Introduction to the COLERAINE stock assessment model



Arni Magnusson

Overview

MODEL

- generalized equations
- Excel user interface
- Bayesian parameter estimation
- AD Model Builder optimization

DATA

- catch by age/length, biomass indices
- stratification by sex and gears
- priors for estimated parameters
- values of fixed parameters

OUTPUT

- max likelihood point estimates
- MCMC posterior likelihood profiles
- projections of harvest strategies

APPLICATION

Icelandic cod data

Background

Ray Hilborn

Professor, Univ. of Wash.

Theory & application

Mark Maunder

Ph.D. student, Univ. of Wash.

Programming & application

Ana Parma

Int. Pac. Halibut Comm.

Theory

Billy Ernst

Ph.D. student, Univ. of Wash.

Programming & application

John Payne

Ph.D. student, Univ. of Wash.

User interface

Paul Starr

New Zeal, Seaf, Ind. Council

Application

Generalized model

Coleraine can be used to model a wide variety of stocks. The model may or may not be

- sex-specific
- gear-specific
- fitted to data from commercial landings and/or research surveys and/or CPUE
- fitted to age and/or length data

Population dynamics

$$N_{a+1,t+1} = N_{a,t} e^{-M} (1 - u_{a,t})$$

N : population size

a:age

t: year

M : natural mortality rate

u: harvest rate

Harvest rate and landings

$$u_{t} = \frac{C_{t}}{e^{-0.5M} \sum_{a} s_{a} N_{a,t} w_{a,t}}$$

u : harvest rate N : population size

t: year s: selectivity

C: landings a: age

M : natural mortality rate w : weight

Age distribution in first year

recruits
$$N_{1,1} = \omega R_0$$

$$N_{a,1} = N_{1,1} e^{-M(a-1)}$$

plus group
$$N_{a,1} = \frac{N_{1,1} e^{-M(A-1)}}{1 - e^{-M}}$$

N: population size

 ω : initial recruitment fraction

 $R_{
m o}$: virgin recruitment

a: age

M : natural mortality rate

 \mathcal{V}_{init} : initial selectivity

 u_{init} : initial harvest rate

A: maximum age

 $N_{1,1} = R_0$

| | Age 1 | Age 2 | Age 3 | Age 4 | Age 5 | Age 6 | Age 7 | Age 8 | Age 9 | Age 10 |
|------|----------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
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$$N_{1,1} = R_0$$

| $N_{a,1} = N_{1,1} e^{-M(a-1)} \prod_{i=1}^{a-1} (1 - u_{i,1})$ |
|---|
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|------|-------|-------|-------|-------|-------|-------|-------|----------|-------|--------|
| | Age 1 | Age 2 | Age 3 | Age A | Age 5 | Age 6 | Age 7 | Age 8 | Age 9 | Age 10 |
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$$N_{1,1} = R_0$$

$$N_{a,1} = N_{1,1} e^{-M(a-1)} \prod_{i=1}^{a-1} (1 - u_{i,1})$$

| N - | $N_{1,1} e^{-M(A-1)} \prod_{i=1}^{A-1} u_{i,1}$ |
|-------------|---|
| $N_{A,1}$ = | $1 - e^{-M} \prod_{i=1}^{A-1} u_{i,1}$ |

| | Age 1 | Age 2 | Age 3 | Age 4 | Age 5 | Age 6 | Age 7 | Age 8 | Age 9 | Age 10 |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| 1990 | 1000 | 793 | 553 | 329 | 196 | 116 | 69 | 41 | 24 | • |
| 1991 | | | | | | | | | | |
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$$N_{1,1} = R_0$$

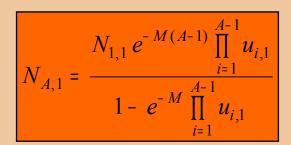
$$N_{a,1} = N_{1,1} e^{-M(a-1)} \prod_{i=1}^{a-1} (1 - u_{i,1})$$

| λ/ - | $N_{1,1} e^{-M(A-1)} \prod_{i=1}^{A-1} u_{i,1}$ |
|-------------|---|
| $N_{A,1}$ = | $\frac{1 - e^{-M} \prod_{i=1}^{A-1} u_{i,1}}{1 - e^{-M} \prod_{i=1}^{A-1} u_{i,1}}$ |

| | Age 1 | Age 2 | Age 3 | Age 4 | Age 5 | Age 6 | Age 7 | Age 8 | Age 9 | Age 10 |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| 1990 | 1000 | 793 | 553 | 329 | 196 | 116 | 69 | 41 | 24 | 80 |
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$$N_{1,1} = R_0$$

$$N_{a,1} = N_{1,1} e^{-M(a-1)} \prod_{i=1}^{a-1} (1 - u_{i,1})$$



| | Age 1 | Age 2 | Age 3 | Age 4 | Age 5 | Age 6 | Age 7 | Age 8 | Age 9 | Age 10 |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| 1990 | 1000 | 793 | 553 | 329 | 196 | 116 | 69 | 41 | 24 | 80 |
| 1991 | | | | 4 | | | | 4 | | |
| 1992 | | | | | 1 | 4 | | | 1 | |
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$$N_{a+1,t+1} = N_{a,t}e^{-M}(1-u_{a,t})$$

$$N_{1,1} = R_0$$

$$N_{a,1} = N_{1,1} e^{-M(a-1)} \prod_{i=1}^{a-1} (1 - u_{i,1})$$

| λ/ - | $N_{1,1} e^{-M(A-1)} \prod_{i=1}^{A-1} u_{i,1}$ |
|-------------|---|
| $N_{A,1}$ = | $\frac{1 - e^{-M} \prod_{i=1}^{A-1} u_{i,1}}{1 - e^{-M} \prod_{i=1}^{A-1} u_{i,1}}$ |

| | Age 1 | Age 2 | Age 3 | Age 4 | Age 5 | Age 6 | Age 7 | Age 8 | Age 9 | Age 10 |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| 1990 | 1000 | 793 | 553 | 329 | 196 | 116 | 69 | 41 | 24 | 80 |
| 1991 | | 783 | 520 | 291 | 173 | 103 | 61 | 36 | 22 | 55 |
| 1992 | | | 536 | 297 | 166 | 99 | 59 | 35 | 21 | 44 |
| 1993 | | | | 334 | 185 | 103 | 62 | 37 | 22 | 40 |
| 1994 | | | | | 223 | 124 | 69 | 41 | 24 | 41 |
| 1995 | | | | | | 138 | 77 | 43 | 25 | 41 |
| 1996 | | | | | | | 73 | 40 | 23 | 35 |
| 1997 | | | | | | | | 43 | 24 | 34 |
| 1998 | | | | | | | | | 24 | 32 |
| 1999 | | | | | | | | | | 30 |
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| 2005 | | | | | | | | | | |

$$N_{a+1,t+1} = N_{a,t}e^{-M}(1-u_{a,t})$$

Recruitment

$$N_{1,\,t+1} = \frac{S_t}{\alpha + \beta S_t}$$

 $N_{\scriptscriptstyle 1}$: recruits

t: years

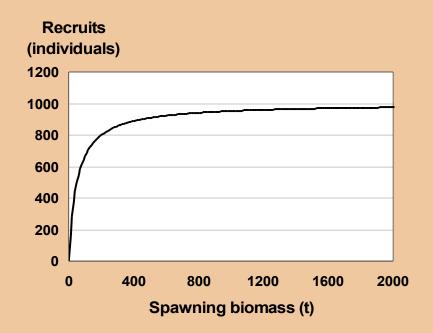
S : spawning biomass

lpha: shape parameter (1 / initial slope)

eta : shape parameter (1 / asymptote)

Beverton-Holt

$$N_{1, t+1} = \frac{S_t}{\alpha + \beta S_t}$$



 $N_{\scriptscriptstyle 1}$: recruits

t: years

S : spawning biomass

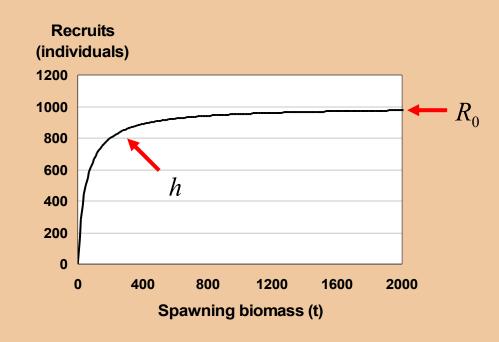
lpha: shape parameter (1 / initial slope)

 β : shape parameter (1 / asymptote)

Reparametrized Beverton-Holt

$$N_{1,\,t+1} = \frac{S_t}{\alpha + \beta S_t}$$

$$N_{1,t+1} = f(S_t | R_0, h)$$



 $R_{
m 0}$: virgin recruitment

h: slope parameter (by definition $0.2 \le h \le 1.0$)

