Technical description of Stock Synthesis assessment program

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Contents

The Stock Synthesis 3 (SS3) assessment program provides a statistical framework for calibration of a population dynamics model using a diversity of fishery and survey data. SS is designed to deal with both age- and size-structure with multiple stock sub-areas and multiple growth patterns. The description here details the most commonly applied features, along with a subset of the more advanced options offered by SS3.

1 Population Model

The factors described here are those that control the rate at which new individuals recruit to the population each time step; the rate at which they die due to fishing and natural mortality; and the rate at which they grow and contribute to the total biomass and reproductive potential of the stock. The total population can be divided among one to many entities. The total of all entities born within a year are referred to as a year-class or cohort. Each of the biologically- or birthseason- delineated entities is referred to as a morph. In addition, each morph can be sub-divided into slow-, medium-, and fast-growing entities termed platoons **(Goodyear 1996; Taylor and Methot 2012)**. The model description here does not include subscripting for morphs or platoons in an attempt for simplicity, but each of these entities is tracked in the population dynamics and biology if the user chooses to invoke these features. Each cohort/morph/platoon is split into males and females if the user invokes a two-sex configuration, and the subscript for gender is included in the description below.

1.1 Initial Numbers-at-Age

The population in the initial year of a SS application can be simply an unfished equilibrium population, a population in equilibrium with an estimated mortality rate that is influenced by data on historical equilibrium catch, or an equilibrium population that has estimable age-specific deviations from this equilibrium for a user-specified number of the younger ages.

The numbers of animals of gender γ in age group a in a virgin state ($y = 0$) is:

$$N_{0,\gamma,a} = \begin{cases} cR_0 e^{-aM_{\gamma,a}} & \text{for } a = 0 \text{ to } A-1 \\ \sum_{a=A}^{3A-1} N_{0,\gamma,a} + \frac{N_{0,\gamma,3A-1} e^{-M_{\gamma,a}}}{1-e^{-M_{\gamma,a}}} & \text{for } a = A \text{ to } 3A-1 \end{cases} \tag{1}$$

where c (I)[1] is a user-defined constant that determines the sex-ratio of recruits[2], $M_{\gamma,a}$ (P)[3] is natural mortality for age a and sex γ, A is the plus-group age, $3A$ is three times the plusgroup age, and R_0 is the number of age-0 fish at unfished equilibrium. The plus group virgin numbers-at-age calculation is based on 3 timex the maximum age to include movement dynamics through age $3A-1$. After calculating the numbers-at-age through age $3A$, the numbers are collapsed to age A for subsequent calculations. Equation 1 use total mortality, $Z_{\gamma,a}$ (see Equation 1.21 XX), rather than $M_{\gamma,a}$ when the initial equilibrium also involves fishing mortality. Although this fishing mortality will reduce spawning biomass, no adjustment to R_0 is made on premise that this reduction has probably not been occurring for enough years to effect this change. R_0 serves as both the starting level of mean recruitment and as the factor that scales the mean spawner-recruitment relationship against which future annual recruitment deviations will act. An estimated offset, R_1, can be applied to R_0. When the initial population involves agespecific deviations, these deviations are an extension of the zero-centered, lognormal recruitment deviations applied to the equilibrium numbers-at-age (see Equation 1.20 XX).

1.2 Recruitment

The number of age-0 fish is related to spawning biomass according to a stock-recruitment relationship. A range of stock-recruitment relationships are available. Here, the Beverton-Holt is described:

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[1] User-specified quantities will henceforth be denoted as "(I)".

[2] The term "recruits" is used to refer to age-0 animals.

[3] Estimated quantities will henceforth be denoted as "(P)".

$$R_y = \frac{4hR_0SB_y}{SB_0(1-h)+SB_y(5h-1)}e^{-0.5b_y\sigma_R^2+\tilde{R}_y} \quad \tilde{R}_y \sim N(0;\sigma_R^2) \tag{2}$$

where R_0 (P) is the unfished equilibrium recruitment, SB_0 is the unfished equilibrium spawning biomass (or spawning output) corresponding to R_0, SB_y is the spawning biomass at the start of the spawning season during year y, h (P) is the steepness parameter, b_y (I) is the bias adjustment fraction applied during year y, σ_R (P) is the standard deviation among recruitment deviations in log space, and $\tilde{R}_y$ (P) is the lognormal recruitment deviation for year y. The bias-adjustment factor **(Methot and Taylor 2011)** ensures unbiased estimation of mean recruitment even during data-poor eras in which the maximum likelihood estimate of the $\tilde{R}_y$ is near 0.0.

