CALCULUS 2 FORMULA SHEET

Integrals/Antiderivatives

$\int u^n\,du = \frac{u^{n+1}}{n+1} + c$

$\int sec\, u\, tan\, u\, du = sec\, u + c$

$\int sin\, u\, du = -cos\, u + c$

$\int cos\, u\, du = sin\, u + c$

$\int sec^2 u\, du = tan\, u + c$

$\int csc^2 u\, du = -cot\, u + c$

$\int csc\, u\, cot\, u\, du = -csc\, u + c$

$\int tan\, u\, du = ln\,|sec\, u| + c$

$\int cot\, u\, du = ln\,|sin\, u| + c$

$\int sec\, u\, du = ln\,|sec\, u + tan\, u| + c$

$\int csc\, u\, du = ln\,|csc\, u - cot\, u| + c$

$\int \frac{1}{u} du = ln\,|u| + c$

$\int e^u\, du = e^u + c$

$\int a^u\, du = \frac{a^u}{ln\, a} + c$

Inverse Trigonometric Functions

$\int \frac{du}{\sqrt{a^2-u^2}} = sin^{-1}\frac{u}{a} + c$

$\int \frac{du}{a^2+u^2} = \frac{1}{a}\, tan^{-1}\frac{u}{a} + c$

$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\, sec^{-1}\frac{u}{a} + c$

$D_x(sin^{-1} u) = \frac{1}{\sqrt{1-u^2}} \cdot u'$

$D_x(tan^{-1} u) = \frac{1}{1+u^2} \cdot u'$

$D_x(sec^{-1} u) = \frac{1}{u\sqrt{u^2-1}} \cdot u'$

$D_x(cos^{-1} u) = \frac{-1}{\sqrt{1-u^2}} \cdot u'$

$D_x(cot^{-1} u) = \frac{-1}{1+u^2} \cdot u'$

$D_x(csc^{-1} u) = \frac{-1}{u\sqrt{u^2-1}} \cdot u'$

Inverse Hyperbolic Functions

$\int \frac{du}{\sqrt{a^2+u^2}} = sinh^{-1}\frac{u}{a} + c = ln\,(u + \sqrt{u^2+a^2}) + c \text{ if } a > 0$

$\int \frac{du}{\sqrt{u^2-a^2}} = cosh^{-1}\frac{u}{a} + c = ln\,(u + \sqrt{u^2-a^2}) + c \text{ if } u > a > 0$

$\int \frac{du}{a^2-u^2} = \frac{1}{a}\, tanh^{-1}\frac{u}{a} + c \text{ if } |u| < a$

$= \frac{1}{a}\, coth^{-1}\frac{u}{a} + c \text{ if } |u| > a$

$= \frac{1}{2a}\, ln\left|\frac{a+u}{a-u}\right| + c$

$y = sinh^{-1} x \text{ IFF } x = sinh\, y \quad y \in R$

$y = cosh^{-1} x \text{ IFF } x = cosh\, y \quad y \geq 0$

$y = tanh^{-1} x \text{ IFF } x = tanh\, y \quad y \in R - \{0\}$

$sinh^{-1} x = ln\,(x + \sqrt{x^2+1}) \quad x \in R$

$cosh^{-1} x = ln\,(x + \sqrt{x^2-1}) \quad x \geq 1$

$tanh^{-1} x = \frac{1}{2}\, ln\frac{1+x}{1-x} \quad |x| < 1$

$coth^{-1} x = \frac{1}{2}\, ln\frac{x+1}{x-1} \quad |x| < 1$

$D_x(sinh^{-1} u) = \frac{1}{\sqrt{u^2+1}} D_x u$

$D_x(cosh^{-1} u) = \frac{1}{\sqrt{u^2-1}} D_x u \quad u > 1$

$D_x(tanh^{-1} u) = \frac{1}{1-u^2} D_x u \quad |u| < 1$

$D_x(coth^{-1} u) = \frac{1}{1-u^2} D_x u \quad |u| > 1$

Hyperbolic Functions

$sinh\, x = \frac{e^x - e^{-x}}{2}$

$cosh\, x = \frac{e^x + e^{-x}}{2}$

$D_x(sinh\, u) = cosh\, u \cdot D_x u$

$D_x(cosh\, u) = sinh\, u \cdot D_x u$

$\frac{1}{coth\, x} = tanh\, x = \frac{sinh\, x}{cosh\, x}$

$coth\, x = \frac{cosh\, x}{sinh\, x}$

$sech\, x = \frac{1}{cosh\, x}$

$csch\, x = \frac{1}{sinh\, x}$

Hyperbolic Identities

$cosh^2 x - sinh^2 x = 1$

$1 - tanh^2 x = sech^2 x$

$1 - coth^2 x = -\, csch^2 x$

$cosh\, x + sinh\, x = e^x$

$cosh\, x - sinh\, x = e^{-x}$

$sinh\,(x + y) = sinh\, x\, cosh\, y + cosh\, x\, sinh\, y$

$cosh\,(x + y) = cosh\, x\, cosh\, y + sinh\, x\, sinh\, y$

$sinh\, 2x = 2\, sinh\, x\, cosh\, x$

$cosh\, 2x = cosh^2 x + sinh^2 x$

FORMULAS:

