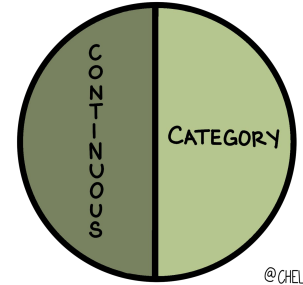


PREDICT



@CHELSEA PARLETT

# Linear Regression II

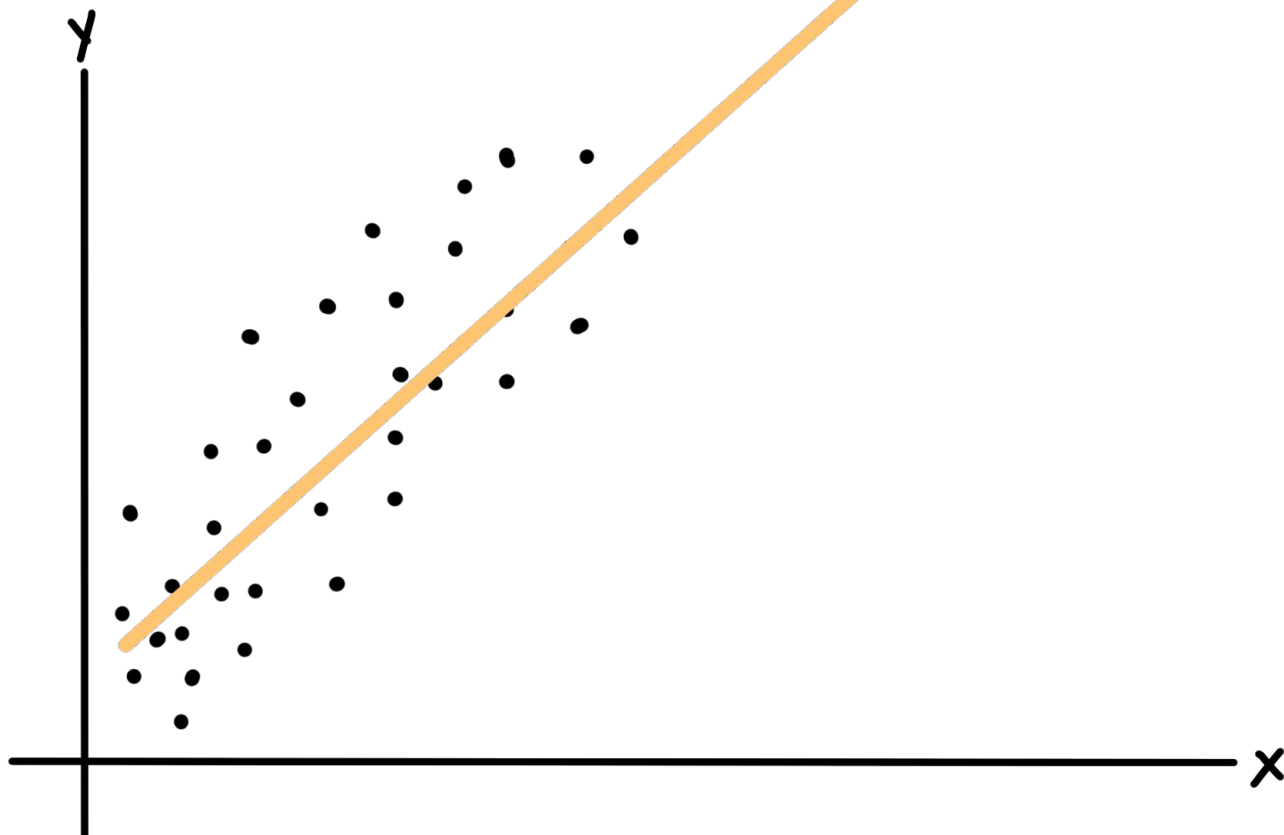
Dr. Chelsea Parlett-Pelleriti

# Linear Regression

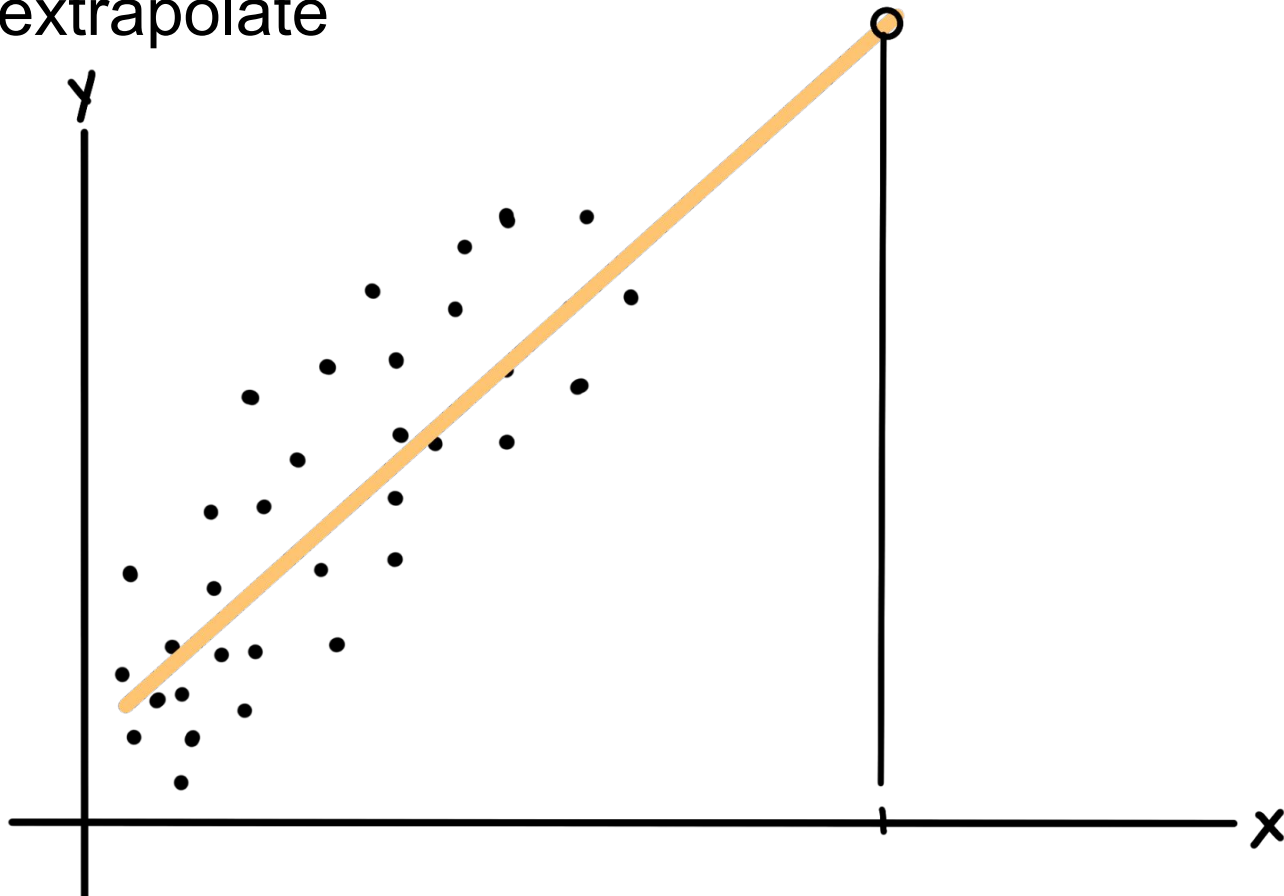
- Extrapolation
- Feature Engineering
- “Non-Linear” Linear Regression
  - Polynomial Regression
  - Interactions
  - Step Functions
  - Basis Functions
  - Regression Splines
  - GAMs

