Lab 2 - Convex vs. Concave Algorithm

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In [1]:
```python
import numpy as np
import matplotlib.pyplot as plt
import math
import time
import pandas as pd
from scipy.stats import linregress
import random
```

In [2]:
```python
def cross_product(p1, p2):
        return p1[0] * p2[1] - p1[1] * p2[0]
```

In [3]:
```python
def is_convex(dot_list):
    if len(dot_list) < 3:
        return "ERROR"

    prev_cross_product = 0

    for i in range(len(dot_list)):
        i1 = dot_list[i]
        i2 = dot_list[(i + 1) % len(dot_list)]
        i3 = dot_list[(i + 2) % len(dot_list)]

        edge1 = (i2[0] - i1[0], i2[1] - i1[1])
        edge2 = (i3[0] - i2[0], i3[1] - i2[1])

        current_cross_product = cross_product(edge1, edge2)
        # Skip collinear points
        if current_cross_product == 0:
            continue

        if prev_cross_product == 0:
            prev_cross_product = current_cross_product

        # Cross products with different signs indicate concavity
        elif prev_cross_product * current_cross_product < 0:
            return False

    return True
```

In [4]:
```python
def generate_random_convex_shape(num_points):
    if num_points < 3:
        raise ValueError("Number of points should be at least 3 for a shape.")

    angles = np.linspace(0.25, 2 * np.pi, num_points)
    x = np.cos(angles)
    y = np.sin(angles)

    if is_convex(list(zip(x, y))):
        return list(zip(x, y))
```

```
    else:
        return None
```

In [17]:
```
def generate_random_concave_shape(num_points):
    if num_points < 4:
        raise ValueError("Number of points should be at least 4 for a concave shape.")

    radius = 1.0
    theta = np.linspace(0, 2 * np.pi, num_points, endpoint=False)
    x = radius * np.cos(theta)
    y = radius * np.sin(theta)

    x[num_points - 2] = 0.0
    y[num_points - 2] = 0.0

    if not is_convex(list(zip(x, y))):
        return list(zip(x, y))
    else:
        return None
```

In [6]:
```
def create_random_shapes(num_points_list):
    shape_list = []
    for num_points in num_points_list:
        convex_shape = generate_random_convex_shape(num_points)
        concave_shape = generate_random_concave_shape(num_points)

        plt.figure(figsize=(8, 4))
        plt.subplot(1, 2, 1)
        x, y = zip(*convex_shape)
        plt.plot(*zip(*convex_shape), linestyle='-', color='green')
        plt.plot([x[-1], x[0]], [y[-1], y[0]], linestyle='-', color='green')
        plt.title(f"Convex Shape ({num_points} points)")

        plt.subplot(1, 2, 2)
        x, y = zip(*concave_shape)
        plt.plot(*zip(*concave_shape), linestyle='-', color='blue')
        plt.plot([x[-1], x[0]], [y[-1], y[0]], linestyle='-', color='blue')
        plt.title(f"Concave Shape ({num_points} points)")

        plt.tight_layout()
        plt.show()
        shape_list.append({"Convex": convex_shape, "Concave": concave_shape})

    return shape_list
```

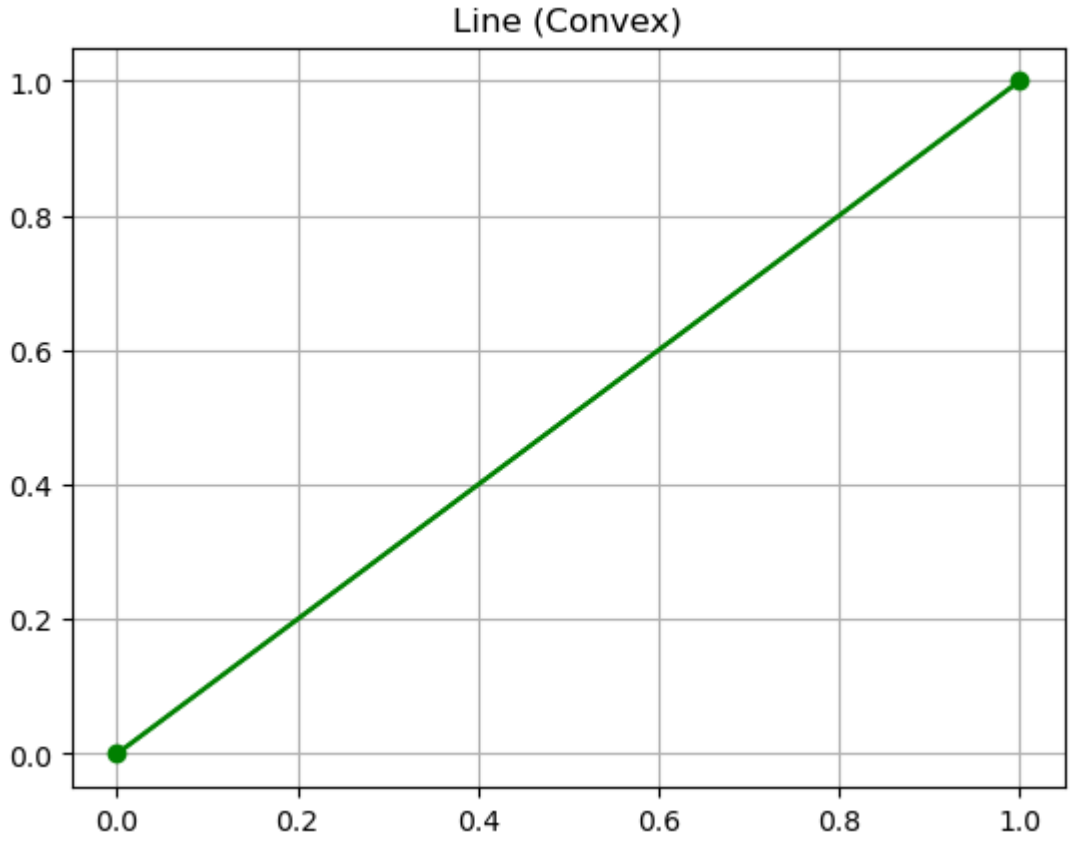
In [18]:
```
def plot_shape(points, shape_name, result, color):
    if(len(points) > 0):
        x, y = zip(*points)
        plt.figure()
        plt.plot(x, y, marker='o', linestyle='-', color=color)
        plt.plot([x[-1], x[0]], [y[-1], y[0]], linestyle='-', color=color)
        plt.title(f"{shape_name} ({'Convex' if result else 'Concave' if result is not
        plt.grid(True)
```

