

Homework #4

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1. Describe sweeping process in one dimension along one angle using the diamond difference scheme.
- a) For $u > 0$, the sweep is in the positive u direction. This means $\psi_{i-\frac{1}{2}}$ is the incoming flux and $\psi_{i+\frac{1}{2}}$ is the outgoing flux. For $u > 0$, "sweeping" consists of using the known incoming flux value to calculate the unknown outgoing flux value. We identify the corresponding relationship based on the known value and then "sweep" on each flux. The diamond difference scheme is utilized in this problem. The diamond difference scheme is different from step difference in that the diamond difference method has $\alpha = 0$ whereas the step difference method sets α to ± 1 . Also the diamond difference method is second order while the step difference is first order. Since we're using the diamond difference scheme, we know that the relation for center cell flux to the outgoing flux is:

$$\psi_{i+\frac{1}{2}} = \frac{2}{1+\alpha} \psi_{ijk} - \frac{1-\alpha}{1+\alpha} \bar{\psi}_{i-\frac{1}{2}} \rightarrow \psi_{ijk} = \frac{1}{2} [(1+\alpha) \psi_{i+\frac{1}{2}} + (1-\alpha) \bar{\psi}_{i-\frac{1}{2}}]$$

Now, we need an expression to find the center cell fluxes.

After we integrate over the mesh cells, we get:

$$\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} dx u \frac{\partial \psi}{\partial x} + \int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}} dy v \frac{\partial \psi}{\partial y} + \int_{z_{k-\frac{1}{2}}}^{z_{k+\frac{1}{2}}} dz w \frac{\partial \psi}{\partial z} + \sum_{ijkl} E \psi_{ijkl} (\vec{r}, E) \text{ constant}$$

and then without energy dependence, it becomes:

$$\frac{u}{\Delta i} (\psi_{i+\frac{1}{2}} - \psi_{i-\frac{1}{2}}) + \frac{v}{\Delta j} (\psi_{j+\frac{1}{2}} - \psi_{j-\frac{1}{2}}) + \frac{w}{\Delta k} (\psi_{k+\frac{1}{2}} - \psi_{k-\frac{1}{2}}) + \sum_{ijkl} \psi_{ijkl} = S_{ijk}$$

$$\underline{\psi_{ijkl}} = \underline{S_{ijk}} - \frac{u}{\Delta i} (\psi_{i+\frac{1}{2}} - \psi_{i-\frac{1}{2}}) - \frac{v}{\Delta j} (\psi_{j+\frac{1}{2}} - \psi_{j-\frac{1}{2}}) - \frac{w}{\Delta k} (\psi_{k+\frac{1}{2}} - \psi_{k-\frac{1}{2}})$$

so we plug the diamond difference relation into the aforementioned eq to get

$$\underline{\psi_{ijkl}} = \underline{S_{ijk}} + \frac{2}{1+\alpha} \left(\frac{|u|}{\Delta i} \bar{\psi}_{i-\frac{1}{2}} + \frac{|v|}{\Delta j} \bar{\psi}_{j-\frac{1}{2}} + \frac{|w|}{\Delta k} \bar{\psi}_{k-\frac{1}{2}} \right)$$

Getting back to cell centered fluxes, we can now calculate them using the aforementioned equation at $\alpha = 0$:

$$\Psi_{ijk} = S_{ijk} + \frac{2}{1+\alpha} \left(\frac{1_{ul}}{\Delta i} \bar{\Psi}_{i+\frac{1}{2}} + \frac{1_{nl}}{\Delta j} \bar{\Psi}_{j+\frac{1}{2}} + \frac{1_{el}}{\Delta k} \bar{\Psi}_{k+\frac{1}{2}} \right)$$

$$S_{ijk} + \frac{2}{1+\alpha} \left(\frac{1_{ul}}{\Delta i} + \frac{1_{nl}}{\Delta j} + \frac{1_{el}}{\Delta k} \right)$$

and then to determine outgoing fluxes, we use:

$$\Psi_{i+\frac{1}{2}} = 2\Psi_{ijk} - \Psi_{i-\frac{1}{2}}$$

Hence, we use incoming flux to solve for center flux and then solve for outgoing flux. If there are no external sources, fission or scattering, then only conduct one sweep. But if the aforementioned conditions are not met, then the entire process must be done repetitively for each angle's flux.

- b) for $u < 0$, the sweep is in the negative u direction so the conditions are reversed of what we performed in part a. Hence, $\Psi_{i+\frac{1}{2}}$ is now the incoming flux so we use the cell centered outgoing flux relation:
- $$\Psi_{i-\frac{1}{2}} = \frac{2}{1-\alpha} \Psi_{ijk} - \frac{1+\alpha}{1-\alpha} \Psi_{i+\frac{1}{2}} \rightarrow \Psi_{ijk} = \frac{1}{2} [(1+\alpha) \Psi_{i+\frac{1}{2}} + (1-\alpha) \Psi_{i-\frac{1}{2}}]$$