# Extrapolation

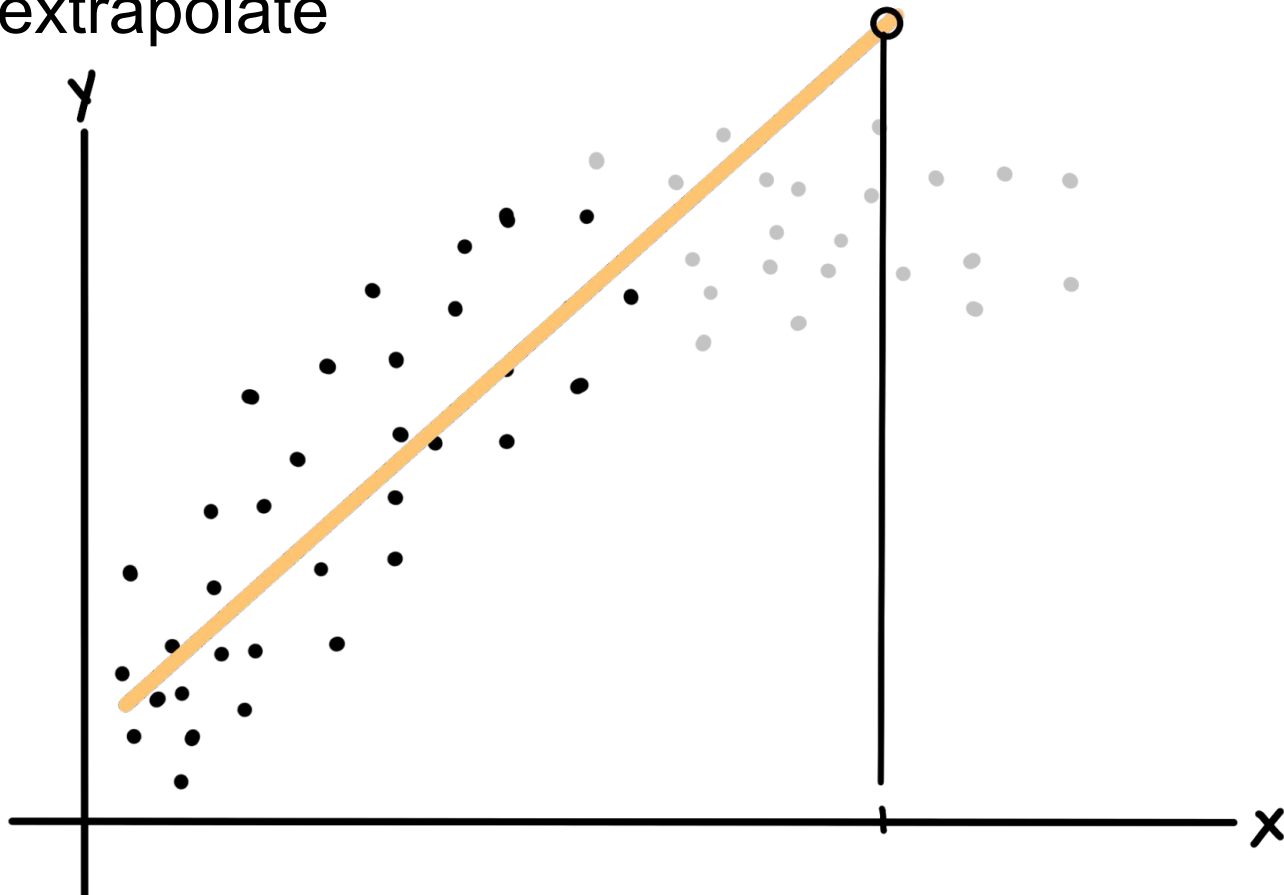
Don't extrapolate



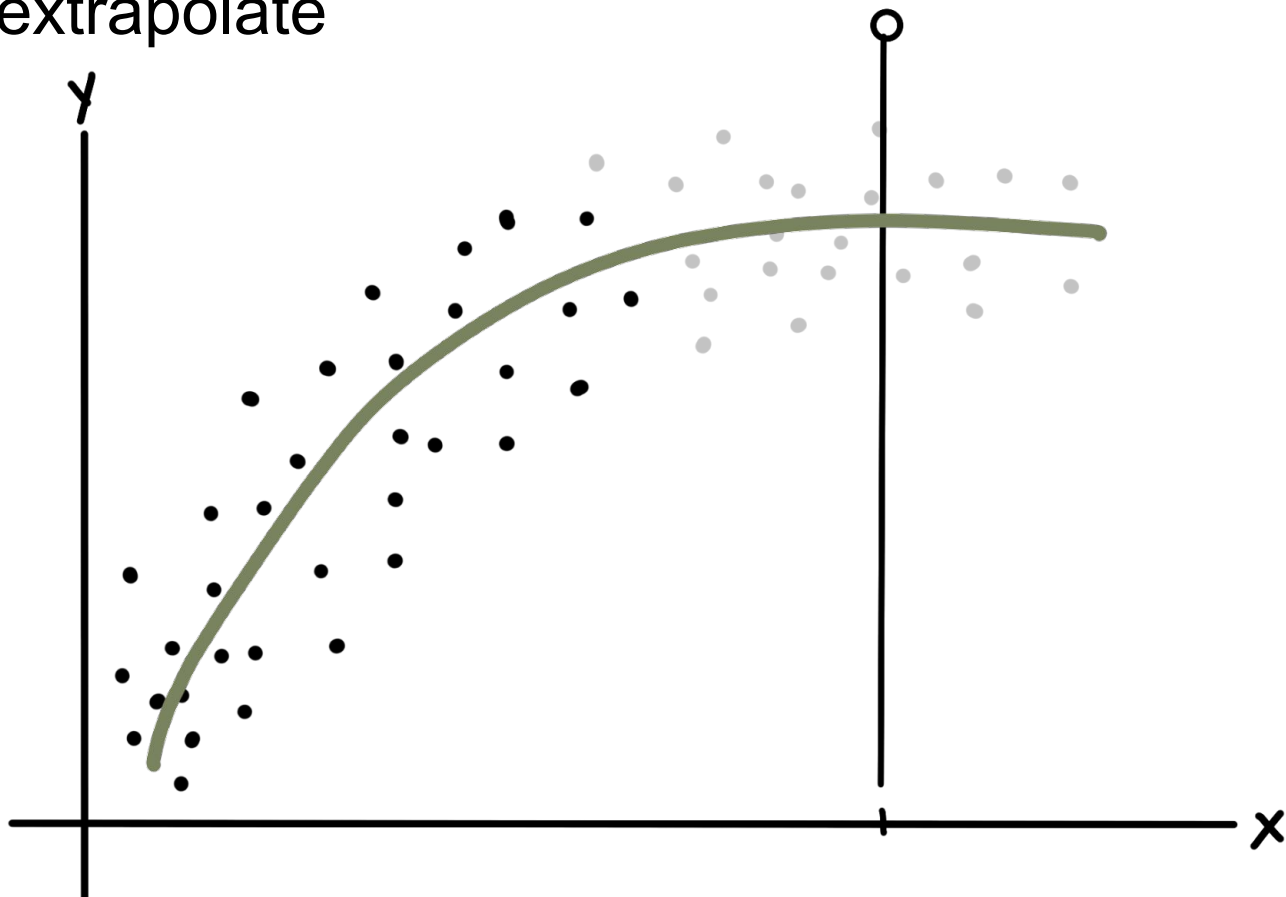
Don't extrapolate



Don't extrapolate



Don't extrapolate



# Don't Extrapolate

Nova Walking Example



# Feature Engineering

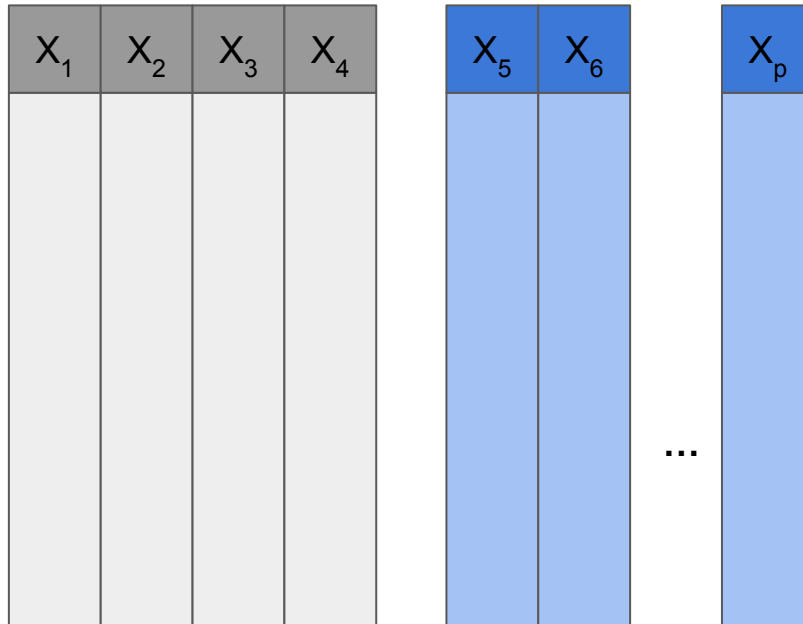
# “Feature Engineering”

## But what if there IS non-linearity?

$X_1$	$X_2$	$X_3$	$X_4$

# “Feature Engineering”

But what if non-linearity?