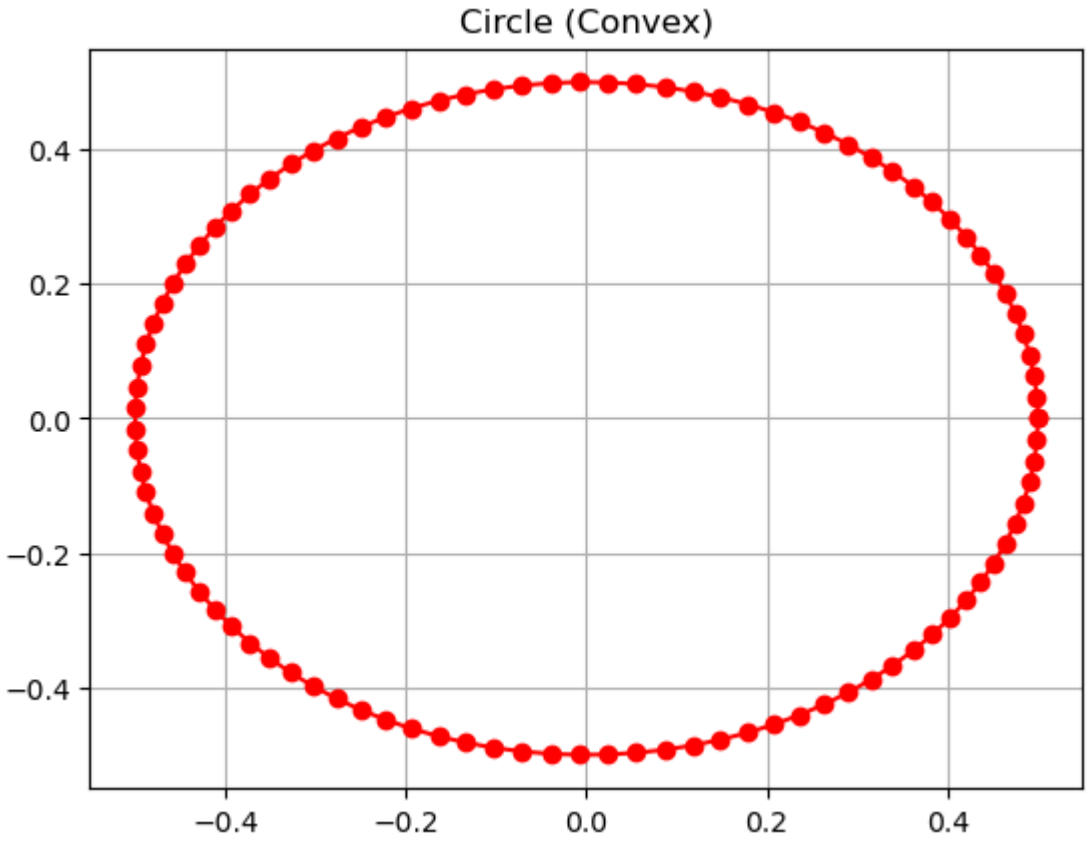
In [19]:
```
shapes = {
    "Empty List": [],
    "Line": [(0, 0), (1, 1)],
```

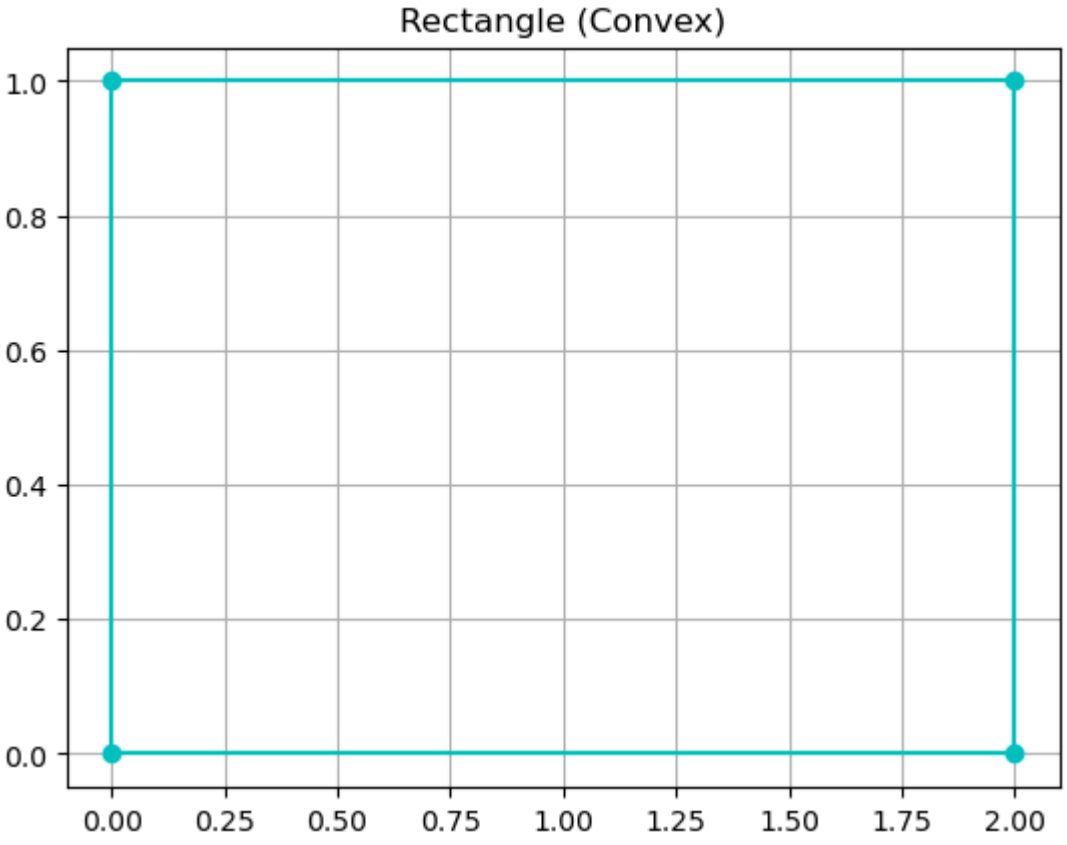
```
    "Circle": [(0.5 * np.cos(theta), 0.5 * np.sin(theta)) for theta in np.linspace(0,
    "Rectangle": [(0, 0), (2, 0), (2, 1), (0, 1)],
    "Star": [(0, 0.5), (0.118, 0.118), (0.5, 0), (0.118, -0.118), (0, -0.5),
         (-0.118, -0.118), (-0.5, 0), (-0.118, 0.118), (0, 0.5)],
    "Pentagon": [(0, 0.5), (0.4755, 0.1545), (0.2939, -0.4045), (-0.2939, -0.4045), (-
    "Nothing Polygon": [(0, 0), (0, 0), (0, 0)],
    "Concave Triangle": [(0, 0), (1, 0), (0.5, 1)],
    "Concave Crescent Moon": [(0, 0), (2, 0), (1.5, 1), (1, 0.5), (0.5, 1)],
    "Convex Polygon": [(0, 0), (1, 1), (3, 1), (4, 0), (3, 2), (1, 2)],
    "Collinear Points": [(0, 0), (1, 1), (2, 2), (3, 3)]
}

colors = ['b', 'g', 'r', 'c', 'm', 'y', 'k']
```