Again, we start at the boundary where we know $\Psi_{i+\frac{1}{2}}$ is the incoming flux and then calculate the cell centered fluxes w/ $\alpha = 0$ to get:

$$\Psi_{ijk} = S_{ijk} + 2 \left(\frac{1_{ul}}{\Delta i} \bar{\Psi}_{i+\frac{1}{2}} + \frac{1_{nl}}{\Delta j} \bar{\Psi}_{j+\frac{1}{2}} + \frac{1_{el}}{\Delta k} \bar{\Psi}_{k+\frac{1}{2}} \right)$$

$$S_{ijk} + 2 \left(\frac{1_{ul}}{\Delta i} + \frac{1_{nl}}{\Delta j} + \frac{1_{el}}{\Delta k} \right)$$

and then compute outgoing fluxes using: $\Psi_{i-\frac{1}{2}} = 2\Psi_{ijk} - \Psi_{i+\frac{1}{2}}$

Again only one sweep is needed if no scatter, fission or reaction. But, otherwise, a repeat of the aforementioned steps is required for each angle.

c) At the reflecting boundary on the right edge including how to transition from $u > 0$ to $u < 0$, we assume reflecting boundary conditions. We start out just as we would if we were solving for $u > 0$ or $u < 0$. The relation we use is determined by our boundary values, so we start there and sweep until the reflecting boundary and we can apply $\Psi_{N+1, i+1/2} = \Psi_{N+1, i-1/2}$. In this case, N is the number of mesh cells. If the first sweep is in the opposing direction to the second condition, then the application of $\Psi_{N+1, i+1/2} = \Psi_{N+1, i-1/2}$ applies. After each two sweep set, S_{ijk} updates, followed by repetition. If there are reflective boundary conditions on both sides, the initial incoming flux will be estimated and updated as the sweep cycle is repeated.

d) If using the angular flux to generate moments during solution process, we would need to store only the flux moments from one iteration to the next in order to generate S_{ijk} .

2. Look at truncation error in the diamond differ method by examining the 1-D case. Consider uncolided neutrons with a zero group source moment along angle α : $M_\alpha \frac{d\psi_\alpha}{dx} + \sum_t \Phi_t(x) = 0$

a) for $u_\alpha > 0$, write an expression for flux at location x' in terms of flux at location x

$$M_\alpha \frac{d\psi_\alpha}{dx} + \sum_t \Phi_t(x) = 0 \Rightarrow M_\alpha \frac{d\psi_\alpha}{dx} = -\sum_t \Phi_t$$

$$\begin{aligned} \frac{\partial \Psi_a}{\partial x} &= -\frac{\Sigma_t \Psi_a(x)}{m_a} \\ \frac{\partial \ln(\Psi_a)}{\partial x} &= \left(-\frac{\Sigma_t \Psi_a(x)}{m_a} \right) \frac{1}{\Psi_a} \\ \left(\frac{1}{\Psi_a} \right) \frac{d\Psi_a}{dx} &= -\frac{\Sigma_t \Psi_a(x) dx}{m_a} \left(\frac{1}{\Psi_a} \right) \\ \frac{d\Psi_a}{\Psi_a} &= -\frac{\Sigma_t dx}{m_a} \end{aligned}$$

Since the locations are x , and x' :

$$\begin{aligned} \int_x^{x'} \frac{1}{\Psi_a(x)} d\Psi_a &= \int_x^{x'} -\frac{\Sigma_t}{m_a} dx \\ \left[\ln(\Psi_a(x)) \right]_x^{x'} &= -\frac{\Sigma_t}{m_a} x' \Big|_x \end{aligned}$$

evaluate from x to x' to get:

$$\begin{aligned} \ln(\Psi_a(x')) - \ln(\Psi_a(x)) &= -\frac{\Sigma_t}{m_a} (x' - x) \\ e^{\ln(\frac{\Psi_a(x')}{\Psi_a(x)})} &= e^{-\frac{\Sigma_t}{m_a} (x' - x)} \\ \frac{\Psi_a(x')}{\Psi_a(x)} &= e^{-\frac{\Sigma_t}{m_a} (x' - x)} (\Psi_a(x)) \end{aligned}$$

$$\boxed{\Psi_a(x') = \Psi_a(x) e^{-\frac{\Sigma_t}{m_a} (x' - x)} \text{ when } x' > x}$$

- b) Now we impose a cartesian grid with mesh index i . From Lewis and Miller, page 131, we see that for spatial truncation error, we can assume we are traveling to the right ($u_n > 0$) since $x' > x$. And since we know this is a cartesian grid, we know $x = x_{i+1/2}$ and $x = x_{i-1/2}$, and since we're moving right, $\Psi_a(x') = \Psi_{a,i+1/2}$ and $\Psi_a(x) = \Psi_{a,i-1/2}$.

We plug our values for x, x' , $\Phi_a(x')$ and $\Phi_a(x)$ in our expression in part a

$$\begin{aligned} \Phi_{a,i+\frac{1}{2}} &= \Phi_{a,i-\frac{1}{2}} + e^{-\frac{\epsilon_b}{\mu_a}((i+\frac{1}{2}) - (i-\frac{1}{2}))} \\ \Phi_{a,i-\frac{1}{2}} &= \Phi_{a,i+\frac{1}{2}} e^{-\frac{\epsilon_b}{\mu_a}(\Delta i)} \\ \ln \frac{\Phi_{a,i+\frac{1}{2}}}{\Phi_{a,i-\frac{1}{2}}} &= \ln \left(e^{-\frac{\epsilon_b}{\mu_a}(\Delta i)} \right) (\Phi_{a,i+\frac{1}{2}}) \\ \left(\Phi_{a,i+\frac{1}{2}} \right) e^{\ln \left(\frac{\Phi_{a,i+\frac{1}{2}}}{\Phi_{a,i-\frac{1}{2}}} \right)} &= e^{-\frac{\epsilon_b}{\mu_a}(\Delta i)} (\Phi_{a,i-\frac{1}{2}}) \\ \Phi_{a,i+\frac{1}{2}} &= \Phi_{a,i-\frac{1}{2}} e^{-\frac{\epsilon_b}{\mu_a}(\Delta i)} \end{aligned}$$