# “Non-linear” Linear Regression

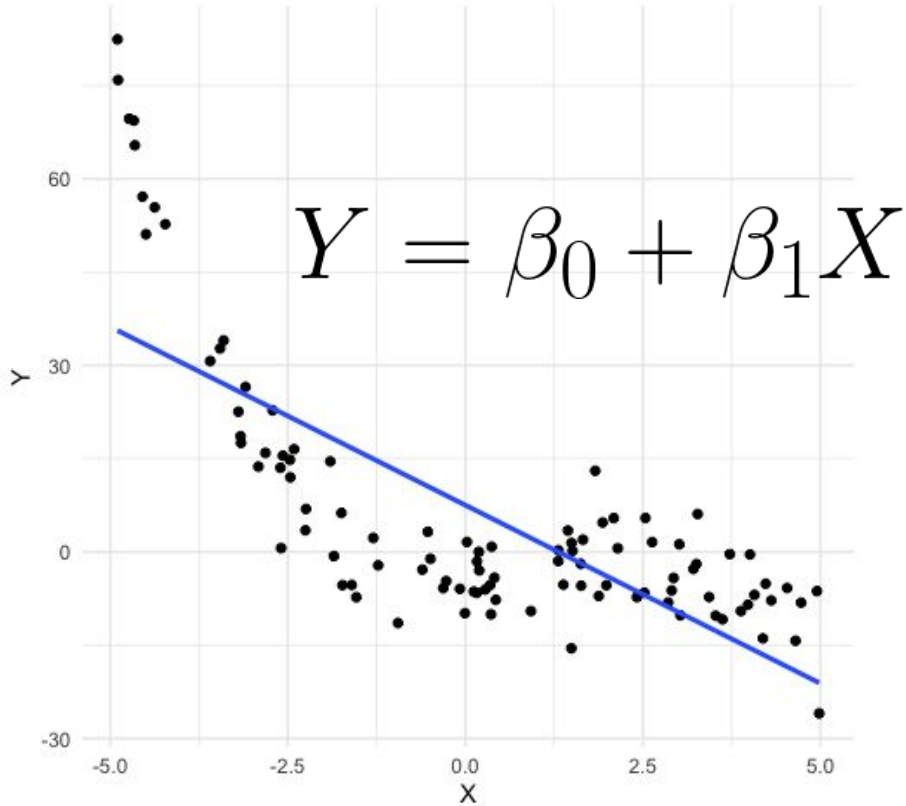
# Creating New Features

Two Ways to Use “Linear Regression” But Get Non-Linearity

1. Create New Features (Polynomial, GAMs...)★
2. Link Functions

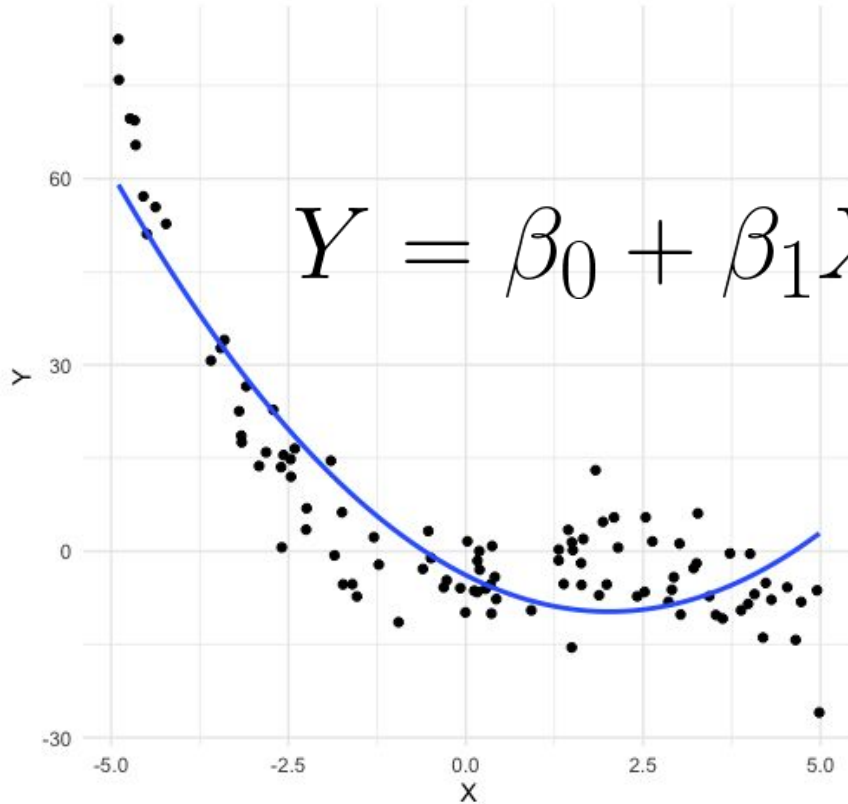
# Polynomial Regression

Linear Regression



# Polynomial Regression

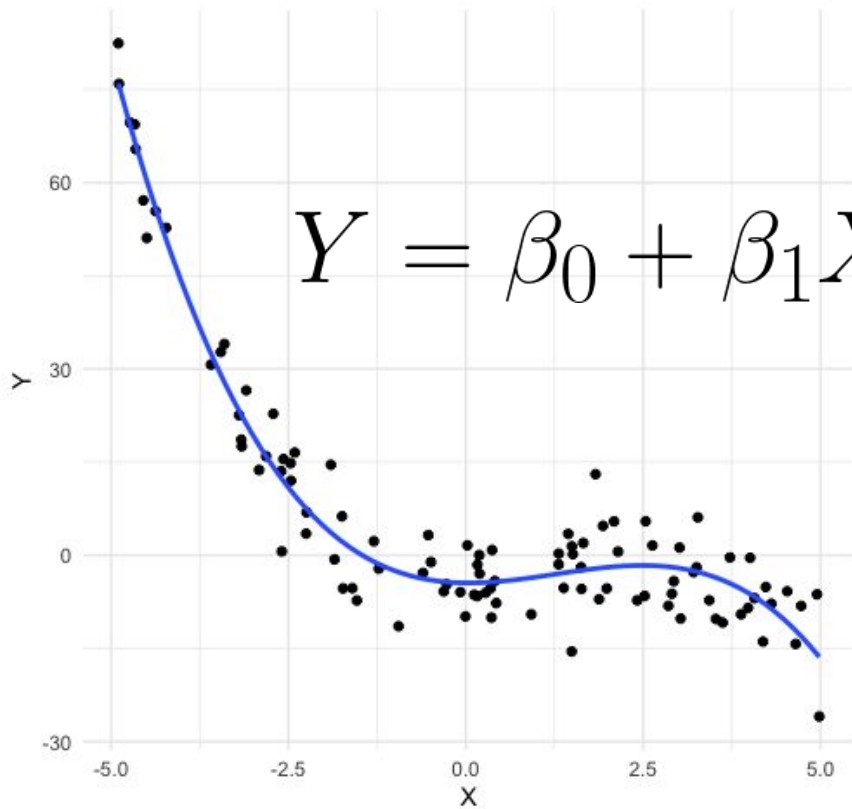
Polynomial Regression (d = 2)



$$Y = \beta_0 + \beta_1 X + \beta_2 X^2$$

# Polynomial Regression

Polynomial Regression (d = 3)

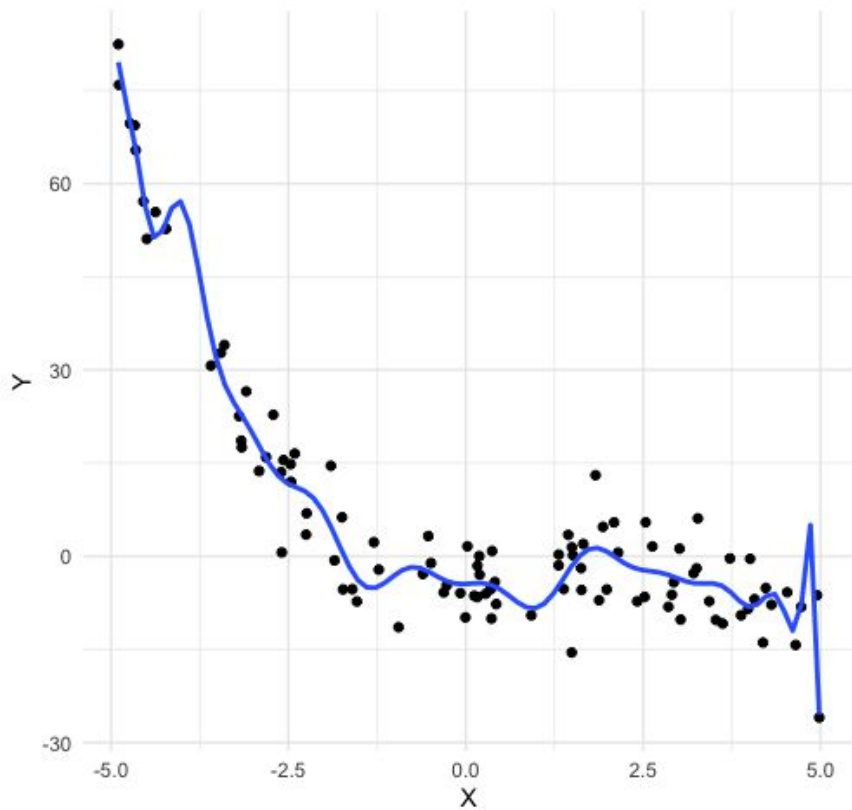


$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 X^3$$



# Polynomial Regression

Polynomial Regression (d = 25)