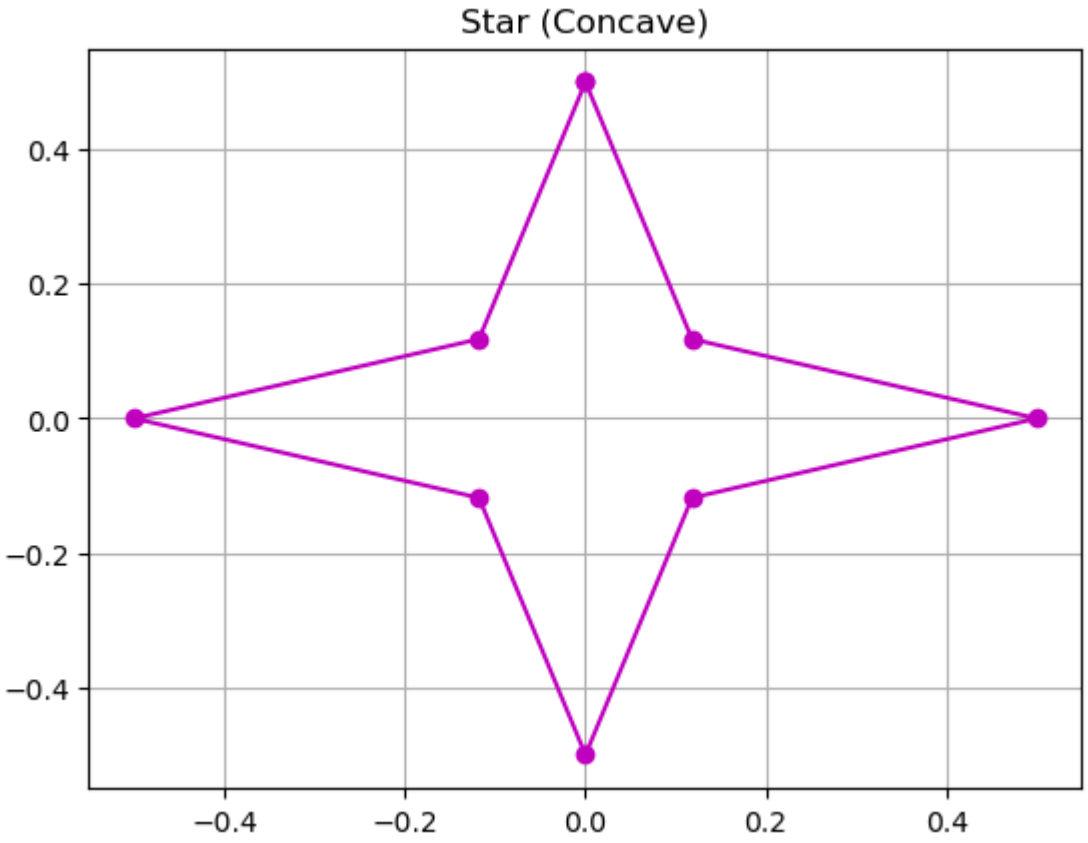
In [9]:

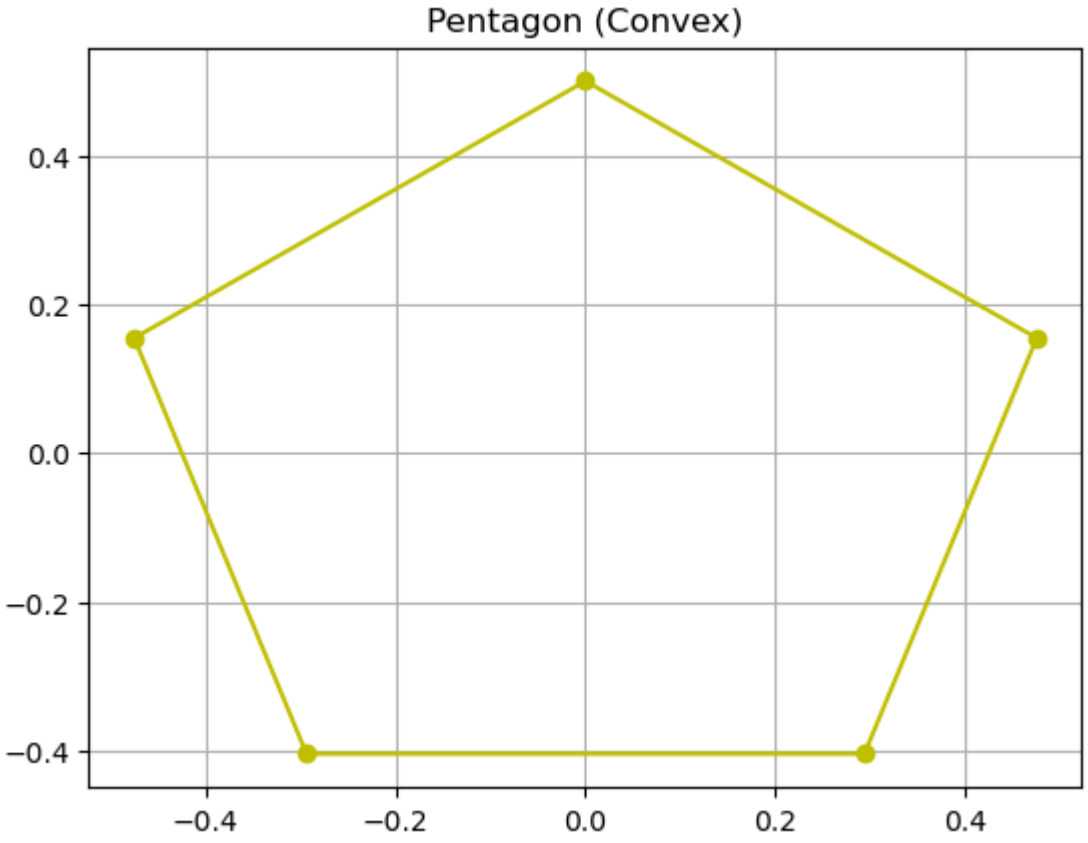
```
for i, (shape_name, shape_points) in enumerate(shapes.items()):
    result = is_convex(shape_points)
    color = colors[i % len(colors)]
    plot_shape(shape_points, shape_name, result, color)
    plt.show()
```