Spawning biomass

$$S_t = \sum_{a} N_{a,t} \Phi_a w_{a,t}$$

S : spawning biomass

N: population size

a:age

t: years

 Φ : fraction mature

 \mathcal{W} : weight

$$N_{1,1} = R_0$$

$$N_{a,1} = N_{1,1} e^{-M(a-1)} \prod_{i=1}^{a-1} (1 - u_{i,1})$$

| λ/ - | $N_{1,1} e^{-M(A-1)} \prod_{i=1}^{A-1} u_{i,1}$ |
|-------------|---|
| $N_{A,1}$ = | $1 - e^{-M} \prod_{i=1}^{A-1} u_{i,1}$ |

| $N_{1,t+1}$ | = |
|-------------|----------|
| $f(S_t)$ | R_0, h |

| | | Age 1 | Age 2 | Age 3 | Age 4 | Age 5 | Age 6 | Age 7 | Age 8 | Age 9 | Age 10 |
|----------|---------------|------------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| 1 | 990 | 1000 | 793 | 553 | 329 | 196 | 116 | 69 | 41 | 24 | 80 |
| 1 | 991 | • | 783 | 520 | 291 | 173 | 103 | 61 | 36 | 22 | 55 |
| 1 | 992 | / | | 536 | 297 | 166 | 99 | 59 | 35 | 21 | 44 |
| 1 | 993⁄ | • | | | 334 | 185 | 103 | 62 | 37 | 22 | 40 |
| 1 | 994 | • | | | | 223 | 124 | 69 | 41 | 24 | 41 |
| <i>M</i> | 995 | • | | | | | 138 | 77 | 43 | 25 | 41 |
| | 996 | • | | | | | | 73 | 40 | 23 | 35 |
| 1 | 997 | • | | | | | | | 43 | 24 | 34 |
| M T | 998 | • | | | | | | | | 24 | 32 |
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$$N_{a+1,t+1} = N_{a,t}e^{-M}(1-u_{a,t})$$

$$N_{1,1} = R_0$$

$$N_{a,1} = N_{1,1} e^{-M(a-1)} \prod_{i=1}^{a-1} (1 - u_{i,1})$$

$$N_{A,1} = \frac{N_{1,1} e^{-M(A-1)} \prod_{i=1}^{A-1} u_{i,1}}{1 - e^{-M} \prod_{i=1}^{A-1} u_{i,1}}$$

$$N_{1,t+1} = f(S_t \mid R_0, h)$$

| | Age 1 | Age 2 | Age 3 | Age 4 | Age 5 | Age 6 | Age 7 | Age 8 | Age 9 | Age 10 | |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|---|
| 1990 | 1000 | 793 | 553 | 329 | 196 | 116 | 69 | 41 | 24 | 80 | |
| 1991 | 273 | 783 | 520 | 291 | 173 | 103 | 61 | 36 | 22 | 55 | 1 |
| 1992 | 321 | 215 | 536 | 297 | 166 | 99 | 59 | 35 | 21 | 44 | |
| 1993 | 989 | 256 | 154 | 334 | 185 | 103 | 62 | 37 | 22 | 40 | ı |
| 1994 | 466 | 793 | 189 | 103 | 223 | 124 | 69 | 41 | 24 | 41 | ı |
| 1995 | 723 | 371 | 564 | 117 | 64 | 138 | 77 | 43 | 25 | 41 | ı |
| 1996 | 1860 | 566 | 244 | 298 | 62 | 34 | 73 | 40 | 23 | 35 | ı |
| 1997 | 459 | 1475 | 395 | 145 | 177 | 37 | 20 | 43 | 24 | 34 | ı |
| 1998 | 501 | 362 | 997 | 220 | 81 | 99 | 20 | 11 | 24 | 32 | ı |
| 1999 | 292 | 393 | 239 | 530 | 117 | 43 | 52 | 11 | 6 | 30 | ı |
| 2000 | 433 | 231 | 269 | 137 | 304 | 67 | 25 | 30 | 6 | 21 | ı |
| 2001 | 726 | 339 | 152 | 143 | 73 | 161 | 36 | 13 | 16 | 14 | ı |
| 2002 | 592 | 572 | 229 | 84 | 79 | 40 | 89 | 20 | 7 | 17 | ı |
| 2003 | 760 | 475 | 423 | 153 | 56 | 53 | 27 | 60 | 13 | 16 | |
| 2004 | 1222 | 599 | 321 | 236 | 85 | 31 | 29 | 15 | 33 | 16 | |
| 2005 | 324 | 959 | 397 | 172 | 127 | 46 | 17 | 16 | 8 | 27 | |

Harvest rate

$$N_{a+1,t+1} = N_{a,t}e^{-M}(1-u_{a,t})$$

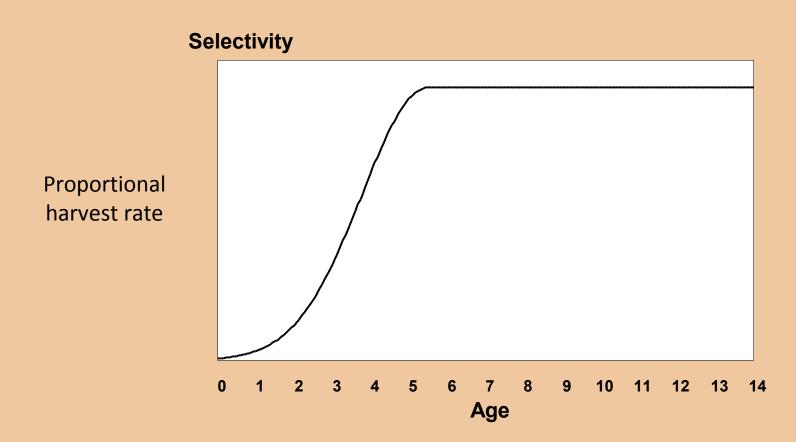
Harvest rate

$$N_{a+1,t+1} = N_{a,t}e^{-M}(1-u_{a,t})$$

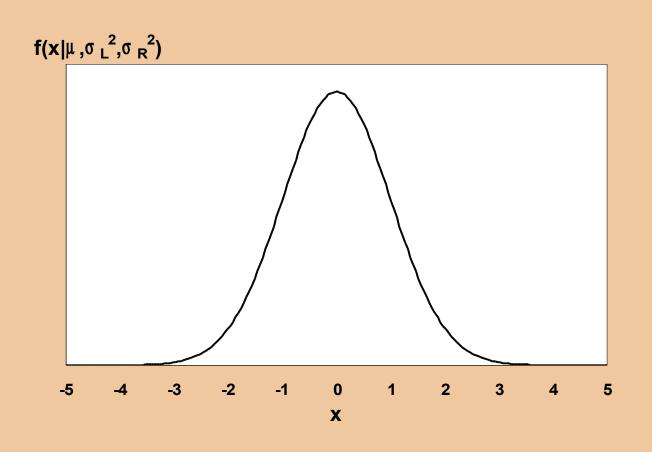
$$u_{a,t} = u_t S_{a,t}$$
 selectivity

$$u_{t} = \frac{Y_{t}}{e^{-0.5M} \sum_{a} s_{a,t} N_{a,t} w_{a,t}}$$

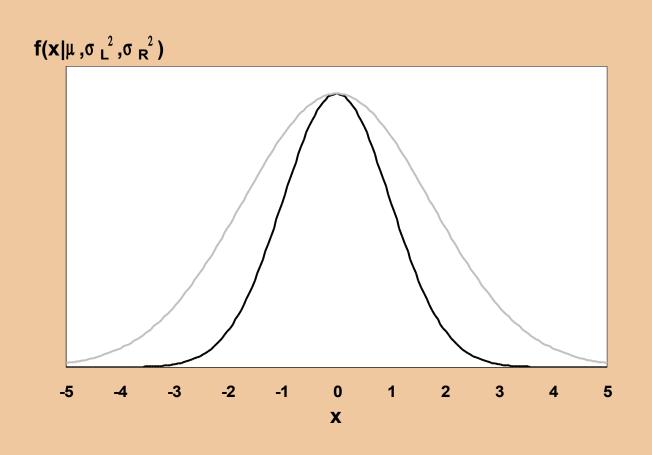
Selectivity



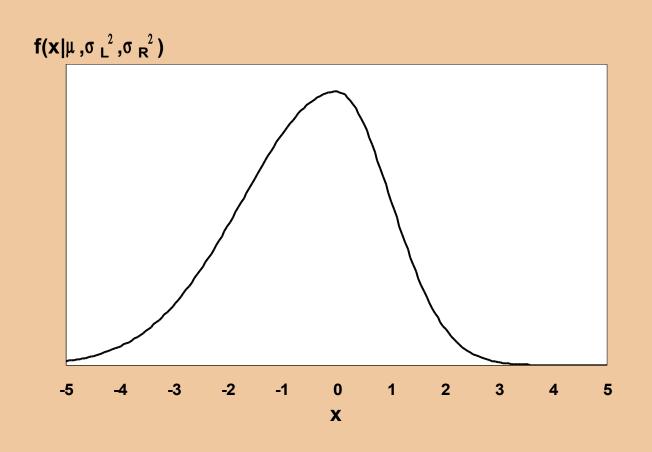
Normal distribution



Different variance

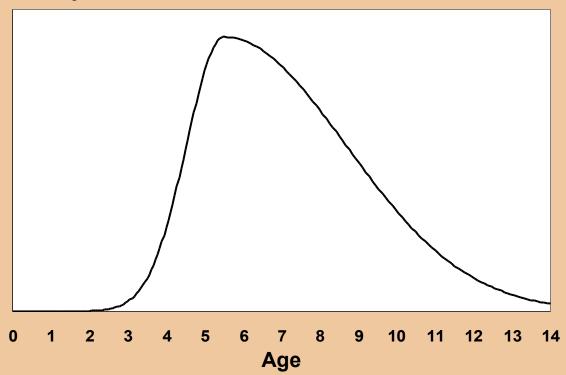


Asymmetric normal

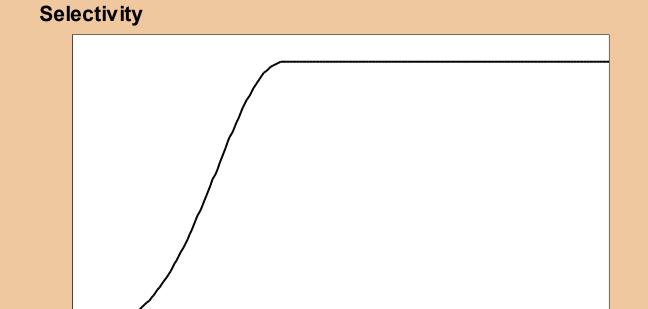


Gillnet

Selectivity



Bottom trawl



Age

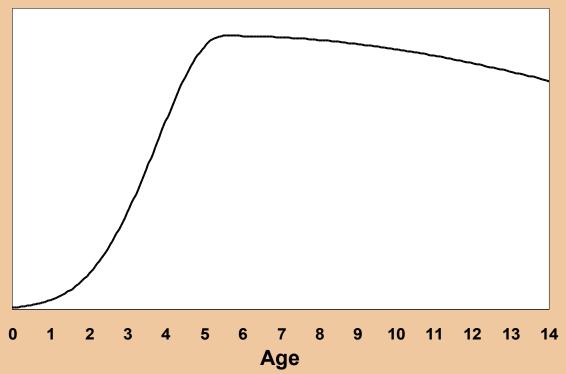
9 10 11 12 13 14

3

5

Bottom trawl





Estimated parameters



$$N_{a,1} = N_{1,1} e^{-M(a-1)} \prod_{i=1}^{a-1} (1 - u_{i,1})$$

$$N_{A,1} = \frac{N_{1,1} e^{-M(A-1)} \prod_{i=1}^{A-1} u_{i,1}}{1 - e^{-M} \prod_{i=1}^{A-1} u_{i,1}}$$