Since $\Delta i = x_i + \frac{1}{2} - x_{i-\frac{1}{2}}$ and $h = \frac{\epsilon_b \Delta i}{2 \mu_a t}$, we get:

$$\boxed{\Phi_{a,i+\frac{1}{2}} = \Phi_{a,i-\frac{1}{2}} e^{-2h}}$$

c) Plug in the relationship you just found in 1D diamond difference equations ($\alpha = 0$). Manipulate to get another expression for $\Phi_{a,i+\frac{1}{2}}$ in terms of $\Phi_{a,i-\frac{1}{2}}$ and h .

To compute outgoing fluxes, using the diamond difference method, we use: $\Phi_{k,i+\frac{1}{2}} = 2\Phi_{i,k} - \Phi_{i-k}$. Here, we can utilize the discretized form of the transport eq:

$$\frac{u}{\Delta i} (\Phi_{i+\frac{1}{2}} - \Phi_{i-\frac{1}{2}}) + \frac{n}{\Delta j} (\Phi_{j+\frac{1}{2}} - \Phi_{j-\frac{1}{2}}) + \frac{\epsilon_b}{\Delta k} (\Phi_{k+\frac{1}{2}} - \Phi_{k-\frac{1}{2}}) + \sum_{l,i,j,k} S_{i,j,k} \Phi_{ijk} = S_{i,j,k}$$

and just plug in $\Phi_{a,i+\frac{1}{2}} = 2\Phi_{ijk} - \Phi_{i-\frac{1}{2}}$ to find how $\Phi_{i+\frac{1}{2}}$ and $\Phi_{i-\frac{1}{2}}$ are connected:

We can neglect source term so $S_{i,j,k} = 0$

$$\begin{aligned} \frac{u}{\Delta i} (\Phi_{i+\frac{1}{2}} - \Phi_{i-\frac{1}{2}}) + \frac{n}{\Delta j} (\Phi_{j+\frac{1}{2}} - \Phi_{j-\frac{1}{2}}) + \frac{\epsilon_b}{\Delta k} (\Phi_{k+\frac{1}{2}} - \Phi_{k-\frac{1}{2}}) + \sum_{l,i,j,k} S_{i,j,k} \Phi_{ijk} &= 0 \\ \Rightarrow \frac{u}{\Delta i} (\Phi_{i+\frac{1}{2}} - \Phi_{i-\frac{1}{2}}) + \sum_{l,i,j,k} S_{i,j,k} \Phi_{ijk} &= 0 \end{aligned}$$

$$\frac{ua}{\Delta i} (\Psi_{a,i} + \frac{1}{2} - \Psi_{a,i-1/2}) + \sum_{a,j \neq i} \Psi_{a,j} = 0$$

we know $\Psi_{ijk} = \frac{1}{2} [(1+\alpha) \Psi_{i+1/2} + (1-\alpha) \Psi_{i-1/2}]$
 from diamond difference convection cell
 flux relation.

for $\alpha=0$, $\Psi_{ijk} = \frac{1}{2} [(1) \Psi_i + \frac{1}{2} + (1) \Psi_{i-1/2}]$, which we can plug in

$$\frac{ua}{\Delta i} (\Psi_{a,i} + \frac{1}{2} - \Psi_{a,i-1/2}) + \xi_i \left(\frac{1}{2} \right) (\Psi_{a,i+1/2} + \Psi_{a,i-1/2}) = 0$$

$$\frac{ua}{\Delta i} (\Psi_{a,i} + \frac{1}{2} - \Psi_{a,i-1/2}) = - \frac{\xi_i}{2} (\Psi_{a,i+1/2} + \Psi_{a,i-1/2})$$

$$\frac{u}{\Delta i} \Psi_{a,i+1/2} - \frac{ua}{\Delta i} \Psi_{a,i-1/2} = - \frac{1}{2} \xi_i \Psi_{a,i+1/2} - \frac{1}{2} \xi_i \Psi_{a,i-1/2}$$

$$\frac{u}{\Delta i} \Psi_{a,i+1/2} + \frac{1}{2} \xi_i \Psi_{a,i+1/2} = + \frac{ua}{\Delta i} \Psi_{a,i-1/2} - \frac{1}{2} \xi_i \Psi_{a,i-1/2}$$

$$\left(\frac{\Delta i}{ua} \right) \Psi_{a,i+1/2} \left(\frac{u}{\Delta i} + \frac{1}{2} \cdot \xi_i \right) = \Psi_{a,i-1/2} \left(\frac{ua}{\Delta i} - \frac{1}{2} \cdot \xi_i \right) \left(\frac{\Delta i}{ua} \right)$$