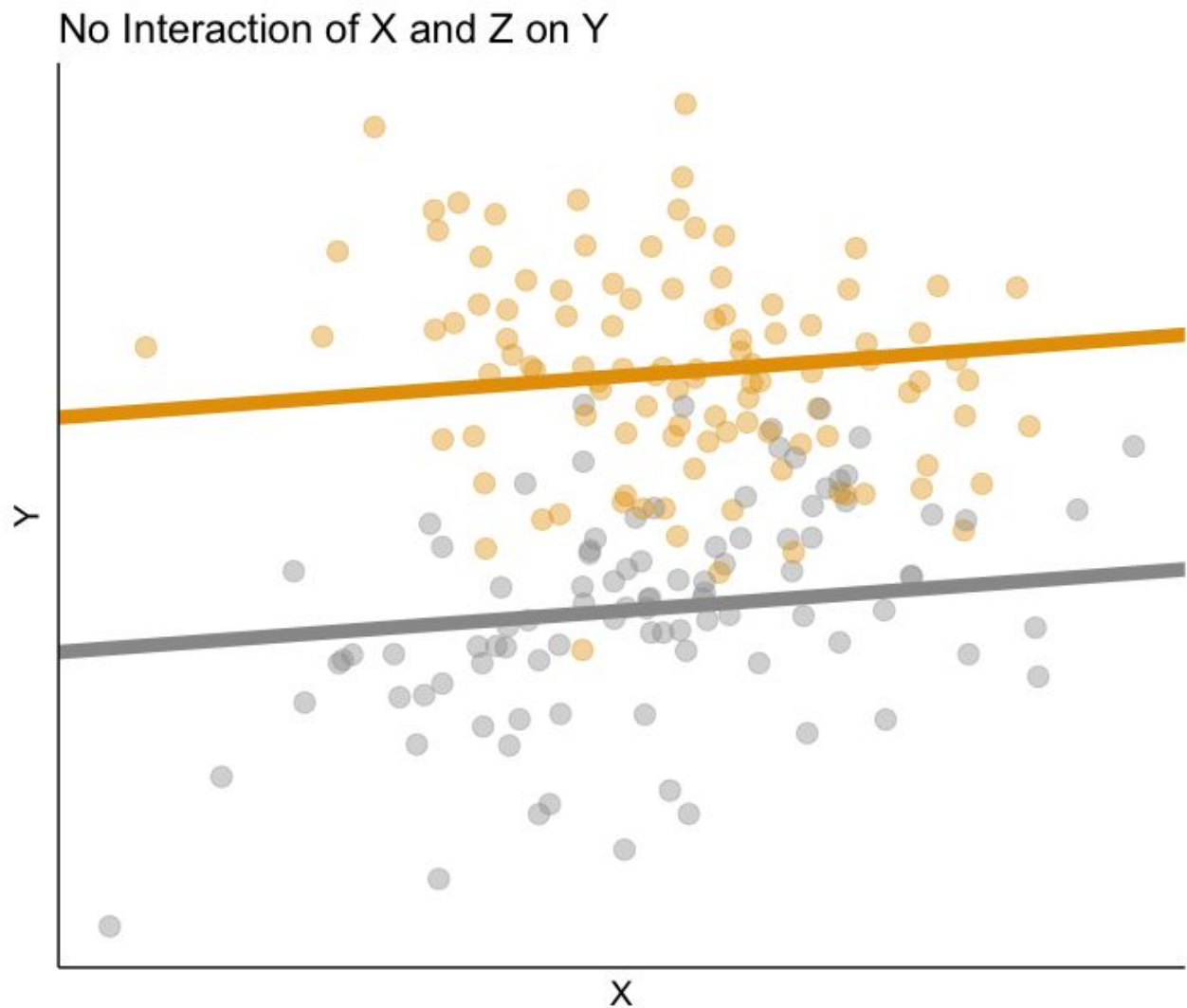
# Polynomial Regression

$x$	$x^2$	$x^3$	...	$x^p$

# Interactions

# Interactions

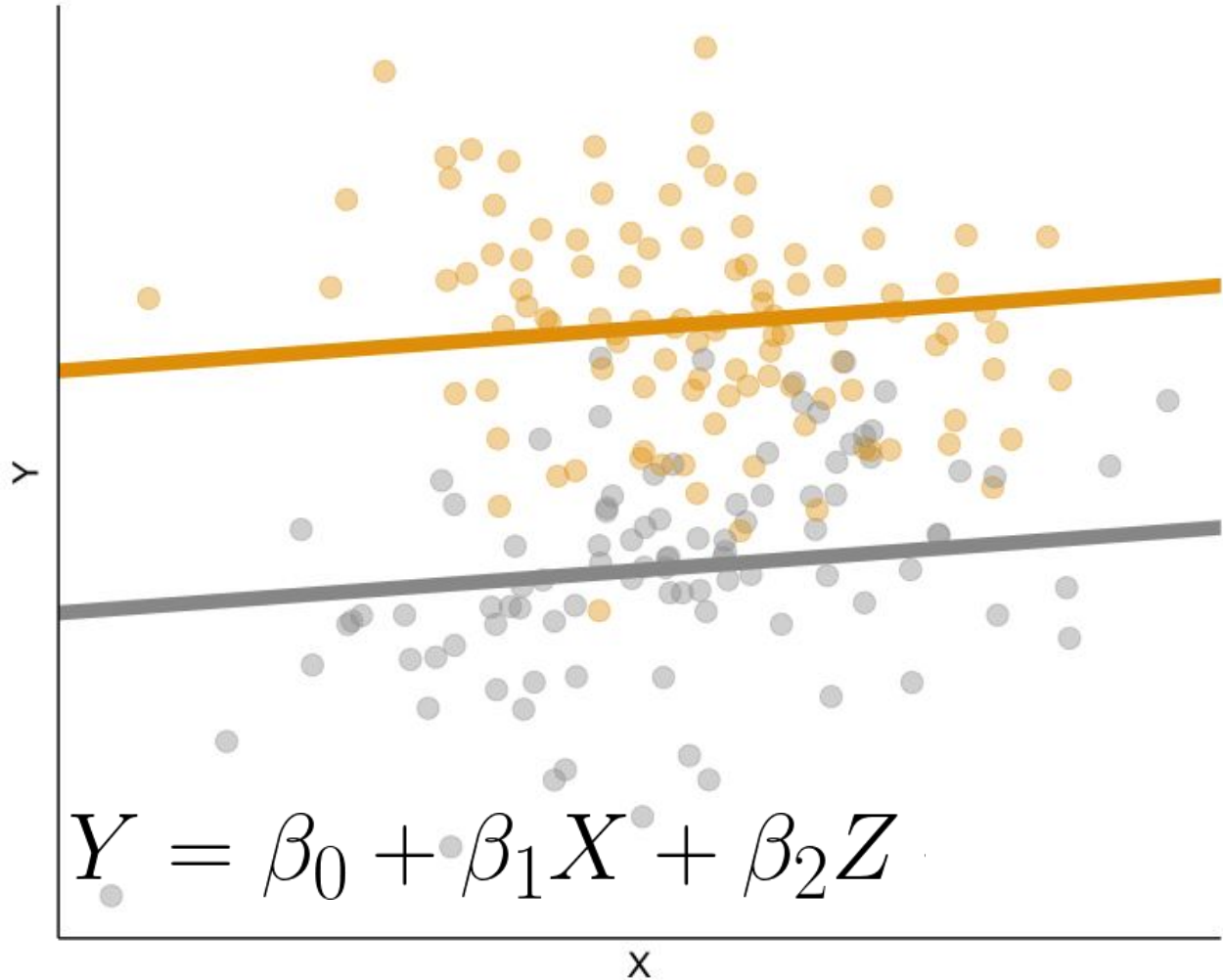
Is the relationship between **X** and **Y** different when you consider the value of **Z**?



# Interactions

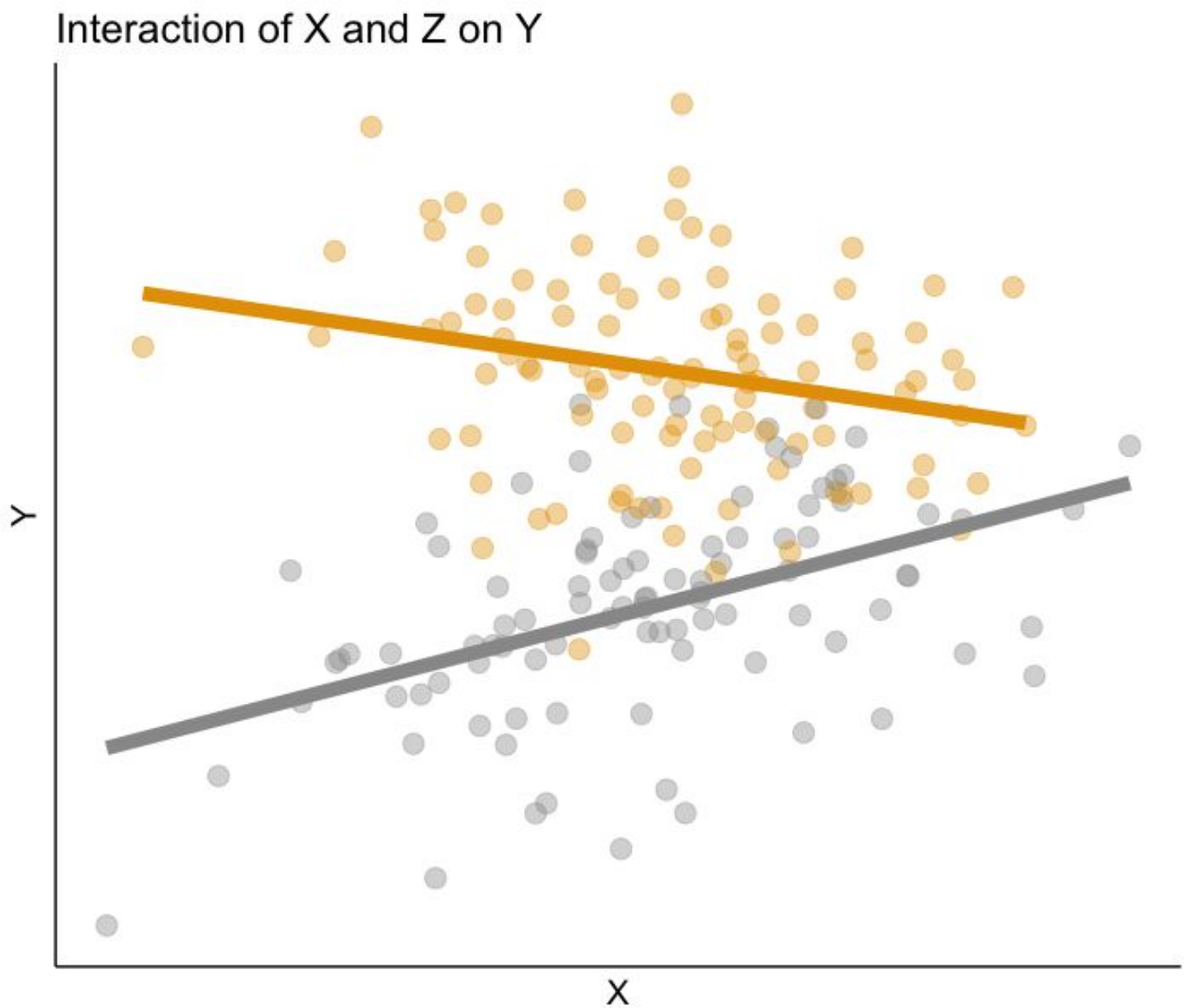
Is the relationship between **X** and **Y** different when you consider the value of **Z**?