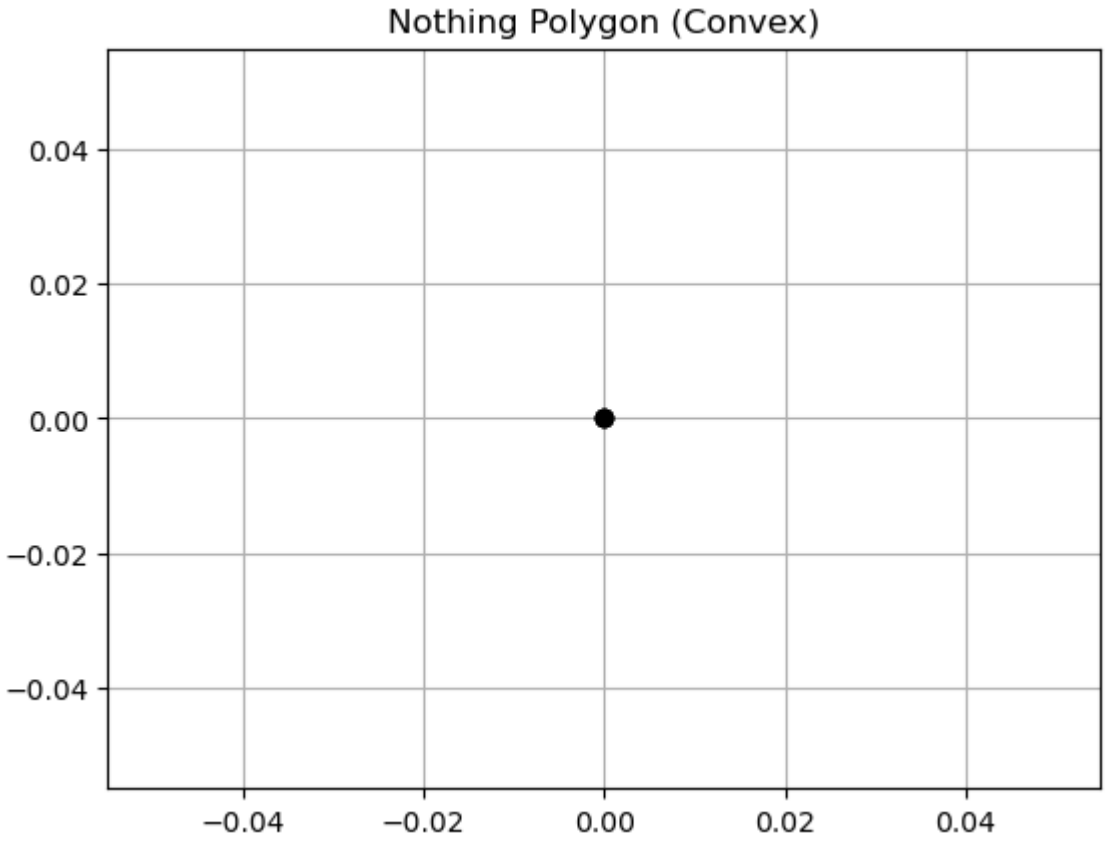


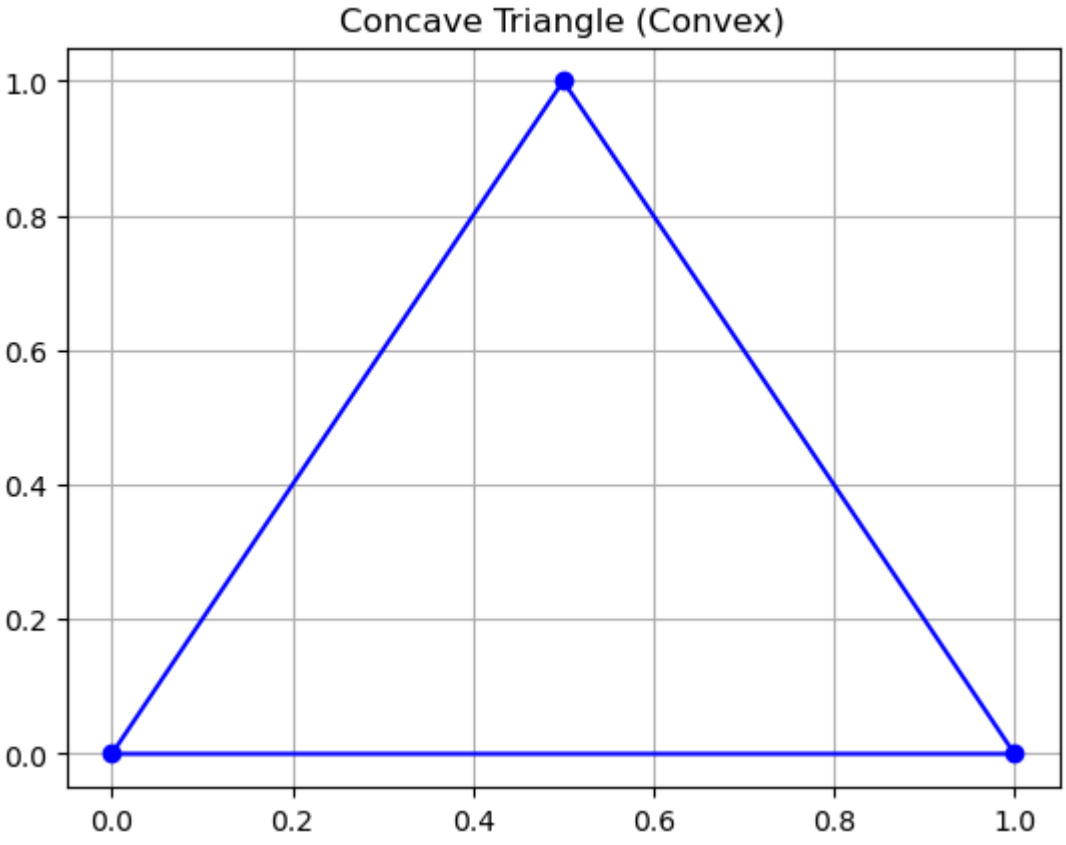

Circle (Convex)
0.4
0.2
0.0
−0.2
−0.4
−0.4
−0.2
0.0
0.2
0.4

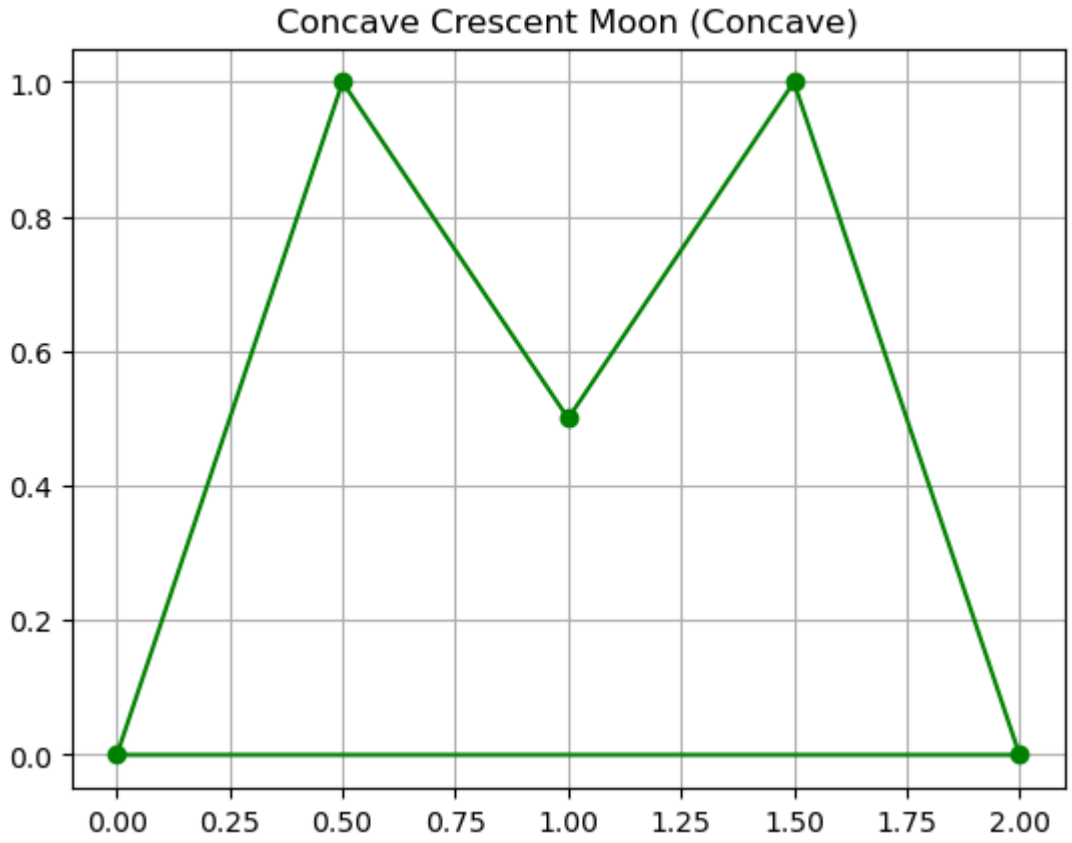

Rectangle (Convex)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

