Equations

Page 76 Eqn. 1: apportionment by area i is equal to the exponential of proportions of recruits to area i divided by the aggregate sum of each of the exponential or proportions of recruits by each area i.

Page 79 Eqn. 2: the rate of movement by area i is equal to the exponential of movement rate for area i divided by the aggregate sum of each of the exponential of movement rates by each area i.

Page 87 Eqn. 3: the expected catch is equal to the ratio of the observed catch to the catch multiplier.

Page 97 Eqn. 4: the seasonal parameter is equal to the input parameter P multiplied by the exponent of the season value.

Page 98 Eqn. 5: the recruits in year y is equal to the product of 4 times steepness times unfished equilibrium recruitment times spawning biomass in year y divided by unfished spawning biomass in equilibrium times open parenthesis 1 minus steepness close parenthesis plus spawning biomass in year y times open parenthesis 5 time steepness minus 1 closed parenthesis. Variation in recruitment generated by multiplying that ratio by the exponential of open parenthesis negative 0.5 time bias in year y times the recruitment sigma squared plus the recruitment variation in year y closed parenthesis. The recruitment variation by year is distributed normally zero centered with standard deviation of the recruitment sigma squared.

Page 99 Eqn. 6: the recruits in year y are equal to the product of unfished recruitment in equilibrium times spawning biomass in year y divided by unfished spawning biomass in equilibrium all multiplied by the exponential of steepness times open parenthesis 1 minus spawning biomass in year y divided by the unfished spawning biomass in equilibrium closed parenthesis times the exponential of negative 0.5 times bias adjustment in year y times recruitment sigma squared plus recruitment variation in year y. The recruitment variation by year is distributed normally zero centered with standard deviation of the recruitment sigma squared.

Page 99 Eqn. 7: the recruits in year y are equal to the joiner function times open parenthesis the minimum recruitment level predicted at a spawning biomass of zero times the unfished recruitment in equilibrium plus the unfished recruitment in equilibrium times spawning biomass in year y divided by steepness times unfished spawning biomass in equilibrium multiplied by open parenthesis 1 minus minimum recruitment closed parenthesis closed parenthesis plus unfished recruitment in equilibrium times open parenthesis 1 minus the joiner function closed parenthesis times exponential of negative 0.5 times bias adjustment in year y times recruitment sigma squared plus recruitment variation in year y. The recruitment variation by year is distributed normally zero centered with standard deviation of the recruitment sigma squared.

Page 99 Eqn. 8: the joiner function is equal to open parenthesis 1 plus the exponential of 1000 plus open parenthesis unfished spawning biomass in equilibrium minus steepness times unfished spawning biomass in equilibrium closed parenthesis divided by unfished spawning biomass in equilibrium closed parenthesis all raised to negative 1.

Page 99 Eqn. 9: z-naught equals negative natural logarithm of survival-naught.

Page 99 Eqn. 10: z-min equals the negative natural logarithm of survival-max which equals the z-naught times open parenthesis 1 minus z-frac.

Page 100 Eqn. 11: recruits in year y equal the spawning biomass in year y times the exponential of open parenthesis negative z-naught plus open parenthesis z-naught minus z-min closed parenthesis times open parenthesis 1 minus open parenthesis spawning biomass in year y divided by unfished spawning biomass in equilibrium closed parenthesis raised to beta closed parenthesis times the exponential of recruitment deviation in year y. The recruitment variation in year y is distributed normally zero centered with standard deviation of recruitment sigma squared.

Page 100 Eqn. 12: z-frac is equal to the natural logarithm of open parenthesis survival-max closed parenthesis minus the natural logarithm of open parenthesis survival-naught closed parenthesis both divided by the negative natural logarithm open parenthesis survival-naught closed parenthesis. This is approximately equal to survival-max minus survival-naught divided by 1 minus survival-naught.

Page 101 Eqn. 13: steepness is equal to 0.2 times the exponential of z-naught times z-frac time open parenthesis 1 minus 0.2 raised to beta closed parenthesis. This is less than 0.2 times the exponential of z-naught which is equal to 1 divided by 5 times survival-naught which is equal to unfished spawning biomass in equilibrium divided by 5 times unfished recruitment in equilibrium.

