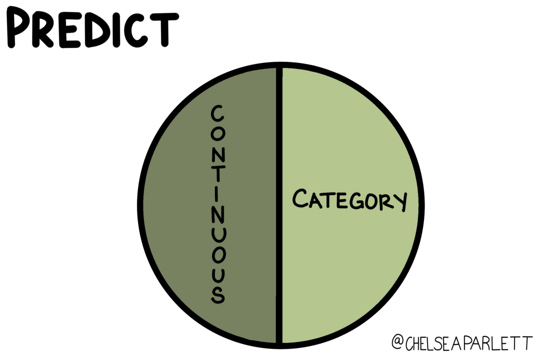


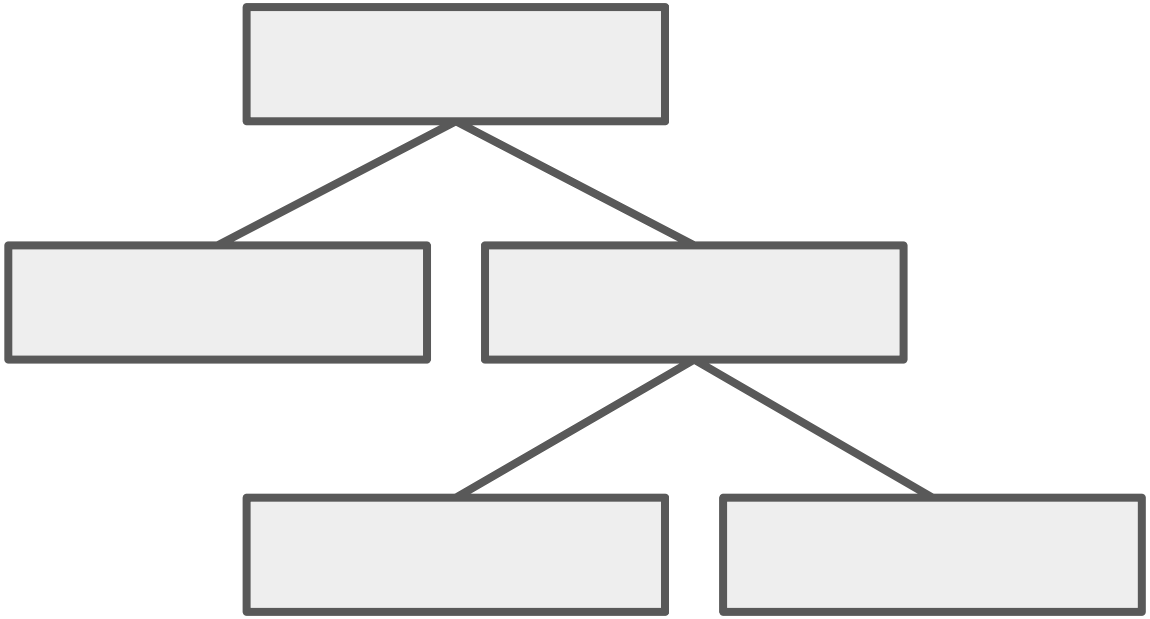
🌲 Based Models

Dr. Chelsea Parlett-Pelleriti

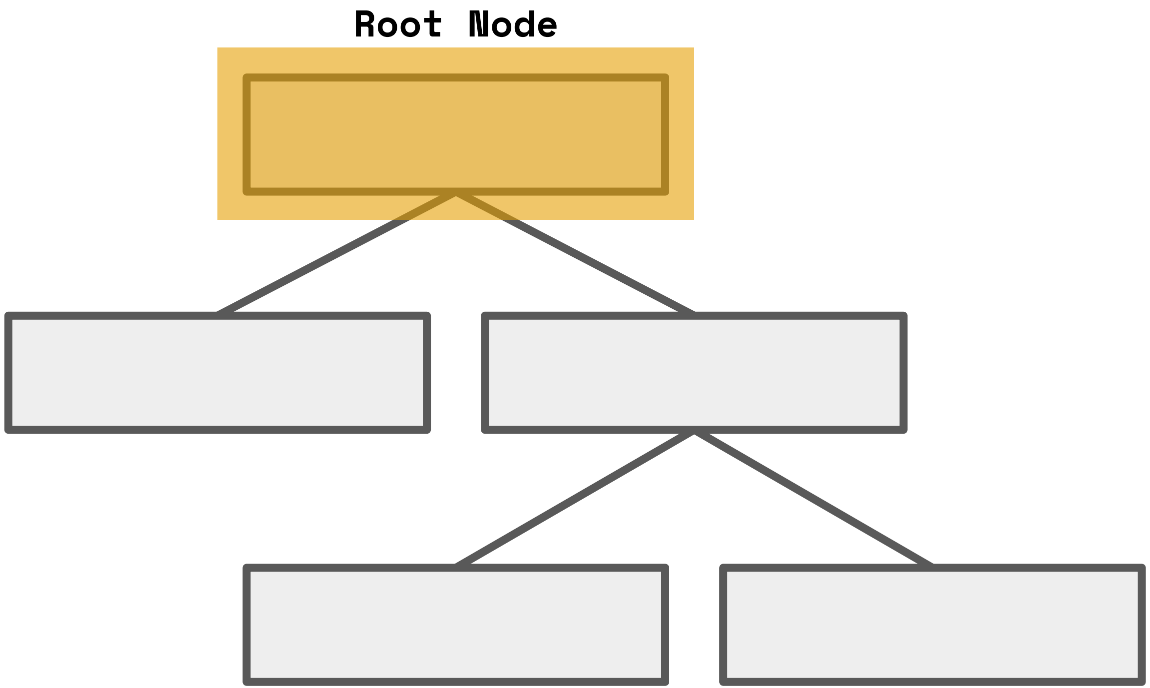
Decision Trees


I BE-LEAF
IN YOU
@CHELSEAPARLETT

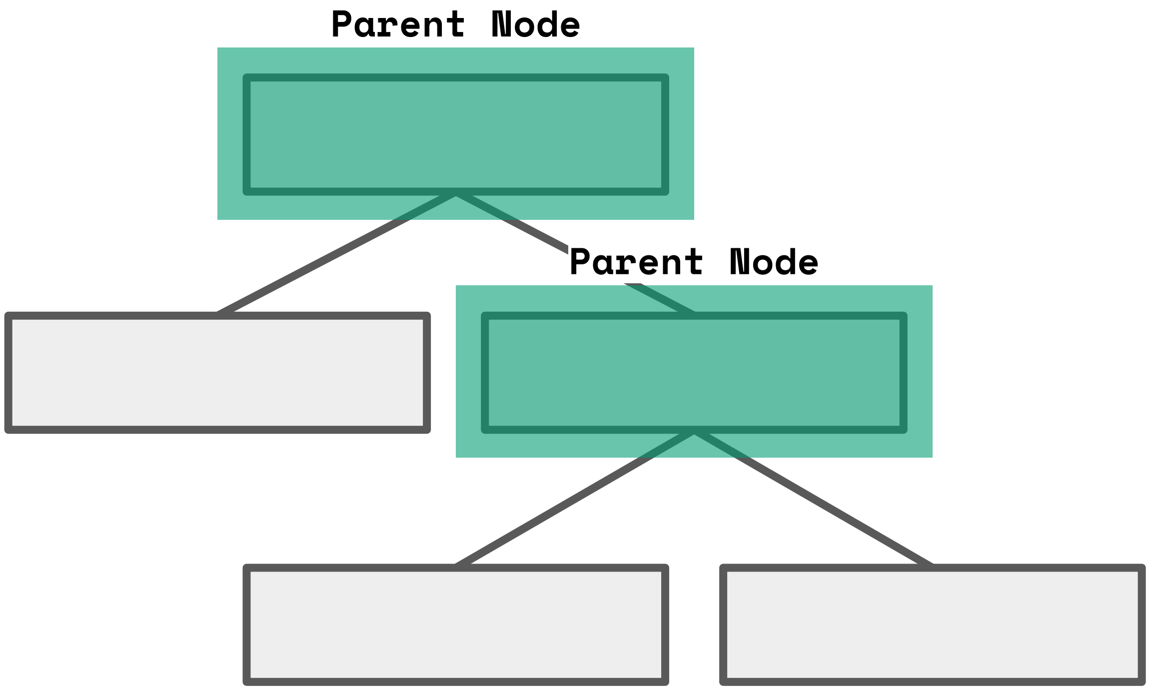
Tree Vocabulary



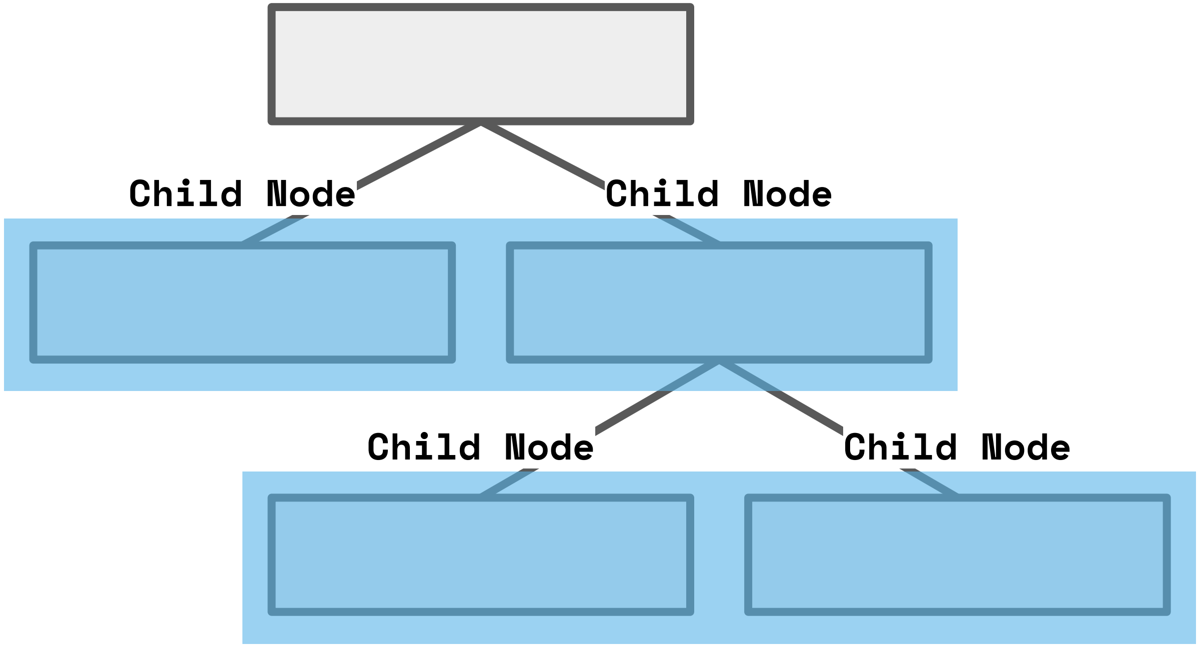
Tree Vocabulary



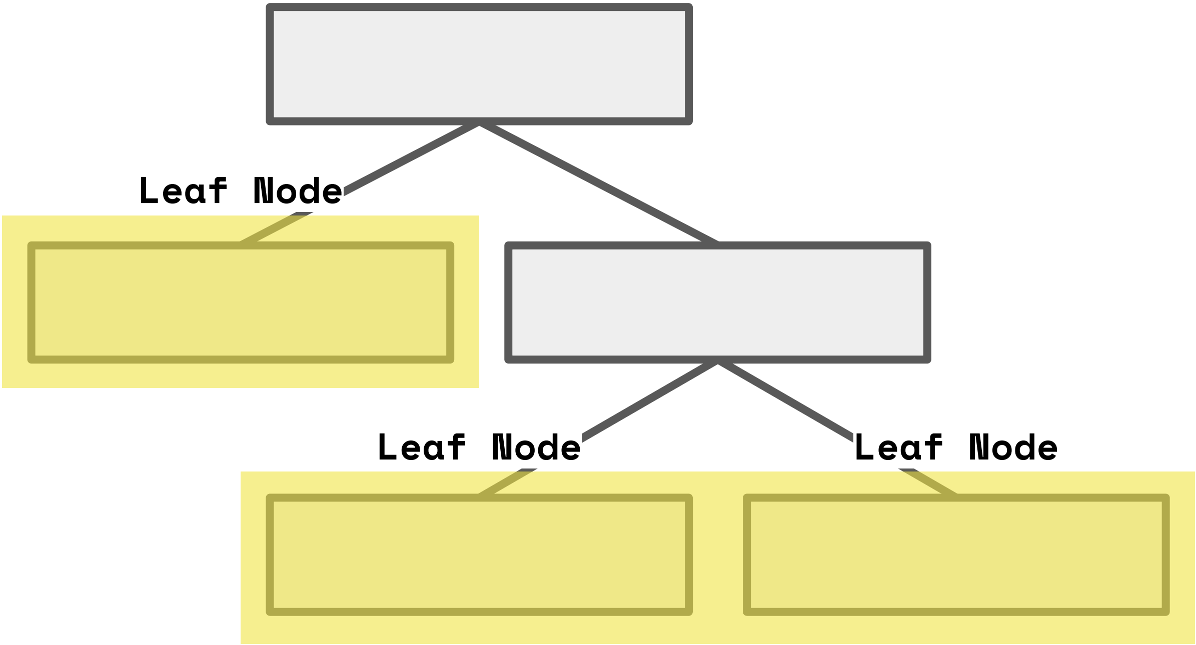
Tree Vocabulary



Tree Vocabulary

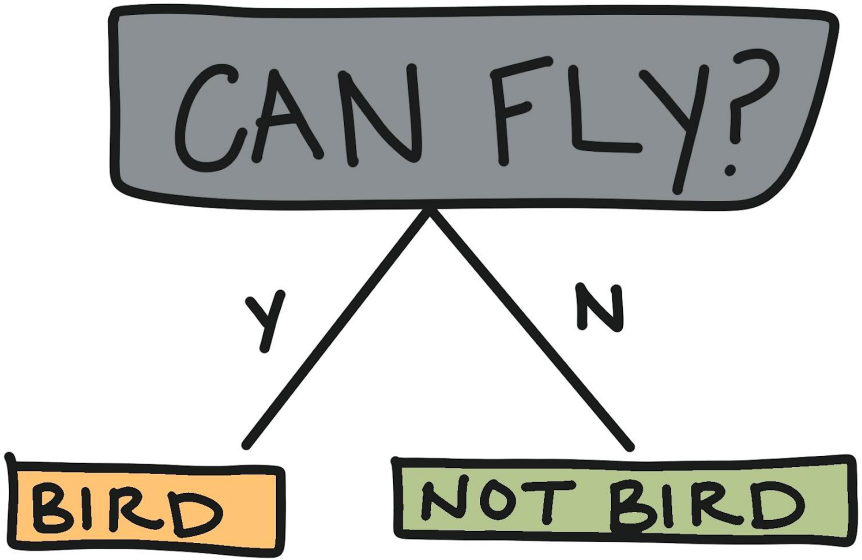