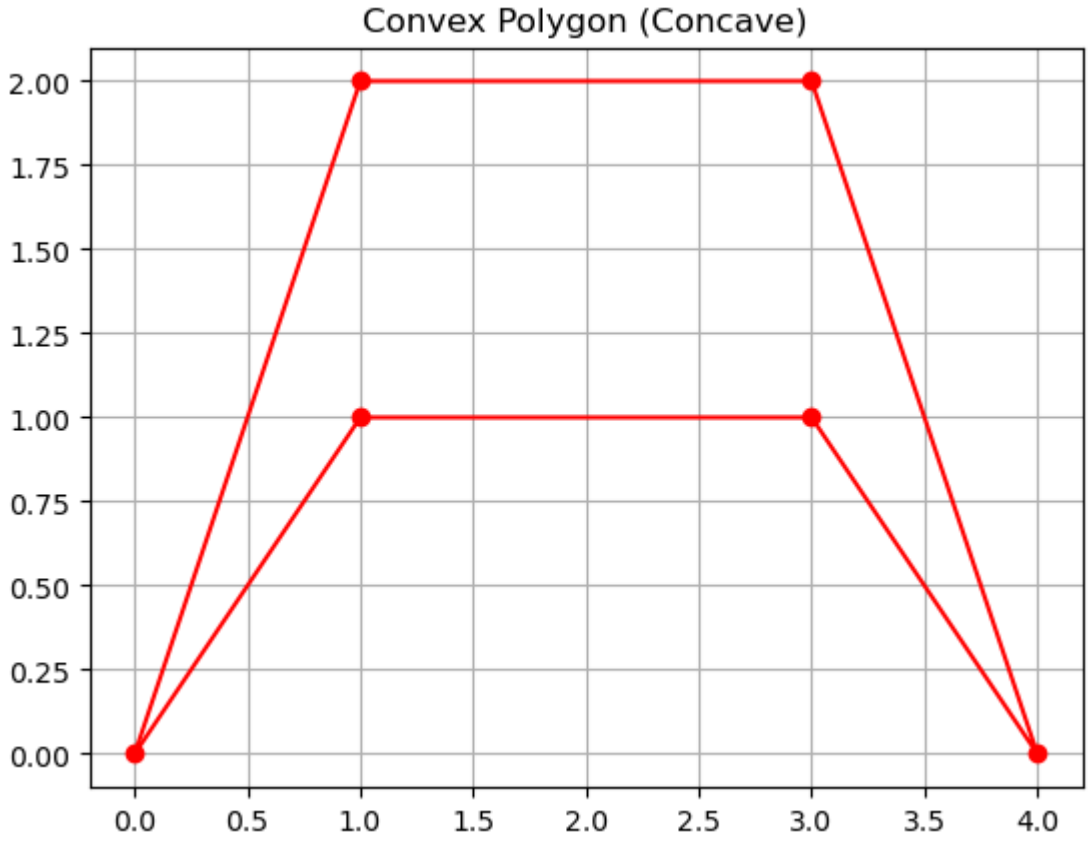

Star (Concave)
0.4
0.2
0.0
−0.2
−0.4
−0.4
−0.2
0.0
0.2
0.4

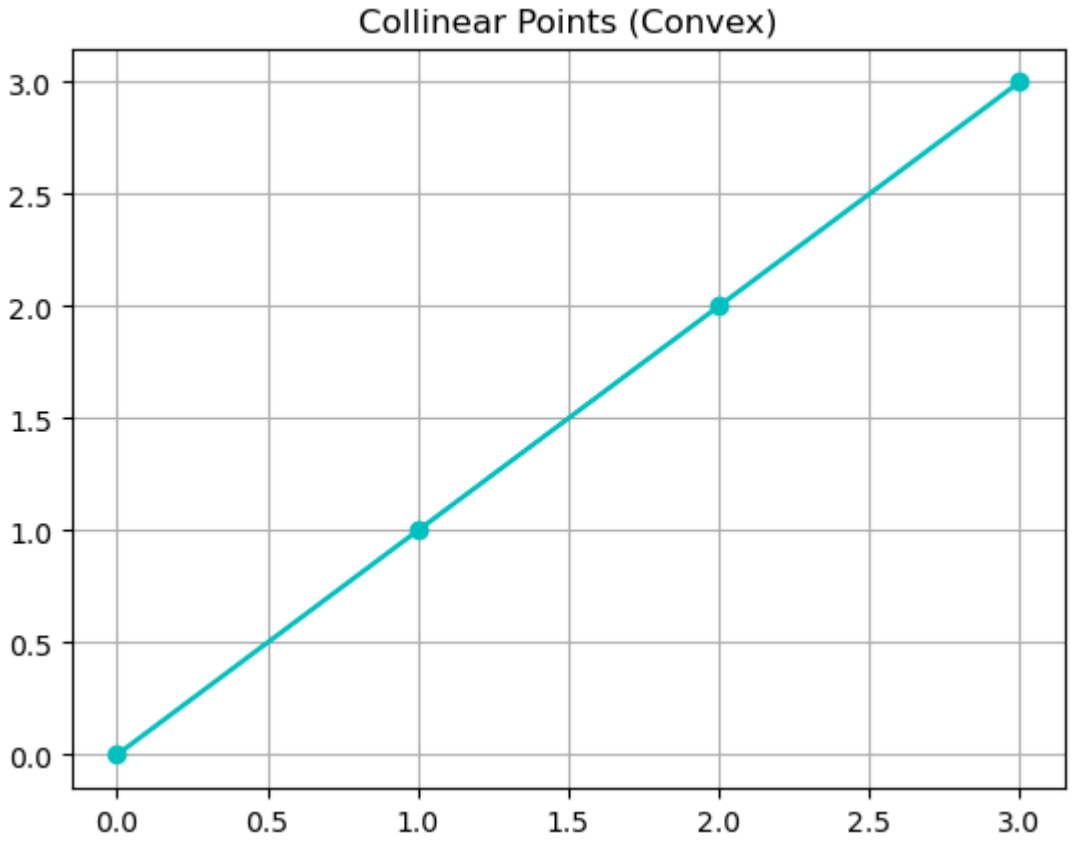

Pentagon (Convex)
0.4
0.2
0.0
−0.2
−0.4
−0.4
−0.2
0.0
0.2
0.4


Nothing Polygon (Convex)
0.04
0.02
0.00
−0.02
−0.04