Page 101 Eqn. 14: recruits in year y is equal to open parenthesis spawning biomass in year y divided by unfished spawning recruitment in year y closed parenthesis times the product of 5 times steepness adjusted time unfished recruitment in equilibrium times open parenthesis 1 minus 0.2 raised to c closed parenthesis all divided by open parenthesis 1 minus 5 times steepness adjusted times 0.2 raised to c closed parenthesis plus open parenthesis 5 times steepness adjusted minus 1 closed parenthesis times spawning biomass in year y divided by unfished spawning biomass in equilibrium closed parenthesis raised to c times exponential of negative 0.5 times bias adjustment in year y times recruitment sigma squared plus recruitment variation in year y. The recruitment variation by year is distributed normally zero centered with standard deviation of the recruitment sigma squared.

Page 101 Eqn. 15: steepness adjusted is equal to 0.2 plus open parenthesis steepness minus 0.2 all divided by 0.8 closed parenthesis times open parenthesis 1 divided by 5 times 0.2 raised to c all minus 0.2 closed parenthesis.

Page 101 Eqn. 16: recruits in year y is equal to unfished recruitment in equilibrium times open parenthesis 1 minus temp parameter closed parenthesis times the exponential of natural logarithm open parenthesis 5 times steepness closed parenthesis times open parenthesis 1 minus spawning biomass in year y divided by unfished spawning biomass in equilibrium closed parenthesis raise to gamma all divided by 0.8 raised to gamma.

Page 101 Eqn. 17: temp is equal to 1 minus spawning biomass in year y divided by unfished spawning biomass in equilibrium if 1 minus spawning biomass in year y divided by unfished spawning biomass in equilibrium is greater than 1 and equal to 0.001 if 1 minus spawning biomass in year y divided by unfished spawning biomass in equilibrium is less than or equal to 0.

Page 108 Eqn. 18: the standard error of the variance in recruitment by year y squared plus the standard deviation of variance in recruitment squared is equal to open parenthesis open parenthesis 1 divided by sigma-d squared plus 1 divided by sigma-R squared closed parenthesis raised to the negative ½ closed parenthesis raised to 2 plus open parenthesis sigma-R squared divided by open parenthesis sigma-R squared plus sigma-d squared closed parenthesis raised to ½ closed parenthesis raised to 2 which is equal to sigma-R squared.

Page 109 Eqn. 19: the bias in year y is equal to the expected value of open parenthesis standard deviation of variance in recruitment in year y closed parenthesis raised to 2 which is equal to 1 minus the standard error of the variance in recruitment in year y squared divided by sigma-R squared.

Page 109 Eqn. 20: x is equal to the sum of the natural exponential of each parameter p, y is equal to the N-cycles, and the penalty is equal to 100000 times open parenthesis x – y closed parenthesis squared.

Page 123 Eqn. 21: selectivity at length l is equal to 1 divided by 1 plus natural exponential of open parenthesis negative natural logarithm of 19 times open parenthesis length bin l minus p1 closed parenthesis divided by p2 closed parenthesis.

Page 124 Eqn. 22: temp is equal to 9 minus the maximum value of p at age a. The selectivity at age a is equal to 1 divided by 1 plus the natural exponential of open parenthesis negative open parenthesis p at age a plus 1 plus temp closed parenthesis closed parenthesis.

Page 124 Eqn. 23: selectivity at age a is equal to the natural exponential of open parenthesis s prime at age a minus s prime max closed parenthesis.

Page 124 Eqn. 24: s prime at age a is equal to the sum from i equal Amin to a of p at i.

Page 124 Eqn. 25: s prime max is equal to maximum value of s prime at age a.

Page 128 Eqn. 26: peak is equal to the minimum of the length bins plus p2 times open parenthesis maximum of the length bins minus the minimum of the length bins closed parenthesis.

Page 128 Eqn. 27: selectivity at length l is equal to the natural exponential of open parenthesis p3 times p1 time open parenthesis peak minus the length bin l closed parenthesis closed parenthesis divided by open parenthesis 1 minus p3 times open parenthesis 1 minus natural exponential of p1 times open parenthesis peak minus length bin l closed parenthesis closed parenthesis

Page 134 Eqn. 28: retention at length l is equal to open parenthesis p3 divided by 1 plus natural exponential of negative open parenthesis length bin l minus open parenthesis p1 plus p4 time male offset closed parenthesis closed parenthesis divided by p2 closed parenthesis times open parenthesis 1 minus the ratio of 1 divided by 1 plus the natural exponential of negative open parenthesis length bin l minus open parenthesis p5 plus p7 times male offset closed parenthesis closed parenthesis divided by p6 closed parenthesis.

