Backpropagation Classwork

Dr. Chelsea Parlett-Pelleriti

All We Want in Life is Gradients

$$\begin{bmatrix} \frac{\partial h}{\partial b_1} \\ \frac{\partial h}{\partial b_2} \\ \frac{\partial h}{\partial w_1} \\ \frac{\partial h}{\partial w_2} \end{bmatrix}$$

Partial Derivatives

$$f(x, y) = x^2 + xy + y^2$$

$$\frac{\partial f}{\partial y} = x + 2y \qquad \frac{\partial f}{\partial x} = 2x + y$$

For a function of with multiple variables, how does the *function change* when you change a *single* variable

The Chain-Rule

$$f(x) = cos(x)$$

$$g(x) = x^2$$

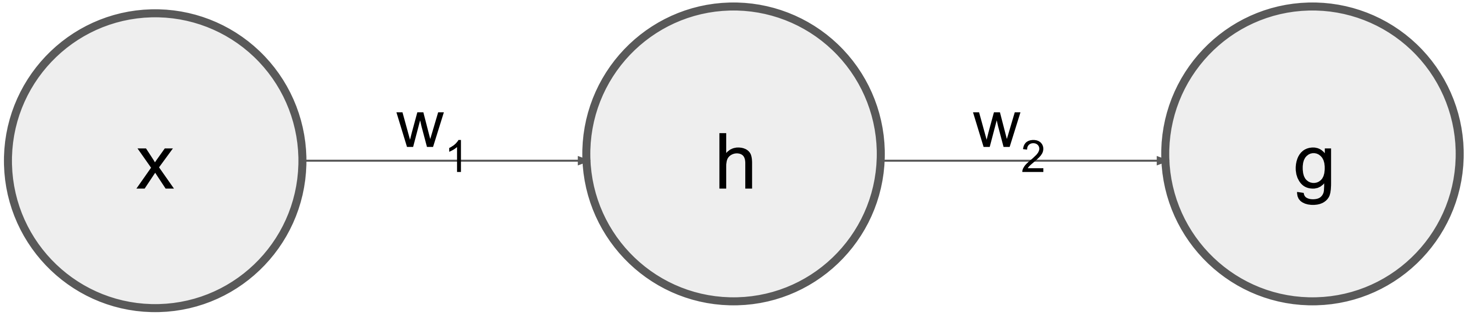
$$f(g(x)) = cos(x^2)$$

If we want to know how changing x affects **f(g(x))** we *first* need to think about how changing x affects **g(x)** *then* how changing **g(x)** affects **f(g(x))**

$$\frac{\partial f(g(x))}{\partial x} = \frac{\partial f}{\partial g}\frac{\partial g}{\partial x}$$

