CS621/CSL611
Quantum Computing For Computer Scientists

The Leap from Classical to Quantum

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Quantum Systems

Enter the world of the quantum

- Let us generalize our states and graphs.
- For our states, rather than insisting that the sum of the entries in the column vector is 1, we shall require that the sum of the modulus squared of the entries be 1.
- This makes sense because we are considering the probability as the modulus squared.
- An example of such a state is

$$X = \left[\frac{1}{\sqrt{3}}, \frac{2i}{\sqrt{15}}, \sqrt{\frac{2}{5}}\right]^T$$

- Rather than talking about graphs with real number weights, we shall talk about graphs with complex number weights.
- Instead of insisting that the adjacency matrix of such a graph be a doubly stochastic matrix, we ask instead that the adjacency matrix be unitary.

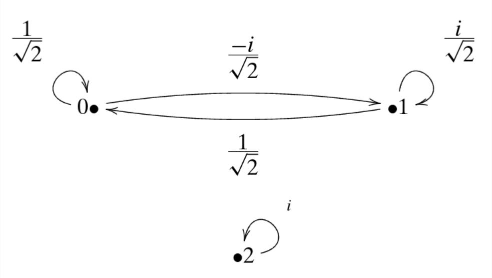
Definition (Unitary Mattrix)

A matrix U is unitary if

$$U \star U^{\dagger} = I = U^{\dagger} \star U$$

Transition

- Consider the graph and corresponding unitary adjacency matrix.



$$U = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{-i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & i \end{bmatrix}$$

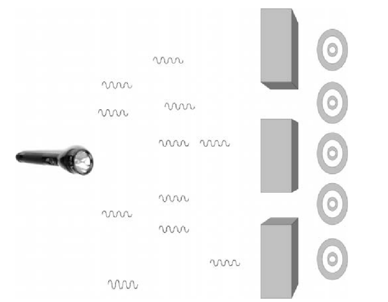
- How unitary matrices act on states?

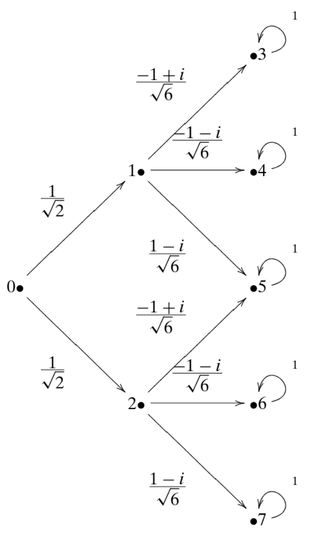
$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{-i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & i \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{2i}{\sqrt{15}} \\ \sqrt{\frac{2}{5}} \end{bmatrix} = \begin{bmatrix} \frac{5+2i}{\sqrt{30}} \\ \frac{-2-\sqrt{5}i}{\sqrt{30}} \\ \sqrt{\frac{2}{5}}i \end{bmatrix}.$$

- Notice that the sum of the modulus squares of Y is 1.

HomeWork

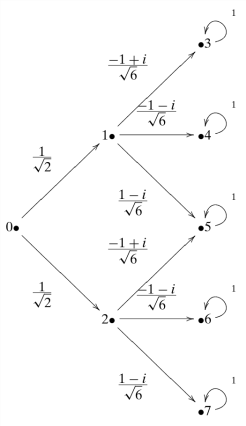
Prove that a unitary matrix preserves the sum of the modulus squares of a column vector multiplied on its right.





- Each slit has a 50% chance of the photons passing through it
- This is captured by $\frac{1}{\sqrt{2}}$ in the graph since $\left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$

The adjacency matrix, P



$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1+i}{\sqrt{6}} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1-i}{\sqrt{6}} & \frac{-1+i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1-i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

- Note: P is not unitary. Why?
- Aim: To demonstrate the phenomenon of interference