−0.04
−0.02
0.00
0.02
0.04

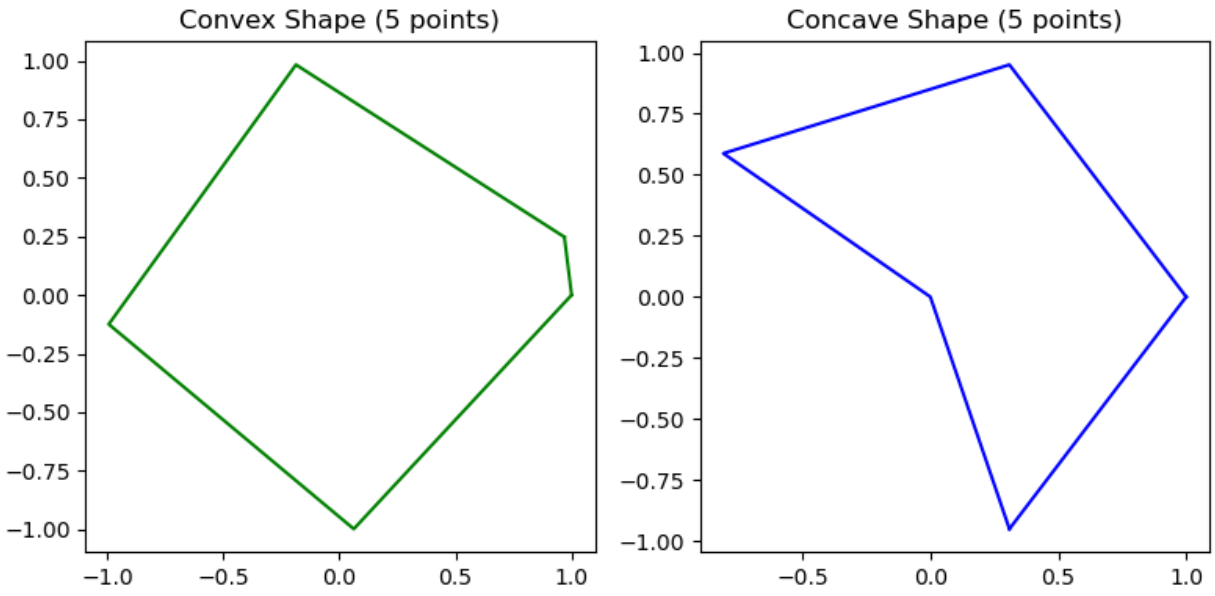

Concave Triangle (Convex)
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

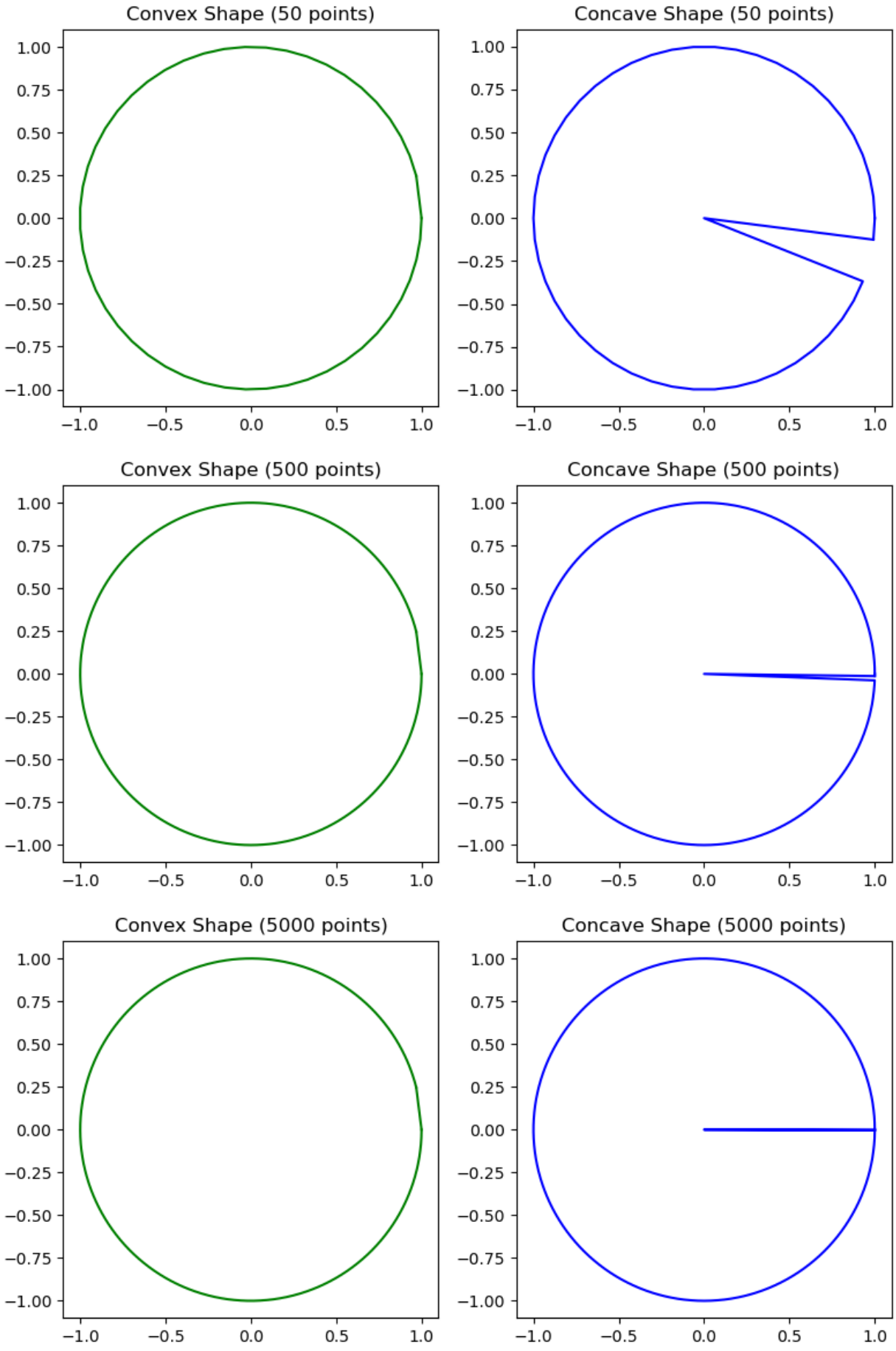

Concave Crescent Moon (Concave)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