Simple Network

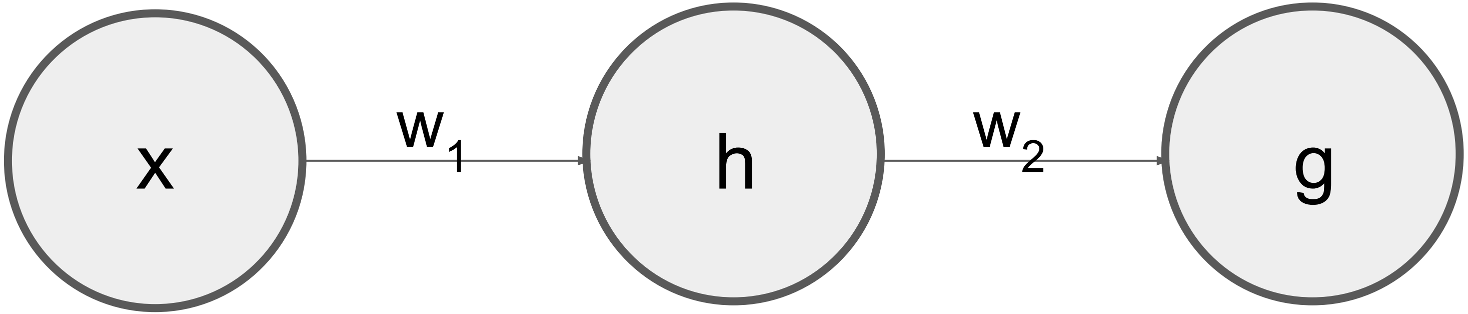
Forward Pass



$$h = x * w_1 + b_1$$

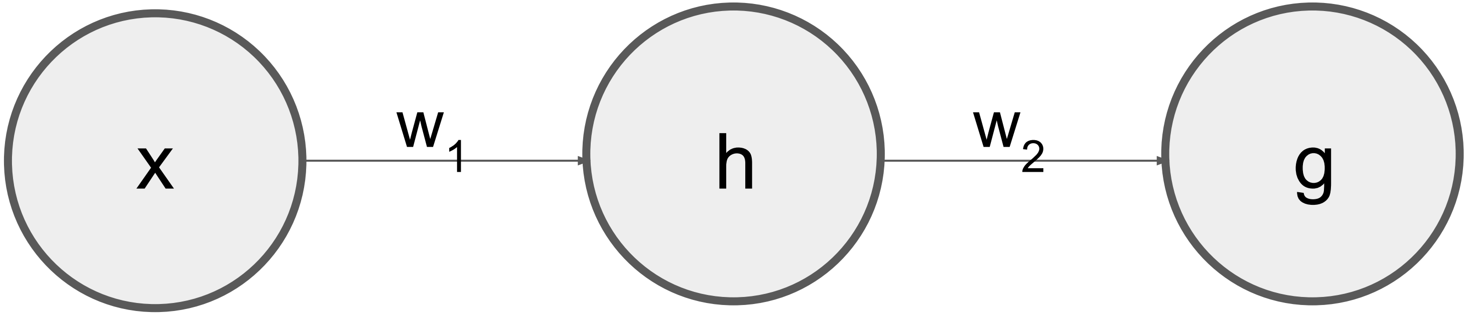
$$g = h * w_2 + b_2$$

Loss



$$L = \frac{1}{2}(y - g)^2$$

Backwards Pass



$$L = \frac{1}{2}(y - g)^2$$

$$\boxed{\frac{\partial L}{\partial g}} = 2 * \frac{1}{2} * (y - g)(-1)$$

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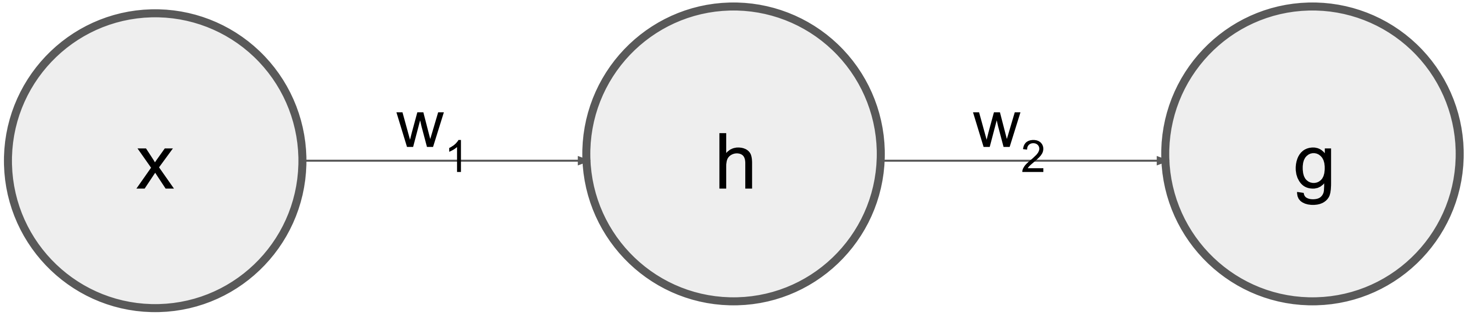
Backwards Pass



$$g = h * w_2 + b_2$$

$$\boxed{\frac{\partial g}{\partial h}} = w_2$$

$$\boxed{\frac{\partial L}{\partial g}} = 2 * \frac{1}{2} * (y - g)(-1)$$

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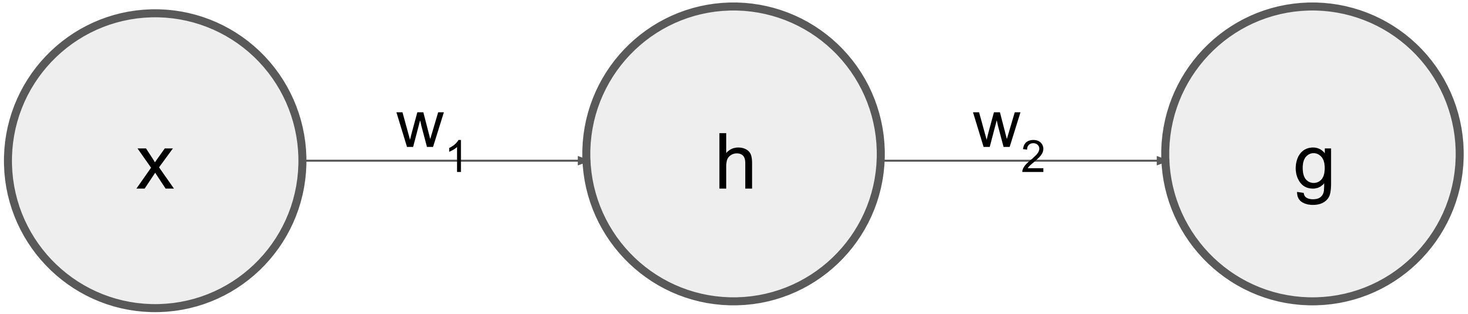
Backwards Pass



$$h = x * w_1 + b_1$$

$$\boxed{\frac{\partial h}{\partial w_1}} = x$$

Backwards Pass

$$\boxed{\frac{\partial L}{\partial g}} = 2 * \frac{1}{2} * (y - g)(-1)$$

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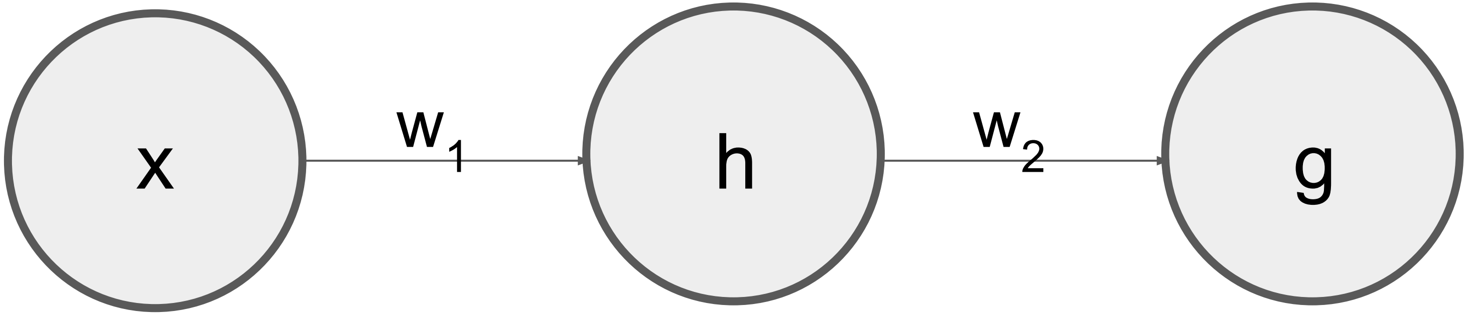

$$\frac{\partial L}{\partial w_1} = \boxed{\frac{\partial L}{\partial g}} * \boxed{\frac{\partial g}{\partial h}} * \boxed{\frac{\partial h}{\partial w_1}}$$