Tree Vocabulary

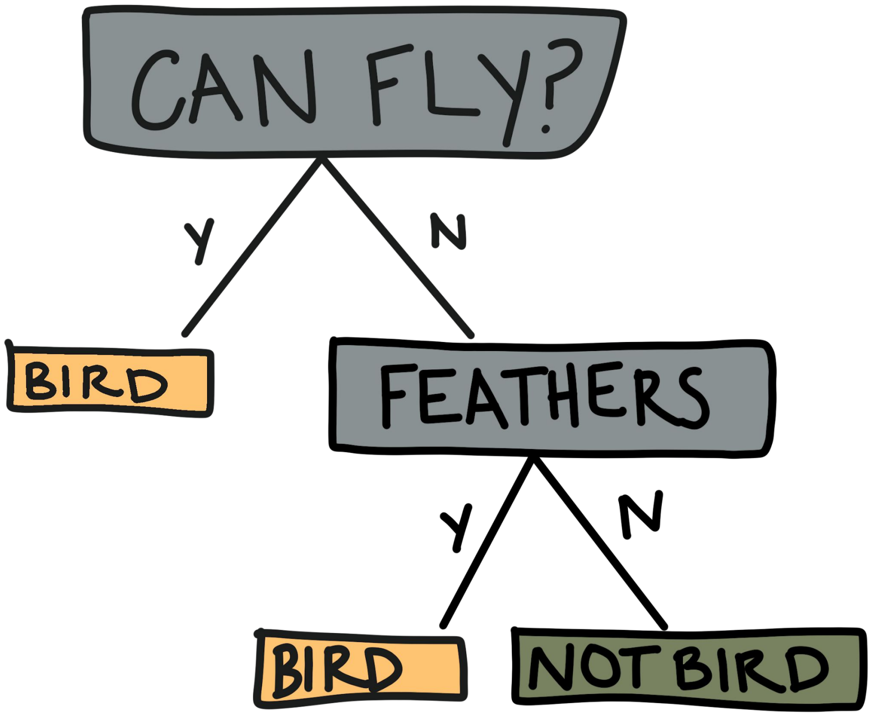


Twenty Questions

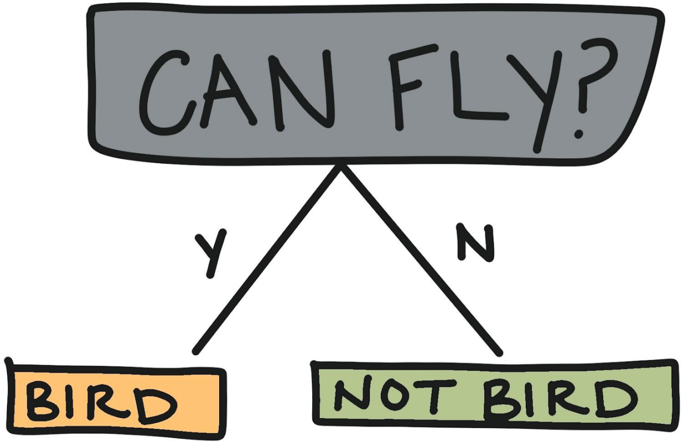
Simple Tree

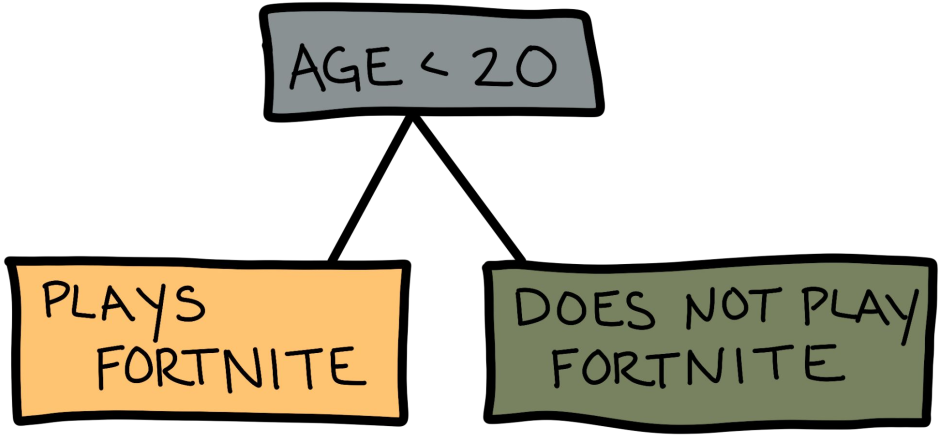


More Complicated Tree



Data Types


CAN FLY?
Y
N
BIRD
NOT BIRD

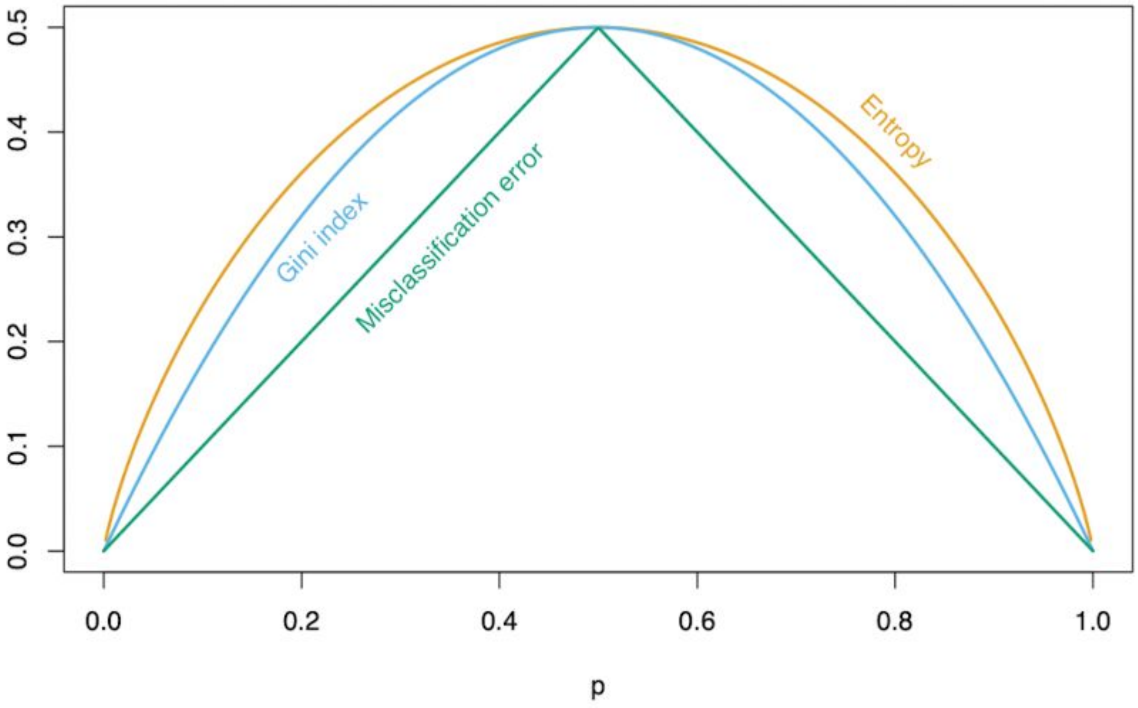

AGE < 20
PLAYS FORTNITE
DOES NOT PLAY FORTNITE

Gini Impurity and Entropy

$$\mathrm{GI} = 1 - \sum_{i=1}^{n} (p_i)^2 \qquad \mathrm{E} = -\sum_{i=1}^{n} p_i * log(p_i)$$

