

Abstract	

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## Preface

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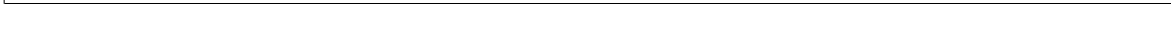
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Notation



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# 1 Introduction

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## 2 Theory

In three dimensions, the matrix  $\mathbf{L}$  is defined as

$$\mathbf{L} = \begin{pmatrix} \nabla^2 C^{-1}[\xi_3, \xi_3, \xi_3, \xi_3] & 2\frac{\lambda_3 - \lambda_1}{\lambda_1 \lambda_3} \langle \xi_1, (\nabla \xi_3) \xi_3 \rangle & 2\frac{\lambda_3 - \lambda_2}{\lambda_2 \lambda_3} \langle \xi_2, (\nabla \xi_3) \xi_3 \rangle \\ 2\frac{\lambda_3 - \lambda_1}{\lambda_1 \lambda_3} \langle \xi_1, (\nabla \xi_3) \xi_3 \rangle & 2\frac{\lambda_3 - \lambda_1}{\lambda_1 \lambda_3} & 0 \\ 2\frac{\lambda_3 - \lambda_2}{\lambda_2 \lambda_3} \langle \xi_2, (\nabla \xi_3) \xi_3 \rangle & 0 & 2\frac{\lambda_3 - \lambda_2}{\lambda_2 \lambda_3} \end{pmatrix}, \quad (2.1)$$

where the dependence of  $\mathbf{L}$  and its constituent parts on  $t$ ,  $t_0$  and  $\mathbf{x}_0$  has been omitted for brevity. The distinction between so-called strong and weak hyperbolic LCSs is based upon whether or not  $\mathbf{L}$  is positive definite (Haller 2010; Farazmand and Haller 2011). By Sylvester's theorem, a square matrix is positive definite if and only if all of the leading principal minors of  $\mathbf{L}$  are positive. The term *leading principal minors* is defined as follows (Gilbert 1991):

**Definition 1** (Principals of square matrices). Let  $\mathbf{A}$  be an  $n \times n$  matrix. For  $1 \leq k \leq n$ , the  $k^{\text{th}}$  *principal submatrix* of  $\mathbf{A}$  is the  $k \times k$  submatrix formed from the first  $k$  rows and first  $k$  columns of  $\mathbf{A}$ . Its determinant is the  $k^{\text{th}}$  principal minor.

Thus, in three dimensions, this amounts to the simultaneous fulfillment of the three requirements

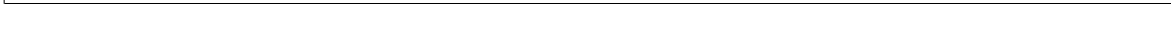
$$\nabla^2 C[\xi_3, \xi_3, \xi_3, \xi_3] > 0 \quad (2.2a)$$

$$2\frac{\lambda_3 - \lambda_1}{\lambda_1 \lambda_3} \nabla^2 C[\xi_3, \xi_3, \xi_3, \xi_3] - 4\left(\frac{\lambda_3 - \lambda_1}{\lambda_1 \lambda_3}\right)^2 \langle \xi_1, (\nabla \xi_3) \xi_3 \rangle^2 > 0 \quad (2.2b)$$

$$\det(\mathbf{L}) > 0 \quad (2.2c)$$



**3 Method**



<b>4 Results</b>
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## 5 Discussion

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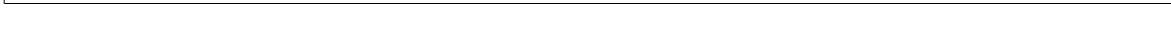
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## 6 Conclusions



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## References

- Farazmand, M. and Haller, G. (2011). "Erratum and addendum to 'A variational theory of hyperbolic Lagrangian coherent structures' [Physica D 240 (2011) 547–598]". In: *Physica D: Nonlinear Phenomena* 241.4, pp. 439–441. ISSN: 0167-2789.
- Gilbert, G. T. (1991). "Positive Definite Matrices and Sylvester's Criterion". In: *The American Mathematical Monthly* 98.1, pp. 44–46.
- Haller, G. (2010). "A variational theory of hyperbolic Lagrangian Coherent Structures". In: *Physica D: Nonlinear Phenomena* 240.7, pp. 547–598. ISSN: 0167-2789.

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<b>A Appendix A</b>