Backwards Pass

$$\boxed{\frac{\partial L}{\partial g}} = 2 * \frac{1}{2} * (y - g)(-1)$$

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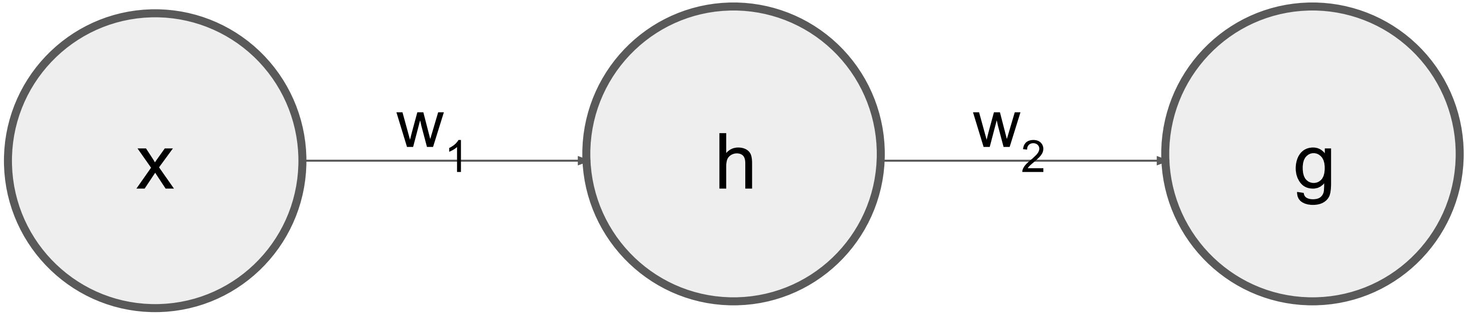

$$\frac{\partial L}{\partial w_1} = \boxed{\frac{\partial L}{\partial g}} * \boxed{\frac{\partial g}{\partial h}} * \boxed{\frac{\partial h}{\partial w_1}}$$

$$\boxed{-(y - g)} * \boxed{w_2} * \boxed{x}$$

Backwards Pass

$$\boxed{\frac{\partial L}{\partial g}} = 2 * \frac{1}{2} * (y - g)(-1)$$

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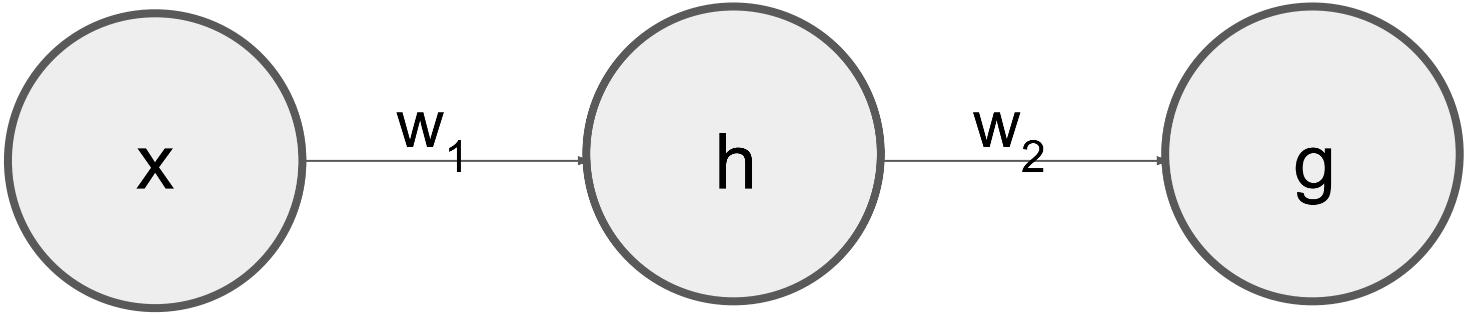

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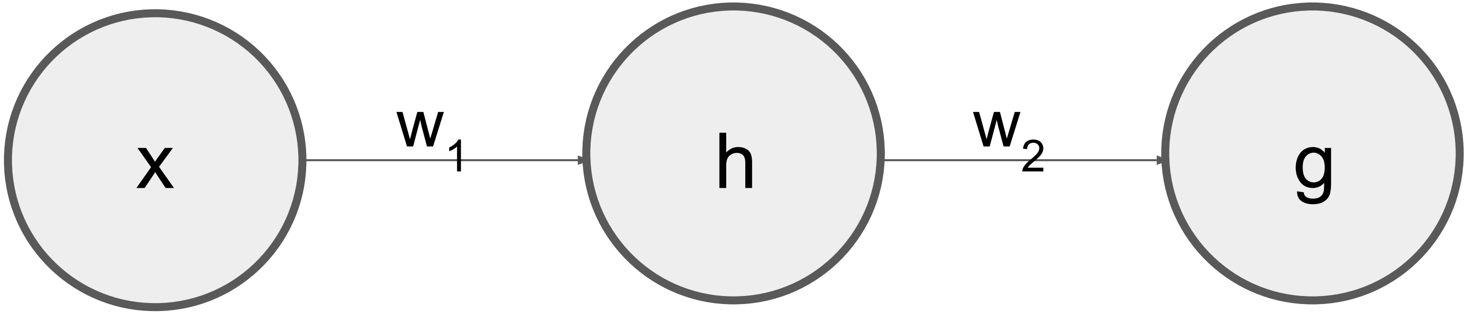