No Interaction of X and Z on Y



# Interactions

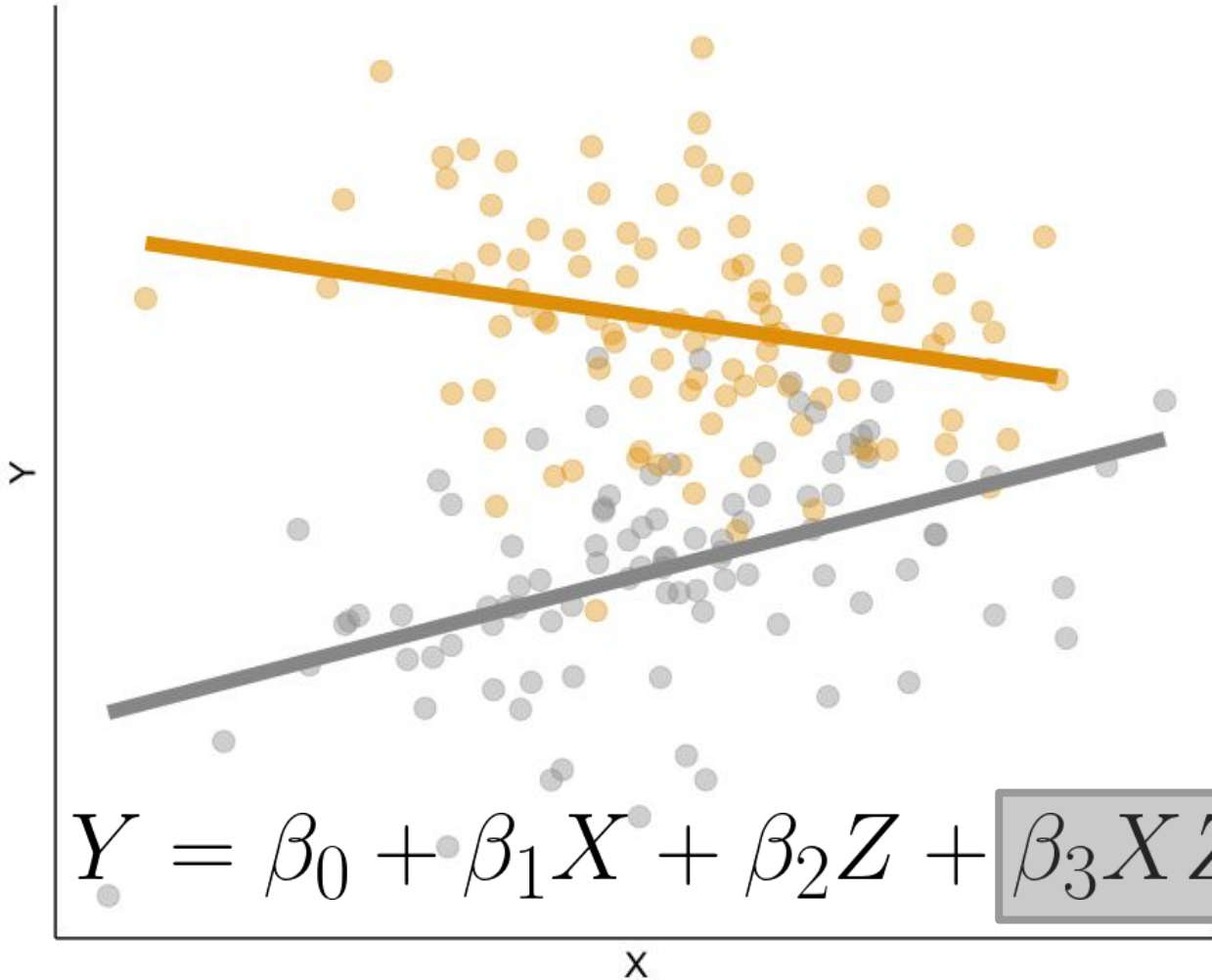
Is the relationship between **X** and **Y** different when you consider the value of **Z**?



# Interactions

Is the relationship between **X** and **Y** different when you consider the value of **Z**?