$$N_{1,t+1} = f(S_t \mid R_0, h)$$

| | Age 1 | Age 2 | Age 3 | Age 4 | Age 5 | Age 6 | Age 7 | Age 8 | Age 9 | Age 10 |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| 1990 | 1000 | 793 | 553 | 329 | 196 | 116 | 69 | 41 | 24 | 80 |
| 1991 | 273 | 783 | 520 | 291 | 173 | 103 | 61 | 36 | 22 | 55 |
| 1992 | 321 | 215 | 536 | 297 | 166 | 99 | 59 | 35 | 21 | 44 |
| 1993 | 989 | 256 | 154 | 334 | 185 | 103 | 62 | 37 | 22 | 40 |
| 1994 | 466 | 793 | 189 | 103 | 223 | 124 | 69 | 41 | 24 | 41 |
| 1995 | 723 | 371 | 564 | 117 | 64 | 138 | 77 | 43 | 25 | 41 |
| 1996 | 1860 | 566 | 244 | 298 | 62 | 34 | 73 | 40 | 23 | 35 |
| 1997 | 459 | 1475 | 395 | 145 | 177 | 37 | 20 | 43 | 24 | 34 |
| 1998 | 501 | 362 | 997 | 220 | 81 | 99 | 20 | 11 | 24 | 32 |
| 1999 | 292 | 393 | 239 | 530 | 117 | 43 | 52 | 11 | 6 | 30 |
| 2000 | 433 | 231 | 269 | 137 | 304 | 67 | 25 | 30 | 6 | 21 |
| 2001 | 726 | 339 | 152 | 143 | 73 | 161 | 36 | 13 | 16 | 14 |
| 2002 | 592 | 572 | 229 | 84 | 79 | 40 | 89 | 20 | 7 | 17 |
| 2003 | 760 | 475 | 423 | 153 | 56 | 53 | 27 | 60 | 13 | 16 |
| 2004 | 1222 | 599 | 321 | 236 | 85 | 31 | 29 | 15 | 33 | 16 |
| 2005 | 324 | 959 | 397 | 172 | 127 | 46 | 17 | 16 | 8 | 27 |