$$h = x * w_1 + b_1$$

$$\boxed{\frac{\partial h}{\partial b_1}} = 1$$

Backwards Pass

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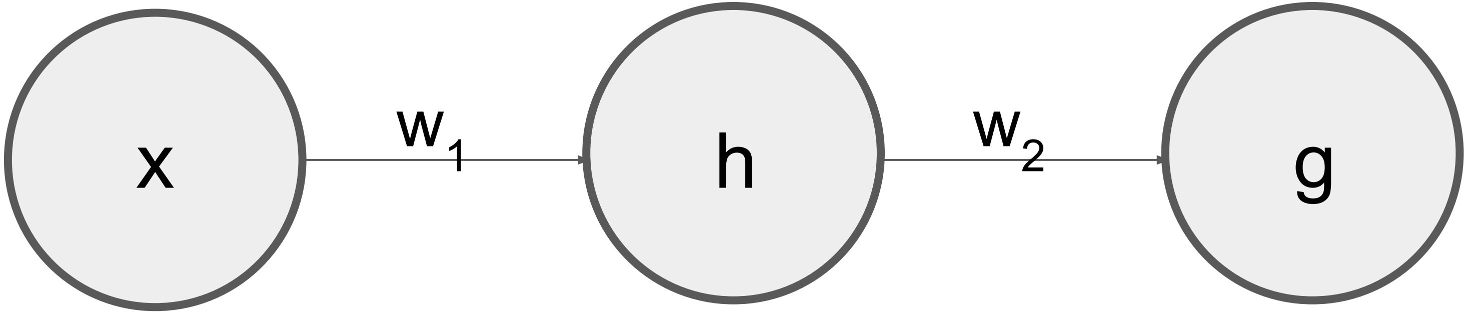

$$\frac{\partial L}{\partial b_1} = \boxed{\frac{\partial L}{\partial g}} * \boxed{\frac{\partial g}{\partial h}} * \boxed{\frac{\partial h}{\partial b_1}}$$

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$$\boxed{\frac{\partial L}{\partial g}} = 2 * \frac{1}{2} * (y - g)(-1)$$

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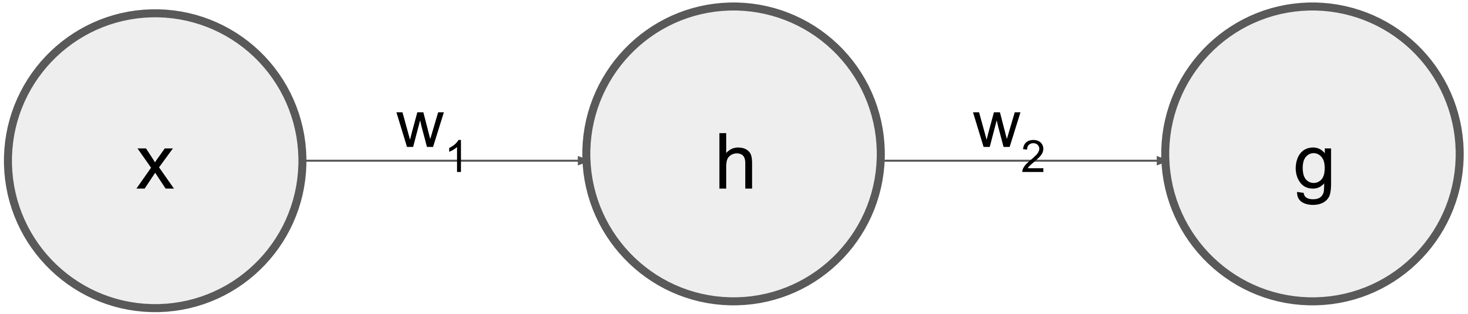

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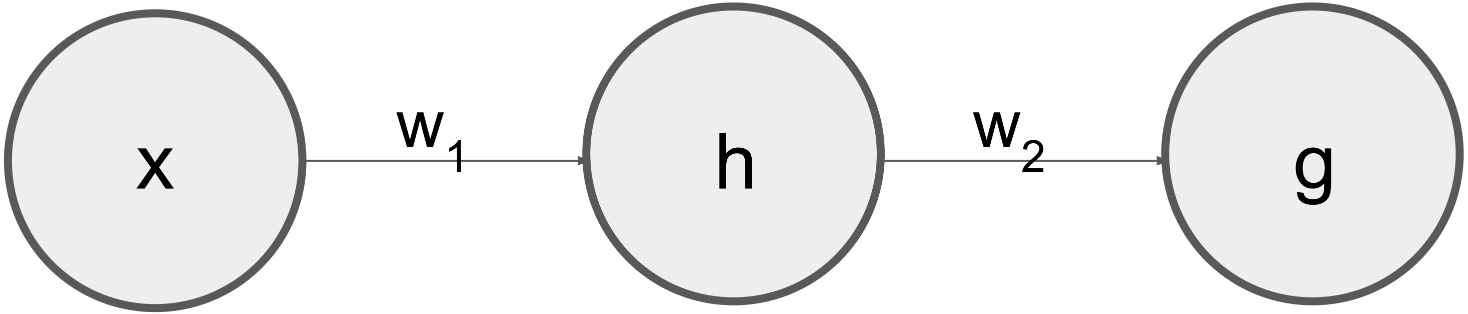

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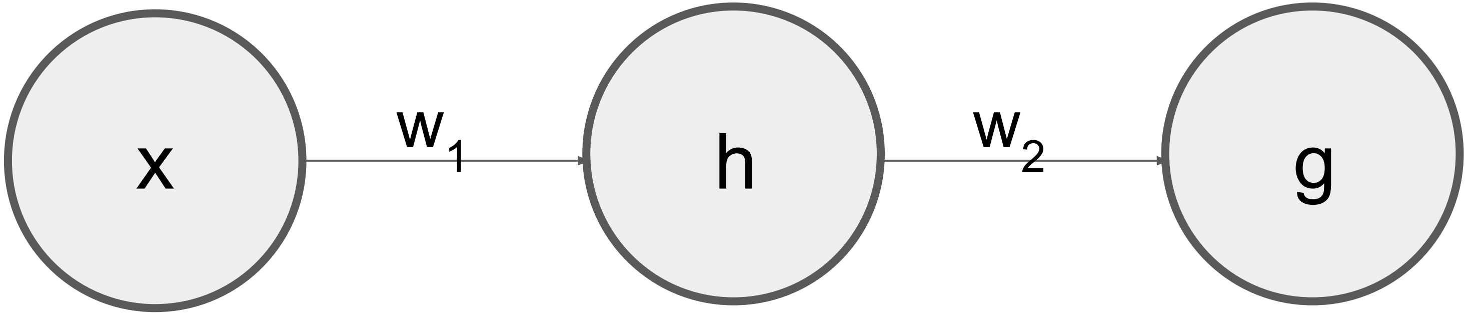

