I mars 2011 traff Tohoku-tsunamien den japanske kysten. Foruten det store antallet dødsfall medførte tsunamien, sammen med jordskjelvet som forårsaket den, en katastrofal kjernefysisk nedsmelting i Fukushima-reaktoren (Atomic Energy Society of Japan 2015). I november 2017 medførte en rekke vulkanutbrudd i den indonesiske provinsen Bali omfattende evakuering av lokalbefolkningen, i tillegg til at mange nærliggende flyplasser ble stengt. De mange kansellerte flyvningene etterlot tusenvis av passasjerer strandet på bakken (Kapoor 2017). I januar 2018 kolliderte den iranskeide oljetankeren Sanchi med et fraktskip fra Hong Kong i Øst-Kina-havet (Obayashi og Mason 2018). Tankeren havarerte, store mengder olje spredtes over havoverflaten og enda større mengder sank til havbunnen sammen med vrakrestene, hvor oljen truer med å forurense også dypet av havet dersom beholderne gir etter for det enorme undersjøiske trykket.

En fellesnevner for de tre ovennevnte naturkatastrofene er at materiale ble sluppet ut i naturen fra det som kan betraktes som punktkilder. Å predikere hvor de omsluttende havstrømmene eller de atmosfæriske vindsystemene fraktet forurensningene var — og er — uhyre viktig for å kunne begrense potensielle humanitære så vel som naturtragedier.

Når en ønsker å analysere komplekse dynamiske system, som for eksempel de ikkelineære mangepartikkelproblemene som beskriver ulike transportfenomen som havstrømmer eller atmosfæriske vindmønstre, er den konvensjonelle tilnærmingen til prediksjon av fremtidige tilstander ved å simulere banene til fasepunkter ofte utilstrekkelig. Dette er fordi prediksjonene denn

At the turn of the millennium, the concept of Lagrangian coherent structures saw the light of day, emerging from the intersection between nonlinear dynamics, that is, the underlying mathematical principles of chaos theory, and fluid dynamics (Haller og Yuan 2000). These provide a new framework for understanding transport phenomena in conceptual fluid flow systems. Lagrangian coherent structures can be described as time-evolving 'landscapes' in a multidimensional space, which dictate macroscopical flow patterns in dynamical systems. In particular, such structures define the interfaces of dynamically distinct, invariant regions. An invariant region in fluid dynamics is characterized as a domain where all particle trajectories that originate within the region, remain in it, although the region itself can move and deform with time. So, simply put; Lagrangian coherent structures enable us to make predictions regarding the future states of flow systems.

There are two possible perspectives regarding the description of fluid flow. The Eulerian approach is to consider the properties of a flow field at a set of fixed points in time and space. An example is the concept of velocity fields, which produce the local and instantaneous velocities at all points within their domains. The Lagrangian point of view, on the other hand, concerns the developing velocity of each fluid element along their paths, as they are transported by the flow. Unlike the Eulerian perspective, the Lagrangian mindset is objective, as in frame-invariant. That is, properties of Lagrangian fields are unchanged by time-dependent translations and rotations of the reference frame. For unsteady flow systems, which are more common than steady reference. Thus, any transport-dictating dynamical structures should hold for *any* choice of reference frame. This is the main rationale for which *Lagrangian*, rather than *Eulerian*, coherent structures have been pursued.

A generic flow system can be described as a structure whose state depends on flowing streams of energy, material or information. Conventional examples of flow systems include the transport of physical properties such as pressure, temperature or matter in fluids, and the transport of charge in electrical currents. A large variety of phenomena can reasonably be modelled as flow systems, such as the classical harmonic oscillator, or the interaction between predator and prey in closed systems (Strogatz 2014, parts I–II). In doing so, valuable pieces of insight can be obtained from well-understood properties of generic flows. In recent years, analyses based upon Lagrangian coherent structures have been conducted for a variety of naturally occuring phenomena which are not commonly considered as flow systems. Two prominent examples are how Olascoaga et al. (2008) used Lagrangian coherent structures in order to forecast the development of toxic algae in the ocean, and Ali og Shah (2007) used Lagrangian coherent structures to predict the formation and stability of human crowd patterns. As these examples emphasize, the theory of Lagrangian coherent structures is applicable to a wide range of systems.

Although the framework for detecting Lagrangian coherent structures is mathematically valid for any number of dimensions, the focus of this project work has been two-dimensional flow systems. Many natural processes can reasonably be described as two-dimensional, perhaps most notably the transport of debris and contaminations, such as garbage patches or the remnants of an oil spill, by means of oceanic surface currents. Being able to successfully predict where such particles will be taken by the flow could enable us to isolate them and accelerate the cleanup process before the particles are able to reach the coastline, thus mitigating potential humanitarian and natural calamities.

At the heart of detecting Lagrangian coherent structures lies the advection of (generalized) fluid elements by means of a velocity field, which describes the system under consideration. This is true both for test cases, where the velocity profile is known analytically, and for real-life systems, typically described by means of some model for the instantaneous velocity field. For this project work, the topic of interest is how the detection of Lagrangian coherent structures depends on the choice of numerical integration method used in order to compute the aforementioned particle transport. In particular, four singlestep methods and four embedded, adaptive step length methods, each with different properties, were used in order to advect a collection of fluid elements by means of an analytically known, two-dimensional, unsteady velocity field.

This thesis is structured based on the idea that readers possessing at least an undergraduate level of knowledge of physics and mathematics, in addition to a rudimentary understanding of programming, should be able to understand and repeat the numerical investigations which have been conducted. To this end, the immediately forthcoming chapter contains a description of the various numerical integration schemes that were utilized, in addition to a generic yet brief mathematical description of the kind of flow systems considered and the Lagrangian coherent structures situated therein, the latter based on variational theory. In the ensuing chapter, we present a transport model frequently used in literature — whose Lagrangian coherent structures are documented — in addition to a description of how the variational principles of Lagrangian coherent structure detection were implemented numerically. Lastly, the results are presented and discussed, before the conclusions of the project as a whole are drawn.

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