Page 134 Eqn. 29: discard mortality at length l is equal to 1 minus the ratio of 1 minus p3 divided by 1 plus the natural exponential of negative open parenthesis length bin l minus open parenthesis p1 plus p4 times male offset closed parenthesis closed parenthesis divided by p2.

Page 139 Eqn. 30: the tagging report rate is equal to the natural exponential of the input parameter divided by 1 plus the natural exponential of the input parameter

Page 149 Eqn. 31: the total like is equal to the sum across objective function components i sum across fleets f of the weight by component and fleet times the likelihood value for component i by fleet f plus weight of recruitment times the likelihood of recruitment plus the sum of all prior weights times the likelihood of the priors plus the sum of all penalty weights times the likelihood of penalties.

Page 180 Eqn. 32: temp is equal to negative 0.5 times the natural exponential of open parenthesis ratio of the lower bound of parameter p minus the upper bound of parameter p plus 0.0000002 all divided by the parameter value minus the lower bound of the parameter p plus 0.0000001 all subtract 1 close parenthesis.

Page 180 Eqn. 33: temp1 is equal to the lower bound of the parameter p plus the ratio of the upper bound of the parameter p minus lower bound of parameter p divided by 1 plus the natural exponential of negative 2 times temp

Page 180 Eqn. 34: selectivity at age and time is equal to selectivity at age times the natural exponential of two-dimensional first-order autoregressive parameter by age and time.

Page 180 Eqn. 35: the vector of two-dimensional first-order autoregressive parameter is multivariate normal distributed zero centered with the standard deviation of sigma-S squared R-total covariance matrix.

Page 180 Eqn. 36: The R-total is equal to the kronecker product of the two correlation matrices for among-age, R, and among-year processes, R-tilde, auto-regressive processes.

Page 181 Eqn. 37: R subscript a and a-tilde is equal to rho subscript a raised to the absolute value of a minus a-tilde.

Page 181 Eqn. 38: R-tilde subscript t and t-tilde is equal to rho of t raised to the absolute value of t minus t-tilde.

Page 181 Eqn. 39: the selectivity deviation subscript a and t is normal distributed zero center with standard deviation of sigma-S squared.

Page 181 Eqn. 40: sigma-s squared is equal to the standard deviation of the selectivity deviation squared plus 1 divided by open parenthesis a maximum minus a minimum plus 1 closed parenthesis times open parenthesis t maximum minus t minimum plus 1 closed parenthesis all multiplied by the total sum across a values the total sum across t values of the squared value of the standard error of the selectivity deviation subscript a,t

Page 181 Eqn. 41: b is equal to 1 minus 1 divided by open parenthesis a maximum minus a minimum plus 1 closed parenthesis times open parenthesis t maximum minus t minimum plus 1 closed parenthesis. That ratio is multiplied by the sum of a and sum of t standard error of the selectivity deviation subscript a, t squared. This product is divided by sigma-S squared.

Page 186 Eqn. 42: mu is equal to the negative of the prior standard deviation times the natural logarithm of open parenthesis the upper bound value of the parameter plus the parameter lower bound divided by 2 with minus the parameter lower bound closed parenthesis minus the parameter standard deviation time the natural logarithm of 0.50.

Page 187 Eqn. 43: the prior likelihood is equal to negative mu minus parameter standard deviation times the natural logarithm of open parenthesis the parameter value minus the parameter lower bound plus 0.0001 closed parenthesis minus the parameter standard deviation times the natural logarithm open parenthesis 1 minus the ratio of the parameter value minus the parameter lower bound minus 0.0001 all divided by the parameter upper bound minus the parameter lower bound closed parenthesis.

Page 187 Eqn. 44: mu is equal to the ratio of the parameter prior minus the parameter lower bound all divided by the parameter upper bound minus the parameter lower bound.

Page 187 Eqn. 45: tau is equal to the ratio of open parenthesis the parameter prior minus the parameter lower bound closed parenthesis times open parenthesis the parameter upper bound minus the parameter prior closed parenthesis divided by the parameter standard deviation squared all minus 1.