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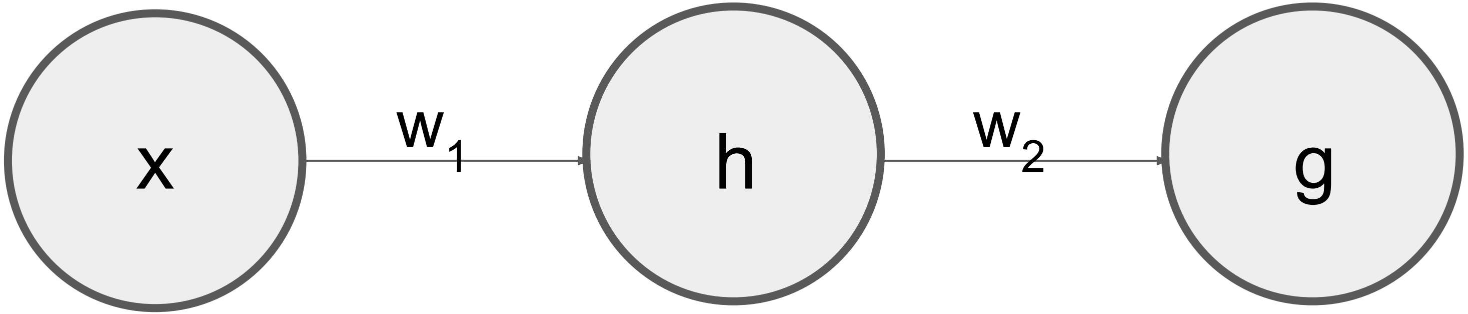

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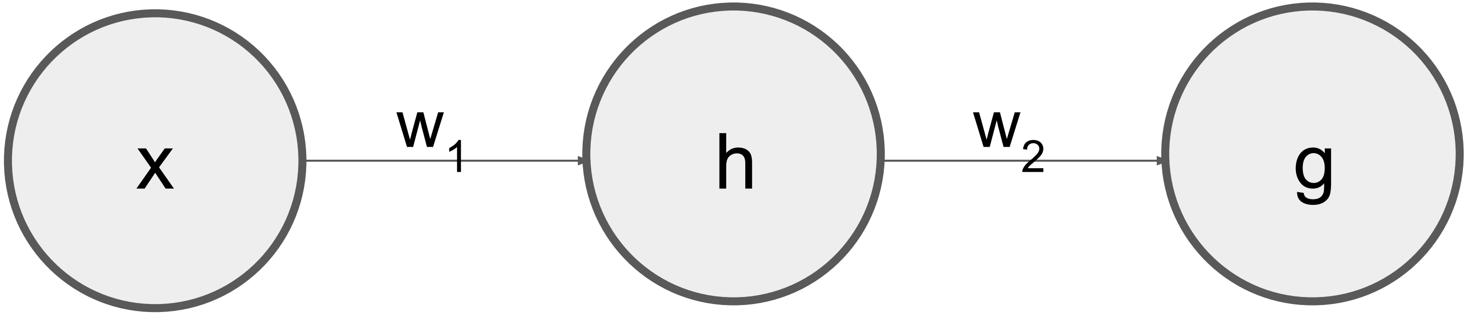

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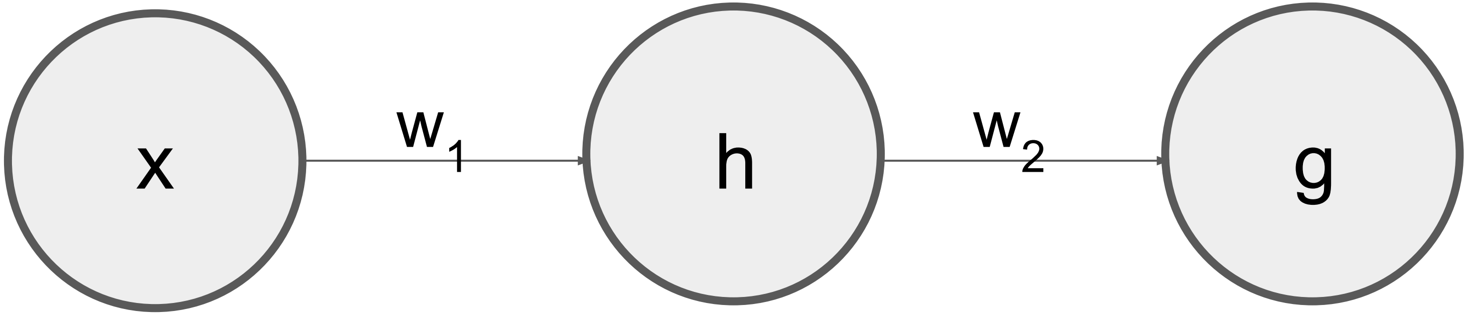

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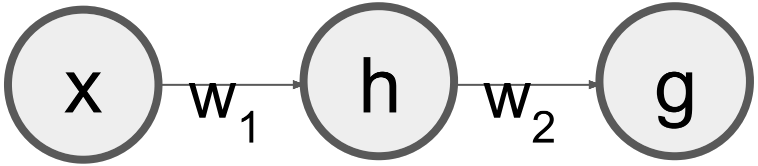
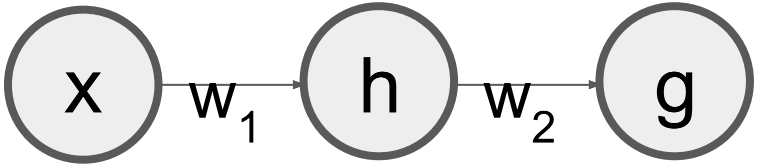


