Likelihood

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Statistical Modeling in R
Universidad de Concepción
19–23 January 2026

Outline

Likelihood

relative probability, support, combine data sources

Estimation

MLE, log likelihood, confidence interval

Normal distribution

N(μ, σ)*, dnorm*

Profile likelihood

procedure, uncertainty

Likelihood concepts

Relative probability

$$P(A_1) = 0.5 \qquad P(A_2) = 0.3 \qquad P(A_3) = 0.2$$

$$L(A_1) = 500 \qquad L(A_2) = 300 \qquad L(A_3) = 200$$

$$L(A_1) = 0.005 \qquad L(A_2) = 0.003 \qquad L(A_3) = 0.002$$

Likelihood concepts

Expresses how well the data **support** some parameter value
or hypothesis

$$L\left(\theta \middle| \text{data}\right)$$

Like *RSS* but even more useful:

not only point estimate, but also **uncertainty**

Likelihood concepts

We can fit a model to many types of data at once and
combine the likelihood components with simple multiplication

$$L \quad = \quad L_1 \;\times\; L_2 \;\times\; \cdots$$

Unified framework, for simple or complex models

Likelihood concepts

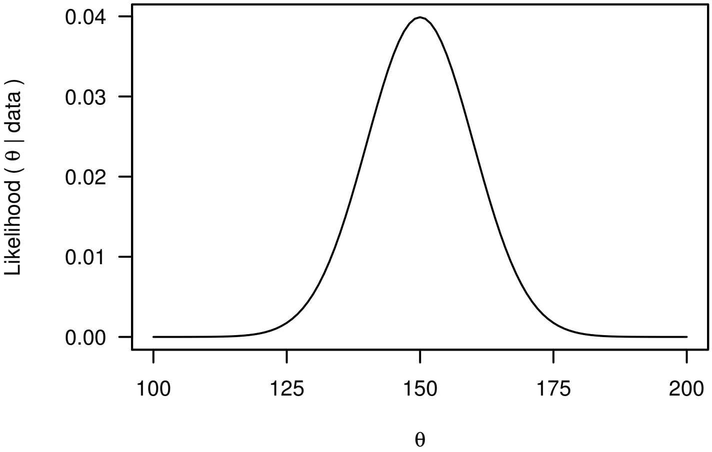
Choose between models with different number of parameters

$$2\log\frac{L_1}{L_0} \quad \sim \quad \chi^2_{\Delta df}$$

$$\text{AIC} \quad = \quad -2\log L \;+\; 2k$$

$$\text{BIC} \quad = \quad -2\log L \;+\; \log(n)k$$

Maximum likelihood estimation



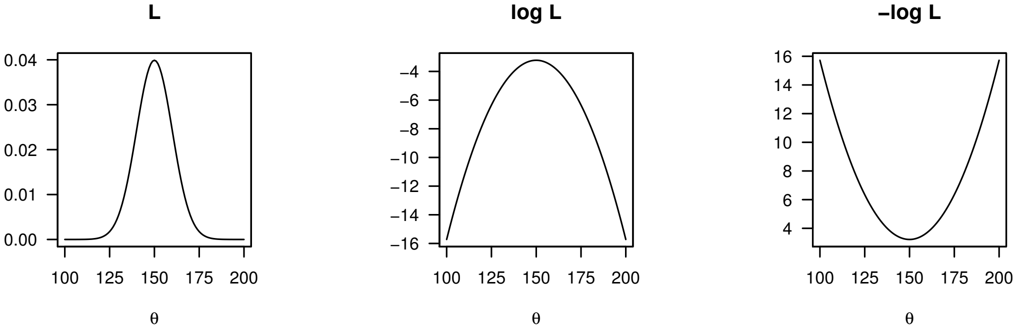
Log likelihood

Log transformation makes things easier

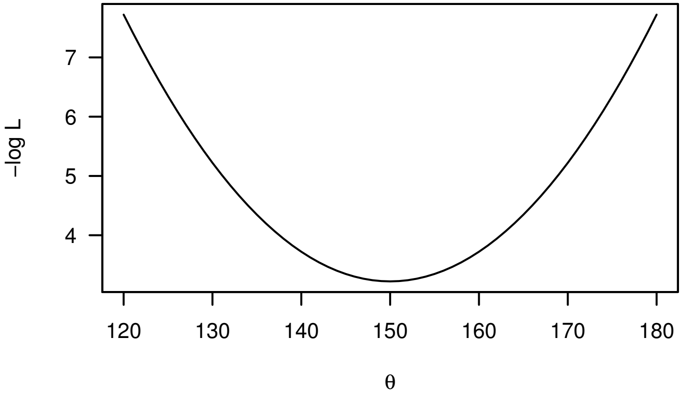
$$\begin{aligned} L(\theta|\text{data}) &= p(\text{data}|\theta) \\ & p(y_1, \ldots, y_n|\theta) \\ & p(y_1|\theta) \times \cdots \times p(y_n|\theta) \\ & \prod p(y_i|\theta) \end{aligned}$$

