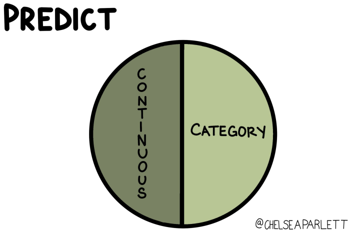
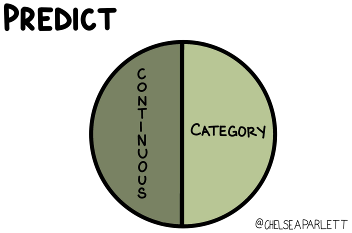


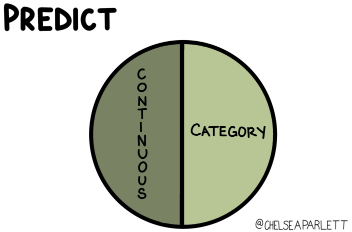
Logistic Regression

Dr. Chelsea Parlett-Pelleriti

Linear Regression in Disguise



Predictions



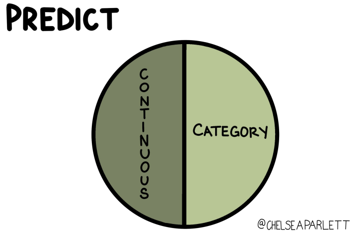
Linear Regression

?

Logistic Regression

Continuous Variable (can be -∞ to ∞)

Binary Categorical Variable (can be 0 or 1)

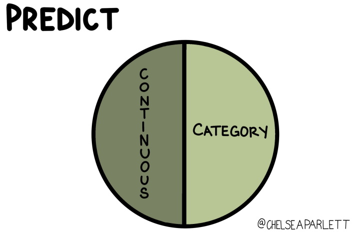


Getting from Binary to Continuous

Linear Models don't predict categorical variables…so what's the closest thing? PROBABILITIES!

1. We want Probabilities
   - But probabilities are bounded 0-1🙁
2. Convert Probabilities to Odds
   - But Odds only go down to 0, and are not symmetric🙁
3. Convert Odds to Log Odds

Getting from Binary to Continuous



1. We want Probabilities
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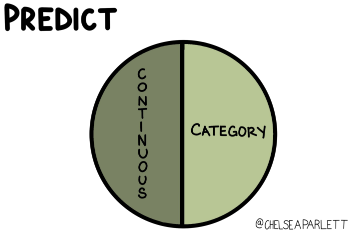
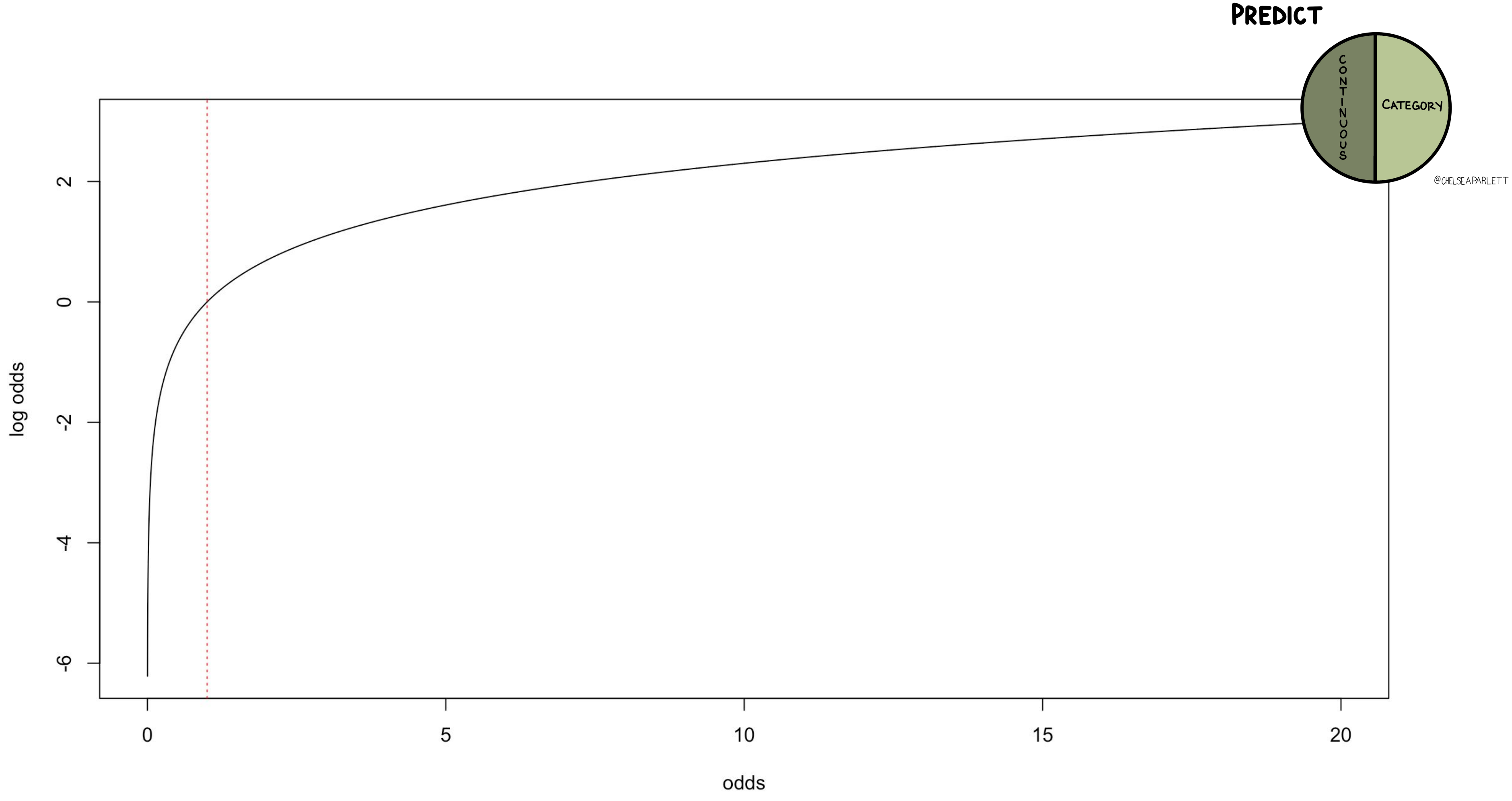
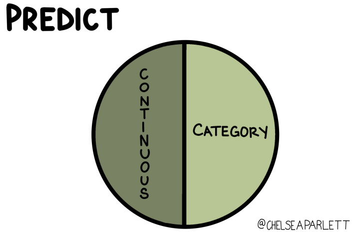
Getting from Binary to Continuous