Goal: choose split so that GI (or entropy) is minimized

Gini Impurity and Entropy



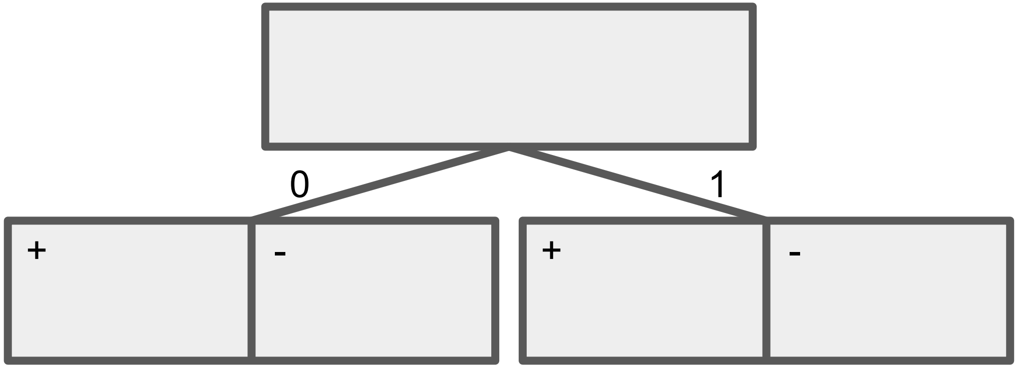
$$\mathrm{GI} = 1 - \sum_{i=1}^{n} (p_i)^2 \qquad \mathrm{E} = - \sum_{i=1}^{n} p_i * log(p_i)$$

Figure from the Elements of Statistical Learning (Hastie et al)

$$1 - \sum_{i=1}^{n} (p_i)^2$$

Categorical

cats	pet	wfh	children	income
1	0	1	1	34
0	1	0	1	58.3
1	1	1	0	71.5
0	0	0	1	74.9
0	0	0	1	75.3
1	0	0	1	75.6
0	0	0	1	81
1	1	1	0	82.3
1	1	1	0	85.6
1	1	1	1	95.4



