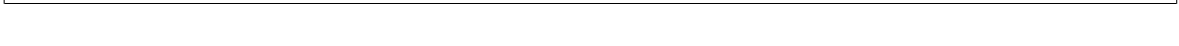


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<b>Abstract</b>



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Sammendrag

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## Preface

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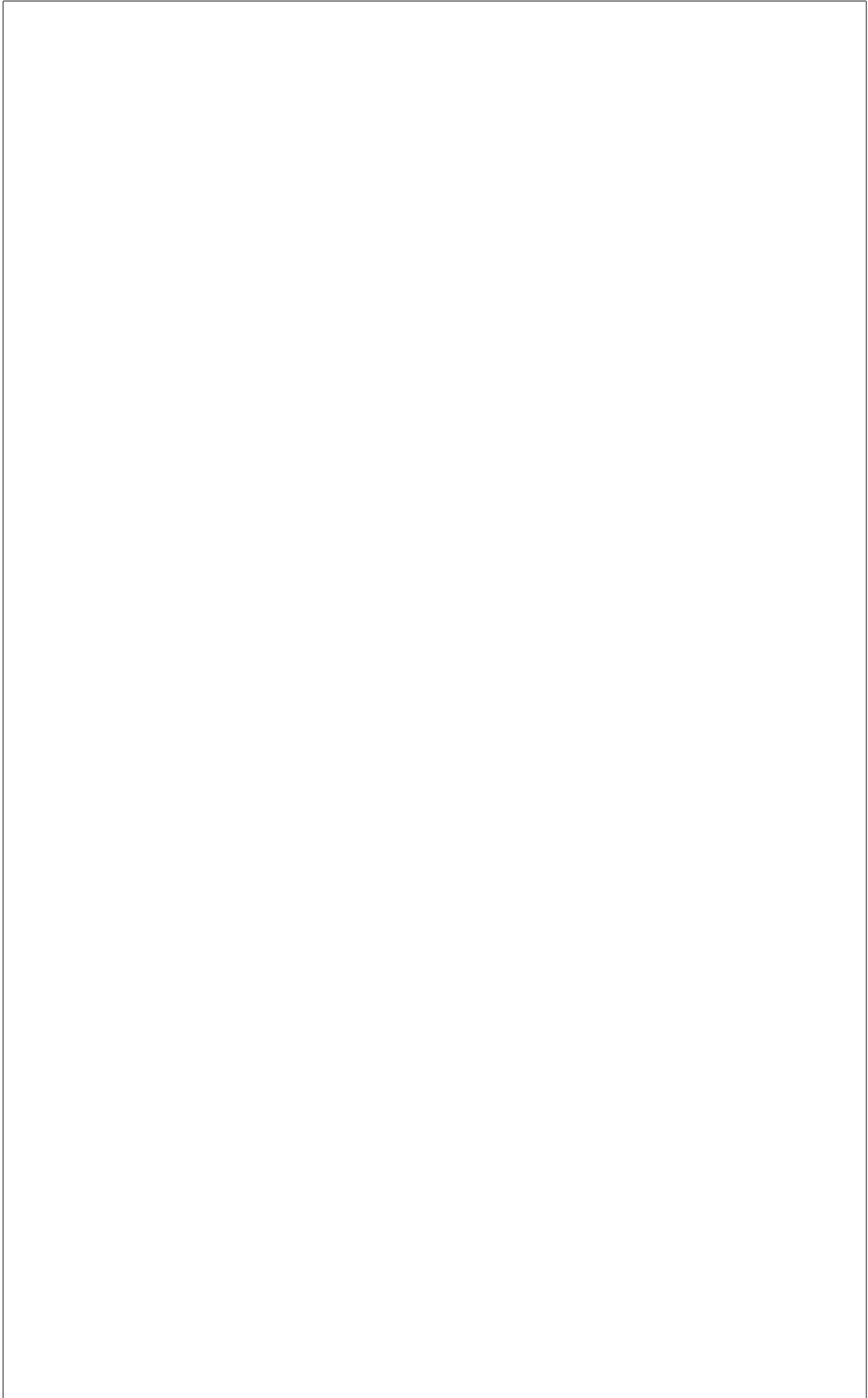
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<b>List of Figures</b>