Selectivity parameters: Sleft, Sfull, Sright

$$N_{a+1,t+1} = N_{a,t}e^{-M}(1-u_{a,t})$$
 $u_{a,t} = u_t s_{a,t}$

User interface

Excel

Maincode.xls

Template.xls

DATA INPUT

Model specifications
Priors for estimated parameters
Values of fixed parameters

Graph.xls

Tracker.xls

MODEL OUTPUT

Parameter estimates Derived statistics Diagnostic graphs





ASCII input file





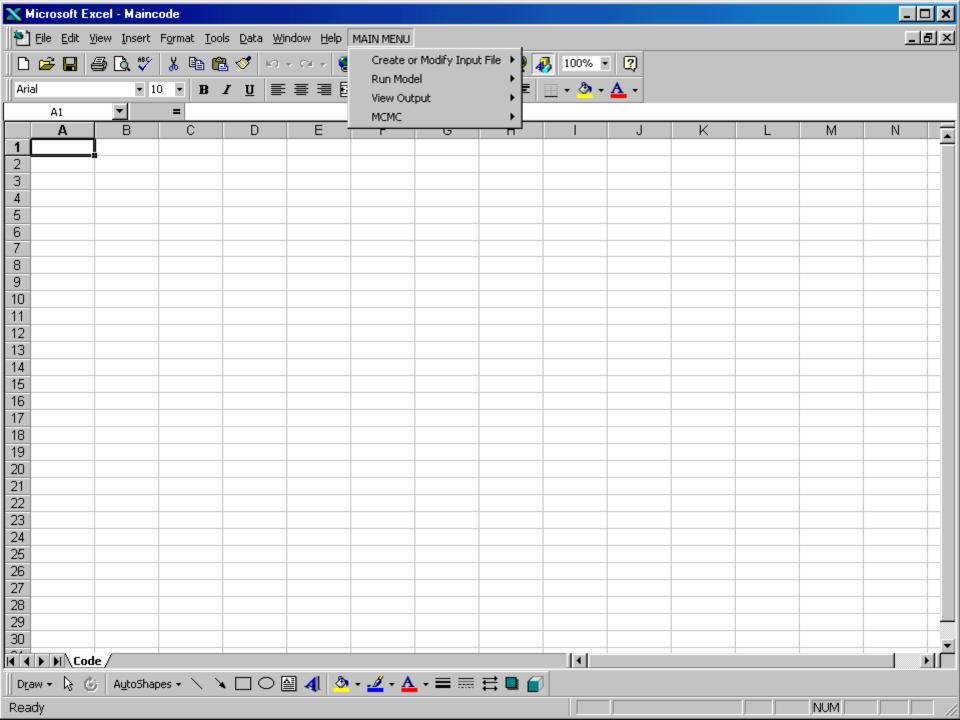


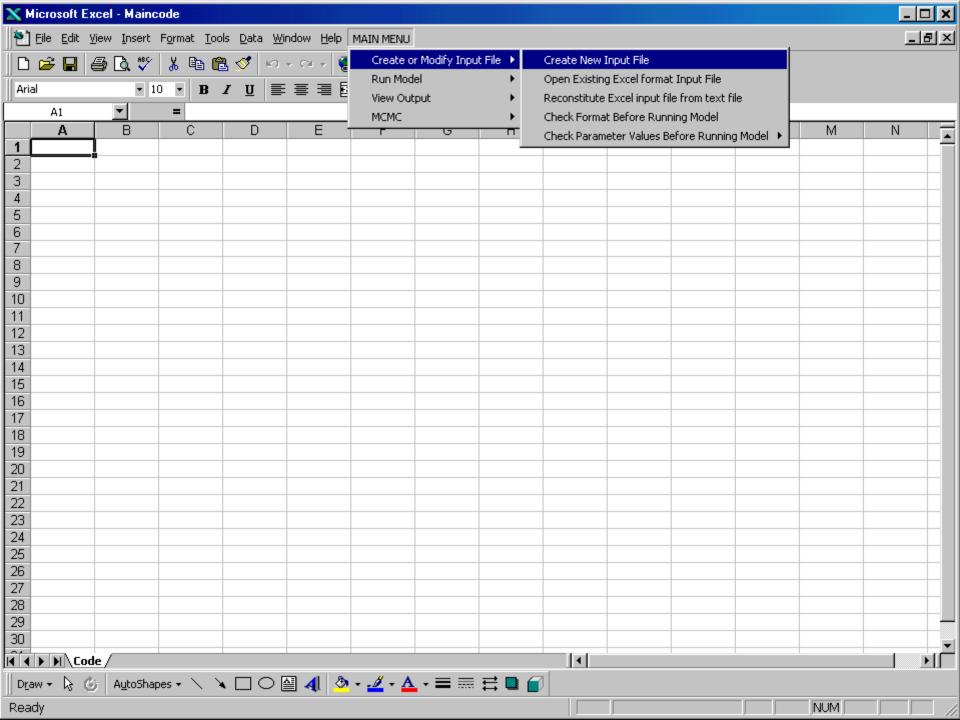


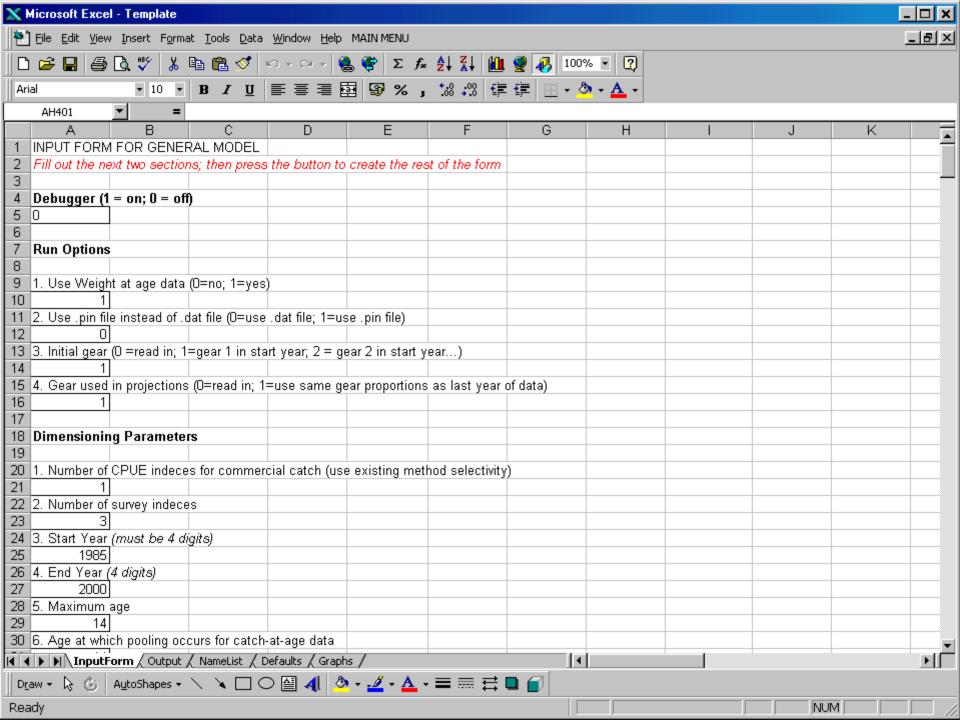
ASCII output files

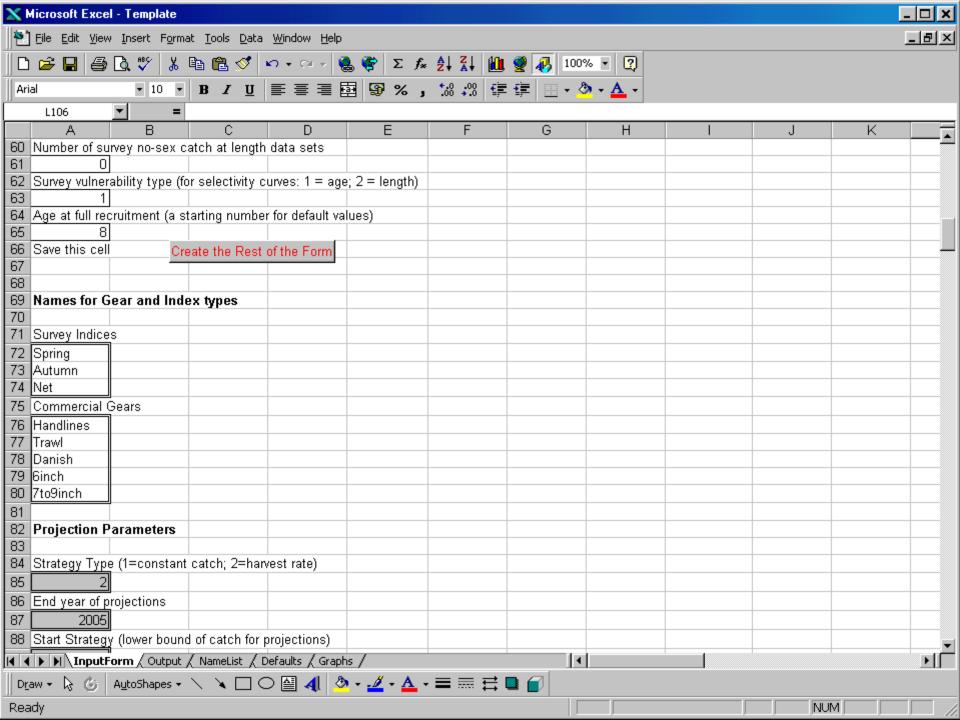


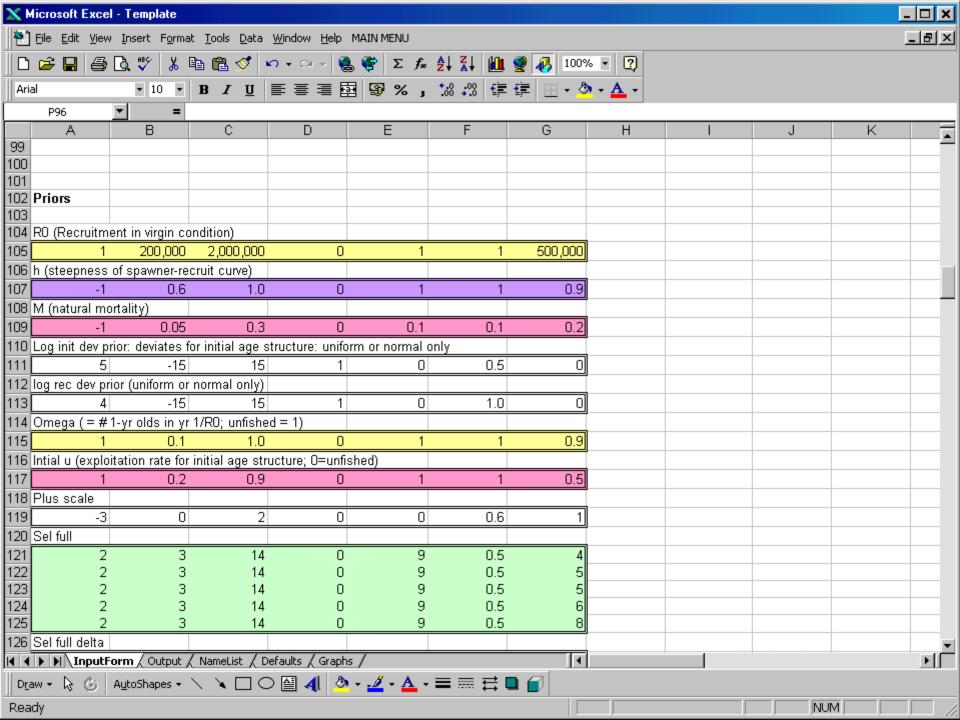
colera.exe

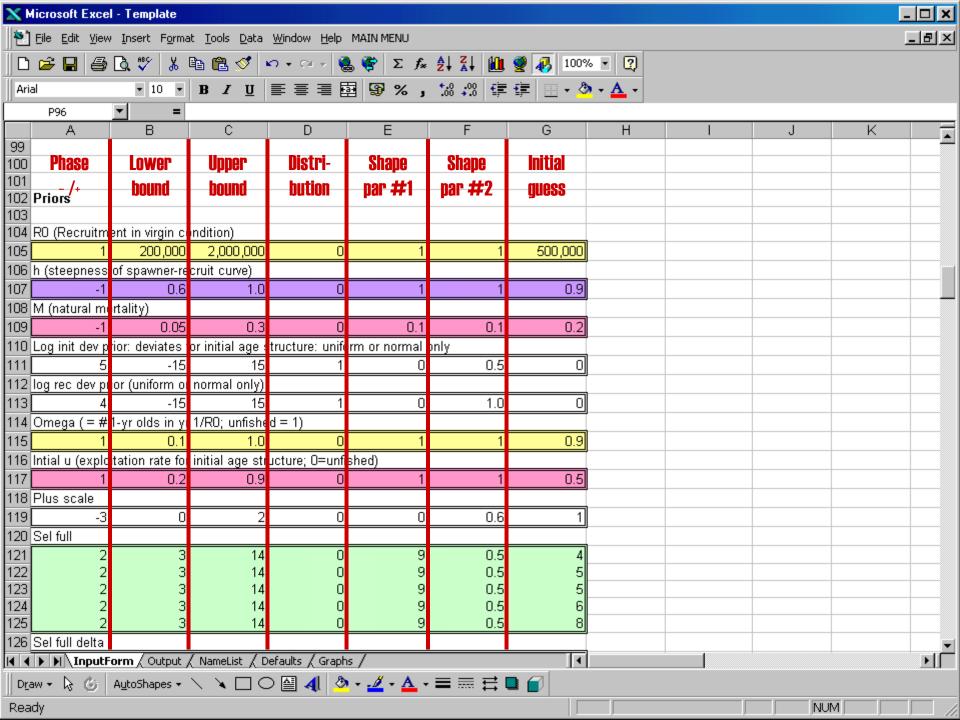


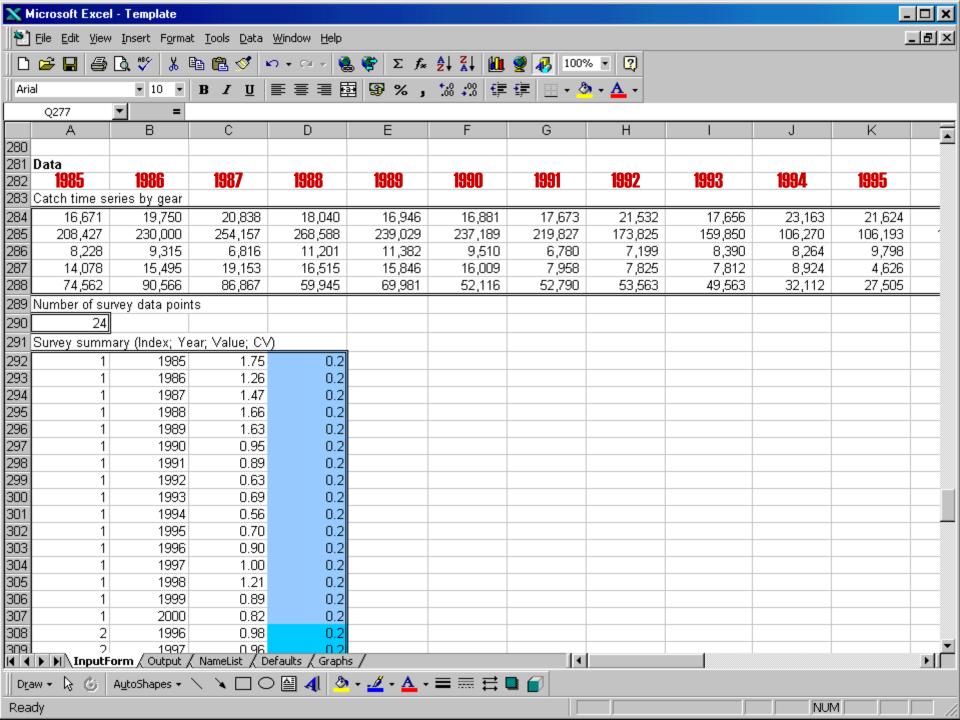


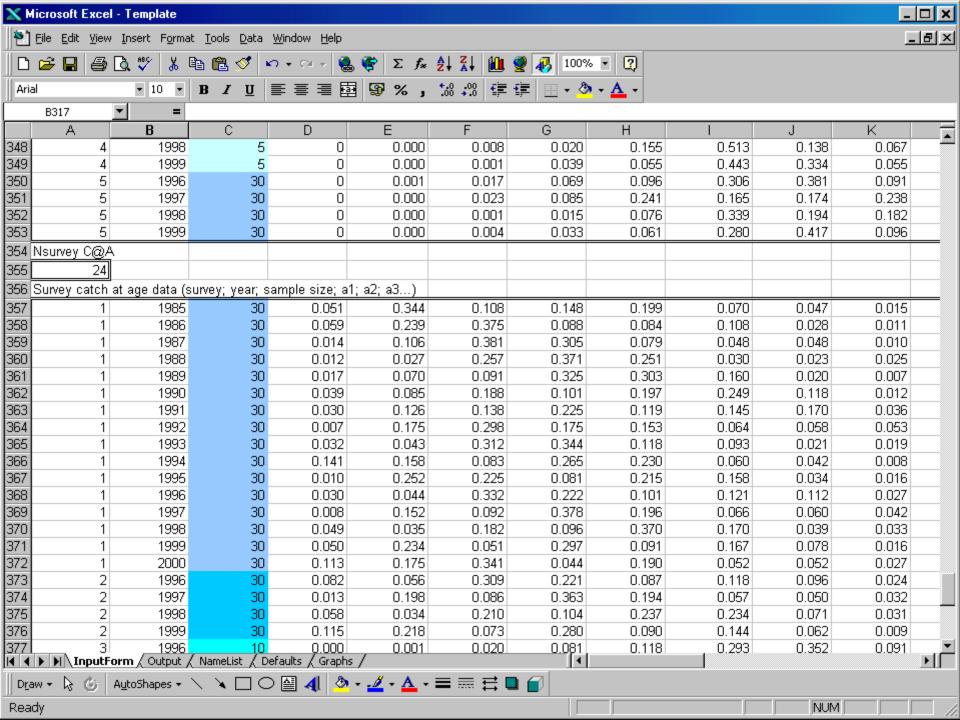


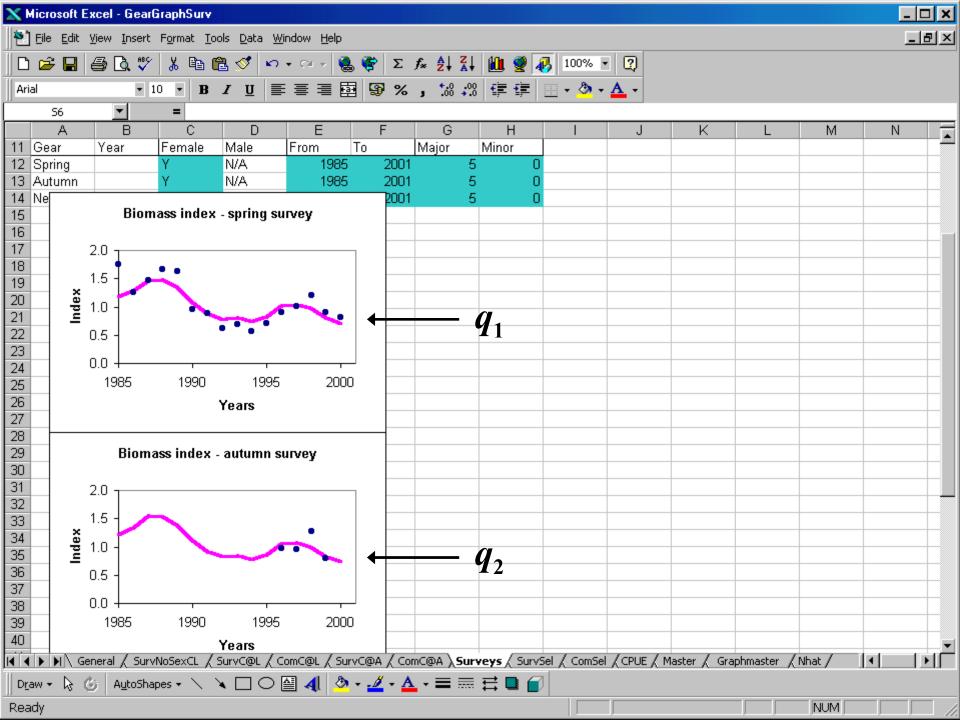




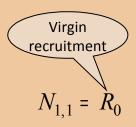








Estimated parameters



$$N_{a,1} = N_{1,1} e^{-M(a-1)} \prod_{i=1}^{a-1} (1 - u_{i,1})$$

$$N_{A,1} = \frac{N_{1,1} e^{-M(A-1)} \prod_{i=1}^{A-1} u_{i,1}}{1 - e^{-M} \prod_{i=1}^{A-1} u_{i,1}}$$

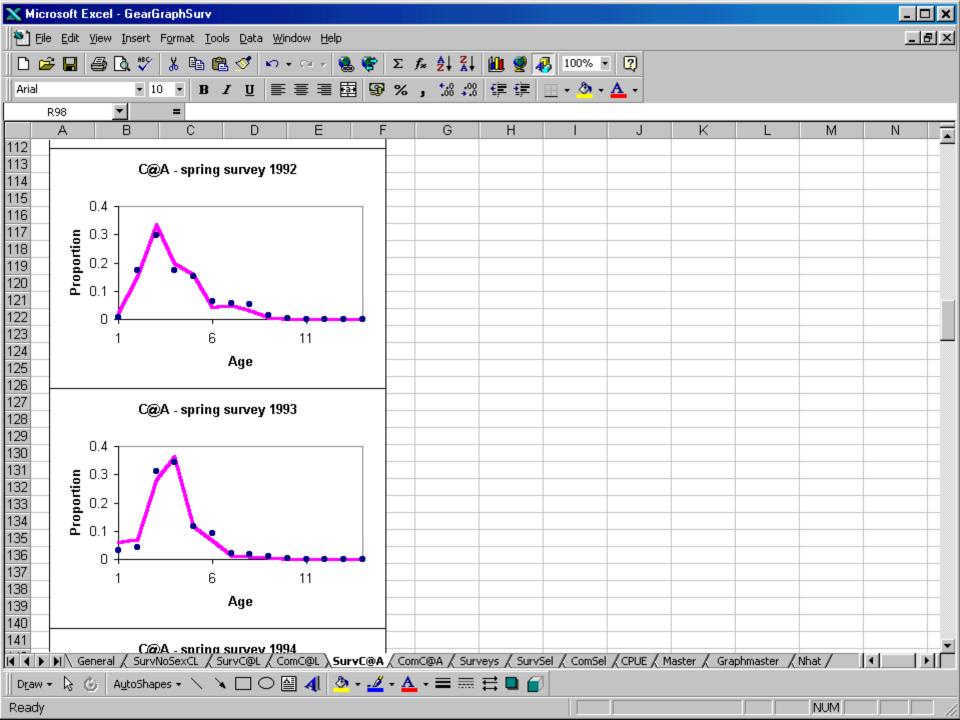
$$N_{1,t+1} = f(S_t \mid R_0, h)$$

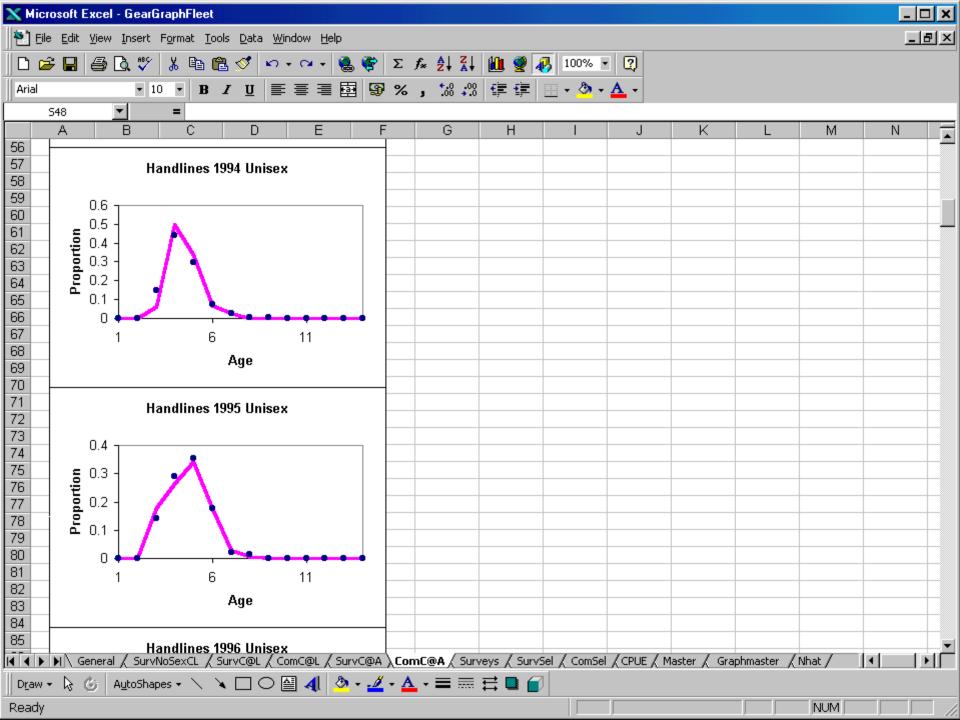
| | Age 1 | Age 2 | Age 3 | Age 4 | Age 5 | Age 6 | Age 7 | Age 8 | Age 9 | Age 10 |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| 1990 | 1000 | 793 | 553 | 329 | 196 | 116 | 69 | 41 | 24 | 80 |
| 1991 | 273 | 783 | 520 | 291 | 173 | 103 | 61 | 36 | 22 | 55 |