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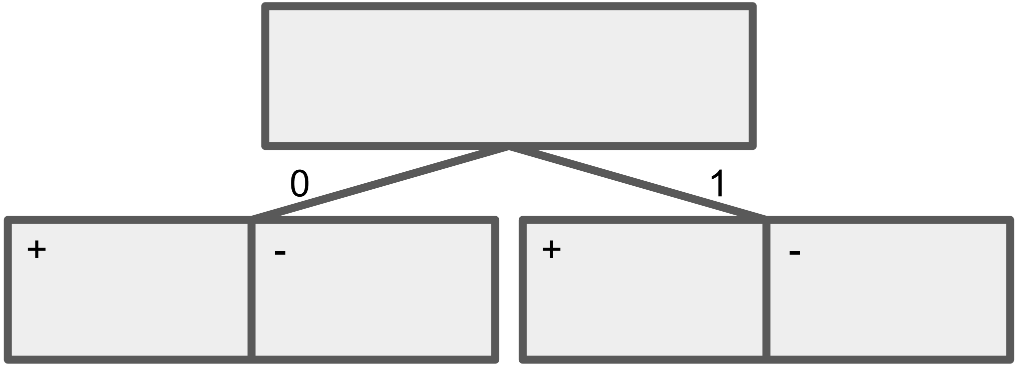

Continuous

$$1 - \sum_{i=1}^{n} (p_i)^2$$

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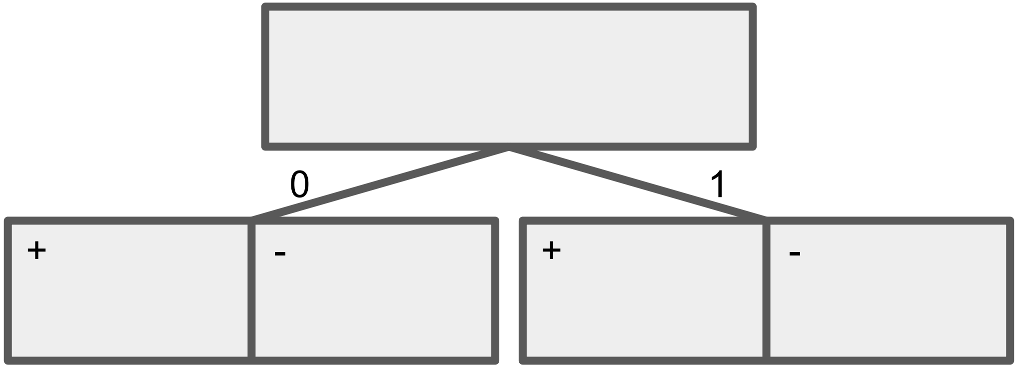

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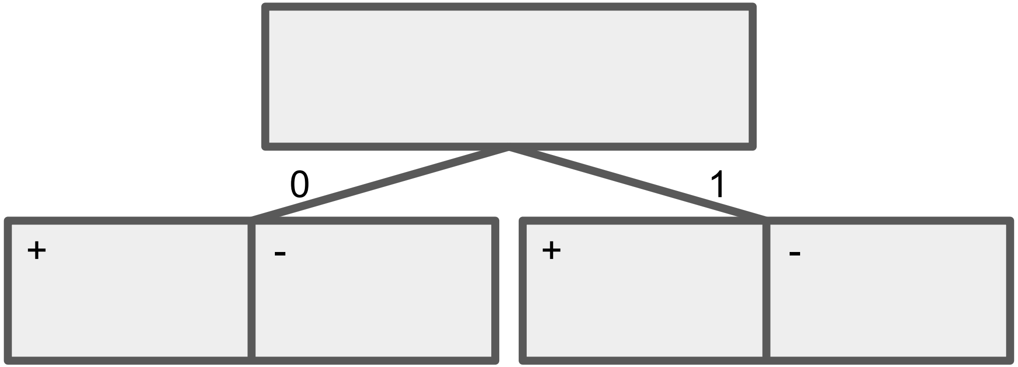


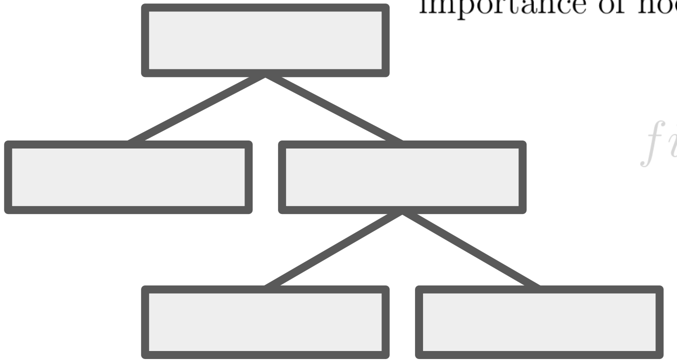
Basic Steps

1. Calculate Gini Impurity (or Entropy/Information Gain) for each node
2. Choose Node with lowest score
3. If the parent node has the lowest score, it is a leaf.

Variable Importance

1. How much does this feature reduce node impurity?
2. If we shuffle the values of this feature, how much does it reduce the performance?

$$\underbrace{\text{imp}_j}_{\text{importance of node } j} = \overbrace{w_j C_j}^{\text{weighted parent node impurity}} - \underbrace{(w_{\text{left}_j} C_{\text{left}_j} + w_{\text{right}_j} C_{\text{right}_j})}_{}$$



$$fi_i = \frac{\sum_{j \in S_i} \text{imp}_j}{\sum_{k \in S_{all}} \text{imp}_k}; S_i \text{ is set of all nodes that split on feature}_i$$

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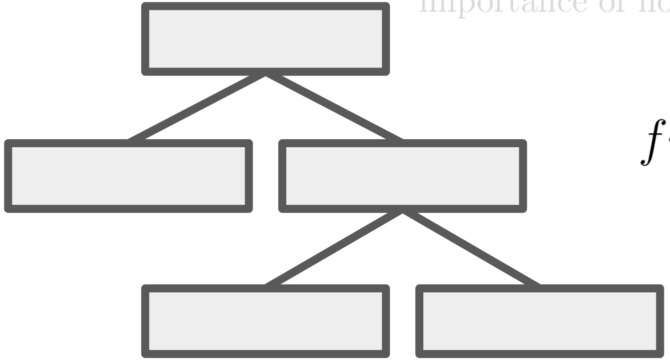

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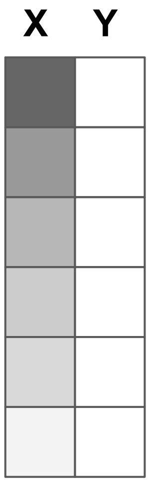

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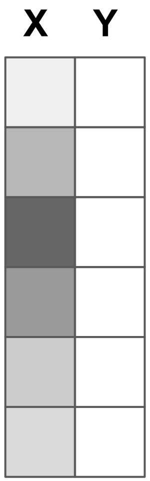

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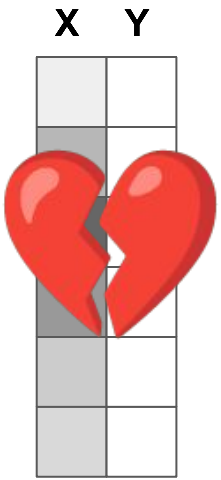
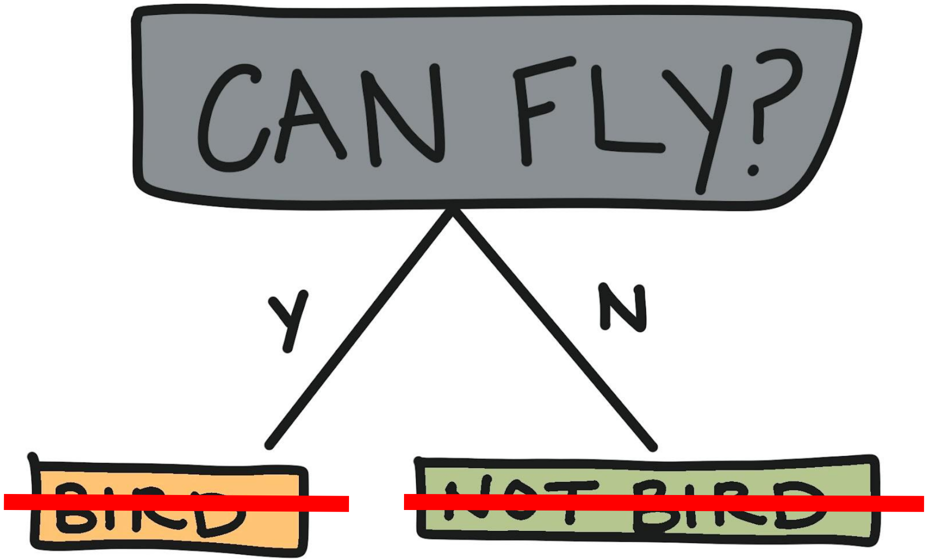


