(row x col)

CSC4601/5601 Thoery of MLProblem Set 1: Linear Algebra Review Name: Hudson Arney

1) For each of the following, indicate if it is a vector or matrix and its dimensions:

1. $u \in \mathbb{R}^4$ 4D Vector - (4 x 1)
2. $p \in \mathbb{R}^{1\times 6}$ (1x6) Matrix
3. $A \in \mathbb{R}^{5\times 3}$ (5x3) Matrix
4. $A^T$ where $A \in \mathbb{C}^{6\times 3}$ (3x6) Matrix

   Transform

2) Given the values of the entries indicated below.

$$U = \begin{bmatrix} 2 & 4 & 6 \\ 9 & 15 & 19 \\ 11 & 13 & 21 \end{bmatrix}$$

$$v = \begin{bmatrix} 1 \\ 3 \\ 5 \\ 7 \\ 9 \end{bmatrix}$$

1. $U_{1,1}$ = 2
2. $U_{2,3}$ = 13
3. $v_4$ = 7
4. The diagonal elements of $U$. 2, 15, 21

3) Linear algebra operations require that the shapes of the matrices and/or vectors match up. For each operation below, indicate if it is valid. If it is valid, give the dimensions of the resulting object. Note that $N \times 1$ and $1 \times N$ are used to indicate column and row vectors, respectively.

X = invalid

√ = valid

1. $u \cdot v$ where $u, v \in \mathbb{R}^{5\times 1}$ √, you would end up with a scalar
2. $uv$ X, cannot multiply due to dimensions (5x1)(5x1)
3. $u^T v$ √, would end up with a scalar (1x1)
4. $uv^T$ √, would end up with a (5x5) matrix
5. $u + v$ √, just add rows
6. $UV$ where $U \in \mathbb{R}^{5\times 6}, V \in \mathbb{R}^{6\times 7}$ √, would end up with a (5x7) matrix
7. $U^T V$ X, dimensions (6x5)(6x7) cannot multiply
8. $UV^T$ X, dimensions (5x6)(7x6) cannot multiply

4) Perform the following linear algebra operations and write the result.

$\begin{bmatrix}2\\4\\6\end{bmatrix}\begin{bmatrix}9 & 15 & 19\end{bmatrix} =$

$$u = \begin{bmatrix}2\\4\\6\end{bmatrix} \quad 1.\ (3\times1)(3\times1) \quad 2.\ (3\times1)(1\times3) = (3\times3)$$

$$v = \begin{bmatrix}9\\15\\19\end{bmatrix} \quad 3.\ (1\times3)(3\times1) = (1\times1)$$

$\begin{bmatrix}2\\4\\6\end{bmatrix}\cdot\begin{bmatrix}9\\15\\19\end{bmatrix} = \begin{bmatrix}2\cdot9\\4\cdot15\\6\cdot19\end{bmatrix} = \begin{bmatrix}18\\60\\114\end{bmatrix}$

$\begin{bmatrix}2 & 4 & 6\end{bmatrix}\begin{bmatrix}9\\15\\19\end{bmatrix} = \begin{bmatrix}18+60+114\end{bmatrix}$

1. $u \cdot v$
2. $uv^T = \begin{bmatrix}18 & 30 & 38\\36 & 60 & 76\\54 & 90 & 114\end{bmatrix}$
3. $u^Tv = \begin{bmatrix}192\end{bmatrix}$
4. $UV$ where $U = \begin{bmatrix}2 & 4 & 6\\9 & 15 & 19\\11 & 13 & 21\end{bmatrix}$ and $V = \begin{bmatrix}2 & 4 & 6\\8 & 10 & 12\\14 & 16 & 18\end{bmatrix}$ $= \begin{bmatrix}(2\cdot2+4\cdot8+6\cdot14) & (2\cdot4+4\cdot10+6\cdot16) & \dots\\ \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot\end{bmatrix} = \begin{bmatrix}120 & 144 & 168\\404 & 490 & 576\\420 & 510 & 600\end{bmatrix}$

5) Give the vectors $\beta$ and $x$ that make the following equations equivalent.

$$y = \beta_0 + \beta_1x_1 + \beta_2x_2 + \beta_3x_3$$

$$y = x^T\beta$$

$\beta = \begin{bmatrix}\beta_0\\\beta_1\\\beta_2\\\beta_3\end{bmatrix} \quad x = \begin{bmatrix}1\\x_1\\x_2\\x_3\end{bmatrix}$

6) Norms and distance.

1. Write the squared norm $\|v\|^2$ of a vector $v$ in terms of a dot product. $= v\cdot v$
2. Convert the equation for the Euclidean distance between two vectors $u$ and $v$ into vector notation using vector arithmetic and norms.

$$d(u,v) = \sqrt{(u_1 - v_1)^2 + (u_2 - v_2)^2 + \cdots + (u_n - v_n)^2}$$

$= \|u - v\|$