| 1992 | 321 | 215 | 536 | 297 | 166 | 99 | 59 | 35 | 21 | 44 |
| 1993 | 989 | 256 | 154 | 334 | 185 | 103 | 62 | 37 | 22 | 40 |
| 1994 | 466 | 793 | 189 | 103 | 223 | 124 | 69 | 41 | 24 | 41 |
| 1995 | 723 | 371 | 564 | 117 | 64 | 138 | 77 | 43 | 25 | 41 |
| 1996 | 1860 | 566 | 244 | 298 | 62 | 34 | 73 | 40 | 23 | 35 |
| 1997 | 459 | 1475 | 395 | 145 | 177 | 37 | 20 | 43 | 24 | 34 |
| 1998 | 501 | 362 | 997 | 220 | 81 | 99 | 20 | 11 | 24 | 32 |
| 1999 | 292 | 393 | 239 | 530 | 117 | 43 | 52 | 11 | 6 | 30 |
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| 2001 | 726 | 339 | 152 | 143 | 73 | 161 | 36 | 13 | 16 | 14 |
| 2002 | 592 | 572 | 229 | 84 | 79 | 40 | 89 | 20 | 7 | 17 |
| 2003 | 760 | 475 | 423 | 153 | 56 | 53 | 27 | 60 | 13 | 16 |
| 2004 | 1222 | 599 | 321 | 236 | 85 | 31 | 29 | 15 | 33 | 16 |
| 2005 | 324 | 959 | 397 | 172 | 127 | 46 | 17 | 16 | 8 | 27 |

$$\hat{I}_t = q\hat{B}_t$$
Survey/CPUE catchability

Selectivity parameters: Sleft, Sfull, Sright

$$N_{a+1,t+1} = N_{a,t}e^{-M}(1-u_{a,t})$$
 $u_{a,t} = u_t s_{a,t}$





Catch-at-age likelihood

$$\log L_{\rm C}^{g} = -0.5 \sum_{t=1}^{N_{years}} \sum_{a=1}^{A} \log \left[\left(P_{a,t}^{g} (1 - P_{a,t}^{g}) + 0.1 / A \right) \right] + \sum_{t=1}^{N_{years}} \sum_{a=1}^{A} \log \left[\exp \left\{ \frac{-\left(\tilde{P}_{a,t}^{g} - P_{a,t}^{g} \right)^{2}}{2\left(P_{a,t}^{g} (1 - P_{a,t}^{g}) + 0.1 / A \right) \tau^{g}} \right\} + 0.01 \right]$$