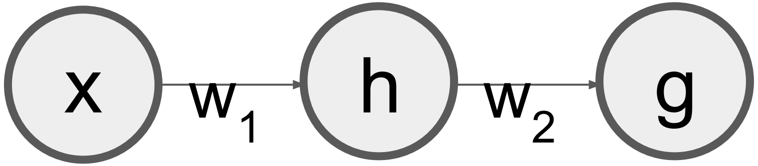
$$\nabla L = \begin{bmatrix} \frac{\partial L}{\partial w_1} \\ \frac{\partial L}{\partial b_1} \\ \frac{\partial L}{\partial w_2} \\ \frac{\partial L}{\partial b_2} \end{bmatrix}$$

Backwards Pass



$$\begin{bmatrix} w_1 \\ b_1 \\ w_2 \\ b_2 \end{bmatrix}_t = \begin{bmatrix} w_1 \\ b_1 \\ w_2 \\ b_2 \end{bmatrix}_{t-1} - \alpha * \begin{bmatrix} \frac{\partial L}{\partial w_1} \\ \frac{\partial L}{\partial b_1} \\ \frac{\partial L}{\partial w_2} \\ \frac{\partial L}{\partial b_2} \end{bmatrix}$$

Backwards Pass



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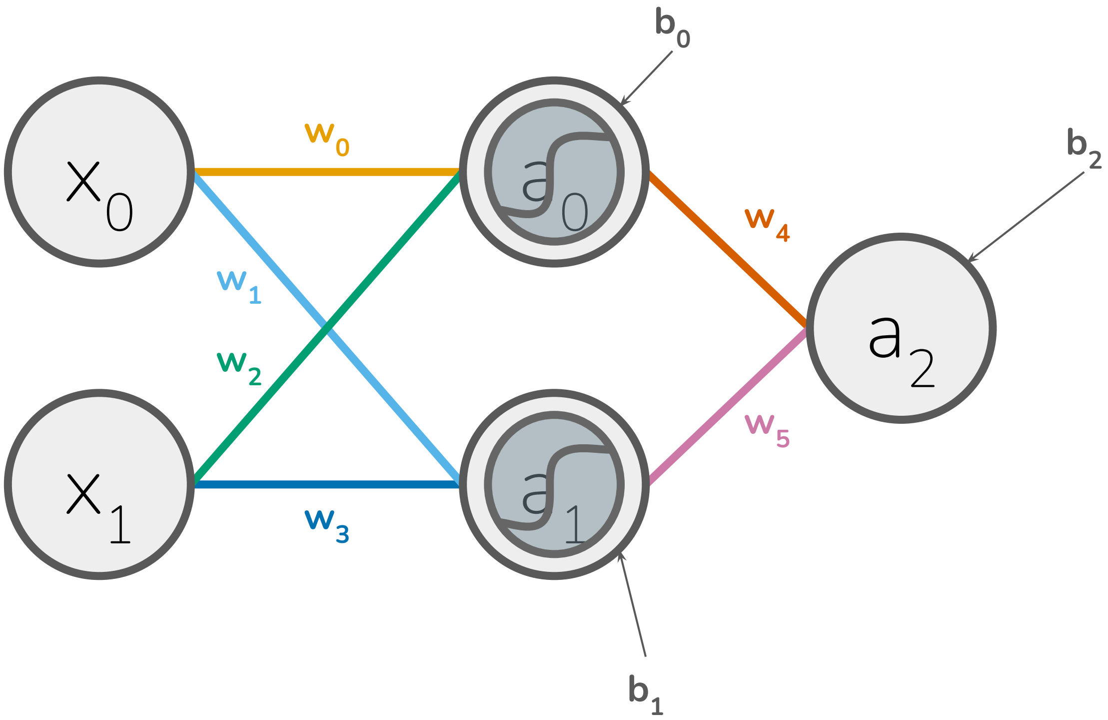
Adjustments to make to each parameter

Note: We have only 1 hidden layer and NO activation functions...