Interaction of X and Z on Y



$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 X Z$$

# Interactions

X	Z	xz



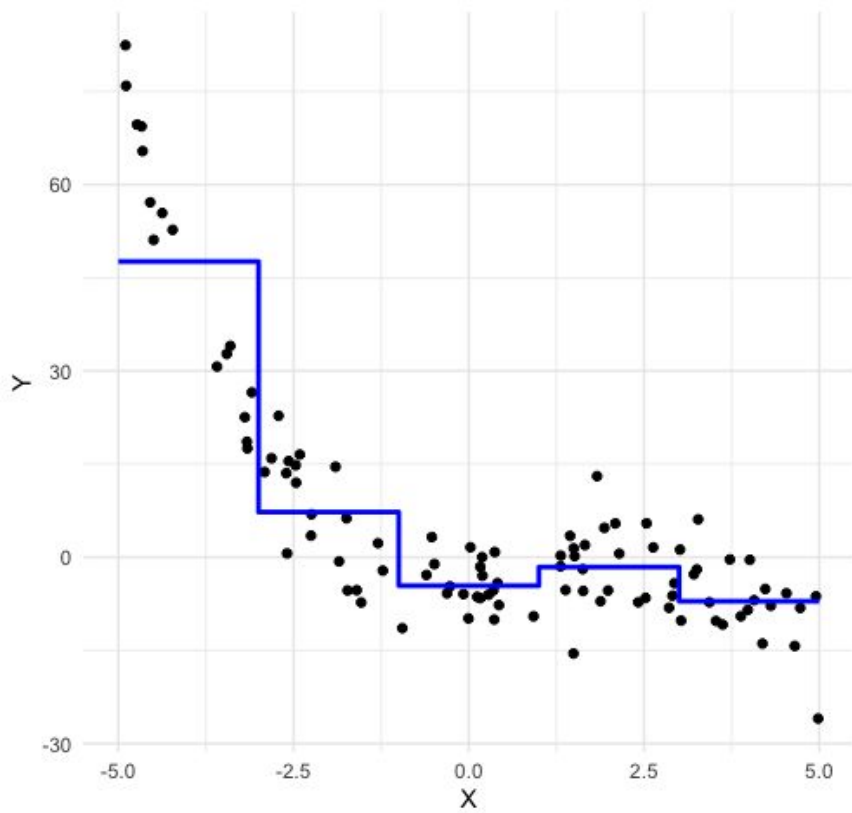
# Interactions

W	X	Z	WX	WZ	XZ	...	WXZ

# Step Functions

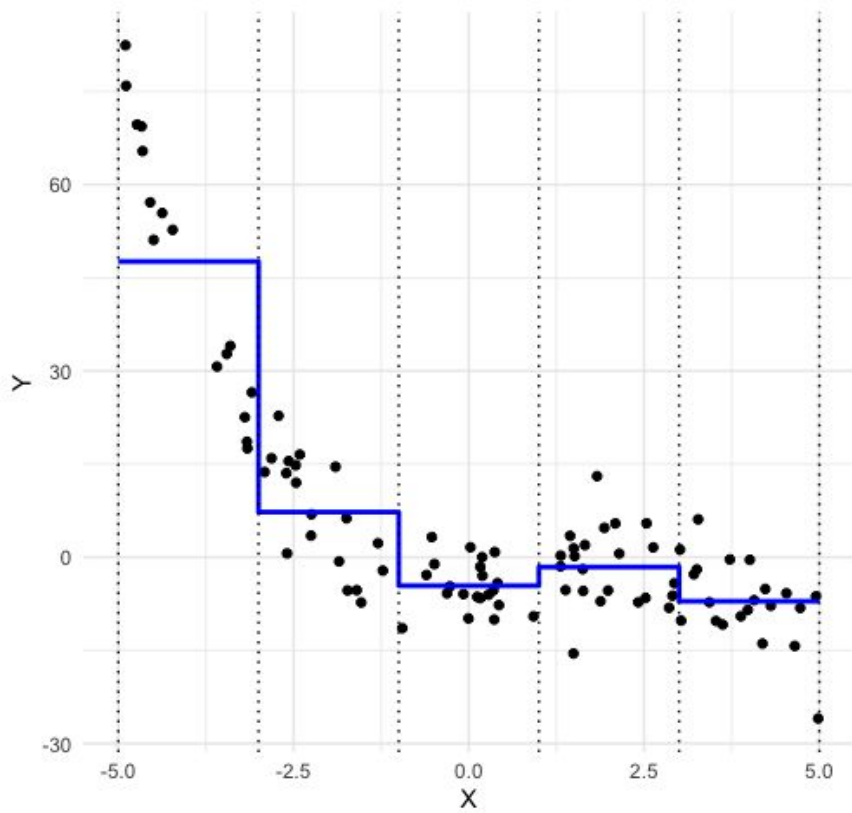
# Step Functions

Step Function

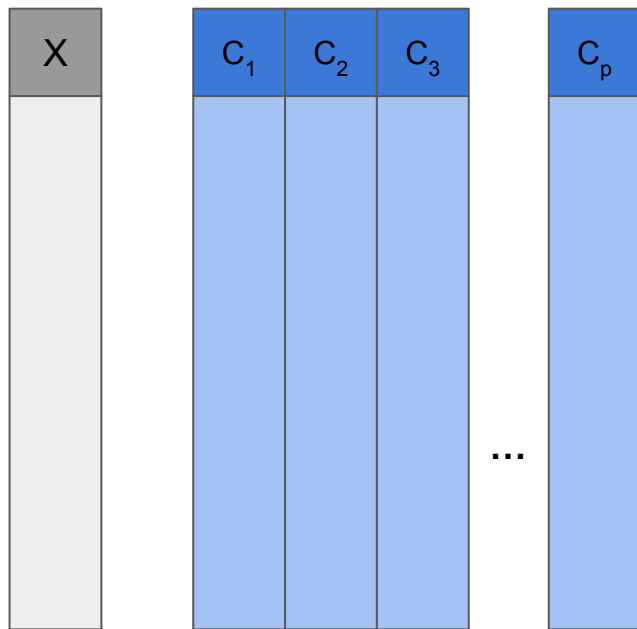


# Step Functions

Step Function



# Step Functions



# Basis Functions

## Basis Functions

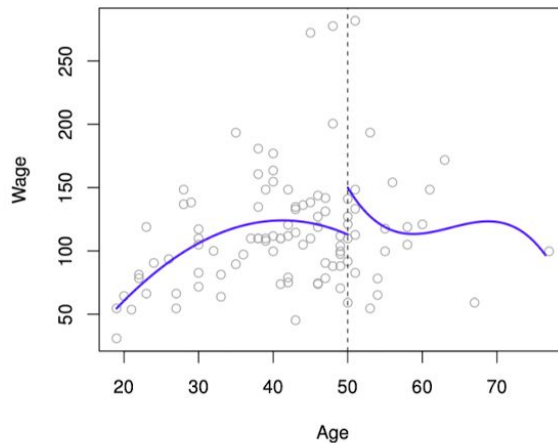
$$Y = \beta_0 + \beta_1 f_1(X_1) + \beta_2 f_2(X_2) + \beta_3 f_3(X_3) + \dots + \beta_p f_p(X_p)$$

# Regression Splines



# Regression Splines

**Piecewise Cubic**



**Continuous Piecewise Cubic**

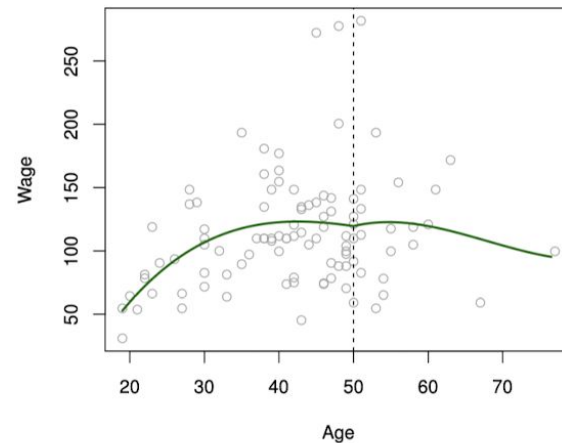
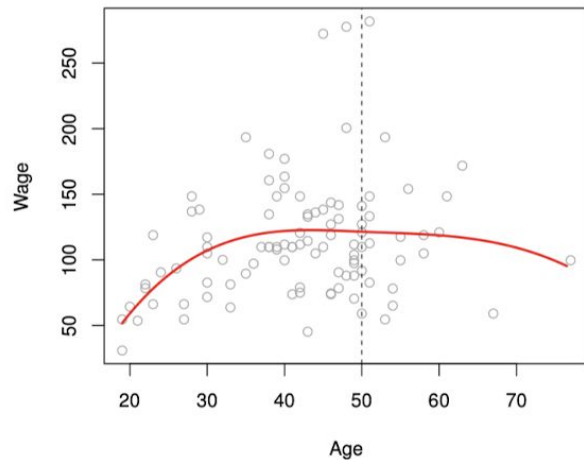
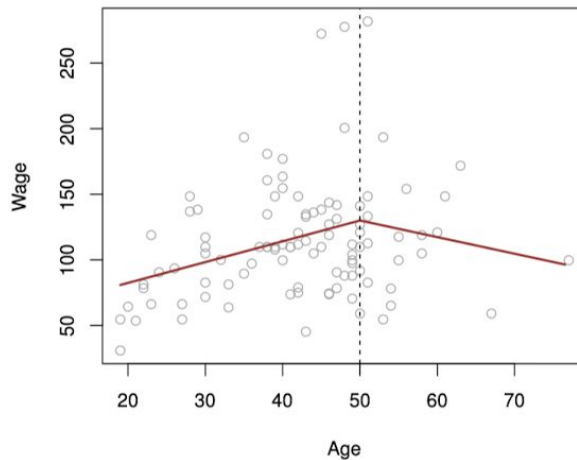