1. We want Probabilities
2. Convert Probabilities to Odds
3. Convert Odds to Log Odds
4.

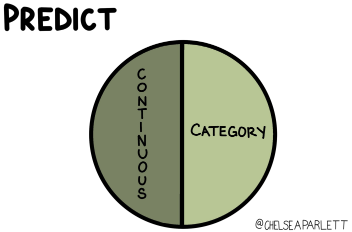

PREDICT
CONTINUOUS
CATEGORY
@CHELSEAPARLETT
log odds
2
0
-2
-4
-6
0
5
10
15
20
odds



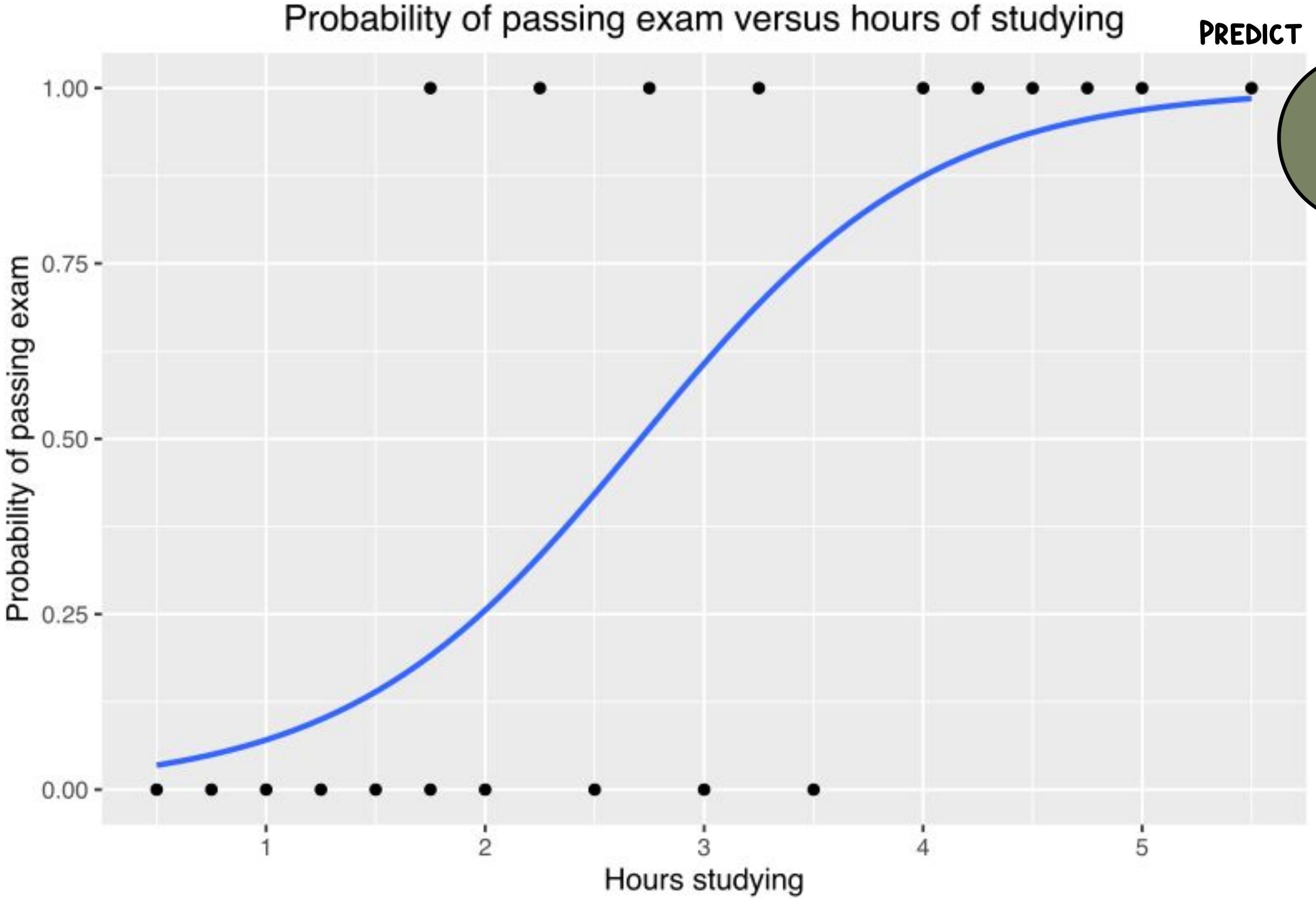
Getting from Binary to Continuous

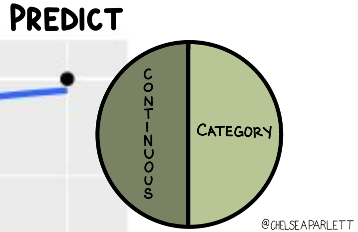
1. We want Probabilities
2. Convert Probabilities to Odds
3. Convert Odds to Log Odds
4.

The Final Formula



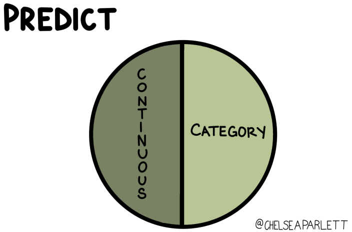
$$\log(p/1-p) = mx + b$$





https://en.wikipedia.org/wiki/Logistic_regression

Assessing Model Performance



- Mainly worried about real world performance: did it make the correct prediction? (accuracy, sensitivity, specificity, F1 score)
- ROC/AUC

Accuracy

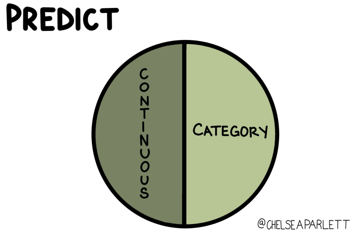
Correct Predictions

True Positive **(TP)** + True Negative **(TN)**

All Predictions

False Negative **(FN)** + False Positive **(FP)** +

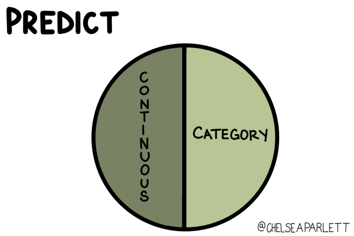
True Positive **(TP)** + True Negative **(TN)**



"How often is the model correct?"

	Predicted Positive	***Predicted*** Negative
Actual Positive	True Positive **(TP)**	False Negative **(FN)**
