TFY4510 Specialization Project in Physics

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I've spent the first few weeks of the project work studying the dynamics, i.e. the transport properties, of an analytically known velocity field, in detail. The velocity field is a simplified dynamical model of a periodic, two dimensional double gyre system, first described by Shadden et al. (2005). It is defined on the domain $[0, 2] \times [0, 1]$, and described mathematically by the stream function

$$\psi(\mathbf{x}, t) = A \sin\big(\pi f(x, t)\big) \sin(\pi y) \tag{1}$$

where

$$f(x, t) = a(t)x^2 + b(t)x \tag{1a}$$
$$a(t) = \epsilon \sin(\omega t) \tag{1b}$$
$$b(t) = 1 - 2\epsilon \sin(\omega t) \tag{1c}$$

and the parameters A, ϵ and ω adjust the properties of the system.

1.1 Error estimation, various integrators

Table 1: Euler, $t = 5$

Euler \ Δt	0.1	0.01	0.001
Avg. abs. err.	$5 \cdot 10^{-3}$	$4 \cdot 10^{-4}$	$4 \cdot 10^{-5}$
Max. abs. err.	$7 \cdot 10^{-2}$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-3}$

Table 2: Heun, $t = 5$

Heun \ Δt	0.1	0.01	0.001
Avg. abs. err.	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
Max. abs. err.	$5 \cdot 10^{-3}$	$5 \cdot 10^{-5}$	$5 \cdot 10^{-7}$

Table 3: Kutta, $t = 5$

Kutta \ Δt	0.1	0.01	0.001
Avg. abs. err.	$4 \cdot 10^{-6}$	$4 \cdot 10^{-9}$	$4 \cdot 10^{-12}$
Max. abs. err.	$2 \cdot 10^{-4}$	$2 \cdot 10^{-7}$	$3 \cdot 10^{-10}$

Table 4: RK4, $t = 5$

RK4 \ Δt	0.1	0.01
Avg. abs. err.	$7 \cdot 10^{-8}$	$7 \cdot 10^{-12}$
Max. abs. err.	$4 \cdot 10^{-6}$	$4 \cdot 10^{-10}$

References

Shadden, S. C., Lekien, F., and Marsden, J. E. (2005). Definition and properties of Lagrangian coherent structures from finite-time Lyapunov exponents in two-dimensional aperiodic flows. *Physica D: Nonlinear Phenomena*, 212(3):271–304.