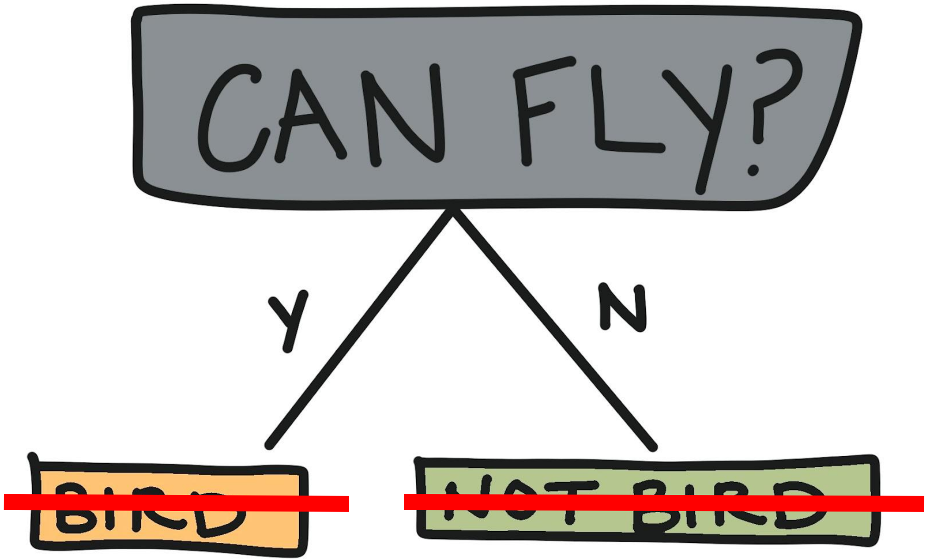
Regression Trees

Regression Trees



Regression Trees

Instead of checking if splits decrease Gini Impurity, we check if they decrease MSE