Image from: Introduction to Statistical Learning with applications in Python (Figure 7.3)

**Cubic Spline**



**Linear Spline**



# Regression Splines

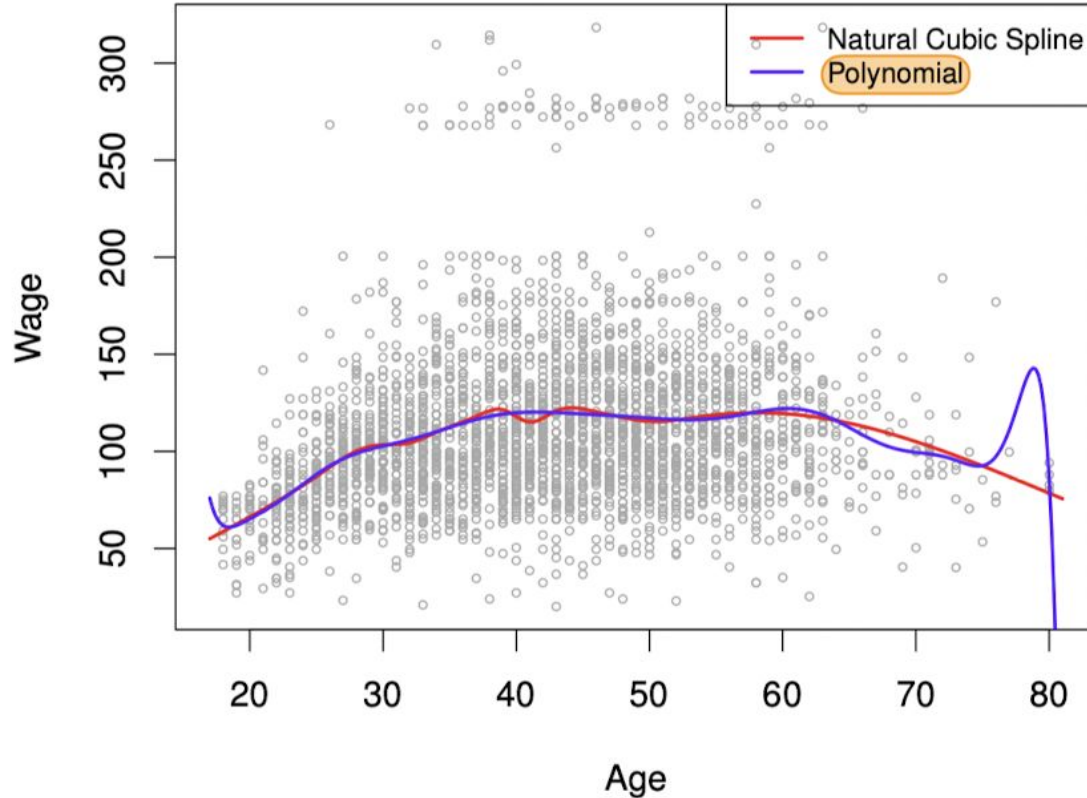


Image from:  
Introduction to  
Statistical Learning  
with applications in  
Python (Figure 7.7)

# Generalized Additive Models

## Generalized Additive Models

$$Y = \beta_0 + \beta_1 f_1(X_1) + \beta_2 f_2(X_2) + \beta_3 f_3(X_3) + \dots + \beta_p f_p(X_p)$$

# Generalized Additive Models

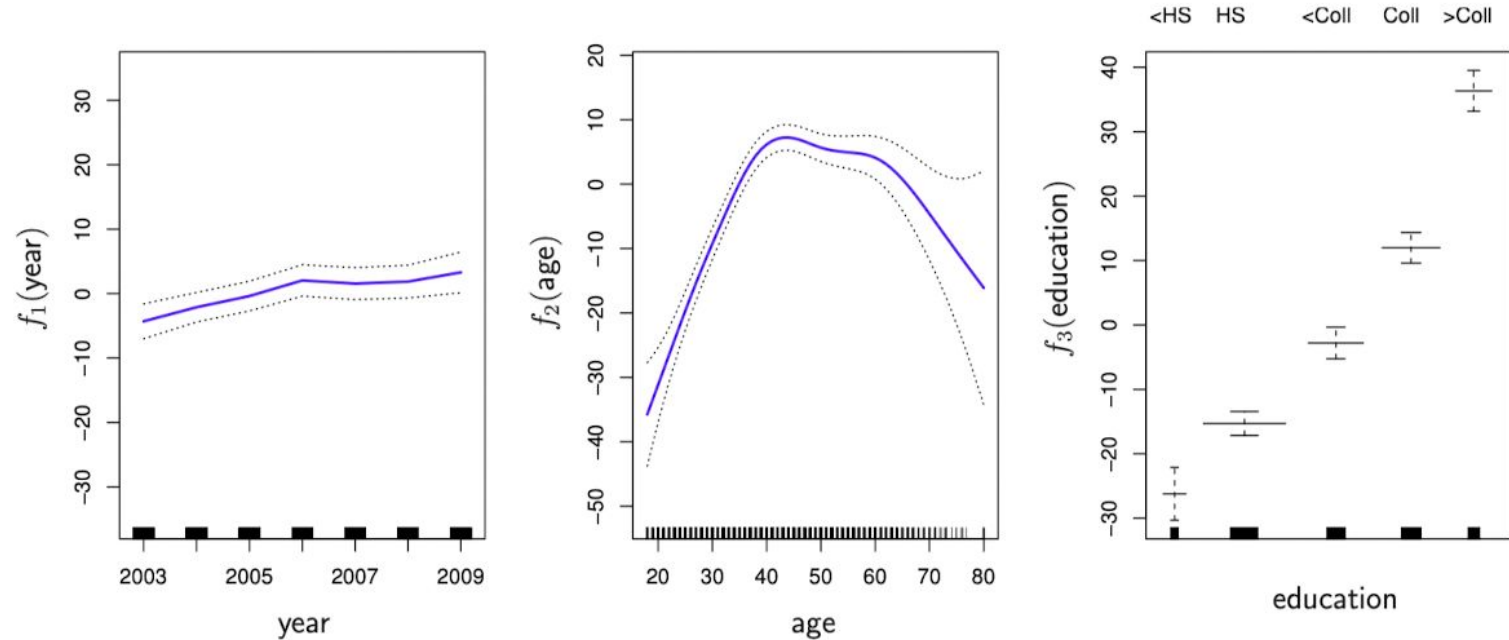


Image from:  
Introduction to  
Statistical Learning  
with applications in  
Python (Figure 7.12)