limited by the number of threads allowed in a thread block
  - It is 512. So the number allowed is <23



# Extend to Arbitrary Sized Square Matrices

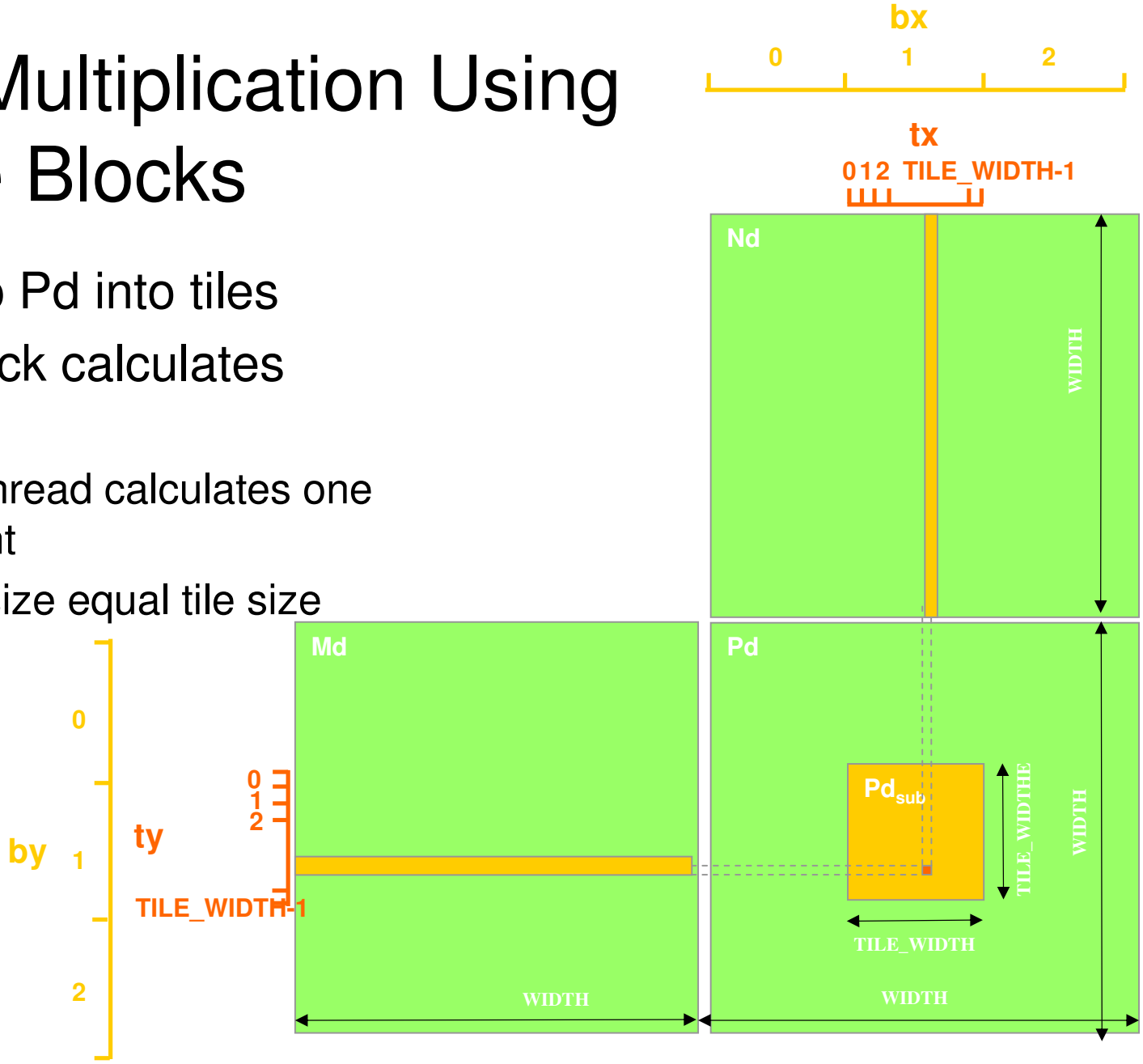
- Use more than one block
- Have each 2D thread block to compute a $(TILE\_WIDTH)^2$ sub-matrix (tile) of the result matrix
  - Each has $(TILE\_WIDTH)^2$ threads
- Generate a 2D Grid of $(WIDTH/TILE\_WIDTH)^2$ blocks

You still need to put a loop around the kernel call for cases where WIDTH/TILE_WIDTH is greater than max grid size (64K)!