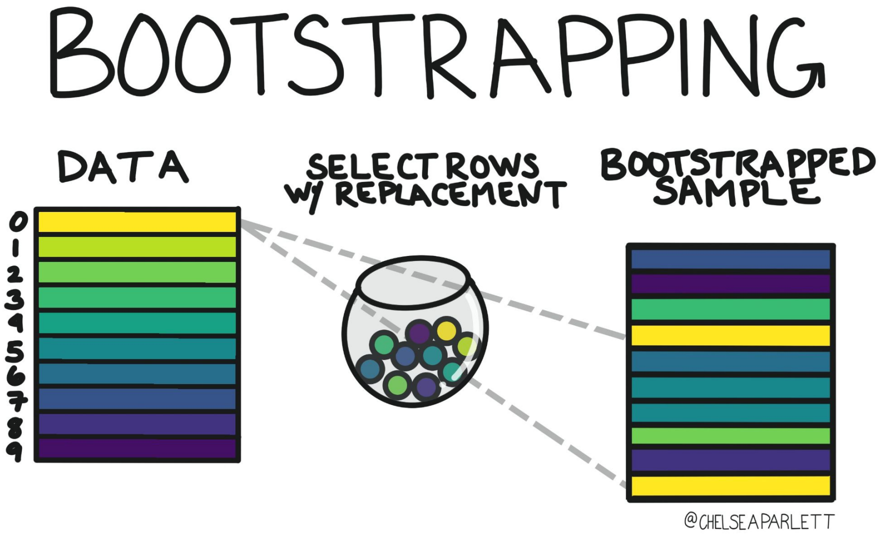
Random Forests

ONE vs. MANY

Random Forests

- Bootstrap Aggregating (**Bagging**)
- **Random Feature Selection**

Random Forests



Random Forests

- Bootstrap Aggregating (**Bagging**)
- **Random Feature Selection**

Random Forests

Random Forests

Random Forests

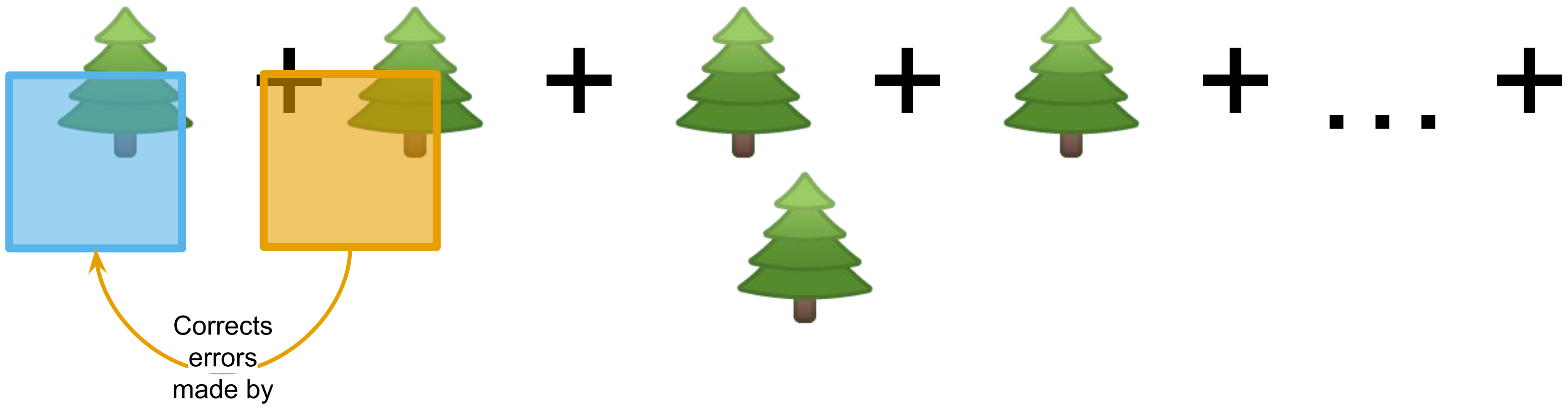
Important Hyperparameters

- # of trees
- # of features per tree

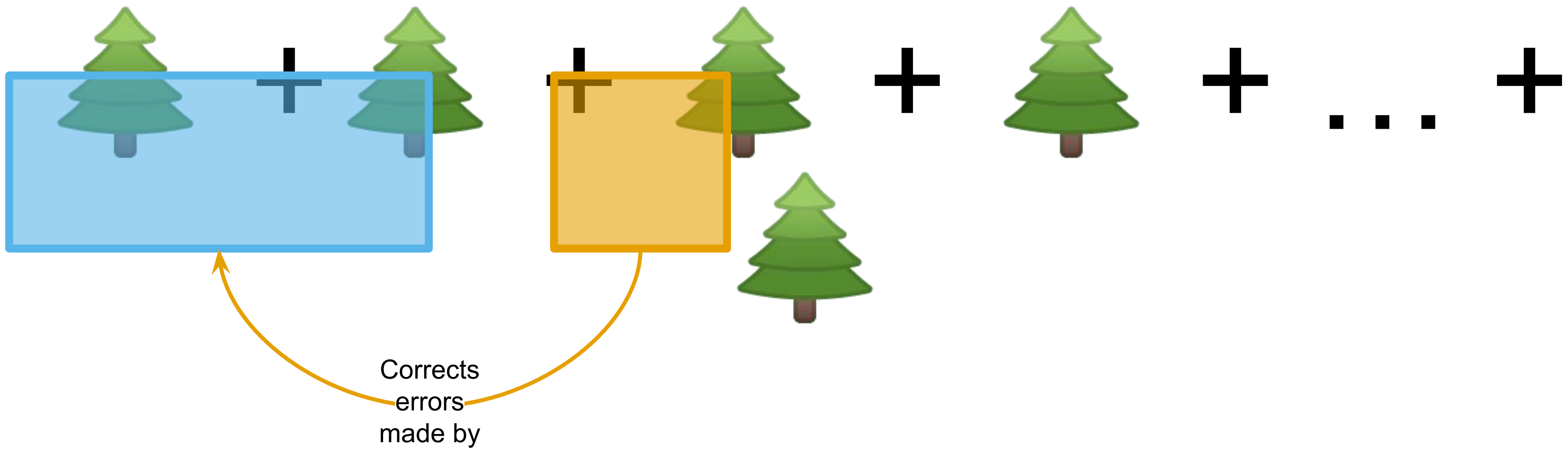
Gradient Boosting Trees

Gradient Boosting Trees

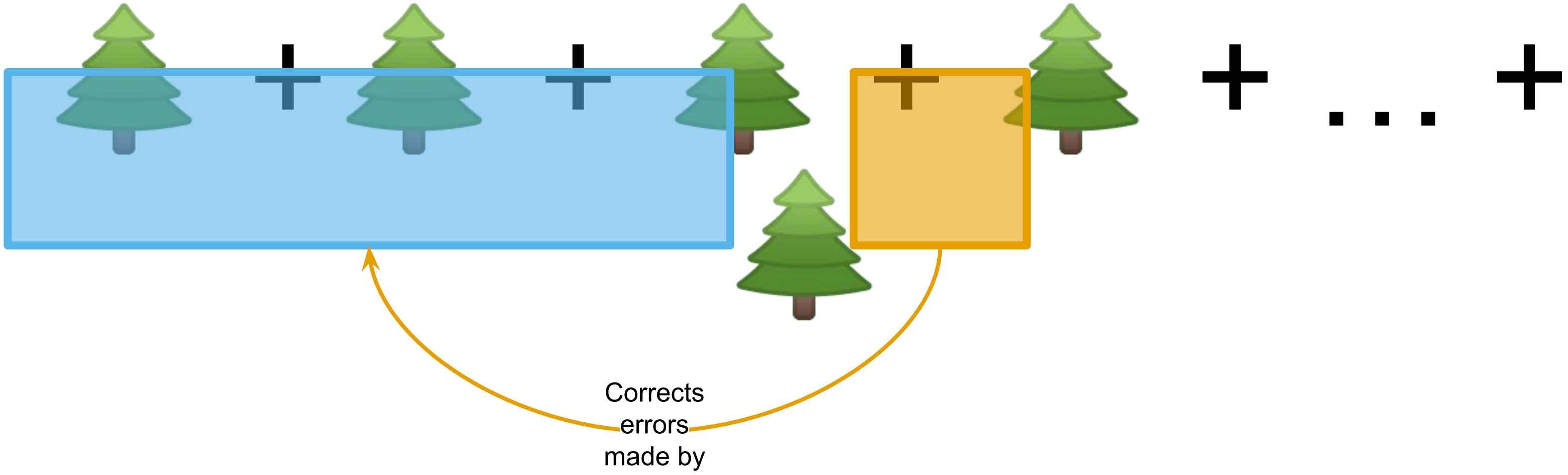
Gradient Boosting Trees



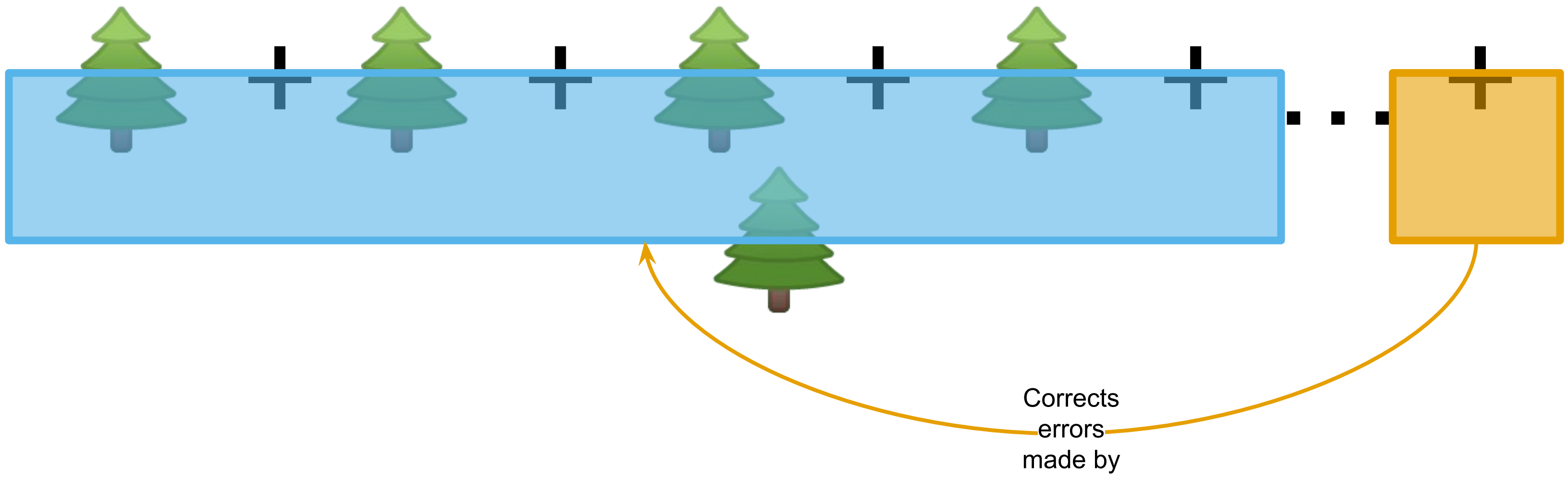
Gradient Boosting Trees



Gradient Boosting Trees



Gradient Boosting Trees



Gradient Boosting Tree

	Age	Initial Guess	Residual
Person 1	20		
Person 2	19		
Person 3	21		
Person 4	20		

Gradient Boosting Tree

	Age	Initial Guess	Residual
Person 1	20	20	
Person 2	19	20	
Person 3	21	20	
Person 4	20	20	

Gradient Boosting Tree

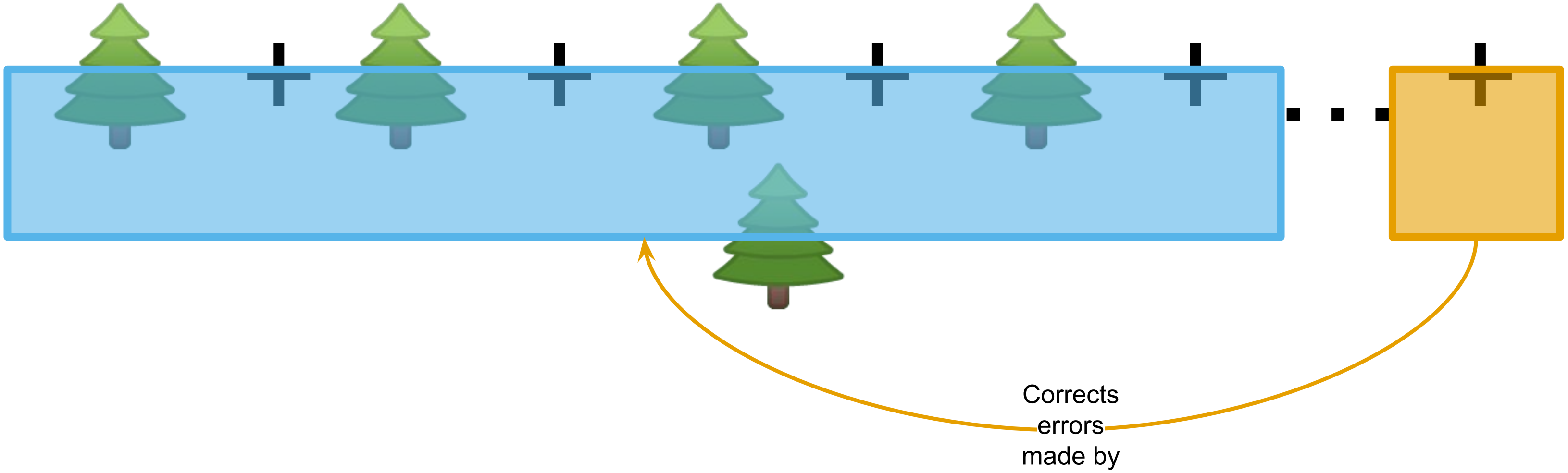
	Age	Initial Guess	Residual
Person 1	20	20	0
Person 2	19	20	-1
Person 3	21	20	1
Person 4	20	20	0