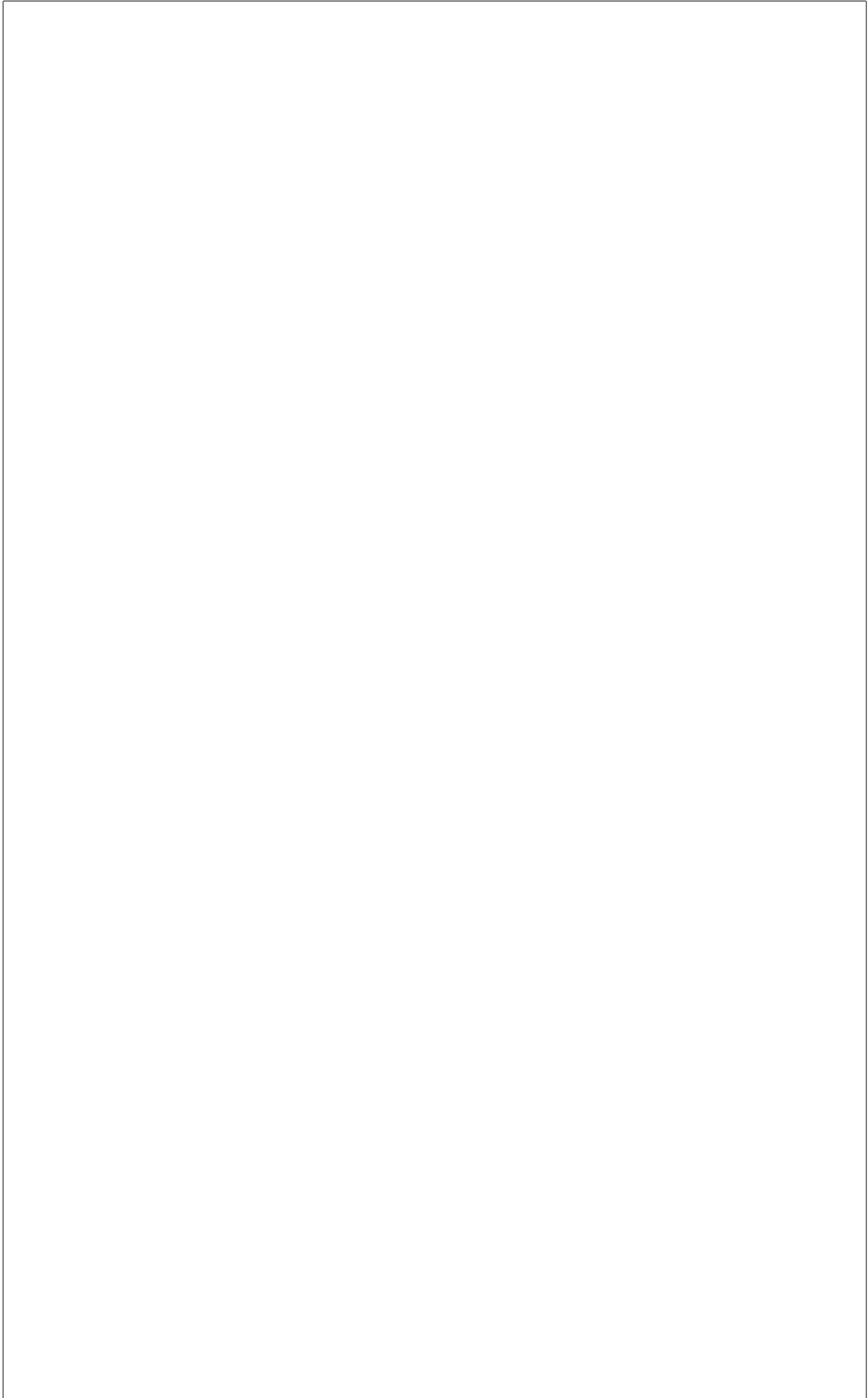
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Notation



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# 1 Introduction

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## 2 Theory

In three dimensions, the matrix  $\mathbf{L}$  is defined as

$$\mathbf{L} = \begin{pmatrix} \nabla^2 C^{-1}[\xi_3, \xi_3, \xi_3, \xi_3] & 2\frac{\lambda_3 - \lambda_1}{\lambda_1 \lambda_3} \langle \xi_1, (\nabla \xi_3) \xi_3 \rangle & 2\frac{\lambda_3 - \lambda_2}{\lambda_2 \lambda_3} \langle \xi_2, (\nabla \xi_3) \xi_3 \rangle \\ 2\frac{\lambda_3 - \lambda_1}{\lambda_1 \lambda_3} \langle \xi_1, (\nabla \xi_3) \xi_3 \rangle & 2\frac{\lambda_3 - \lambda_1}{\lambda_1 \lambda_3} & 0 \\ 2\frac{\lambda_3 - \lambda_2}{\lambda_2 \lambda_3} \langle \xi_2, (\nabla \xi_3) \xi_3 \rangle & 0 & 2\frac{\lambda_3 - \lambda_2}{\lambda_2 \lambda_3} \end{pmatrix}, \quad (2.1)$$