$$\Rightarrow \left(1 + \underbrace{\left[\frac{\xi_i \Delta i}{2ua} \right]}_h \right) (\Psi_{a,i+1/2}) = \left(1 - \underbrace{\left[\frac{\xi_i \Delta i}{2ua} \right]}_h \right) (\Psi_{a,i-1/2})$$

$$\text{hence } \frac{(1+h)}{(1+h)} \Psi_{a,i+1/2} = \frac{(1-h)}{(1+h)} \Psi_{a,i-1/2}$$

$$\Rightarrow \boxed{\Psi_{a,i+1/2} = \frac{(1-h) \Psi_{a,i-1/2}}{(1+h)}}$$

We plug our values for $x, x^1, \Phi_a(x^1)$ and $\Phi_a(x')$ our expression in part a

$$\Phi_{a,i+\frac{1}{2}} = \Phi_{a,i-\frac{1}{2}} e^{-\frac{\epsilon t}{\Delta u} ((x_i + \frac{1}{2}) - (x_i - \frac{1}{2}))}$$

$$\Phi_{a,i+\frac{1}{2}} = \Phi_{a,i-\frac{1}{2}} e^{-\frac{\epsilon t}{\Delta u} (\Delta i)} \quad \ln \Phi_{a,i+\frac{1}{2}} = \ln \left(\frac{\Phi_{a,i+\frac{1}{2}}}{\Phi_{a,i-\frac{1}{2}}} \right) = \frac{-\epsilon t}{\Delta u} (\Delta i)$$

$$\left(\Phi_{a,i-\frac{1}{2}} \right) e^{\ln \left(\frac{\Phi_{a,i+\frac{1}{2}}}{\Phi_{a,i-\frac{1}{2}}} \right)} = e^{-\frac{\epsilon t}{\Delta u} (\Delta i)} (\Phi_{a,i-\frac{1}{2}})$$

$$\Phi_{a,i+\frac{1}{2}} = \Phi_{a,i-\frac{1}{2}} e^{-\frac{\epsilon t}{\Delta u} (\Delta i)}$$

Since $\Delta i = x_i + \frac{1}{2} - x_i - \frac{1}{2}$ and $h = \frac{\epsilon t \Delta i}{|x_i - x_{i+1}|}$, we get:

$$\boxed{\Phi_{a,i+\frac{1}{2}} = \Phi_{a,i-\frac{1}{2}} e^{-2h}}$$

- c) Plug in the relationship you just found in 1D diamond difference equations ($\alpha=0$). Manipulate to get another expression for $\Phi_{a,i+\frac{1}{2}}$ in terms of $\Phi_{a,i-\frac{1}{2}}$ and h .

To compute outgoing fluxes using the diamond difference method, we use: $\Phi_{a,i+\frac{1}{2}} = 2\Phi_{ijk} - \Phi_{i-k}$
here, we can utilize the discretized form of the transport eq:

$$\frac{u}{\Delta i} (\Phi_{i+\frac{1}{2}} - \Phi_{i-\frac{1}{2}}) + \frac{n}{\Delta j} (\Phi_{j+\frac{1}{2}} - \Phi_{j-\frac{1}{2}}) + \frac{k}{\Delta k} (\Phi_{k+\frac{1}{2}} - \Phi_{k-\frac{1}{2}}) + \sum_{t,ijk} \Phi_{ijk} = S_{ijk}$$

and just plug in $\Phi_{a,i+\frac{1}{2}} = 2\Phi_{ijk} - \Phi_{i-k}$ to find how $\Phi_{i+\frac{1}{2}}$ and $\Phi_{i-\frac{1}{2}}$ are connected:

we can neglect source term so $S_{ijk} = 0$

$$\frac{u}{\Delta i} (\Phi_{i+\frac{1}{2}} - \Phi_{i-\frac{1}{2}}) + \frac{n}{\Delta j} (\Phi_{j+\frac{1}{2}} - \Phi_{j-\frac{1}{2}}) + \frac{k}{\Delta k} (\Phi_{k+\frac{1}{2}} - \Phi_{k-\frac{1}{2}}) + \sum_{t,ijk} \Phi_{ijk} = 0$$

$$\Rightarrow \frac{u}{\Delta i} (\Phi_{i+\frac{1}{2}} - \Phi_{i-\frac{1}{2}}) + \sum_{t,ijk} \Phi_{ijk} = 0$$

$$\frac{ua}{\Delta i} (\Psi_{a,i+1/2} - \Psi_{a,i-1/2}) + \xi_{i+1/2} \Psi_{a,i+1/2} = 0$$

we know $\Psi_{ijk} = \frac{1}{2} [(1+\alpha) \Psi_{i+1/2} + (1-\alpha) \Psi_{i-1/2}]$
from diamond difference cent in cell
flux relation.