Gradient Boosting Tree

Actual Value = Prediction + Residual

	Age	Initial Guess	Residual
Person 1	20	20	0
Person 2	19	20	-1
Person 3	21	20	1
Person 4	20	20	0

Gradient Boosting Trees



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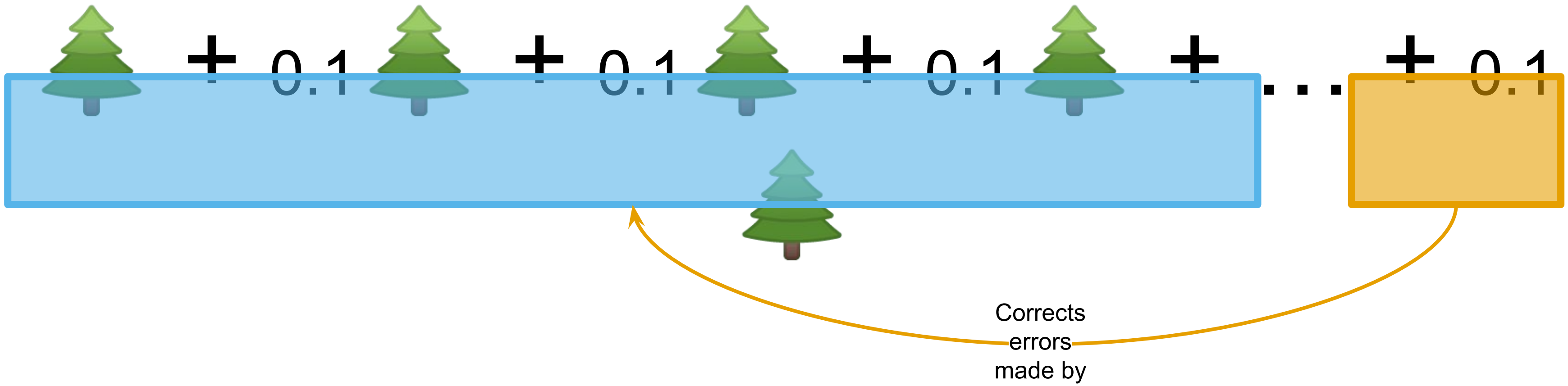

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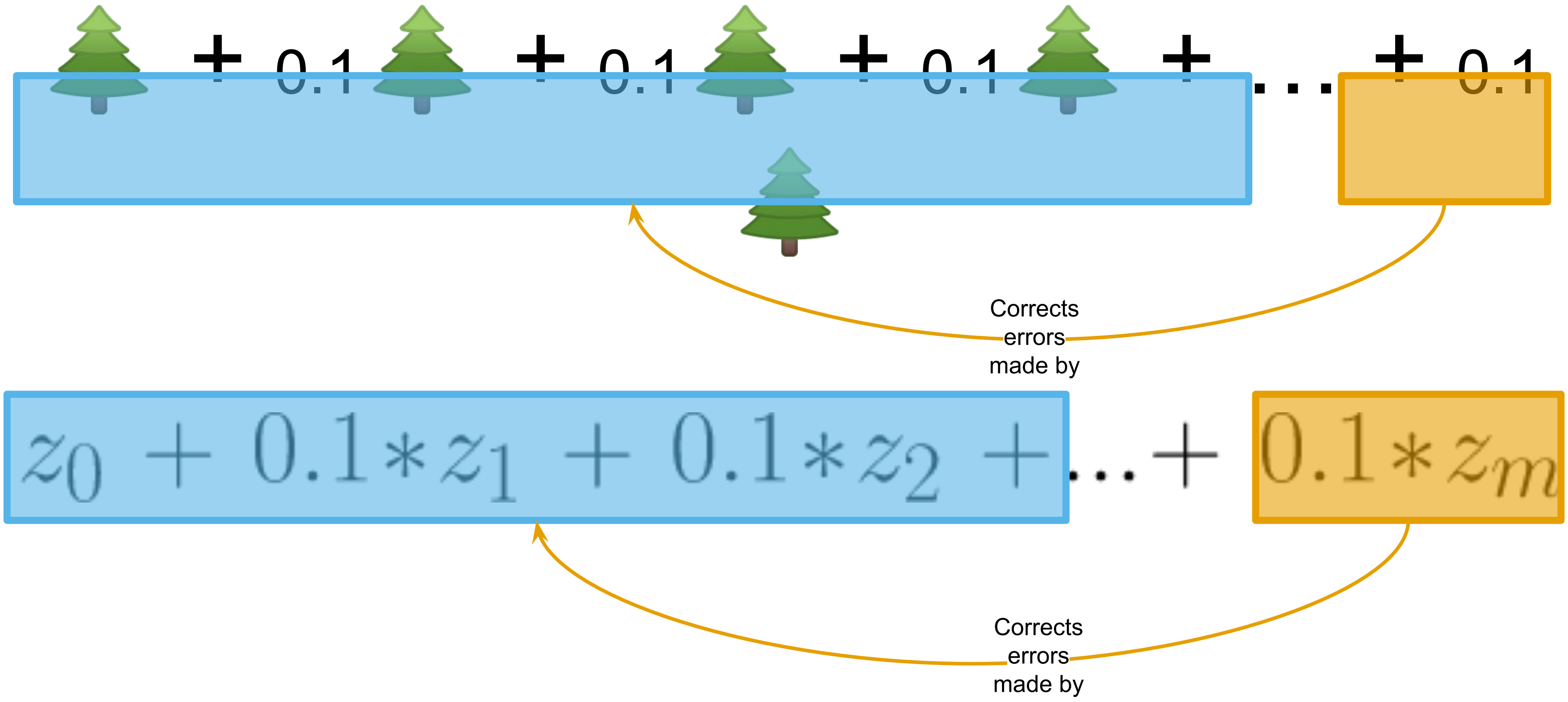
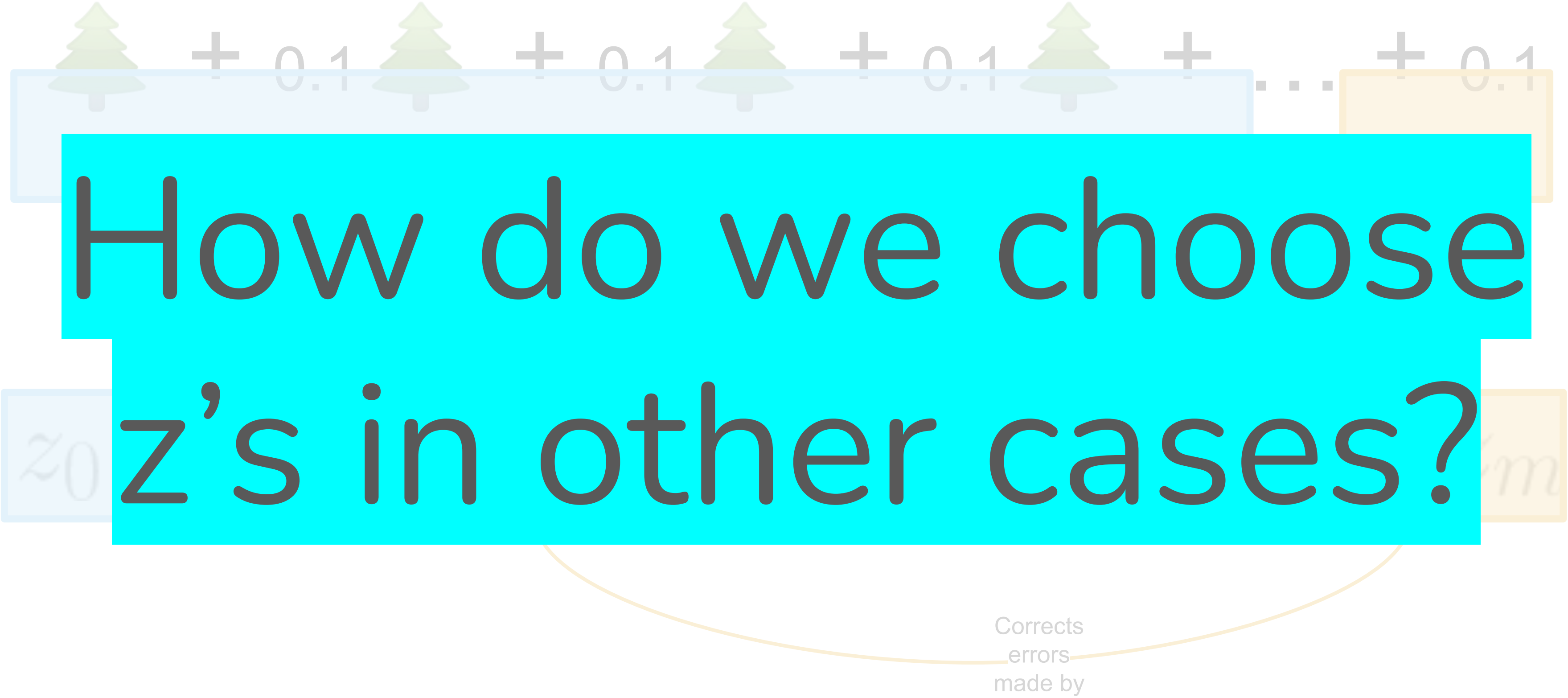