$$\log L(\theta|\text{data}) = \sum \log p(y_i|\theta)$$

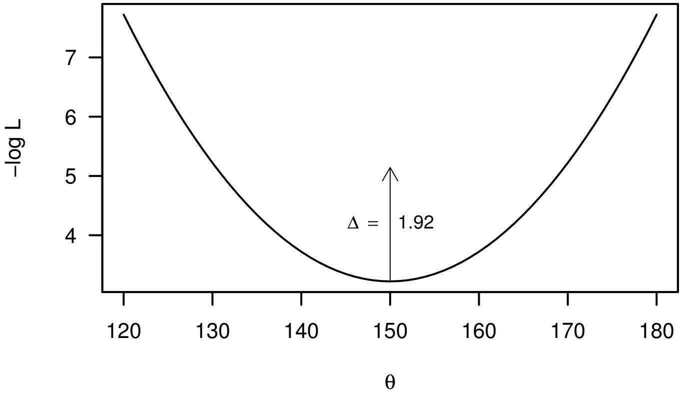
Log likelihood



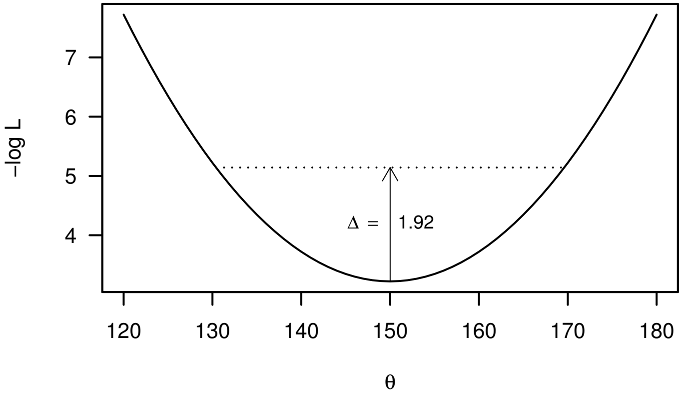
Confidence interval



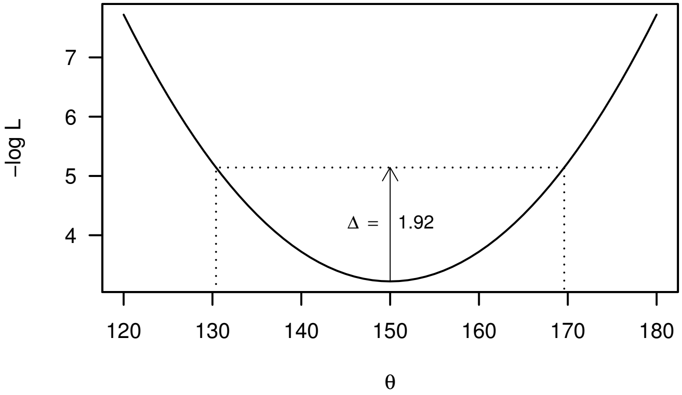
Confidence interval



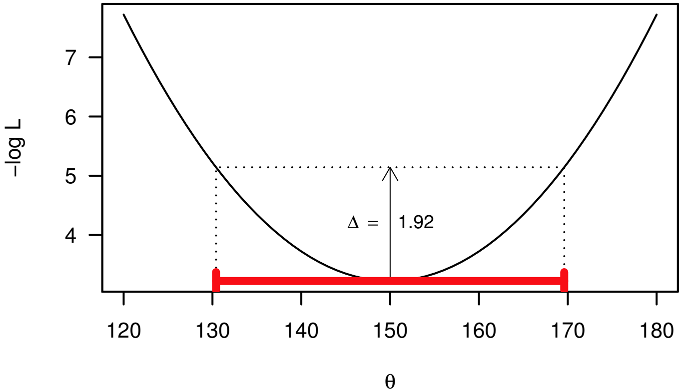
Confidence interval



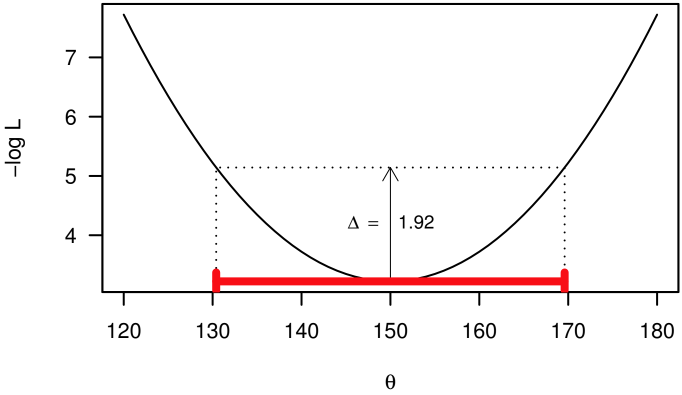
Confidence interval



Confidence interval



Confidence interval



$0.5\chi^2_{df=1} = 1.92$ for 95% confidence interval

Normal distribution

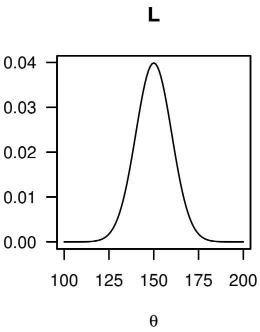
$$
\begin{aligned}
f(x) &= \frac{1}{\sigma\sqrt{2\pi}}\ e^{-\frac{(x-\mu)^2}{2\sigma^2}} \\
p(y_i|\theta) &= \frac{1}{\sigma\sqrt{2\pi}}\ e^{-\frac{(y_i-\mu_i)^2}{2\sigma^2}} \\
L(\theta|y) &= \prod\left(\frac{1}{\sigma\sqrt{2\pi}}\ e^{-\frac{(y_i-\mu_i)^2}{2\sigma^2}}\right) \\
