Strong Gravitational Lensing

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1 Introduction

Einstein's theory of general relativity states that space-time is warped in the presence of massive objects. This interpretation can be used to explain the orbits of the planets around the sun and is also the basis for explaining strong gravitational lensing. A photon can be bent by a gravitational field in the same way a planet can be. In strong gravitational lensing, a very massive foreground object such as a galaxy or a cluster of galaxies bends the light from a background object such as another galaxy or a quasar thereby acting like a lens (magnifying and distorting the image in the background). The image created by the lens is determined by the foreground galaxies mass, mass profile, and its orientation with the background source. The important result of strong gravitational lensing is that the angular seperation of the image produced is directly related to the mass of the lensing galaxy or galaxies. Therefore, the mass of an entire galaxy or galaxy cluster can be measured very accurately at large distances. This is useful for astronomers because it is a way of measuring the dark matter present in galaxies.

2 Project Description

The goal of my program is to allow users to familiarize themselves with the effects of strong gravitational lensing. With so many parameters influencing strong lensing, it is difficult to visualize the subsequent lensed image. My program allows users to manipulate multiple sources and lens galaxies and see a pixelized view of the resulting lensed image.

The user is required to input the number of lensing galaxies to evaluate (up to three). Each of these galaxies are parameterized by their size (i.e. Einstein radius), x and y position coordinates, axis ratio, and position angle. The user is asked to input these values. To keep things simple, the mass profiles of the lens galaxies are assumed to be singular isothermal ellipsoids. The user will also be required to input the number of source galaxies to evaluate (also a maximum of three). Each of these galaxies are parameterized by their amplitude, size, position in the source plane, axis ratio, and position angle. The user must also input these values. For simplicity the source galaxies are assumed to have a gaussian surface brightness profile.

After all the input parameters are given, the program calculates the deflections caused by each of the lens galaxies and evalutes the lensing effect this has on the source galaxy or galaxies. The program then writes this pixelized image to the specified file in pgm format.

3 Manual

The Strong Gravitational Lensing program can be run by typing 'lens filename.pgm'. Where filename.pgm is the name of the file the output will be written to. It need not exits, if it does, it will be written over. The program is fairly self-explanitory. It procedes by prompting the user to enter the desired parameter. The user may quit at any time by entering 'q'. The following summarizes the input parameters of the program.

3.1 Einstein Radius (Lens Size)

The size of the lens galaxies, often called the Einstein radius, is in units of pixels and can be anything between 0 and 100. An Einstein radius of zero will have no effect on the source galaxy(ies) and can be entered to see what the un-lensed sources would look like (see Test Procedure below). Any negative input will default to zero and any input greater than 100 will be set to the maximum.

3.2 X and Y Position

The positions of the lens and source galaxies can be placed anywhere between $(-100, 100)$ in the x and y pixel coordinates. Any input values outside of this boudary will be set to 0.

3.3 Axis Ratio

The axis ratio of a galaxy is the ratio of the apparent, line-of-sight, size of the major axis with respect to the minor axis. Therefore, an axis ratio of 1 corresponds to a perfectly circular galaxy, seen face-on, and an axis ratio of 0 corresponds to a galaxy seen perfectly edge-on. The user may input any value between 0 and 1 for the axis ratio. An input outside of this range will default to 1.

3.4 Position Angle

The position angle is the degrees counter-clockwise from the x-axis in which the major axis points. The input must be in the range $(0 - 360)$, anything outside this range will be set to 0.

3.5 Source Amplitude

The amplitude of the source is analagous to the brightness of the source. The user may input a value between 0 and 100. If the input is greater than 100 it will be set to 100 and a negative input will be set to 0. A source amplitude of 0 is the same as a non-existant source. Note that the scale of the output image is set to the sum of all the source amplitudes.

3.6 Source Radius

The light profile of all the sources are assumed to be Gaussian, therefore, their effective radii are paramaterized by the sigma of a Gaussian distribution. The size, in pixels, of the source sigma (radius) can be anything between 0 and 100. An input above 100 will be set to 100 and any negative inputs will be set to 0. A source size of 0 is the same as a non-existant source.

4 Equations

The deflections produced by a singular isothermal ellipsoid are given by the equations:

$$\vec{x_1}(x,y) = \frac{b}{\sqrt{1-q^2}} \tan^{-1} \frac{\theta_1 \sqrt{1-q^2}}{\sqrt{q\theta_1^2 + \frac{\theta_2^2}{q}}} \tag{1}$$

$$\vec{x_2}(x,y) = \frac{b}{\sqrt{1-q^2}} \tanh^{-1} \frac{\theta_2 \sqrt{1-q^2}}{\sqrt{q\theta_1^2 + \frac{\theta_2^2}{q}}} \tag{2}$$

where b is the Einstein radius (i.e. size) of the lens in angular units, q is the axis ratio of the lens, $\theta_1 = x\cos\phi + y\sin\phi$, $\theta_2 = x\cos\phi - y\sin\phi$, x and y are the distances in cartesian coordinates in the lens plane from the lens center and ϕ is the position angle of the lens.

The surface brightness profile of the source galaxies is given by the equation:

$$\Sigma(R_e) = I_0 exp^{-\frac{1}{2}\left(\frac{x^2 q + \frac{y^2}{q}}{R_e{}^2}\right)} \tag{3}$$

Where I_0 is the amplitude of the source galaxy, q is the axis ratio of the source galaxy, x and y are the distances in cartesian coordinates in the source plane from the source galaxy, R_e is the effective radius of the source galaxy.

4.1 Calculations

As mentioned previously, after all the inputs are provided by the user the program calculates the deflections caused by each of the lens galaxies. This is done by looping over all of the pixels in the lens plane and calculating the deflections according to equations (1) and (2) for each of the lens galaxies. The total deflection due to all of the lens galaxies is simply the sum of the deflections. The program then determines what each source looks like due to these deflections by looping over every pixel in the source plane and moving it according to the previously calculated deflection. The generated image is then written to the specified file and can be viewed using *gimp*.

5 Test Procedure

One way to test that the program is working correctly is to input a lensing galaxy with a size (Einstein radius) of zero. The resulting lensing system should look like the source galaxy, that is, unaffected. Another way to check the program is to input a lensing galaxy with an axis ratio of 1.0 and a source galaxy directly behind the lensing galaxy. The resulting lensed image should be a complete ring, called an “Einstein ring”.