Page 187 Eqn. 46: the beta prior parameter is equal to tau times mu.

Page 187 Eqn. 47: the alpha prior parameter is equal to tau time open parenthesis 1 minus mu closed parenthesis.

Page 188 Eqn. 48: the prior likelihood is equal to open parenthesis 1 minus beta prior parameters closed parenthesis times the natural logarithm open parenthesis 0.0001 plus the parameter minus the parameter lower bound closed parenthesis plus open parenthesis 1 minus alpha prior parameter times natural logarithm open parenthesis 0.0001 plus the parameter upper bound minus the parameter closed parenthesis minus open parenthesis 1 minus beta prior parameter closed parenthesis times natural logarithm open parenthesis 0.0001 plus parameter prior minus parameter lower bound closed parenthesis minus open parenthesis 1 minus alpha prior parameter closed parenthesis times natural logarithm open parenthesis 0.0001 plus parameter upper bound minus parameter prior closed parenthesis.

Page 188 Eqn. 49: the prior likelihood is equal to 1 divided by 2 times open parenthesis natural logarithm of the parameter minus the parameter prior all divided by the parameter prior standard deviation closed parenthesis squared.

Page 188 Eqn. 50: the prior likelihood is equal to 1 divided by 2 times open parenthesis natural logarithm of the parameter minus the parameter prior plus 1 divided by 2 times the parameter prior standard deviation squared all divided by parameter prior standard deviation closed parenthesis squared.

Page 188 Eqn. 51: the scale is equal to the parameter prior standard deviation squared divided by the parameter prior.

Page 188 Eqn. 52: the shape is equal to the parameter prior divided by the scale.

Page 188 Eqn. 53: the prior likelihood is equal to the negative shape value times natural logarithm of the scale minus the natural logarithm of the gamma function of the shape plus open parenthesis shape minus 1 closed parenthesis times the natural logarithm of the parameter minus the ratio of the parameter to the scale.

Page 188 Eqn. 54: the prior likelihood is equal to 1 divided by 2 times open parenthesis parameter minus the parameter prior all divided by the parameter prior standard deviation closed parenthesis squared.

Page 210 Eqn. 55: the expected recruitment is equal to the function of spawning biomass times the natural exponential of open parenthesis beta times the environmental data closed parenthesis times the natural exponential open parenthesis negative 0.5 times sigma-R squared closed parenthesis.

Figure 1. (page 185-jitter)  Illustration of the jitter algorithm. The initial parameter value "Input init" is shown on the x-axis and is transformed by a cumulative normal distribution with 0.001 and 0.999 quantiles set to the "Min" and "Max" parameter bounds. This distribution is illustrated by a black curve with a sigmoidal shape increasing from the lower left (with x, y values of Min, 0.001) to the upper right (with values Max, 0.999). The transformed initial value is about 0.17 on the y-axis. A uniform distribution in the transformed cumulative normal distribution space is used to sample a new jittered initial value and is represented by a histogram on the y-axis. This uniform distribution has bounds determined by the transformed initial value plus or minus the "jitter" input, which in this example is 0.2, resulting in a range of about -0.03 to 0.37 centered around the transformed initial value of 0.17. A second histogram on the x-axis shows how these jittered samples are mapped back into the parameter space. This second histogram shows a skewed distribution, where the mode is larger than "Input init" but the lower tail is longer reflecting the same number of values spread over a larger range. This distribution of back-transformed values shows a spike in the lower tail associated with values from the uniform distribution below 0.001 that get mapped back to one-tenth of the way from the bound to the original initial value. In addition to the histograms showing the distribution of values , the figure shows a single realization using red arrows, where the 0.17 transformed value is jittered to 0.06 by a uniform random value which is then mapped back to a "New init" value partway between the Min value and the "Input init" value.

Figure 2. (page 187-beta prior) The shape of the symmetric beta prior across alternative standard deviation values and the change in the negative log-likelihood. The shape of symmetric beta prior is relatively flat across values with limited increases in the change in the negative log-likelihood at the low and high parameter bounds if the standard deviation value is a low value (>0.5). As the standard deviation of the distribution increases more of the distribution weight is shifted to the center of the parameter bounds, with sharp increases in the negative log-likelihood value at the lower and upper bounds.