Actual Negative	False Positive **(FP)**	True Negative **(TN)**

Sensitivity/Recall



Correctly Predicted Positives

True Positive (**TP**)

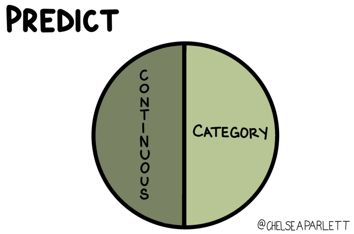
Actual Positives

False Negative **(FN)** + True Positive (**TP**)

"How often is the model correct for Positive Cases?"

	Predicted Positive	***Predicted*** Negative
Actual Positive	True Positive **(TP)**	False Negative **(FN)**
Actual Negative	False Positive **(FP)**	True Negative **(TN)**

Specificity



Correctly Predicted Negatives

True Negative **(TN)**

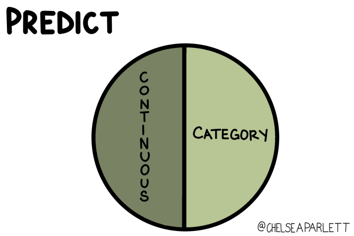
Actual Negatives

True Negative **(TN)** + False Positive **(FP)**

"How often is the model correct for Negative Cases?"

	Predicted Positive	***Predicted*** Negative
Actual Positive	True Positive **(TP)**	False Negative **(FN)**
Actual Negative	False Positive **(FP)**	True Negative **(TN)**

Precision



Correctly Predicted Positives

True Positive (**TP**)

Actual Positives

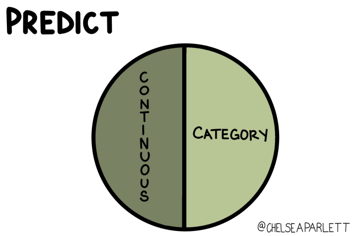
False Positive **(FP)** + True Positive (**TP**)

"How many of the predicted Positives are correct?"