$D_x(tanh\, u) = sech^2 u \cdot D_x u$

$D_x(coth\, u) = -\, csch^2 u \cdot D_x u$

$D_x(sech\, u) = -\, sech\, u\, tanh\, u \cdot D_x u$

$D_x(csch\, u) = -\, csch\, u\, coth\, u \cdot D_x u$

$\int \sinh u\, du = \cosh u + c$

$\int \cosh u\, du = \sinh u + c$

$\int \operatorname{sech}^2 u\, du = \tanh u + c$

$\int \operatorname{csch}^2 u\, du = -\coth u + c$

$\int \operatorname{sech} u \tanh u\, du = -\operatorname{sech} u + c$

$\int \operatorname{csch} u \coth u\, du = -\operatorname{csch} u + c$

Integration By Parts (IBP)

$\int u dv = uv - \int v du$

Exponential functions

$D_x(e^u) = e^u \cdot D_x u$

$D_x(a^u) = a^u \cdot \ln a \cdot D_x u$

Logarithmic Functions

$D_x(\ln u) = \frac{1}{u} \cdot D_x u$

$D_x(\log_a u) = \frac{1}{(\ln a)\, u} \cdot D_x u$

Laws of Logarithm

$\ln xy = \ln x + \ln y$

$\ln x^r = r \ln x$

$\ln \frac{x}{y} = \ln x - \ln y$

$a^x = (e^{\ln a})^x$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$\sec^2 x = 1 + \tan^2 x$

$\csc^2 x = 1 + \cot^2 x$

Double Angle Identities

$\sin 2x = 2 \sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 1 - 2\sin^2 x$

$= 2\cos^2 x - 1$

$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

Power Reducing Identities

$\sin^2 x = \frac{1 - \cos 2x}{2}$

$\cos^2 x = \frac{1 + \cos 2x}{2}$

$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$

Basic Reciprocal Identities

$\csc(x) = \frac{1}{\sin x}$

$\sin(x) = \frac{1}{\csc x}$

$\sec(x) = \frac{1}{\cos x}$

$\cos(x) = \frac{1}{\sec x}$

$\cot(x) = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$

$\tan(x) = \frac{1}{\cot x} = \frac{\sin x}{\cos x}$

Integrals Involving Sine-Cosine Products

$\sin mx \sin nx = \frac{1}{2}(\cos[(m-n)x] - \cos[(m+n)x])$

$\sin mx \cos nx = \frac{1}{2}(\sin[(m-n)x] + \sin[(m+n)x])$

$\cos mx \cos nx = \frac{1}{2}(\cos[(m-n)x] + \cos[(m+n)x])$

Special Integration Formulas

$\int \sqrt{a^2 - u^2}\, du = \frac{1}{2}\left(u\sqrt{a^2 - u^2} + a^2 \arcsin \frac{u}{a}\right) + C$

$\int \sqrt{u^2 - a^2}\, du = \frac{1}{2}\left(u\sqrt{u^2 - a^2} - a^2 \ln\left|u + \sqrt{u^2 - a^2}\right|\right) + C,$ $u > a$

$\int \sqrt{u^2 + a^2}\, du = \frac{1}{2}\left(u\sqrt{u^2 + a^2} + a^2 \ln\left|u + \sqrt{u^2 + a^2}\right|\right) + C$

Trigonometric Substitution

$\sqrt{a^2 - u^2} = a \cos\theta \qquad u = a \sin\theta$

$\sqrt{a^2 + u^2} = a \sec\theta \qquad u = a \tan\theta$

$\sqrt{u^2 - a^2} = a \tan\theta \qquad u = a \sec\theta$

Partial Fractions

1. When $N(x) > D(x)$, then

$$\frac{N(x)}{D(x)} = (a\, polynomial) + \frac{remainder}{D(x)}$$

2. Factor denominator into factors in the form

$$(px + q)^m \text{ and } (ax^2 + bx + c)^n$$

3. Linear Factors: $(px + q)^m$

$$\frac{A_1}{(px+q)} + \frac{A_2}{(px+q)^2} + \cdots + \frac{A_m}{(px+q)^m}$$

4. Quadratic Factors: $(ax^2 + bx + c)^n$

$$\frac{B_1x + C_1}{ax^2 + bx + c} + \frac{B_2x + C_2}{(ax^2 + bx + c)^2} + \cdots + \frac{B_nx + C_n}{(ax^2 + bx + c)^n}$$