 $L_{
m C}$: catch-at-age likelihood

g: gear

a: age

t: years

T: number of years

a:age

 ${\cal A}$: number of age groups

P : proportional catch at age

au: 1 / catch sample size

Biomass index likelihood

$$\log L_I^g = \sum_t \log \left[\exp \left(-0.5 \frac{I^g \mathcal{E}_t^2}{I^g \sigma_t^2} \right) + 0.01 \right]$$

 $L_{\scriptscriptstyle I}$: biomass index likelihood

g: gear

I: biomass index

t: years

 \mathcal{E} : log-normal error term

 σ : survey variance

Total likelihood

$$\log L = \sum_{g} \log L_{I}^{g} + \sum_{g} \log L_{C}^{g} + \sum_{g} \log L_{S}^{g}$$

L : total likelihood

g: gear

 $L_{\scriptscriptstyle I}$: biomass index likelihood

 $L_{\it C}$: commercial catch-at-age likelihood

 $L_{\scriptscriptstyle S}$: survey catch-at-age likelihood

Penalties

$$Pen_p = 0.5 \frac{\varepsilon_p^2}{\sigma_p^2}$$

For example, recruitment deviates or time-varying selectivity

Pen: penalty for deviation

p: parameter

 ${\mathcal E}$: residual from prior $\,\mu$

 σ : variance of prior distribution

Objective function

$$f = \log L - \sum_{p} Pen_{p}$$

L : total likelihood

Pen: penalty sum of square

p: parameter

$$N_{1,1} = R_0$$

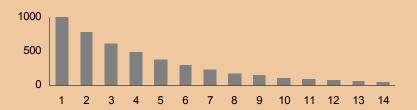
$$N_{a,1} = N_{1,1} e^{-M(a-1)} \prod_{i=1}^{a-1} (1 - u_{i,1})$$

| λ/ - | $N_{1,1} e^{-M(A-1)} \prod_{i=1}^{A-1} u_{i,1}$ |
|-------------|---|
| $N_{A,1}$ = | $\frac{1 - e^{-M} \prod_{i=1}^{A-1} u_{i,1}}{1 - e^{-M} \prod_{i=1}^{A-1} u_{i,1}}$ |

| $N_{1,t+1}$ | = |
|-------------|---------|
| $f(S_t)$ | R_0,h |

| | Age 1 | Age 2 | Age 3 | Age 4 | Age 5 | Age 6 | Age 7 | Age 8 | Age 9 | Age 10 |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| 1990 | 1000 | 793 | 553 | 329 | 196 | 116 | 69 | 41 | 24 | 80 |
| 1991 | 273 | 783 | 520 | 291 | 173 | 103 | 61 | 36 | 22 | 55 |
| 1992 | 321 | 215 | 536 | 297 | 166 | 99 | 59 | 35 | 21 | 44 |
| 1993 | 989 | 256 | 154 | 334 | 185 | 103 | 62 | 37 | 22 | 40 |
| 1994 | 466 | 793 | 189 | 103 | 223 | 124 | 69 | 41 | 24 | 41 |
| 1995 | 723 | 371 | 564 | 117 | 64 | 138 | 77 | 43 | 25 | 41 |
| 1996 | 1860 | 566 | 244 | 298 | 62 | 34 | 73 | 40 | 23 | 35 |
| 1997 | 459 | 1475 | 395 | 145 | 177 | 37 | 20 | 43 | 24 | 34 |
| 1998 | 501 | 362 | 997 | 220 | 81 | 99 | 20 | 11 | 24 | 32 |
| 1999 | 292 | 393 | 239 | 530 | 117 | 43 | 52 | 11 | 6 | 30 |
| 2000 | 433 | 231 | 269 | 137 | 304 | 67 | 25 | 30 | 6 | 21 |
| 2001 | 726 | 339 | 152 | 143 | 73 | 161 | 36 | 13 | 16 | 14 |
| 2002 | 592 | 572 | 229 | 84 | 79 | 40 | 89 | 20 | 7 | 17 |
| 2003 | 760 | 475 | 423 | 153 | 56 | 53 | 27 | 60 | 13 | 16 |
| 2004 | 1222 | 599 | 321 | 236 | 85 | 31 | 29 | 15 | 33 | 16 |
| 2005 | 324 | 959 | 397 | 172 | 127 | 46 | 17 | 16 | 8 | 27 |

$$N_{a+1,t+1} = N_{a,t}e^{-M}(1-u_{a,t})$$



$$N_{1,1} = R_0$$

| $N_{1,t+1}$ | = |
|--------------|----------|
| $f(S_t \mid$ | R_0, h |

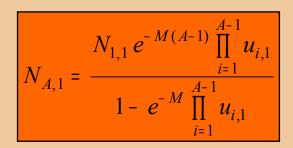
| | Age 1 | Age 2 | Age 3 | Age 4 | Age 5 | Age 6 | Age 7 | Age 8 | Age 9 | Age 10 |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| 1990 | 1000 | 793 | 553 | 329 | 196 | 116 | 69 | 41 | 24 | 80 |
| 1991 | 273 | 783 | 520 | 291 | 173 | 103 | 61 | 36 | 22 | 55 |
| 1992 | 321 | 215 | 536 | 297 | 166 | 99 | 59 | 35 | 21 | 44 |
| 1993 | 989 | 256 | 154 | 334 | 185 | 103 | 62 | 37 | 22 | 40 |
| 1994 | 466 | 793 | 189 | 103 | 223 | 124 | 69 | 41 | 24 | 41 |
| 1995 | 723 | 371 | 564 | 117 | 64 | 138 | 77 | 43 | 25 | 41 |
| 1996 | 1860 | 566 | 244 | 298 | 62 | 34 | 73 | 40 | 23 | 35 |
| 1997 | 459 | 1475 | 395 | 145 | 177 | 37 | 20 | 43 | 24 | 34 |
| 1998 | 501 | 362 | 997 | 220 | 81 | 99 | 20 | 11 | 24 | 32 |
| 1999 | 292 | 393 | 239 | 530 | 117 | 43 | 52 | 11 | 6 | 30 |
| 2000 | 433 | 231 | 269 | 137 | 304 | 67 | 25 | 30 | 6 | 21 |
| 2001 | 726 | 339 | 152 | 143 | 73 | 161 | 36 | 13 | 16 | 14 |
| 2002 | 592 | 572 | 229 | 84 | 79 | 40 | 89 | 20 | 7 | 17 |
| 2003 | 760 | 475 | 423 | 153 | 56 | 53 | 27 | 60 | 13 | 16 |
| 2004 | 1222 | 599 | 321 | 236 | 85 | 31 | 29 | 15 | 33 | 16 |
| 2005 | 324 | 959 | 397 | 172 | 127 | 46 | 17 | 16 | 8 | 27 |

$$N_{a+1,t+1} = N_{a,t}e^{-M}(1-u_{a,t})$$

Ago 2 | Ago 4 | Ago 5 | Ago 6 | Ago 7 | Ago 8 | Ago 9 | Ago 10

$$N_{1,1} = R_0$$

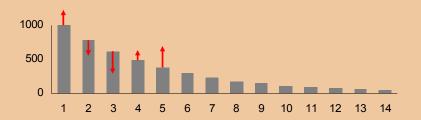
$$N_{a,1} = N_{1,1} e^{-M(a-1)} \prod_{i=1}^{a-1} (1 - u_{i,1})$$



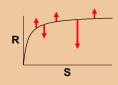


| | Age 1 | Age 2 | Age 3 | Age 4 | Age 5 | Age 6 | Age / | Age 8 | Age 9 | Age 10 |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| 1990 | 1000 | 793 | 553 | 329 | 196 | 116 | 69 | 41 | 24 | 80 |
| 1991 | 273 | 783 | 520 | 291 | 173 | 103 | 61 | 36 | 22 | 55 |
| 1992 | 321 | 215 | 536 | 297 | 166 | 99 | 59 | 35 | 21 | 44 |
| 1993 | 989 | 256 | 154 | 334 | 185 | 103 | 62 | 37 | 22 | 40 |
| 1994 | 466 | 793 | 189 | 103 | 223 | 124 | 69 | 41 | 24 | 41 |
| 1995 | 723 | 371 | 564 | 117 | 64 | 138 | 77 | 43 | 25 | 41 |
| 1996 | 1860 | 566 | 244 | 298 | 62 | 34 | 73 | 40 | 23 | 35 |
| 1997 | 459 | 1475 | 395 | 145 | 177 | 37 | 20 | 43 | 24 | 34 |
| 1998 | 501 | 362 | 997 | 220 | 81 | 99 | 20 | 11 | 24 | 32 |
| 1999 | 292 | 393 | 239 | 530 | 117 | 43 | 52 | 11 | 6 | 30 |
| 2000 | 433 | 231 | 269 | 137 | 304 | 67 | 25 | 30 | 6 | 21 |
| 2001 | 726 | 339 | 152 | 143 | 73 | 161 | 36 | 13 | 16 | 14 |
| 2002 | 592 | 572 | 229 | 84 | 79 | 40 | 89 | 20 | 7 | 17 |
| 2003 | 760 | 475 | 423 | 153 | 56 | 53 | 27 | 60 | 13 | 16 |
| 2004 | 1222 | 599 | 321 | 236 | 85 | 31 | 29 | 15 | 33 | 16 |
| 2005 | 324 | 959 | 397 | 172 | 127 | 46 | 17 | 16 | 8 | 27 |