	Predicted Positive	***Predicted*** Negative
Actual Positive	True Positive **(TP)**	False Negative **(FN)**
Actual Negative	False Positive **(FP)**	True Negative **(TN)**

F1 Score

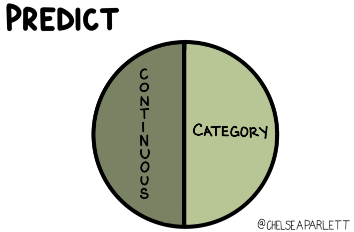
$$\frac{2}{\frac{1}{\text{Precision}} + \frac{1}{\text{Recall}}}$$



Combination of Precision (how often predicted positives ARE positive) and Recall (how often we correctly predict actual positives)

	Predicted Positive	***Predicted*** Negative
Actual Positive	True Positive **(TP)**	False Negative **(FN)**
Actual Negative	False Positive **(FP)**	True Negative **(TN)**

ROC AUC



Correctly Predicted Positives

True Positive (**TP**)

Actual Positives

False Negative **(FN)** + True Positive (**TP**)

	Predicted Positive	***Predicted*** Negative
Actual Positive	True Positive (**TP**)	False Negative **(FN)**
Actual Negative	False Positive **(FP)**	True Negative **(TN)**

1 -

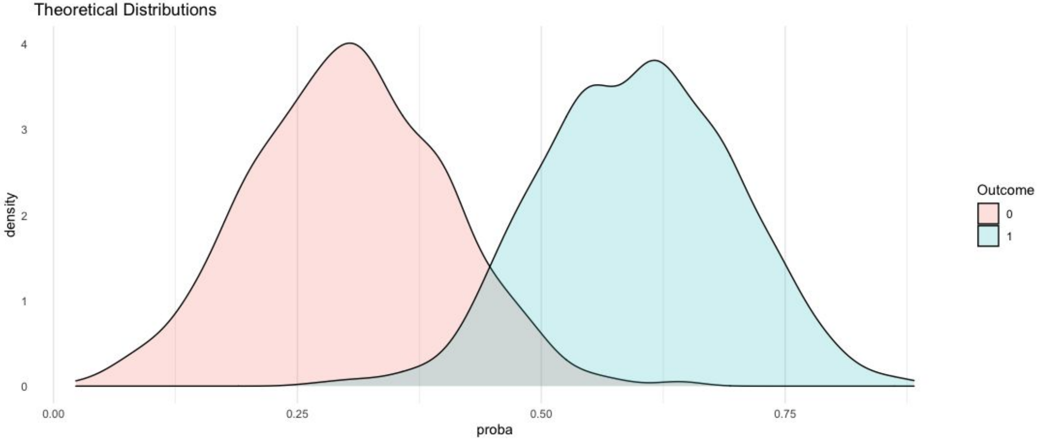
Correctly Predicted Negatives

True Negative **(TN)**

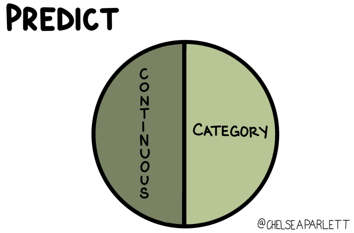
Actual Negatives

True Negative **(TN)** + False Positive **(FP)**

ROC AUC


Theoretical Distributions
density
proba
0.00
0.25
0.50
0.75
0
1
2
3
4
Outcome
0
1

Loss Functions



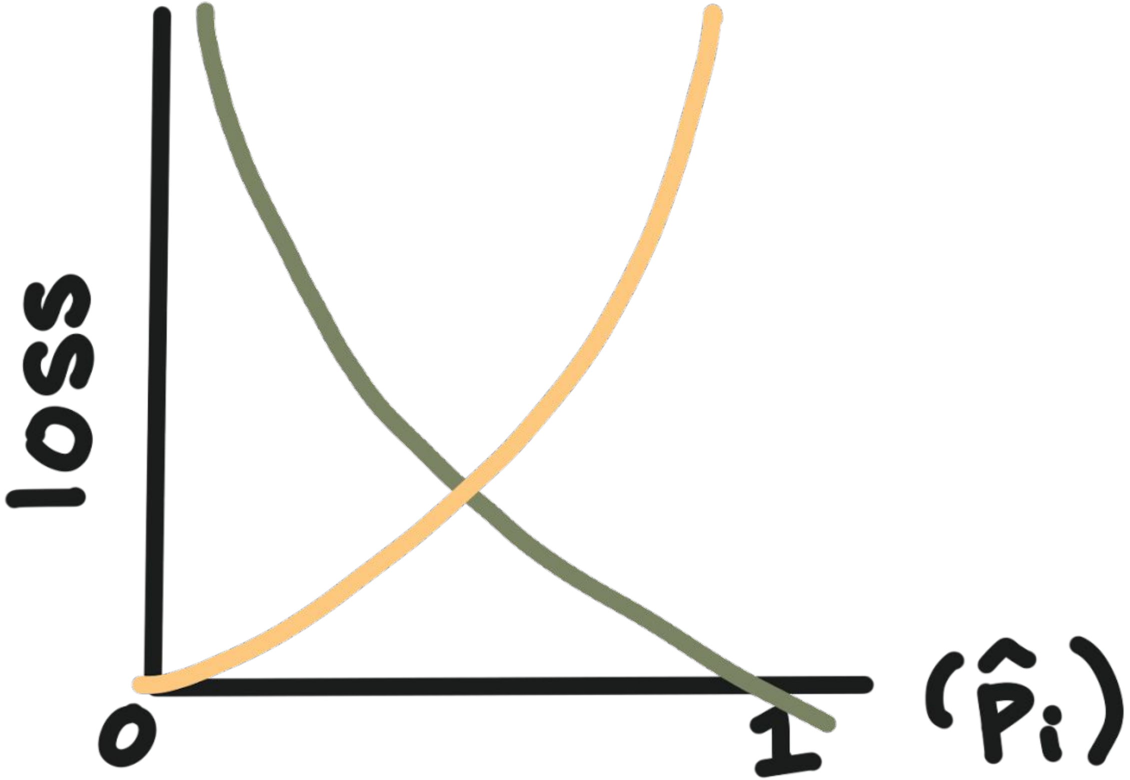
LINEAR:

$$(\hat{y}_i - y_i)^2$$

LOGISTIC:

$$\begin{cases} -\log(\hat{p}_i) & \text{if } y=1 \\ -\log(1-\hat{p}_i) & \text{if } y=0 \end{cases}$$

Loss Functions



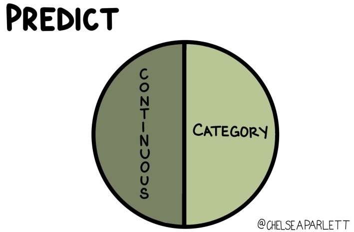
LINEAR:

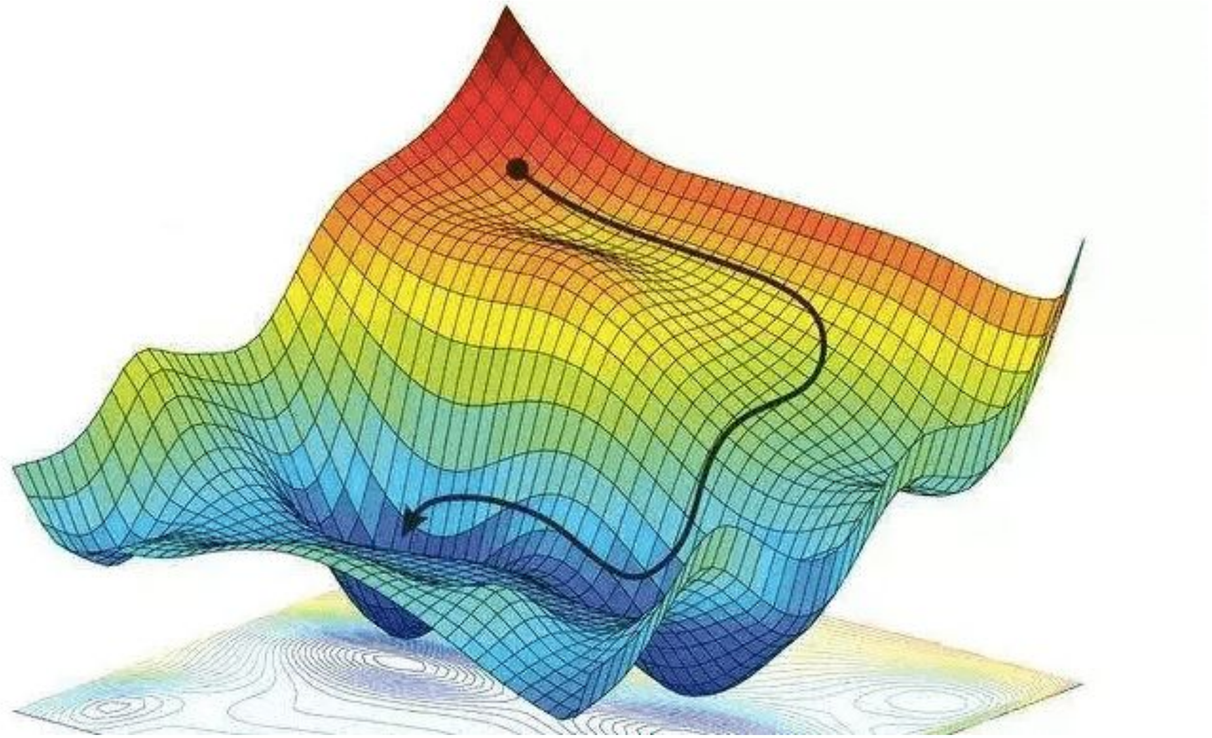
$$(\hat{y}_i - y_i)^2$$

LOGISTIC:

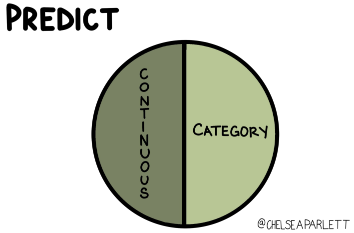
$$\begin{cases} -\log(\hat{p}_i) & \boxed{\text{if } y=1} \\ -\log(1-\hat{p}_i) & \boxed{\text{if } y=0} \end{cases}$$