$$\text{for } \alpha=0, \Psi_{ijk} = \frac{1}{2} [(1) \Psi_i + \frac{1}{2} + (1) \Psi_i - \frac{1}{2}], \text{ which we can plug}$$

$$\frac{ua}{\Delta i} (\Psi_{a,i+1/2} - \Psi_{a,i-1/2}) + \xi_i \left(\frac{1}{2} \right) (\Psi_{a,i+1/2} + \Psi_{a,i-1/2}) =$$

$$\frac{ua}{\Delta i} (\Psi_{a,i+1/2} - \Psi_{a,i-1/2}) = -\xi_i \left(\Psi_{a,i+1/2} + \Psi_{a,i-1/2} \right)$$

$$\frac{ua}{\Delta i} \Psi_{a,i+1/2} + \frac{ua}{\Delta i} \Psi_{a,i-1/2} = -\frac{1}{2} \xi_i \Psi_{a,i+1/2} - \frac{1}{2} \xi_i \Psi_{a,i-1/2}$$

$$\frac{ua}{\Delta i} \Psi_{a,i+1/2} + \frac{1}{2} \xi_i \Psi_{a,i+1/2} = +\frac{ua}{\Delta i} \Psi_{a,i-1/2} - \frac{1}{2} \xi_i \Psi_{a,i-1/2}$$

$$\left(\frac{\Delta i}{ua} \right) \Psi_{a,i+1/2} \left(\frac{u}{\Delta i} + \frac{1}{2} \cdot \xi_i \right) = \Psi_{a,i-1/2} \left(\frac{ua}{\Delta i} - \frac{1}{2} \cdot \xi_i \right) \left(\frac{\Delta i}{ua} \right)$$

$$\Rightarrow \left(1 + \underbrace{\left[\frac{\xi_i \Delta i}{2ua} \right]}_h \right) (\Psi_{a,i+1/2}) = \left(1 - \underbrace{\left[\frac{\xi_i \Delta i}{2ua} \right]}_h \right) (\Psi_{a,i-1/2})$$

$$\text{hence } \frac{(1+h)}{(1-h)} \Psi_{a,i+1/2} = \frac{(1-h)}{(1+h)} \Psi_{a,i-1/2}$$

$$\Rightarrow \boxed{\Psi_{a,i+1/2} = \frac{(1-h)}{(1+h)} \Psi_{a,i-1/2}}$$

- d) Look at part b and expand the exponential in a power series and expand part c expression in a power series through h^2 terms & show same.

We know power series can be represented as:

$$\sum_{n=0}^{\infty} a_n(x-c)^n = a_0 + a_1(x-c)^1 + a_2(x-c)^2 + \dots$$

The expression we found in part b is:

$$\Psi_{a,i+\frac{1}{2}} = \Psi_{a,i-\frac{1}{2}} e^{-2h}$$

Expanding this via power series becomes:

$$\Psi_{a,i+\frac{1}{2}} = \Psi_{a,i-\frac{1}{2}} e^{-2h} \left(1 + (-2h) + \frac{(-2h)^2}{2!} \right) = \Psi_{a,i-\frac{1}{2}} [1 - 2h + 2h^2 + \dots]$$

Expanding the exponential through power series from part b gives us $[1 - 2h - 2h^2 + \dots]$

Expanding part c via power series gives us:

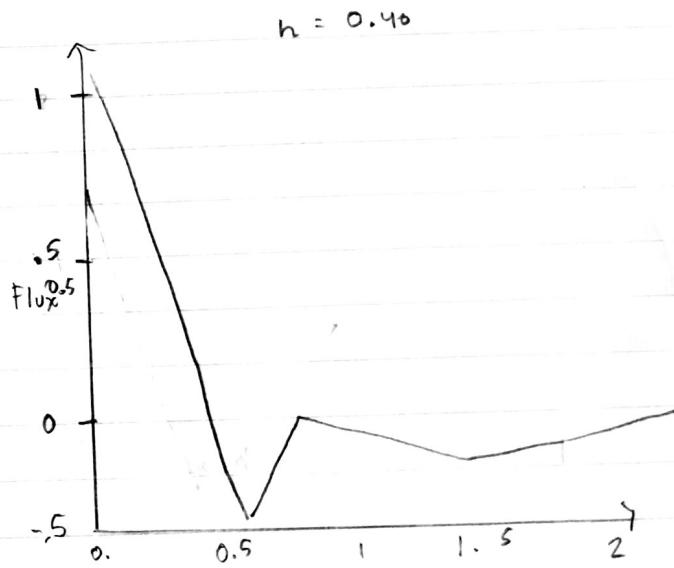
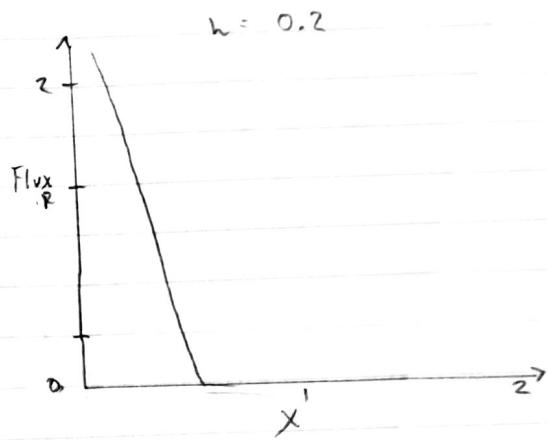
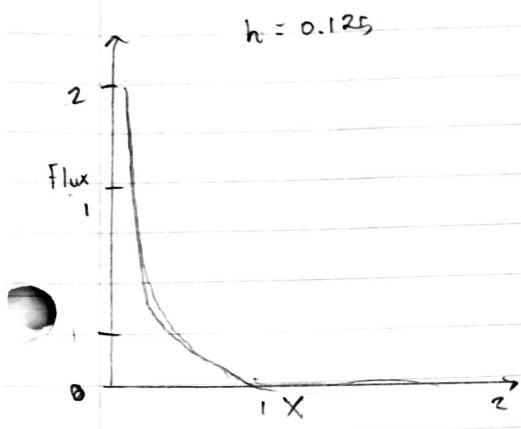
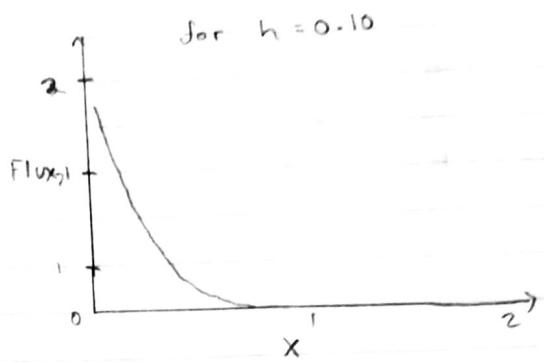
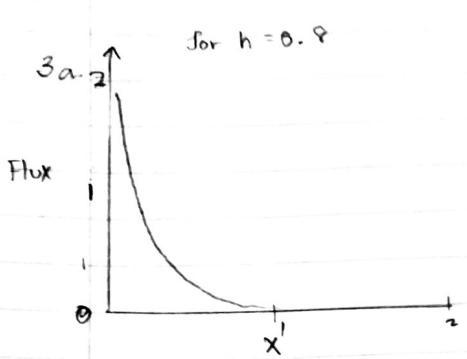
$$\text{The value of } \frac{h+1}{h+1} = [1 - 2h + 2h^2 + \dots]$$

