Problem Set 3 **CS 3851** **Name** Hudson Arney

Submission instructions: Your written answers should be submitted to Canvas as a PDF.

1. Describe in your own words the difference between $\Theta(\cdot)$, $O(\cdot)$, and $\Omega(\cdot)$.

Big Theta = Represents the tight bound of a function. It is used to describe an upper and lower bound that grow at the same rate as the function.

Big O = Represents an upper bound of a function. It is the upper limit of a functions behavior. Can have best and worst case scenarios.

Big Omega = Signifies the lower bound of a function. It represents the lower limit on a functions behavior.

2. Prove the following:

a. $n + 3 \in \Theta(n)$

$n+3$ is an upper bound for $n \leq n+3$ for all $n \geq 0$

$n+3$ is a lower bound for $n+3 \geq c*n$ for $c>0$ and $n \geq 0$

$c=\frac{1}{4} \Rightarrow n+3 \geq \frac{1}{4}*n$

$n+3 \geq \frac{1}{4}n + n$

$n+3 \geq n$ Therefore $n+3 \in \Theta(n)$

b. $n \notin \Theta(n \log n)$ Prove $c_1 * n \log n \leq n \leq c_2 * n \log n$ for $n \geq n_0$

$c_1 \log n \leq 1 \leq c_2 \log n \Rightarrow c_1 \leq \frac{1}{\log n} \leq c_2$

This cannot exist because there are no constants c_1 or c_2 that complete this $\lim_{n \to \infty} c_1 \leq \infty \leq c_2$

therefore $n \notin (n \log n)$

c. $n^2 + n - 25 \in \Theta(n^2)$

$0 \leq c_1 n^2 \leq n^2 + n - 25 \leq c_2 n^2$

$0 \leq c_1 \leq 1 + \frac{1}{n} - \frac{25}{n^2} \leq c_2$

$\lim_{n \to \infty} \left[0 \leq c_1 \leq 1 \leq c_2 \right]$

There are constants such that c_1 is less than 1 but more than 0, and an infinite # of values greater than 1 for c_2 that satisfy $n^2 + n - 25 \in \Theta(n^2)$

d. $n^3 - n + 5 \notin \Theta(n)$

$0 \leq c_1 n \leq n^3 - n + 5 \leq c_2 n$

$0 \leq c_1 \leq n^2 - 1 + \frac{5}{n} \leq c_2$

$\lim_{n \to \infty} \left[0 \leq c_1 \leq \infty \leq c_2 \right]$

c_2 does not have a valid constant greater than ∞ therefore, $n^3 - n + 5 \notin \Theta(n)$

e. $n + 3 \in O(n)$

$0 \leq n + 3 \leq c\,n$

$0 \leq 1 + \frac{3}{n} \leq c$

$\lim_{n \to \infty} \left[1 + \frac{3}{n} \right] = 0 \leq 1 \leq c$

Any number over 1 is a constant to satisfy, $n + 3 \in O(n)$

$f(n) \quad g(n)$

f. $n^2 \notin O(n \log n)$

$0 \leq (n \log n) \leq c_1 n^2$

$0 \leq \log n \leq n * c_1$

There is no value for c_1 that can satisfy this set therefor $n^2 \notin O(n \log n)$

g. $n^2 + n - 25 \in \Omega(n)$

$0 \leq C * n \leq n^2 + n - 25$

$0 \leq c \leq n + 1 - \frac{25}{n}$

$\lim_{n \to \infty} \left[0 \leq c \leq \infty + 1 \right]$ There is an infinite # of constants for statements; therefore, c to fullfill this $n^2 + n - 25 \in \Omega(n)$

h. $n^3 - n + 5 \notin \Omega(n^4)$

$0 \leq cn^4 \leq n^3 - n + 5$

$0 \leq c \leq \frac{1}{n} - \frac{1}{n^3} + \frac{5}{n^4}$

$\lim_{n \to \infty} \left[0 \leq c \leq \frac{1}{n} - \frac{1}{n^3} + \frac{5}{n^4} \right]$

$0 \leq c \leq 0$, there is no such value for c to exist in this problem; therefore, $n^3 - n + 5 \notin \Omega(n^4)$

3. Give an example of a function that is in $O(n^2)$ but not in $\Theta(n^2)$ and then prove it.

$n \in O(n^2)$

$0 \le n \le cn^2$

$0 \le 1 \le cn$

$\lim_{n\to\infty} \boxed{0 \le 1 \le c \cdot \infty}$

c can be any constant that is non-negative and complete this

$n \notin \Theta(n^2)$

$0 \le c_1 n^2 \le n \le c_2 n^2$

$0 \le c_1 n \le 1 \le c_2 n$

$\lim_{n\to\infty} \left[0 \le \infty \le 1 \le \infty \right] \Rightarrow$ Not possible

$f(n) = n \in O(n^2)$ but $\notin \Theta(n^2)$

4. Give an example of a function that is in $\Omega(n^2)$ but not in $\Theta(n^2)$ and then prove it.

$f(n) = n$

$\Omega(n^2)$

$0 \le c \cdot n \le n^2$

$\lim_{n\to\infty} 0 \le c \le n$

$0 \le c \le \infty$

If c is positive this is true

$\Theta(n^2)$

$0 \le c_1 n^2 \le n \le c_2 n^2$

$0 \le c_1 n \le 1 \le c_2 n$

$\lim_{n\to\infty} \left[0 \le \infty \le 1 \le \infty \right] \Rightarrow$ Not possible

$f(n) = n \in \Omega(n^2)$ but $\notin \Theta(n^2)$

5. Prove that $\Theta(g(n)) \subset \Omega(g(n))$.

If there exists some element $x \in \Theta(g(n))$ it would satisfy

$$0 \leq c_1 g(n) \leq x \leq c_2 g(n)$$ where c_1, c_2, and $n_0 \geq 0$

Thus x will also satisfy

$$0 \leq c\, g(n) \leq x$$

Therefore $\Theta(g(n))$ is a subset of $\Omega(g(n))$

Definitions

$\Theta(g(n)) = \{ f(n)$: there exists positive constants c_1, c_2, and n_0 such that

$0 \le c_1\, g(n) \le f(n) \le c_2\, g(n)$ for all $n \ge n_0 \}$

$O(g(n)) = \{ f(n)$: there exists positive constants c and n_0 such that

$0 \le f(n) \le c\, g(n)$ for all $n \ge n_0 \}$

$\Omega(g(n)) = \{ f(n)$: there exist positive constants c and n_0 such that

$0 \le cg(n) \le f(n)$ for all $n \ge n_0 \}$