$$N_{a+1,t+1} = N_{a,t}e^{-M}(1-u_{a,t})$$

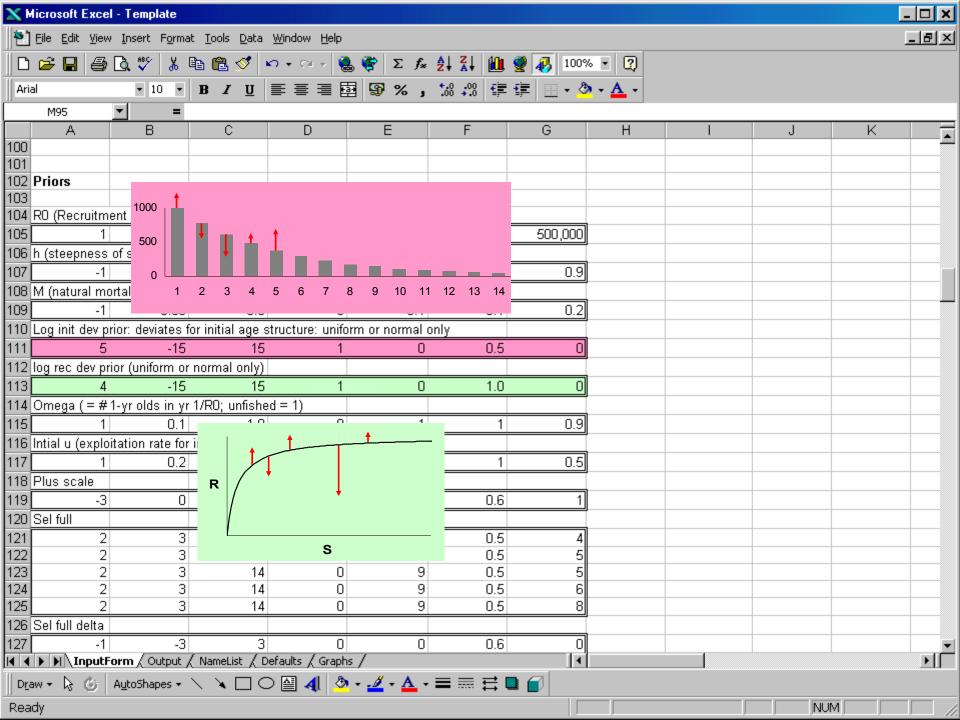


$$N_{1,1} = R_0$$



| | Age 1 | Age 2 | Age 3 | Age 4 | Age 5 | Age 6 | Age 7 | Age 8 | Age 9 | Age 10 |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| 1990 | 1000 | 793 | 553 | 329 | 196 | 116 | 69 | 41 | 24 | 80 |
| 1991 | 273 | 783 | 520 | 291 | 173 | 103 | 61 | 36 | 22 | 55 |
| 1992 | 321 | 215 | 536 | 297 | 166 | 99 | 59 | 35 | 21 | 44 |
| 1993 | 989 | 256 | 154 | 334 | 185 | 103 | 62 | 37 | 22 | 40 |
| 1994 | 466 | 793 | 189 | 103 | 223 | 124 | 69 | 41 | 24 | 41 |
| 1995 | 723 | 371 | 564 | 117 | 64 | 138 | 77 | 43 | 25 | 41 |
| 1996 | 1860 | 566 | 244 | 298 | 62 | 34 | 73 | 40 | 23 | 35 |
| 1997 | 459 | 1475 | 395 | 145 | 177 | 37 | 20 | 43 | 24 | 34 |
| 1998 | 501 | 362 | 997 | 220 | 81 | 99 | 20 | 11 | 24 | 32 |
| 1999 | 292 | 393 | 239 | 530 | 117 | 43 | 52 | 11 | 6 | 30 |
| 2000 | 433 | 231 | 269 | 137 | 304 | 67 | 25 | 30 | 6 | 21 |
| 2001 | 726 | 339 | 152 | 143 | 73 | 161 | 36 | 13 | 16 | 14 |
| 2002 | 592 | 572 | 229 | 84 | 79 | 40 | 89 | 20 | 7 | 17 |
| 2003 | 760 | 475 | 423 | 153 | 56 | 53 | 27 | 60 | 13 | 16 |
| 2004 | 1222 | 599 | 321 | 236 | 85 | 31 | 29 | 15 | 33 | 16 |
| 2005 | 324 | 959 | 397 | 172 | 127 | 46 | 17 | 16 | 8 | 27 |

$$N_{a+1,t+1} = N_{a,t}e^{-M}(1-u_{a,t})$$



Estimated parameters

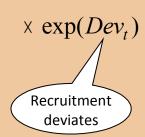


$$\times \exp(Dev_a)$$
 Initial deviates

$$N_{a,1} = N_{1,1} e^{-M(a-1)} \prod_{i=1}^{a-1} (1 - u_{i,1})$$

| λ/ - | $N_{1,1} e^{-M(A-1)} \prod_{i=1}^{A-1} u_{i,1}$ |
|-------------|---|
| $N_{A,1}$ = | $\frac{1 - e^{-M} \prod_{i=1}^{A-1} u_{i,1}}{1 - e^{-M} \prod_{i=1}^{A-1} u_{i,1}}$ |

$$N_{1,t+1} = f(S_t \mid R_0, h)$$

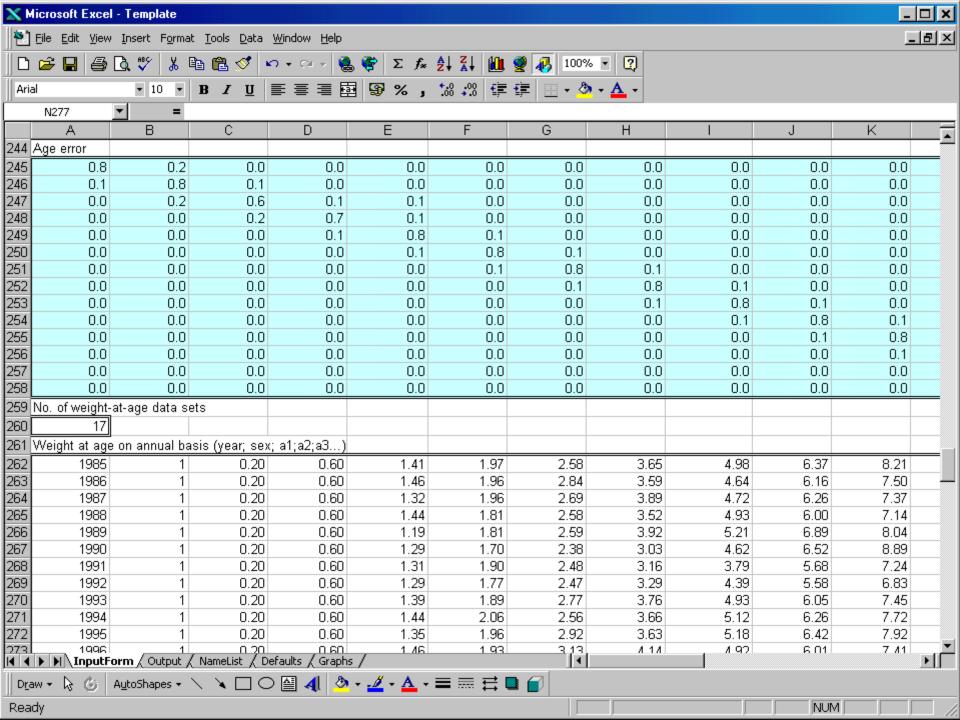


| | Age 1 | Age 2 | Age 3 | Age 4 | Age 5 | Age 6 | Age 7 | Age 8 | Age 9 | Age 10 |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| 1990 | 1000 | 793 | 553 | 329 | 196 | 116 | 69 | 41 | 24 | 80 |
| 1991 | 273 | 783 | 520 | 291 | 173 | 103 | 61 | 36 | 22 | 55 |
| 1992 | 321 | 215 | 536 | 297 | 166 | 99 | 59 | 35 | 21 | 44 |
| 1993 | 989 | 256 | 154 | 334 | 185 | 103 | 62 | 37 | 22 | 40 |
| 1994 | 466 | 793 | 189 | 103 | 223 | 124 | 69 | 41 | 24 | 41 |
| 1995 | 723 | 371 | 564 | 117 | 64 | 138 | 77 | 43 | 25 | 41 |
| 1996 | 1860 | 566 | 244 | 298 | 62 | 34 | 73 | 40 | 23 | 35 |
| 1997 | 459 | 1475 | 395 | 145 | 177 | 37 | 20 | 43 | 24 | 34 |
| 1998 | 501 | 362 | 997 | 220 | 81 | 99 | 20 | 11 | 24 | 32 |
| 1999 | 292 | 393 | 239 | 530 | 117 | 43 | 52 | 11 | 6 | 30 |
| 2000 | 433 | 231 | 269 | 137 | 304 | 67 | 25 | 30 | 6 | 21 |
| 2001 | 726 | 339 | 152 | 143 | 73 | 161 | 36 | 13 | 16 | 14 |
| 2002 | 592 | 572 | 229 | 84 | 79 | 40 | 89 | 20 | 7 | 17 |
| 2003 | 760 | 475 | 423 | 153 | 56 | 53 | 27 | 60 | 13 | 16 |
| 2004 | 1222 | 599 | 321 | 236 | 85 | 31 | 29 | 15 | 33 | 16 |
| 2005 | 324 | 959 | 397 | 172 | 127 | 46 | 17 | 16 | 8 | 27 |

$$\hat{I}_t = q\hat{B}_t$$
Survey/CPUE catchability

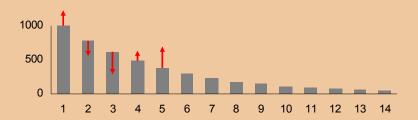
Selectivity parameters: Sleft, Sfull, Sright

$$N_{a+1,t+1} = N_{a,t}e^{-M}(1-u_{a,t})$$
 $u_{a,t} = u_t s_{a,t}$

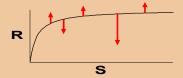


Confronting uncertainty

Initial process error



Recruitment process error



Ageing observation error

| 0.8 | 0.2 | 0.0 | 0.0 |
|-----|-----|-----|-----|
| 0.1 | 0.8 | 0.1 | 0.0 |
| 0.0 | 0.2 | 0.6 | 0.1 |
| 0.0 | 0.0 | 0.2 | 0.7 |
| 0.0 | 0.0 | 0.0 | 0.1 |
| 0.0 | 0.0 | 0.0 | 0.0 |