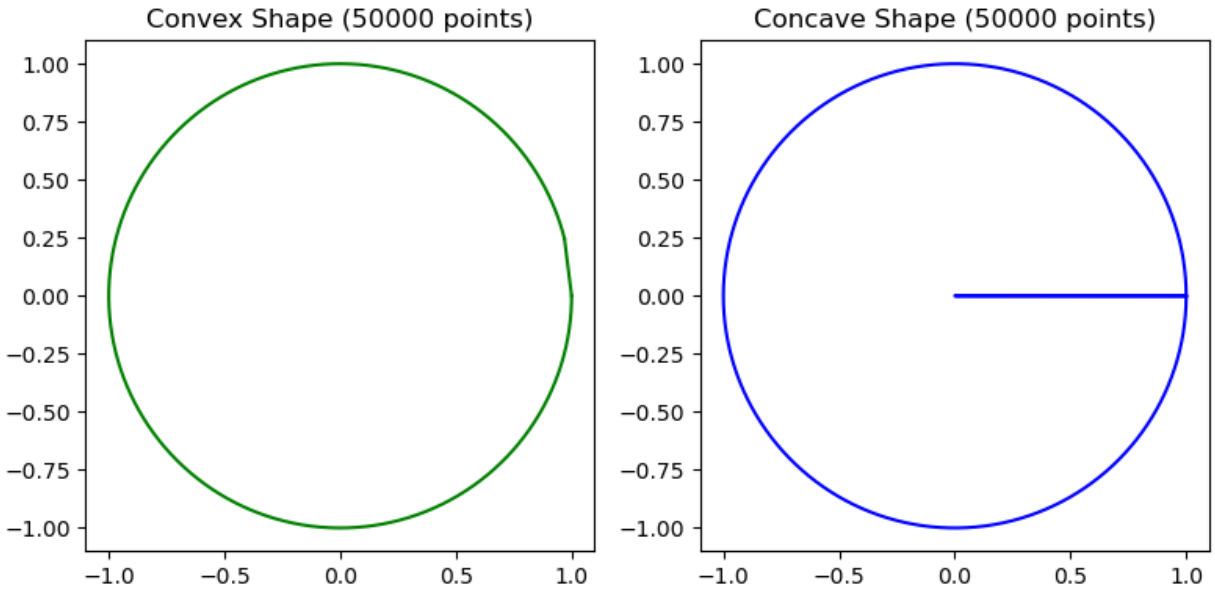

Convex Polygon (Concave)
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0



In [20]:
```
num_points_list = [5, 50, 500, 5000, 50000]
random_shapes = create_random_shapes(num_points_list)
```