The annual bias-adjustment fraction by b_y is the piecewise linear function:

$$b_y = \begin{cases} 0 & \text{for } y \leq y_1^b \\ b_{\max}(1-\frac{y-y_1^b}{y_2^b-y_1^b}) & \text{for } y_1^b < y < y_2^b \\ b_{\max} & \text{for } y_2^b \leq y \leq y_3^b \\ b_{\max}(1-\frac{y_3^b-y}{y_4^b-y_3^b}) & \text{for } y_3^b < y < y_4^b \\ 0 & \text{for } y_4^b \leq y \end{cases} \tag{3}$$

where y_1^b (I) is the first year of the bias ramp up adjustment period, y_2^b (I) is the last year of the bias ramp up adjustment period, y_3^b (I) is the first year of the bias ramp down adjustment period, y_4^b (I) is the last year of the bias ramp down adjustment period, and $b_{\max}$ (I) is the maximum bias adjustment applied to recruitment deviations.

The total annual recruitment can be partitioned among growth morphs and birth seasons and areas according to a design matrix. Each of these entities can be further divided into males and females according to a pre-specified fraction. Finally, each of these entities can be further subdivided into platoons that will have slow, medium or large size-at-age relative to the average size-at-age for the overall morph. For morphs that are designated to recruit in a season after the spawning season, their age 0 for the purposes of growth occurs at the start of that season. Thus, they will have smaller size-at-age relative to morphs of that annual cohort that are born earlier, but will grow towards the same L_∞.

1.2.1 Additional Stock Recruitment Relationship Functional Forms

1.3 Natural Mortality

Natural mortality can take several alternative forms, including age-specific and or sex-specific. Further, natural mortality parameters, in common with growth parameters, can be time-varying or functions of environmental inputs. The most basic and simple form of natural mortality is:

$$M_{\gamma,a} = \text{constant} \tag{4}$$

where the natural mortality rate is constant across ages a beginning at age 0 and equal for sexes γ.

1.3.1 Additional Natural Mortality Functional Forms

1.4 Initial Growth

Growth follows the von Bertalanffy function as re-formulated by **Schnute (1981)**, or by the Richards equation which has an option for a 3rd parameter to govern growth **(ADD**

REFERENCE). Growth is sex-specific. SS3 also allows for additional morphs with different growth patterns.

Mean size-at-age is calculated from growth parameters at the start of the initial year. The mean size-at-age of each morph is progressed forward according to the growth parameters active during that time period for subsequent seasons within that year and for subsequent years.

The sex-specific size-at-age in the initial population using the von Bertalanffy growth function is calculated as:

$$L_{0,\gamma,a} = \begin{cases} L_{\text{min bin}} + ba & \text{for } a \leq a_3 \\ L_{\infty,\gamma} + (L_{1,\gamma} - L_{\infty,\gamma})e^{-k_\gamma(a-a_3)} & \text{for } a > a_3 \text{ to } a = A-1 \end{cases} \tag{5}$$

where $L_{\text{min bin}}$ is the lower limite of the first population bin, b is the linear slope of growth for $a \leq a_3$ calculated as:

1.5 Growth

1.6 Variation in Size-at-Age

1.7 Age-Length Population Structure

1.8 Body Weight

1.9 Maturity and Fecundity

1.10 Population with Fishing Mortality

1.11 Selectivity

1.12 Retention

2 Observation Model

2.1 Survey Observation

2.2 Abundance Indices

2.3 Composition Data

2.3.1 Length Compositions

2.3.2 Age Compositions

3 Statistical Model

3.1 Likelihood Components

3.2 Recruitment Deviaitons

3.3 Parameter Priors

3.4 Parameter Deviations

3.5 Crash Penalties

4 Management Quantities

4.1 Reference Points

4.2 Forecast

4.2.1 U.S. West Coast Groundfish Control Rule

4.2.2 U.S. Alaska Control Rule

5 Advanced Model Options

6 References