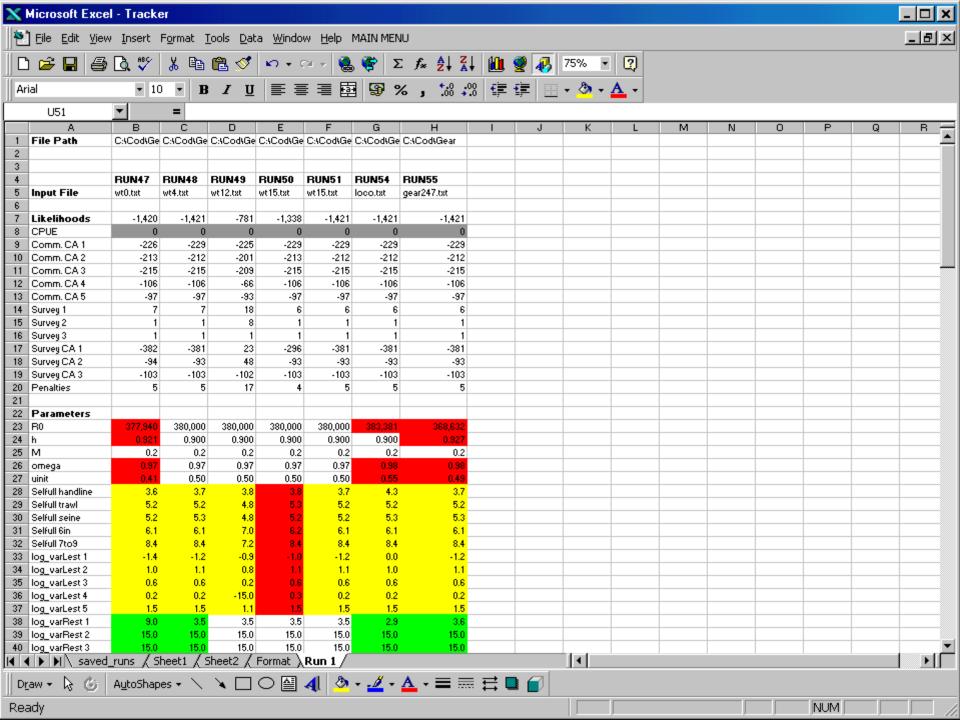
Likelihood function with log-normal error structure

$$\ln L_I^g = \sum_t \ln \left[\exp \left(-0.5 \frac{I^g \varepsilon_t^2}{I^g \sigma_t^2} \right) + 0.01 \right]$$

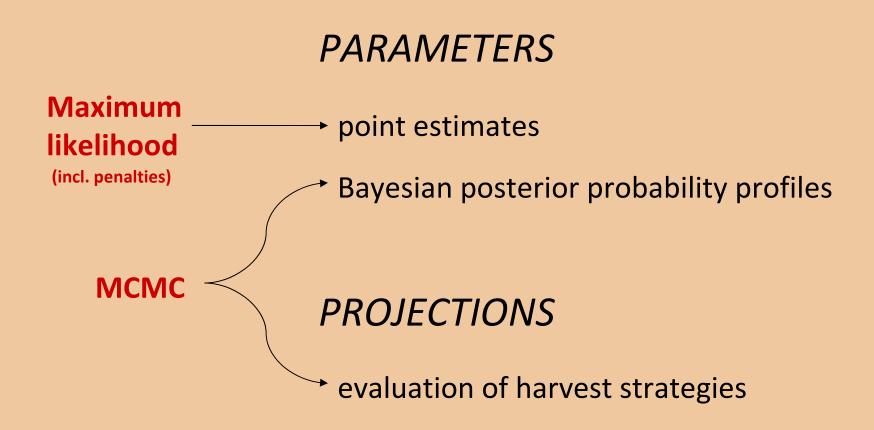
Model output

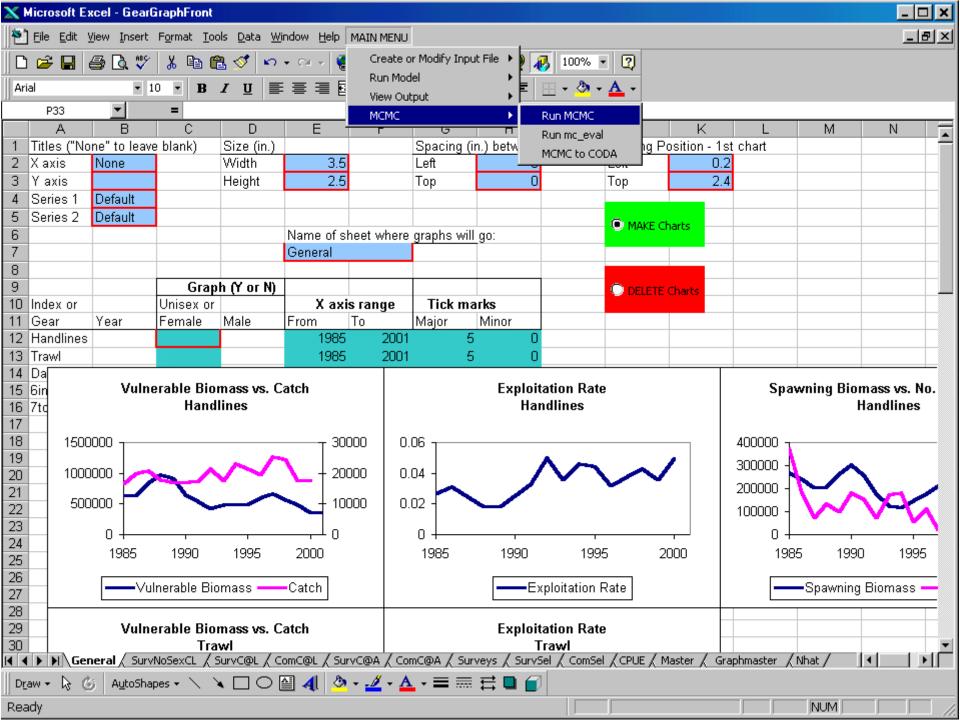
PARAMETERS

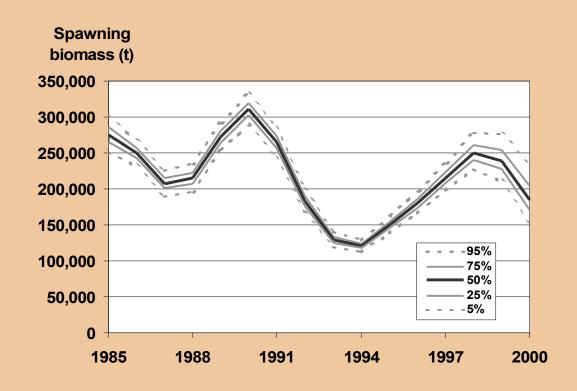
point estimates

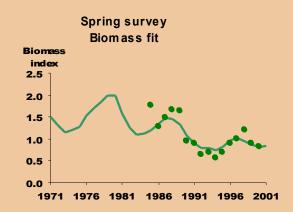


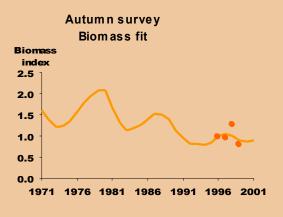
Model output

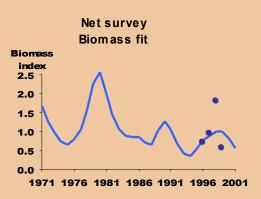


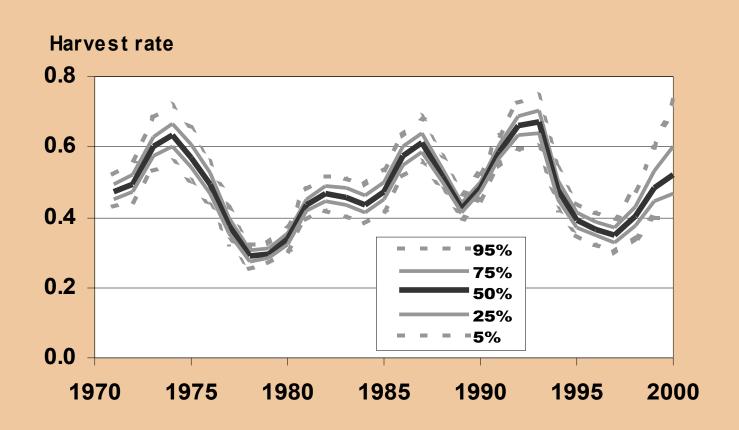


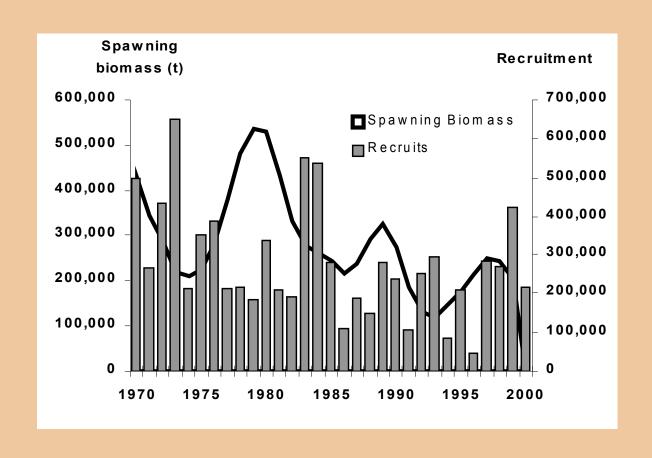




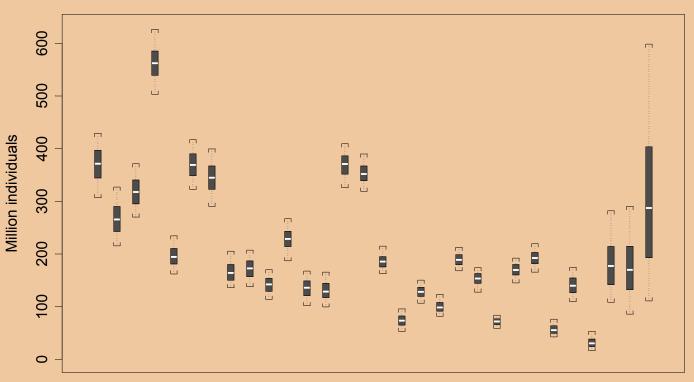








Recruitment (3 yr old) distribution: Percentiles from MCMC runs



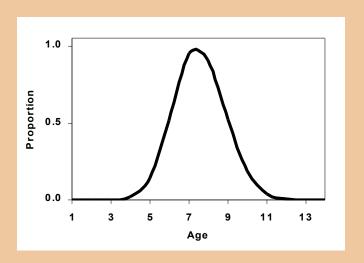
707172737475767778798081828384858687888990919293949596979899

Brood Year

spring survey selectivity



net survey selectivity



Coleraine

Homepage

http://www.fish.washington.edu/research/coleraine

User manual

http://www.fish.washington.edu/Publications/pdfs/0116.pdf

(Hilborn et al. 2001, UW School Aquat. Fish. Sci. Rep.)