Convex Shape (50 points)
Concave Shape (50 points)
Convex Shape (500 points)
Concave Shape (500 points)
Convex Shape (5000 points)
Concave Shape (5000 points)
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.0
−0.5
0.0
0.5
1.0



In [11]:
```python
def benchmark_algorithm(shapes_list):
    execution_times = {"Shape": [], "Points": [], "Time (µs)": []}

    for shape_dict in shapes_list:
        for shape_name, shape_points in shape_dict.items():
            num_points = len(shape_points)

            # Measure execution time
            start_time = time.perf_counter()
            is_convex(shape_points)
            end_time = time.perf_counter()

            execution_time = (end_time - start_time) * 1000000  # microseconds

            # Store the results
            execution_times["Shape"].append(shape_name)
            execution_times["Points"].append(num_points)
            execution_times["Time (µs)"].append(execution_time)

    return pd.DataFrame(execution_times)
```

In [12]:
```python
execution_times_random_shapes = benchmark_algorithm(random_shapes)
execution_times_random_shapes.head(10)
```

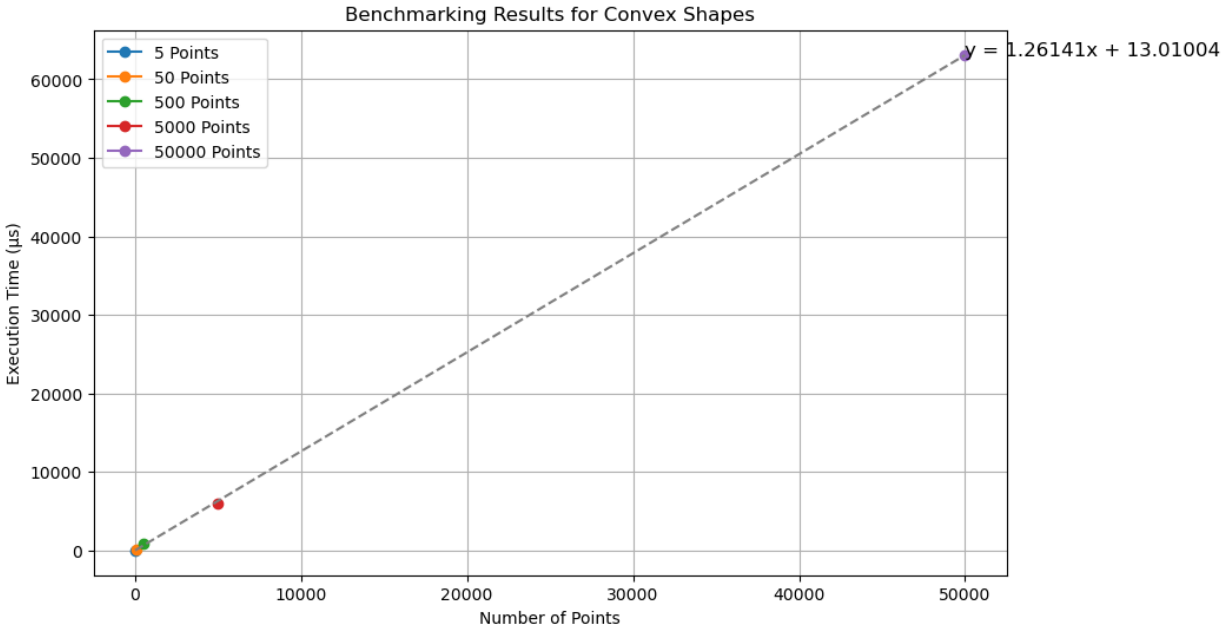
Out[12]:

	Shape	Points	Time (µs)
0	Convex	5	28.199982
1	Concave	5	9.800016
2	Convex	50	100.900012
3	Concave	50	94.200019
4	Convex	500	901.499996
5	Concave	500	642.999978
6	Convex	5000	5999.100016
7	Concave	5000	5926.999991
8	Convex	50000	63113.200013
9	Concave	50000	60957.100010

In [13]:
```
def plot_benchmark_results(execution_times, shape_name):
    plt.figure(figsize=(10, 6))
    plt.title(f"Benchmarking Results for {shape_name}")
    all_x = []
    all_y = []

    for num_points in num_points_list:
        data_subset = execution_times[execution_times['Points'] == num_points]
        x = data_subset['Points']
        y = data_subset['Time (µs)']

        all_x.extend(x)
        all_y.extend(y)

        plt.plot(x, y, marker='o', label=f"{num_points} Points")

    # Fit a linear trendline for all the points
    slope, intercept, _, _, _ = linregress(all_x, all_y)
    trend_equation = f'y = {slope:.5f}x + {intercept:.5f}'

    plt.plot(all_x, slope * np.array(all_x) + intercept, linestyle='--', color='gray')
    plt.text(all_x[-1], slope * all_x[-1] + intercept, trend_equation, fontsize=12)

    plt.xlabel("Number of Points")
    plt.ylabel("Execution Time (µs)")
    plt.legend()
    plt.grid(True)
    plt.show()
```

In [14]:
```
convex_data = execution_times_random_shapes[execution_times_random_shapes['Shape'] ==
concave_data = execution_times_random_shapes[execution_times_random_shapes['Shape'] ==
```

In [15]:
```
plot_benchmark_results(convex_data, "Convex Shapes")
```



In [16]:
```
plot_benchmark_results(concave_data, "Concave Shapes")
```