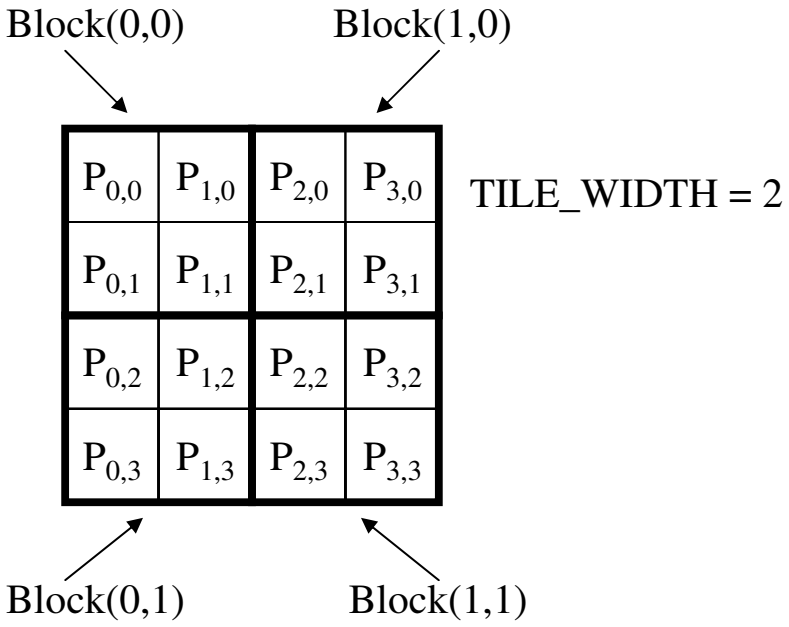


# Matrix Multiplication Using Multiple Blocks

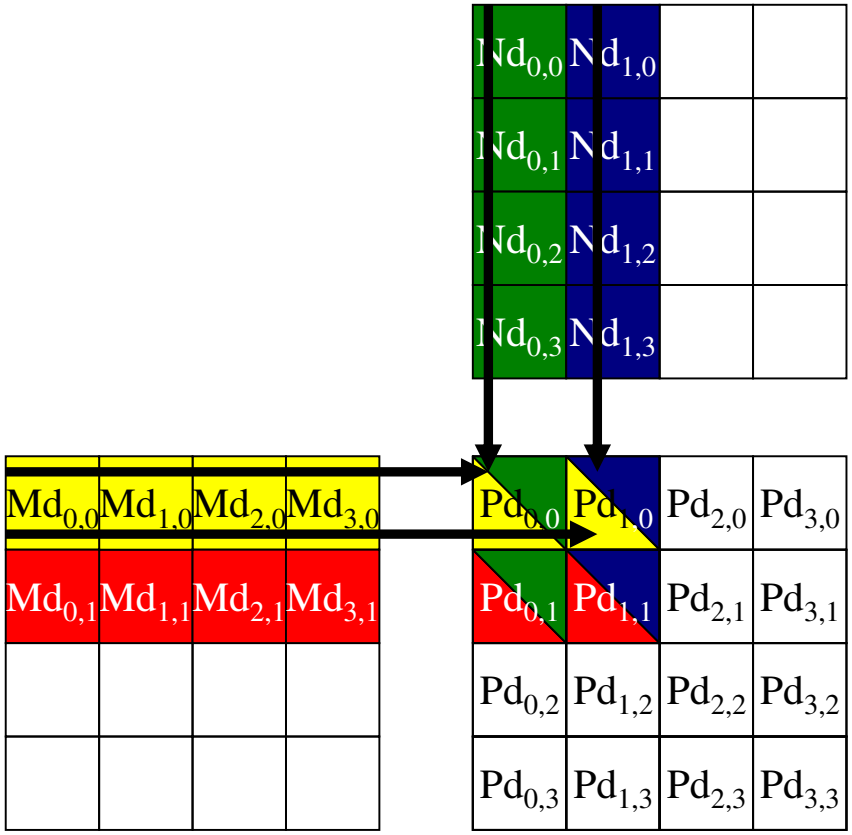
- Break-up Pd into tiles
- Each block calculates one tile
  - Each thread calculates one element
  - Block size equal tile size



# A Small Example



# A Small Example: Multiplication



# Revised Matrix Multiplication Kernel using Multiple Blocks

```
__global__ void MatrixMulKernel(float* Md, float* Nd, float* Pd, int Width)
{
// Calculate the row index of the Pd element and M
int Row = blockIdx.y*TILE_WIDTH + threadIdx.y;
// Calculate the column index of Pd and N
int Col = blockIdx.x*TILE_WIDTH + threadIdx.x;

float Pvalue = 0;
// each thread computes one element of the block sub-matrix
for (int k = 0; k < Width; ++k)
  Pvalue += Md[Row*Width+k] * Nd[k*Width+Col];

Pd[Row*Width+Col] = Pvalue;}
```

Note how the Row and Column indices are computed.