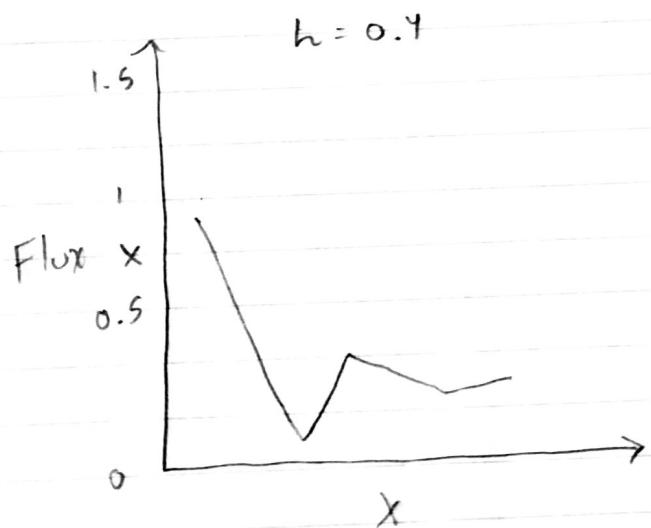
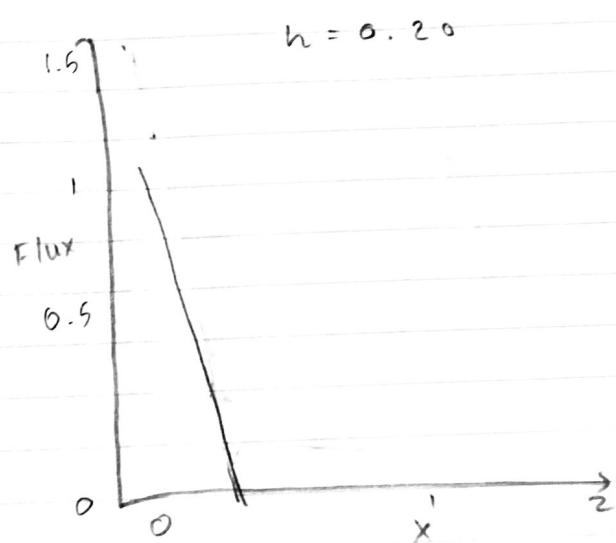
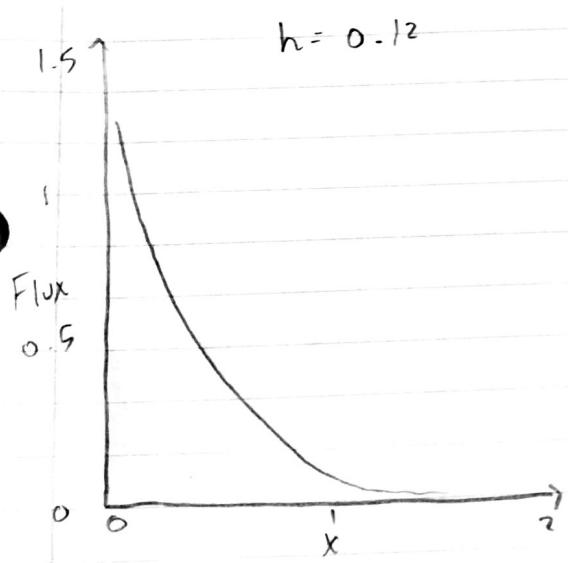
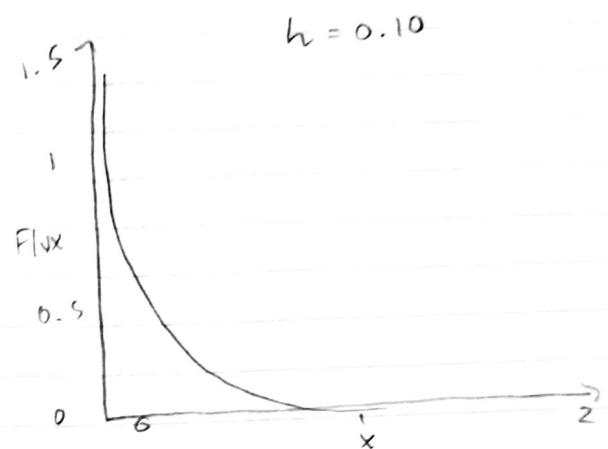
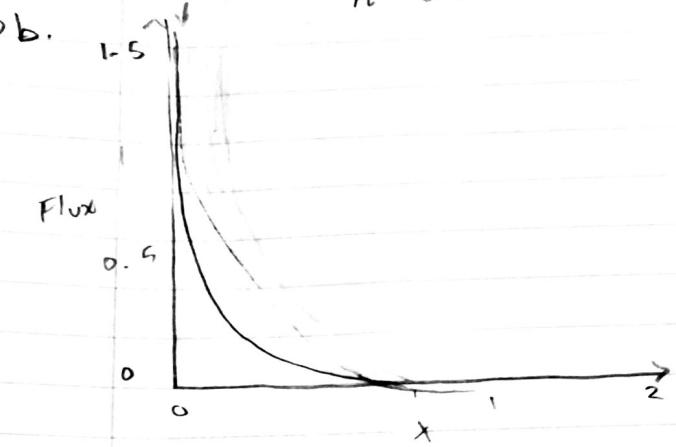
Both are the same through the h^2 term meaning their relation is second order accurate.