Gradient Descent





Getting from Binary to Continuous



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1. We want Probabilities
    - But probabilities are bounded 0-1🙁
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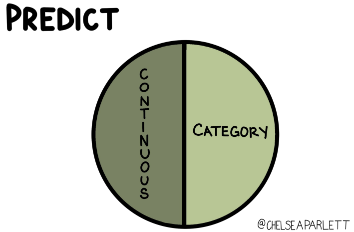
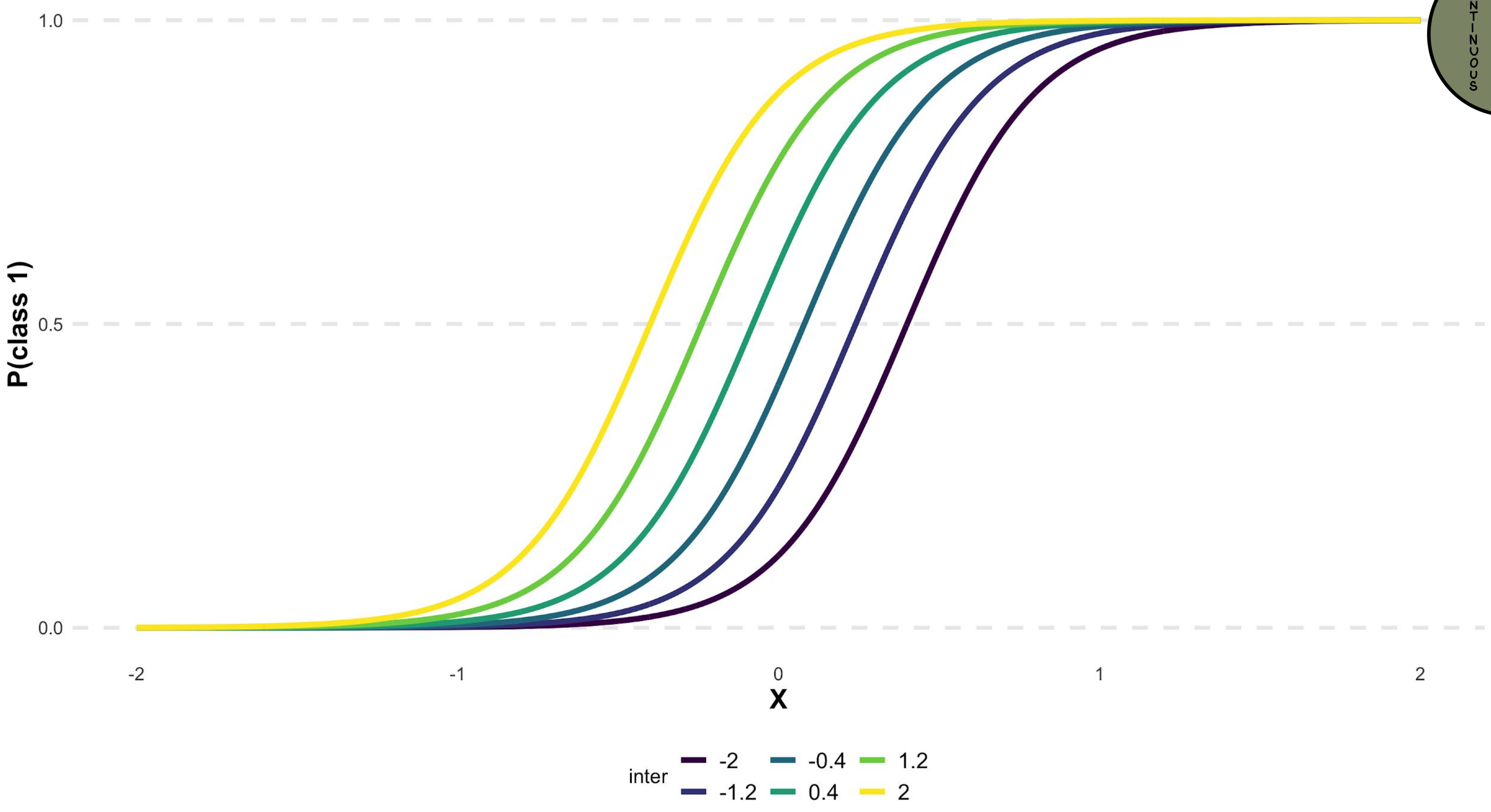
All the Steps

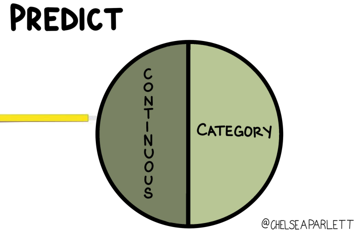


Probability p	**Odds** p/(1-p)	**Log Odds** log(p/1-p)
0.1	0.1111	-2.1972
0.5	**1**	**0**
0.9	9	2.1972