Gradient Boosting Trees



How do we choose z’s in other cases?

Gradient Boosting Trees: Some Extra Math

$$z_i = -\frac{\partial L(y, F_i)}{\partial F_i}$$

Let's generalize that math! Having future trees predict the error works for a regression tree using **(y-p)^2** as it's loss.

Gradient Boosting Trees: Some Extra Math

$$z_i = -\frac{\partial L(y, F_i)}{\partial F_i}$$

But in all cases, subsequent trees will predict the **negative gradient** (z_i) of the Loss with respect to the Ensemble prediction

Gradient Boosting Trees: Some Extra Math

$$z_i = -\frac{\partial L(y, F_i)}{\partial F_i}$$

But in all cases, subsequent trees will predict the **negative gradient** (z_i) of the Loss with respect to the Ensemble prediction

Baseline prediction

$$F_i = \sum_{t=0}^{i} z_t \qquad F_2 = \boxed{z_0} + \boxed{z_1} + \boxed{z_2}$$

Errors

Gradient Boosting Trees: Some Extra Math

$$z_i = -\frac{\partial L(y, F_i)}{\partial F_i}$$

But in all cases, subsequent trees will predict the **negative gradient** (z_i) of the Loss with respect to the Ensemble prediction

$$F_i = \sum_{t=0}^{i} z_t$$

$$F_1 = z_0 + z_1$$
$$F_2 = z_0 + z_1 + z_2$$

$$F_i = F_{i-1} + z_i$$

Gradient Boosting Trees: Some Extra Math

$$z_i = \boxed{-\frac{\partial L(y, F_i)}{\partial F_i}}$$

Negative Gradient of Loss w.r.t. Ensemble Prediction

Gradient Boosting Trees: Some Extra Math

$$z_i = \boxed{-\frac{\partial L(y, F_i)}{\partial F_i}}$$

Negative Gradient of Loss w.r.t. Ensemble Prediction

The Negative Gradient tell us what *adjustments* we should make to our prediction (**F_i**) in order to *decrease* our loss.

Gradient Boosting Trees: Some Extra Math

$$z_i = \boxed{-\frac{\partial L(y, F_i)}{\partial F_i}}$$

Negative Gradient of Loss w.r.t. Ensemble Prediction

The Negative Gradient tell us what *adjustments* we should make to our prediction (**F_i**) in order to *decrease* our loss.

$$L(y,p) = \boxed{(y-p)^2}$$

Let's choose Squared Error as our Loss

$$-\frac{\partial L(y,p)}{\partial p} = \boxed{2(y-p)}$$

Negative Gradient

Gradient Boosting Trees: Some Extra Math

$$z_i = \boxed{-\frac{\partial L(y, F_i)}{\partial F_i}}$$

Negative Gradient of Loss w.r.t. Ensemble Prediction

The Negative Gradient tell us what *adjustments* we should make to our prediction (**F_i**) in order to *decrease* our loss.

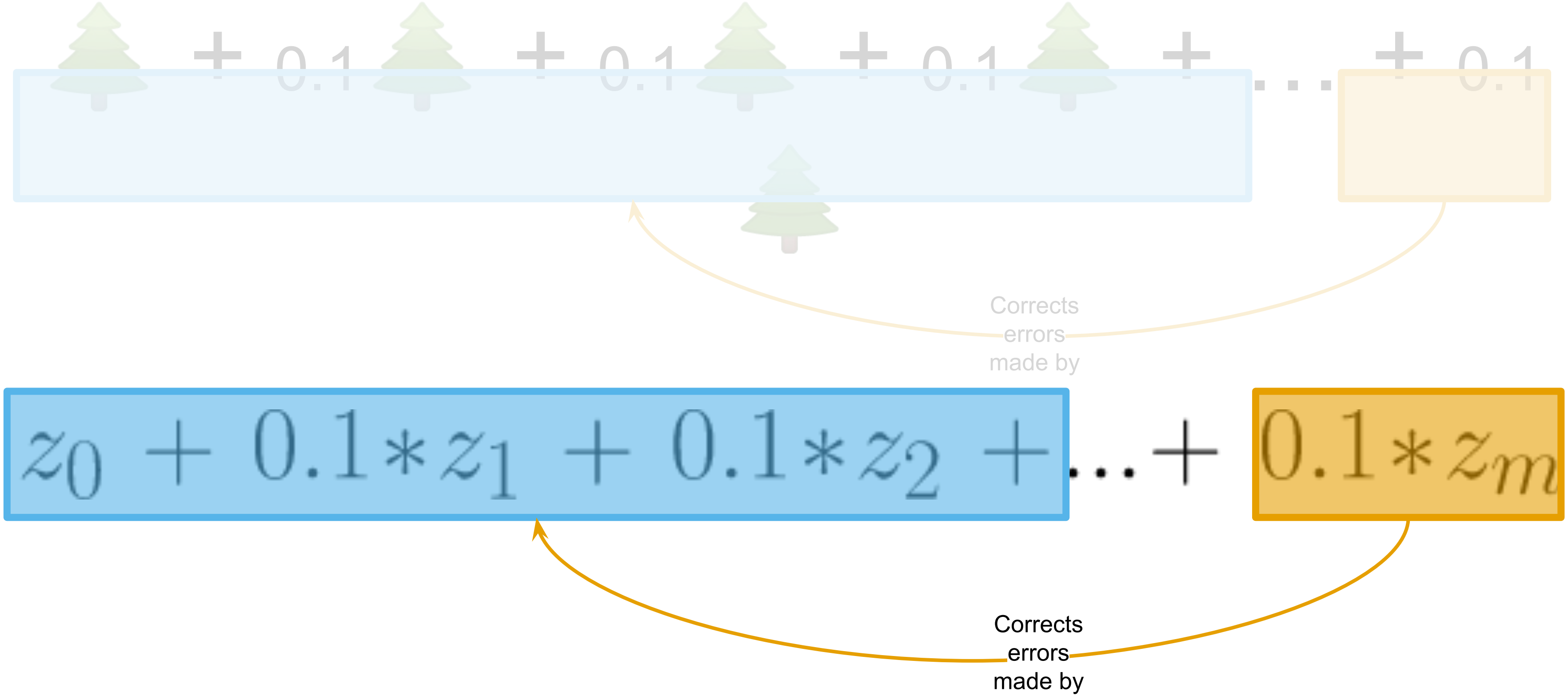
$$L(y,p) = \boxed{(y-p)^2}$$

Let's choose Squared Error as our Loss

$$-\frac{\partial L(y,p)}{\partial p} = 2\boxed{(y-p)}$$

Just the Error!

Gradient Boosting Trees



Gradient Boosting Trees: Some Extra Math

$$z_i = -\frac{\partial L(y, F_i)}{\partial F_i}$$

With squared loss, **error *is* the negative gradient**, but the negative gradient will work in other situations