- e) look at the expression for $\Psi_{a,i+\frac{1}{2}}$. What is a condition on h that would guarantee the flux would be positive and what does that mean about mesh spacing given the smallest Δx in a set and a specific ϵ_t ?

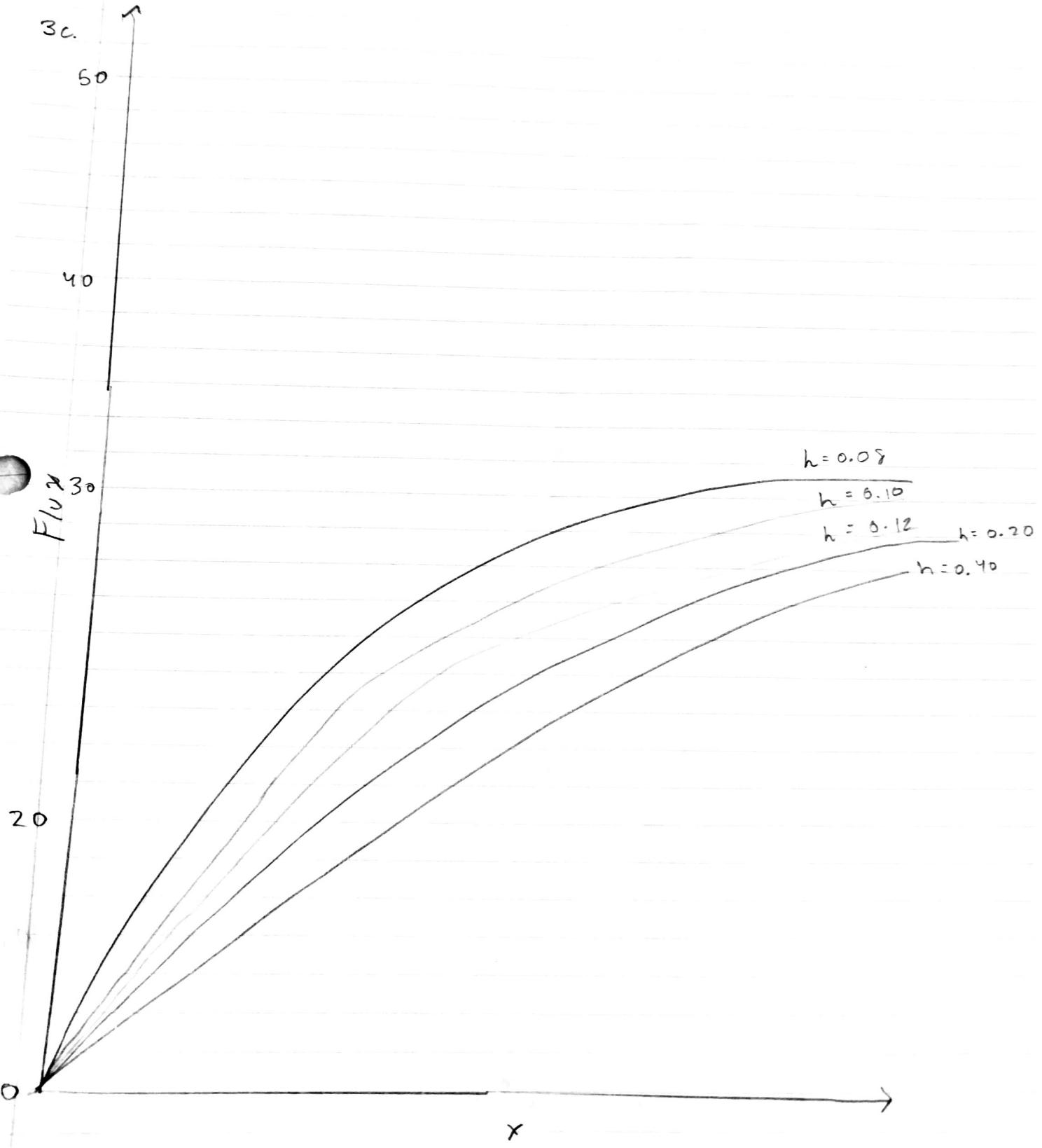
Since we know $\Psi_{i+\frac{1}{2}} = \frac{1-h}{1+h} \Psi_{i-\frac{1}{2}}$, we see that h being larger than 1 could result in a negative $\Psi_{i+\frac{1}{2}}$. In order to guarantee a positive result and avoid a negative flux, $\Delta x < \frac{2\Delta t}{\epsilon_t}$, is required. cont. \rightarrow