Logistic Curves with different Intercepts

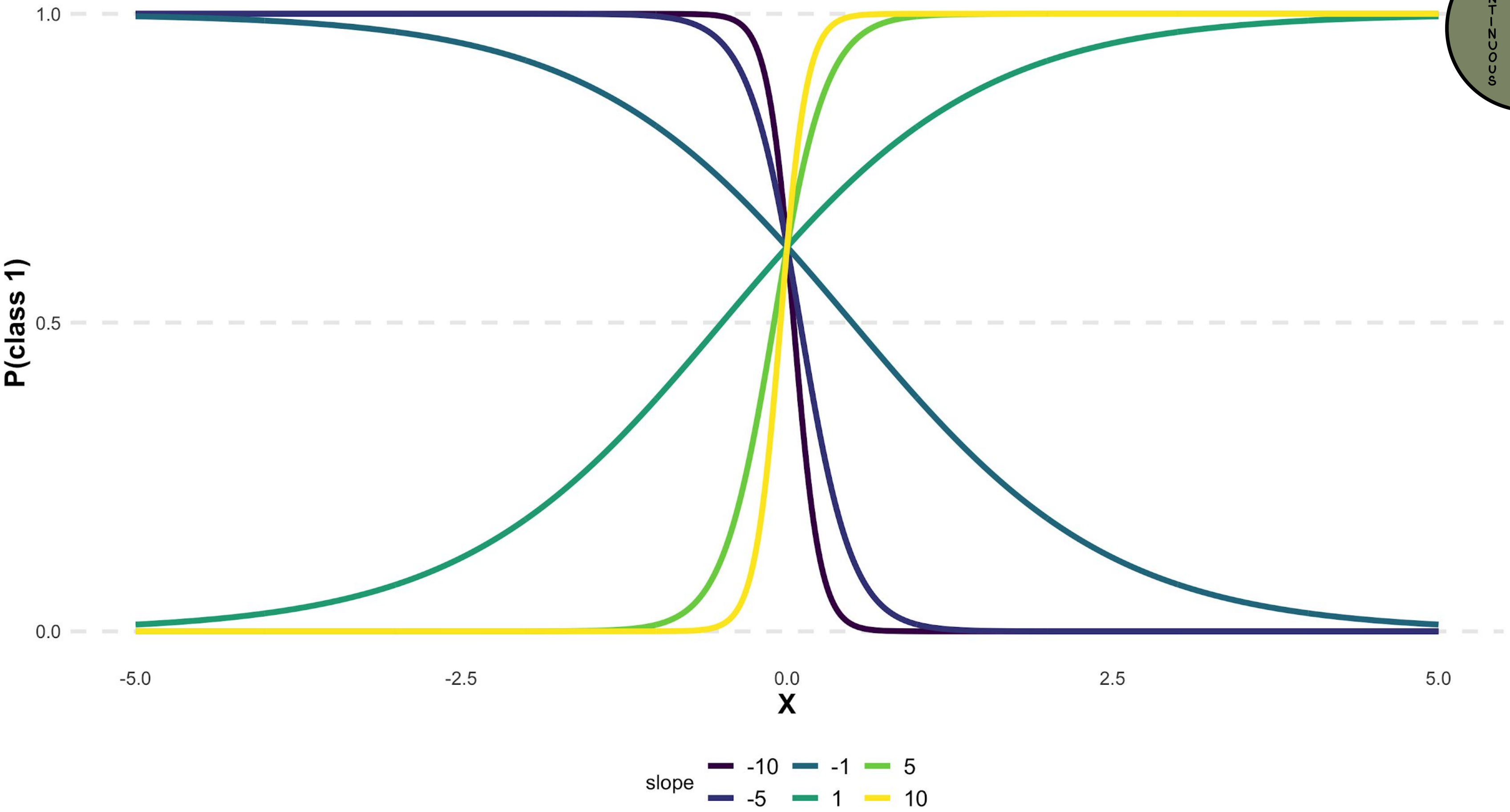
inter + 5x

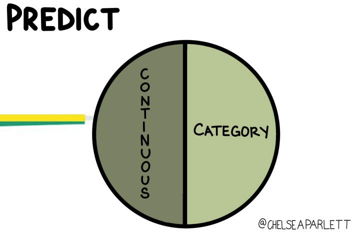


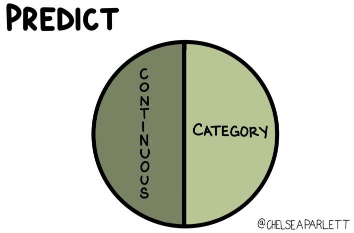


Logistic Curves with different Slopes

0.5 + slope*x







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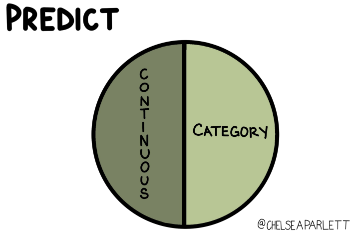
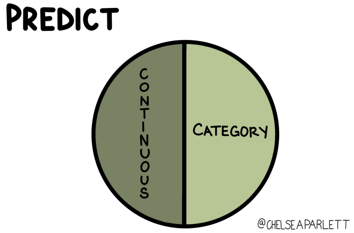
Interpreting Coefficients