-\log L &= \left[0.5n\log(2\pi)\right] \quad + \quad n\log\sigma \quad + \quad \frac{\sum(y_i-\mu_i)^2}{2\sigma^2} \\
&= \left[0.5n\log(2\pi)\right] \quad + \quad n\log\sigma \quad + \quad \frac{RSS}{2\sigma^2}
\end{aligned}
$$

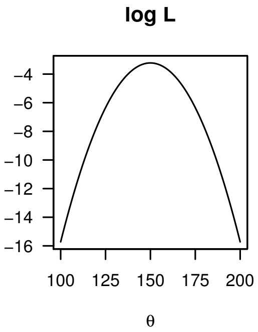
dnorm in R

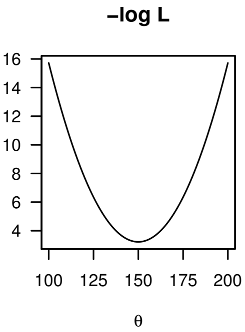
```
L <- prod(dnorm(y, mu, sigma))
```

```
neglogL <- -sum(dnorm(y, mu, sigma, log=TRUE))
```

dnorm in R








```
dnorm(theta,
m=150, s=10)
```

```
dnorm(theta,
m=150, s=10, log=TRUE)
```

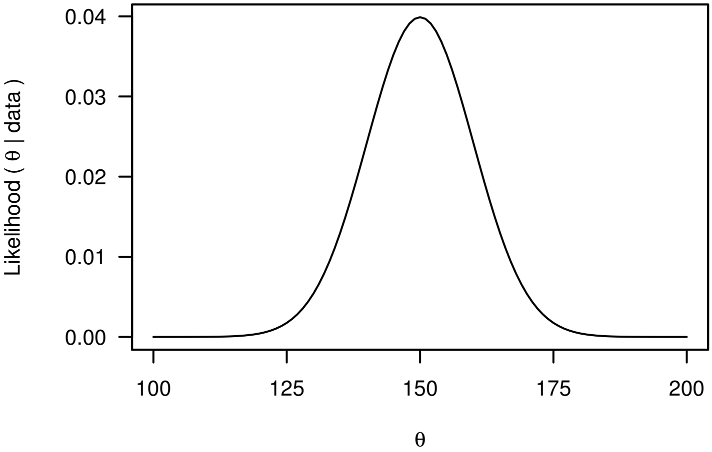
```
-dnorm(theta,
m=150, s=10, log=TRUE)
```

Profile likelihood

1. Fix θ (a parameter of interest) at some value
2. Minimize $-\log L$ by estimating all other parameters
3. Save this value of $-\log L$

Repeat over a range of θ values

Profile likelihood



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