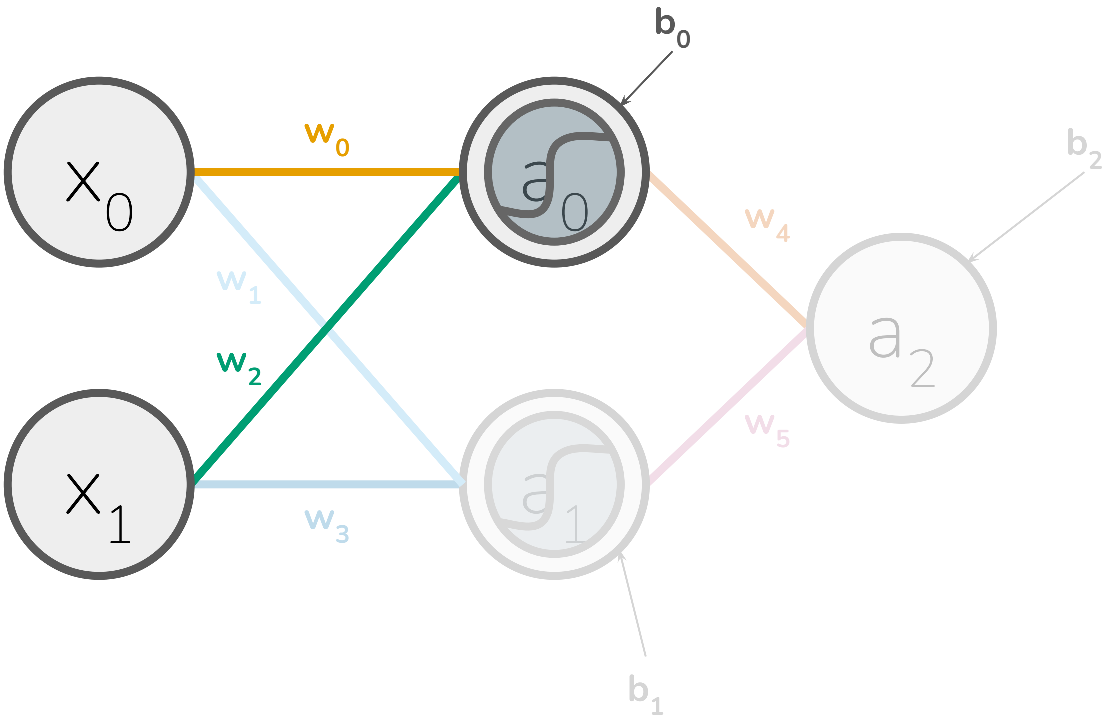
Adjustments to make to each parameter

A More Complicated Network

A More Complicated Network



Forward Pass

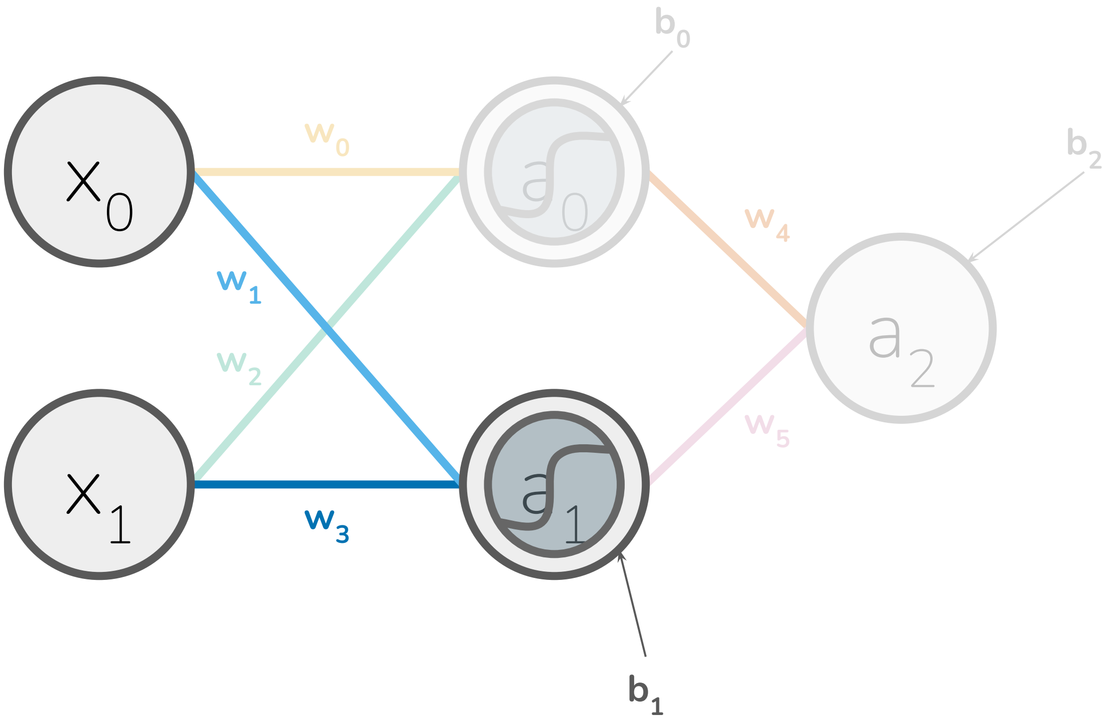


$$z_0 = (x_0 * w_0) + (x_1 * w_2) + b_0$$
$$a_0 = \sigma(z_0)$$

$$z_1 = (x_0 * w_1) + (x_1 * w_3) + b_1$$
$$a_1 = \sigma(z_1)$$

$$a_2 = (a_0 * w_4) + (a_1 * w_5) + b_2$$

Forward Pass



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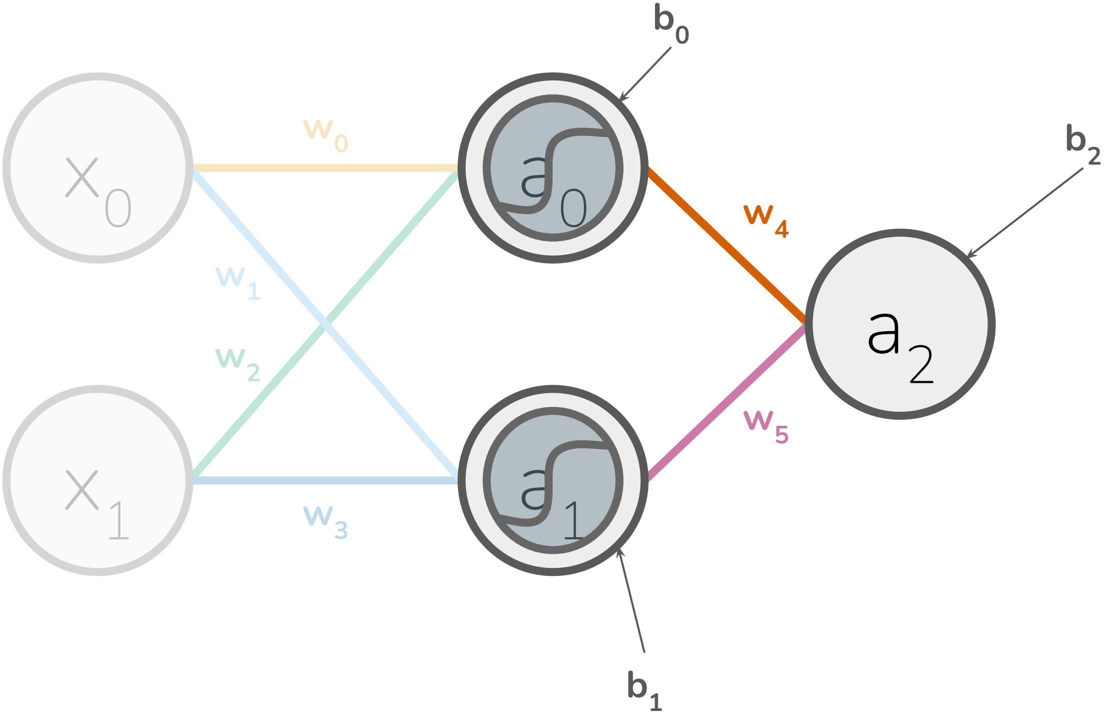