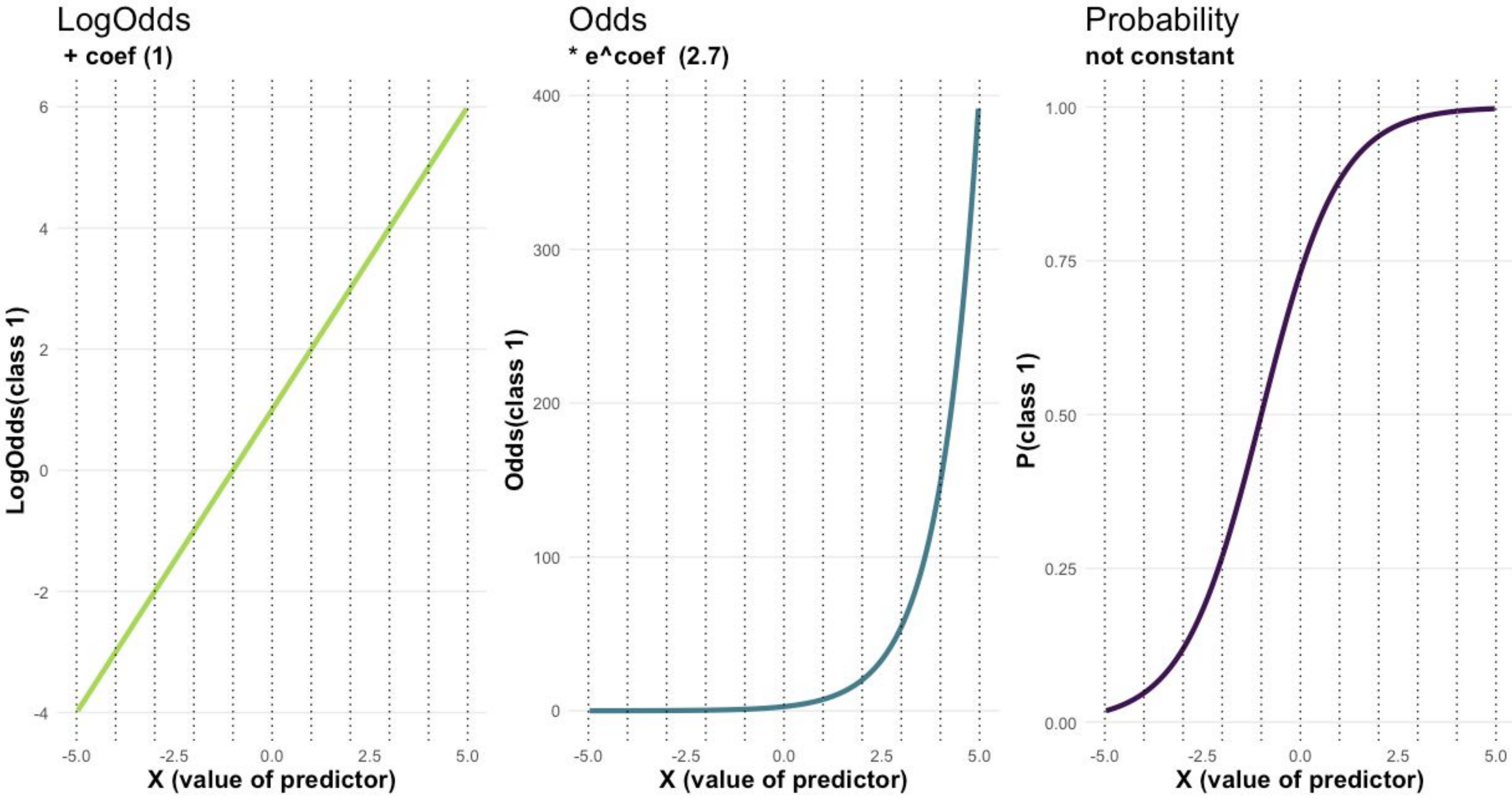


	coef
const	-2.9777
age	0.1445
income	-0.0066
months_subbed	0.0015

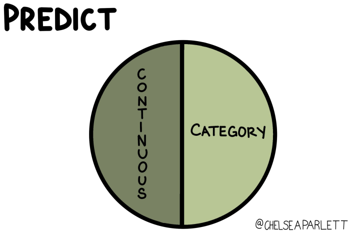
Interpreting Coefficients



	coef	e^{coef}
const	-2.9777	0.05090979
age	0.1445	1.155462
income	-0.0066	0.9934217
months_subbed	0.0015	1.001501


LogOdds
+ coef (1)
LogOdds(class 1)
X (value of predictor)
Odds
* e^coef (2.7)
Odds(class 1)
X (value of predictor)
Probability
not constant
P(class 1)
X (value of predictor)

Interpreting Coefficients



✅ Log odds

✅ Odds

⚠️ Probabilities*