where the dependence of  $\mathbf{L}$  and its constituent parts on  $t$ ,  $t_0$  and  $\mathbf{x}_0$  has been omitted for brevity. The distinction between so-called strong and weak hyperbolic LCSs is based upon whether or not  $\mathbf{L}$  is positive definite (Haller 2010; Farazmand and Haller 2011). By Sylvester's theorem, a square matrix is positive definite if and only if all of the leading principal minors of  $\mathbf{L}$  are positive. The term *leading principal minors* is defined as follows (Gilbert 1991):

**Definition 1** (Principals of square matrices). Let  $\mathbf{A}$  be an  $n \times n$  matrix. For  $1 \leq k \leq n$ , the  $k^{\text{th}}$  *principal submatrix* of  $\mathbf{A}$  is the  $k \times k$  submatrix formed from the first  $k$  rows and first  $k$  columns of  $\mathbf{A}$ . Its determinant is the  $k^{\text{th}}$  principal minor.

Thus, in three dimensions, the matrix  $\mathbf{L}$  being positive definite amounts to the simultaneous fulfillment of the three requirements

$$\nabla^2 C^{-1}[\xi_3, \xi_3, \xi_3, \xi_3] > 0, \quad (2.2a)$$

$$2\frac{\lambda_3 - \lambda_1}{\lambda_1 \lambda_3} \left\{ \nabla^2 C^{-1}[\xi_3, \xi_3, \xi_3, \xi_3] - 2\frac{\lambda_3 - \lambda_1}{\lambda_1 \lambda_3} \langle \xi_1, (\nabla \xi_3) \xi_3 \rangle^2 \right\} > 0, \quad (2.2b)$$

$$\det(\mathbf{L}) > 0. \quad (2.2c)$$

By straightforward algebraic manipulations, the inequality (2.2c) is equivalent to

$$4\frac{(\lambda_3 - \lambda_2)(\lambda_3 - \lambda_1)}{\lambda_1 \lambda_2 \lambda_3^2} \left\{ \nabla^2 C^{-1}[\xi_3, \xi_3, \xi_3, \xi_3] - \left( 2\frac{\lambda_3 - \lambda_1}{\lambda_1 \lambda_3} \langle \xi_1, (\nabla \xi_3) \xi_3 \rangle^2 + 2\frac{\lambda_3 - \lambda_2}{\lambda_2 \lambda_3} \langle \xi_2, (\nabla \xi_3) \xi_3 \rangle^2 \right) \right\} > 0. \quad (2.3)$$

Now, we make use of the following result from Haller (2010):

**Lemma 1.** *At each point of a weak LCS in three dimensions, the following identity holds:*

$$\begin{aligned} \nabla^2 C^{-1}[\xi_3, \xi_3, \xi_3, \xi_3] = & -\frac{1}{\lambda_3^2} \langle \xi_3, \nabla^2 \lambda_3 \xi_3 \rangle + 2\frac{\lambda_3 - \lambda_1}{\lambda_1 \lambda_3} \langle \xi_1, (\nabla \xi_3) \xi_3 \rangle^2 \\ & + 2\frac{\lambda_3 - \lambda_2}{\lambda_2 \lambda_3} \langle \xi_2, (\nabla \xi_3) \xi_3 \rangle^2. \end{aligned} \quad (2.4)$$

*Proof.* See theorem 7 in Haller (2010). □

Using equation (2.4) in conjunction with equation (2.3) and the relationships  $0 \leq \lambda_1 \leq \lambda_2 \leq \lambda_3$ , the inequalities (2.2) can be expressed as follows: Foo

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### 3 Method

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<b>4 Results</b>
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## 5 Discussion

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## 6 Conclusions

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## References

- Farazmand, M. and Haller, G. (2011). "Erratum and addendum to 'A variational theory of hyperbolic Lagrangian coherent structures' [Physica D 240 (2011) 547–598]". In: *Physica D: Nonlinear Phenomena* 241.4, pp. 439–441. ISSN: 0167-2789.
- Gilbert, G. T. (1991). "Positive Definite Matrices and Sylvester's Criterion". In: *The American Mathematical Monthly* 98.1, pp. 44–46.
- Haller, G. (2010). "A variational theory of hyperbolic Lagrangian Coherent Structures". In: *Physica D: Nonlinear Phenomena* 240.7, pp. 547–598. ISSN: 0167-2789.



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<b>A Appendix A</b>