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1+i}{\sqrt{6}} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1-i}{\sqrt{6}} & \frac{-1+i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1-i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 1 \end{bmatrix} . \quad |P[i,j]|^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- Note: $|P[i,j]|^2 = B$ from the probabilistic double-slit experiment due to the choice of complex numbers for P
- Implication: Behavior same as probabilistic case for one time click

$$P^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1+i}{\sqrt{12}} & \frac{-1+i}{\sqrt{6}} & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{-1-i}{\sqrt{12}} & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{-1+i}{\sqrt{6}} & \frac{1-i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 \\ \frac{-1-i}{\sqrt{12}} & 0 & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 0 & 1 & 0 \\ \frac{-1+i}{\sqrt{12}} & 0 & \frac{1-i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad |P^2[i,j]|^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 1 & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & 1 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- How does $|P^2[i,j]|^2$ compare with B^2 in the probabilistic experiment?

$$B^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 1 & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & 1 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad |P^2[i,j]|^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 1 & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & 1 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Where do the matrices differ?

- $B^2[5,0] = \frac{1}{2}\left(\frac{1}{3}\right) + \frac{1}{2}\left(\frac{1}{3}\right) = \frac{1}{3}$
- $P^2[5,0] = \frac{1}{\sqrt{2}}\left(\frac{1-i}{\sqrt{6}}\right) + \frac{1}{\sqrt{2}}\left(\frac{-1+i}{\sqrt{6}}\right) = 0 \implies |P^2[5,0]|^2 = 0$
- Implication: There will be no photon at vertex 5

How to justify $|P^2[5,0]|^2 = 0$?

- Interference?
  - Points to the wave-like nature of light
- What about the photo-electric effect?
  - Points toward a different direction: light is absorbed and emitted in discrete quantities photons

Dual Nature of Light (and Matter)

On some occasions it acts as a beam of particles, and at other times it acts like a wave

How to explain the following?

Experiment can be done with a single photon shot from vertex 0. Even in this scenario, interference will **still** occur.

- Let the state of the system be given by

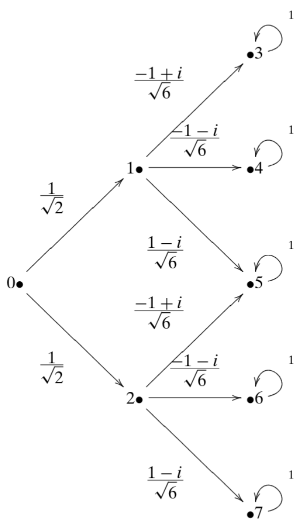
$$X = [c_0, c_1, \cdots, c_{n-1}]^T \in \mathbb{C}_n$$

- **Incorrect**: Prob. of the photon's being in position k is $|c_k|^2$
- To be in state X means that the particle is in some sense in **all** positions **simultaneously**.

- The photon passes through the top slit and the bottom slit simultaneously
- And when it exits both slits, it can cancel itself out.

Superposition

A photon is not in a single position, rather it is in many positions, a **superposition**.



- The Superposition is a private world of the photon
- To observe we must make a *measurement*
- When **measured** the quantum object is no longer in a superposition of states.
- It **collapses** to a single classical state.

Definition (State of a quantum system)

A system is in state X means that after measuring it, it will be found in position i with probability $|c_i|^2$.

- This feature of quantum physics, that a measurement disturbs the system and forces it to choose is another strange phenomenon *with no classical analog*.

- Richard Feynman's comments on the double-slit experiment

  'We choose to examine a phenomenon which is impossible, absolutely impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the only mystery. We cannot make the mystery go away by "explaining" how it works. We will just tell you how it works.' **Feynman, 1963, Vol. III, page 1-1**

Leveraging the Power of Superposition

- Superposition of states is the real power behind quantum computing.
- Classical computers are in one state at every moment.

Ultimate possibility in parallel processing

- Imagine putting a computer in many different classical states simultaneously
- Then processing with all the states at once
- Welcome to Quantum Computing

Take-Away So far

- States in a quantum system are represented by column vectors of complex numbers whose sum of moduli squared is 1.
- The dynamics of a quantum system is represented by unitary matrices and is therefore reversible. The undoing is obtained via the algebraic inverse, i.e., the adjoint of the unitary matrix representing forward evolution.
- The probabilities of quantum mechanics are always given as the modulus square of complex numbers.
- Quantum states can be superposed, i.e., a physical system can be in more than one basic state simultaneously