Figure 3. (page 196 Input vs Eff N) The relationship between the observed sample size (the input sample number) versus the effective sample size for the McAllister-Ianelli data-weighting approach. The effective sample size is the product of the input sample size and the data weighting applied to the data set. The harmonic mean of the effective sample sizes is a horizontal line with the arithmetic mean of the observed sample sizes is a vertical line. The relationship between the observed and effective sample size for each year is indicated on the figure. Tuning these data using the McAllister-Ianelli data-weighting approach will result in the one-to-one line between the observed and effective sample size will cross the harmonic and arithmetic mean lines where they intersect.

Figure 4. The mean length of the length samples for each year from Fleet 1 (called the MexCal S1 NSP fleet) with 95% confidence intervals based on current samples sizes using the Francis data weighting method (referred to as TA1.8). Thinner intervals with capped ends show result of further adjusting sample sizes based on suggested multiplier, 0.2739, with 95% interval (0.1661-0.6305) for length data from Fleet 1. The blue line shows the expected variation of the mean length across years.

Figure 5. Spawner-recruitment relationship for darkblotched rockfish (Gertseva and Thorson, 2013). Red points represent estimated recruitments, the solid black line is the stock-recruit relationship and the green line represents the adjustment to this relationship after adjustment to account for the lognormal distribution associated with each year. The "+" symbol labeled 1915 near the right side represents both the virgin and initial equilibrium of the model. The numerous red points close to the initial conditions correspond to the early years of the model with low harvest rates. The estimates of annual recruitment in the later years of the model, where composition data are informative, are distributed above and below the stock-recruit relationship. The highest estimated recruitment years are 1981, 1995, and 2000 and the lowest estimated recruitments occurred in 1993, 1998, and 2001.

Figure 6. Time series of log recruitment deviations for darkblotched rockfish with 95% uncertainty intervals. The start year of the model is 1915, but recruitment deviations are estimated starting in 1870. The 45 deviation estimates for 1870–1914 inform the age structure used in the start year. The black color for the years 1960–2011 indicates the "main" recruitment deviation vector, while the blue color for the years 1870–1959 and 2012–2024 indicates the "early" and "late/forecast" recruitment deviation vectors, respectively.

Figure 7. Time series of standard error estimates for the log recruitment deviations for darkblotched rockfish with 95% uncertainty intervals. As in Figure 10, the black color indicates the main recruitment period. This period with lower standard error is associated with higher variability among deviations (Figure 10). The red line at 0.75 indicates the value in this model.

Figure 8. Transformation of the standard error estimates for darkblotched rockfish following the approach suggested by Methot et al. (2011). These values were used to set the 5 values controlling the degree of bias adjustment (as a fraction of sigmaR squared) to account for differences in the mean and median of the lognormal distribution from which the recruitment deviations are drawn. The red line indicates a bias adjustment of 0 up to the 1960.75, ramping up to a maximum adjustment level of 0.877 for the period 1990.4–2008.98, and reducing back to 0 starting in 2013.08. Note that these values controlling the bias adjustment need not be integer year values. Also the break points in the bias adjustment function need not match the break points between early, main, and late/forecast recruitment deviation vectors. The blue line indicates a functional form that minimizes the sum of squared differences between the bias adjustment function and the transformed standard error values. The subtle differences between red and blue lines are unlikely to have any appreciable effect on the model results. The value by year of the bias adjustment within the model compared the estimated alternative bias adjustment. The bias adjustment should be low, or at zero, in early years of the model with little to no data and then gradually ramp up when data begin to become informative reaching a maximum plateau value when the most data are available and then decline back to zero at the end of the model.

Figure 9. Comparison of time series of spawning depletion for darkblotched rockfish models with early recruitment deviations (starting in 1870) and without early deviations (only main recruitment deviations starting in 1960). The point estimates are similar, but the 95% uncertainty intervals are substantially different. With no recruitment deviations for the early period, the estimates of spawning depletion in the early years are very precise and uncertainty increases as the stock moves into the data rich period. In contrast, the addition of the early recruitment deviations results in a large uncertainty in spawning depletion for the early years and an increase in precision as the stock moves into the data rich period. In this application, the uncertainty associated with the recent years is independent of the assumptions about early recruitments.