Mesh spacing is most problematic when Δx is small or S_n is large, and can result in negative fluxes. To guarantee positive flux, the more S_n order increases, the more mesh size should decrease.





3c.



4. Starting from the following general system of equations

$$\frac{u_a}{h_i} (\Psi_{a,i+\frac{1}{2}}^s - \Psi_{a,i-\frac{1}{2}}^s) + \sum_{t,i}^2 \Psi_{t,i}^s = 2\pi \sum_{a=1}^N w_a \sum_{g=1}^G \sum_{s,i}^{sg} (\alpha' \rightarrow \alpha) \Psi_{a,i}^{s,i} + \frac{x_3}{2} \sum_{g=1}^G v_g \sum_{s,i}^s \phi_i^s + \frac{1}{2}$$

where ϕ is scalar flux

write 5 coupled equations for a five group problem

Assume neutrons only down scatter from fast groups (1&2) to thermal groups (3, 4 and 5). Assume that thermal group can upscatter into other thermal groups and can downscatter. Assume there is an external and fission source

For 1:

$$\frac{u_a}{h_i} (\Psi_{a,i+\frac{1}{2}}^1 - \Psi_{a,i-\frac{1}{2}}^1) + \sum_{t,i}^1 \Psi_{t,i}^1 = 2\pi \sum_{a=1}^N w_a \sum_{g=1}^1 \sum_{s,i}^{1g} (\alpha' \rightarrow \alpha) \Psi_{a,i}^{1,i} + \frac{x_1}{2} \sum_{g=1}^5 v_g \sum_{s,i}^{1g} \phi_i^s + \dots$$

Something to be noted about Group one is that neutrons cannot scatter in it and neutrons are created by fission in group 1

For 2:

$$\frac{u_a}{h_i} (\Psi_{a,i+\frac{1}{2}}^2 - \Psi_{a,i-\frac{1}{2}}^2) + \sum_{t,i}^2 \Psi_{t,i}^2 = 2\pi \sum_{a=1}^N w_a \sum_{g=1}^2 \sum_{s,i}^{2g} (\alpha' \rightarrow \alpha) \Psi_{a,i}^{2,i} + \frac{x_2}{2} \sum_{g=1}^5 v_g \sum_{s,i}^{2g} \phi_i^s + \dots$$

For group two, neutrons can scatter in it from group one

for 3:

$$\frac{u_a}{n_i} (\Psi_{a,i+\frac{1}{2}}^3 - \Psi_{a,i-\frac{1}{2}}^3) + \sum_{i=1}^3 \Psi_{a,i}^3 \\ = 2\pi \sum_{a=1}^{\infty} w_a \sum_{g=1}^5 \sum_{A,i}^{3g'} \Psi_{a,i}^{3g'} + \frac{x_3}{2} \sum_{g=1}^5 v_g' \sum_{f,i}^{g'} \Phi_{f,i}^{g+\frac{1}{2}}$$

For group 3, neutrons can downscatter from groups one and two into group three and four. In group three

for 4:

$$\frac{u_a}{n_i} (\Psi_{a,i+\frac{1}{2}}^4 - \Psi_{a,i-\frac{1}{2}}^4) + \sum_{i=1}^4 \Psi_{a,i}^4 \\ = 2\pi \sum_{a=1}^{\infty} w_a \sum_{g=1}^5 \sum_{s,i}^{4g'} \Psi_{a,i}^{4g'} + \frac{x_4}{2} \sum_{g=1}^5 v_g' \sum_{f,i}^{g'} \Phi_{f,i}^{g+\frac{1}{2}}$$

For group 4, neutrons can downscatter from groups one, two and three into group four.

for 5:

$$\frac{u_a}{n_i} (\Psi_{a,i+\frac{1}{2}}^5 - \Psi_{a,i-\frac{1}{2}}^5) + \sum_{i=1}^5 \Psi_{a,i}^5 \\ = 2\pi \sum_{a=1}^{\infty} w_a \sum_{g=1}^5 \sum_{s,i}^{5g'} \Psi_{a,i}^{5g'} + \frac{x_5}{2} \sum_{g=1}^5 v_g' \sum_{f,i}^{g'} \Phi_{f,i}^{g+\frac{1}{2}}$$

Lastly, neutrons from groups one, two, three and four can downscatter into group five.