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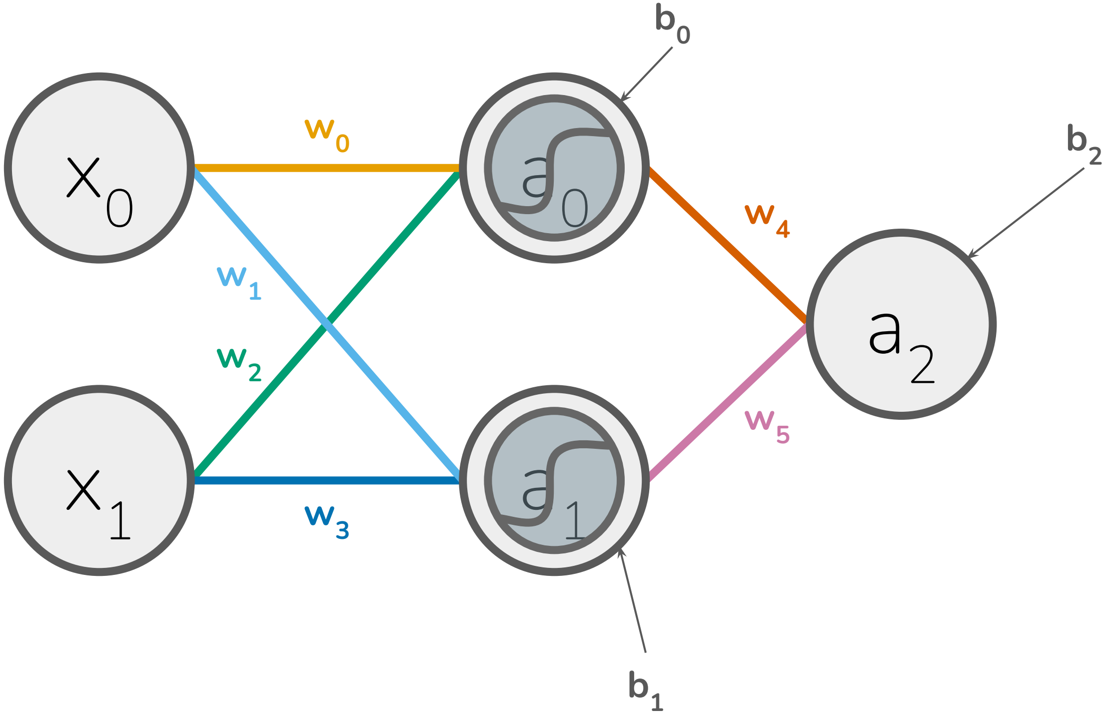

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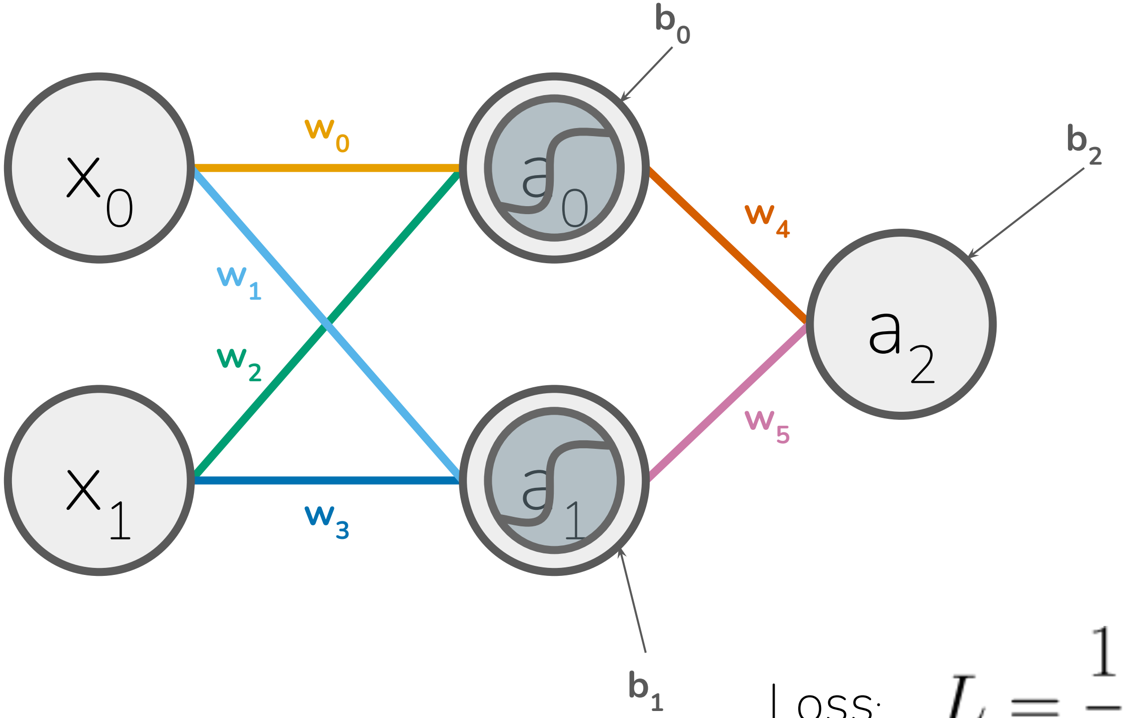
Loss



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Loss: $$L = \frac{1}{2}(y - a_2)^2$$

Questions

1. What is the dimension of our gradient (i.e. how many parameters are there in the model?)
2. How many individual partial derivatives do we need to calculate in order to compute all the terms in the gradient?
3. What is the partial derivative of L with respect to w_4? (Don't calculate the derivatives, just use the chain rule to figure out which partial derivatives you'd need in order to calculate it)
4. What is the partial derivative of L with respect to w_0? (Don't calculate the derivatives, just use the chain rule to figure out which partial derivatives you'd need in order to calculate it)
5. Do earlier weights (nearer the input layer) have *longer* or *shorter* chains of partial derivatives?

Notes:

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