

# TFY4510 SPECIALIZATION PROJECT IN PHYSICS

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I've spent the first few weeks of the project work studying the dynamics, i.e. the transport properties, of an analytically known velocity field, in detail. The velocity field is a simplified dynamical model of a periodic, two dimensional double gyre system, first described by [Shadden et al. \(2005\)](#). It is defined on the domain  $[0, 2] \times [0, 1]$ , and described mathematically by the stream function

$$\psi(\mathbf{x}, t) = A \sin(\pi f(x, t)) \sin(\pi y) \quad (1)$$

where

$$f(x, t) = a(t)x^2 + b(t)x \quad (1a)$$

$$a(t) = \epsilon \sin(\omega t) \quad (1b)$$

$$b(t) = 1 - 2\epsilon \sin(\omega t) \quad (1c)$$

and the parameters  $A$ ,  $\epsilon$  and  $\omega$  adjust the properties of the system.

### 1.1 Error estimation, various integrators

Table 1: Euler,  $t = 5$

Euler \ $\Delta t$	0.1	0.01	0.001
Avg. abs. err.	$5 \cdot 10^{-3}$	$4 \cdot 10^{-4}$	$4 \cdot 10^{-5}$
Max. abs. err.	$7 \cdot 10^{-2}$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-3}$

Table 2: Heun,  $t = 5$

Heun \ $\Delta t$	0.1	0.01	0.001
Avg. abs. err.	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
Max. abs. err.	$5 \cdot 10^{-3}$	$5 \cdot 10^{-5}$	$5 \cdot 10^{-7}$

Table 3: Kutta,  $t = 5$

Kutta \ $\Delta t$	0.1	0.01	0.001
Avg. abs. err.	$4 \cdot 10^{-6}$	$4 \cdot 10^{-9}$	$4 \cdot 10^{-12}$
Max. abs. err.	$2 \cdot 10^{-4}$	$2 \cdot 10^{-7}$	$3 \cdot 10^{-10}$

Table 4: RK4,  $t = 5$ 

RK4 \ $\Delta t$	0.1	0.01
Avg. abs. err.	$7 \cdot 10^{-8}$	$7 \cdot 10^{-12}$
Max. abs. err.	$4 \cdot 10^{-6}$	$4 \cdot 10^{-10}$

## References

Shadden, S. C., Lekien, F., and Marsden, J. E. (2005). Definition and properties of Lagrangian coherent structures from finite-time Lyapunov exponents in two-dimensional aperiodic flows. *Physica D: Nonlinear Phenomena*, 212(3):271–304.