Simple-reinforced-concrete-beam

August 1, 2023

1 General

1.1 Project information

Project number:
Project name:
Date:
Engineer:
Checker:

1.2 Purpose of calculations

Describe the particular use to which the design note is put on this occasion.

1.3 Design note

Name of design note: Simple reinforced concrete beam
Version: PA
Last modified: 2023-07-28
Custodian: MPJ Grégoire, ing.

This design note checks ultimate and serviceability limit states for a single span reinforced concrete beam loaded with a live load at the midspan.

1.4 Citations

Citations such as §, Fig. and Tab. are to the Canadian Highway Bridge Design Code or to its Commentary, unless noted otherwise.

1.5 References

1.5.1 Normative

- CAN/CSA S6:19, Canadian Highway Bridge Design Code, CSA, 2019
- CAN/CSA S6.1:19, Commentary on the Canadian Highway Bridge Design Code, CSA, 2019

1.5.2 Informative

- Manuel de conception des structures, MTQ

1.5.3 Drawings

- none

1.6 Assumptions

1. Plane sections remain plane

1.7 To do:

- Add comparison with M_r
- Add check for over-reinforcement
- Add ULS check for shear
- Add SLS check for cracking
- Add development length for longitudinal bars, effective A_s
- Maybe add SLS check for deflection
- Fix calculation for deflection under point load
- Add more citations maybe

This design note is not a substitute for engineering judgement.

2 Initialisation

[1]:
```
from math import sqrt, pi
import numpy as np

import matplotlib.pyplot as plt

import pandas as pd
pd.options.display.float_format = '{:" " .2f}'.format  # TODO spaces not commas

import pint
ureg = pint.UnitRegistry()
ureg.default_format = '~'
Q_ = ureg.Quantity
ureg.setup_matplotlib(True)

import pint_pandas
pint_pandas.PintType.ureg = ureg
```

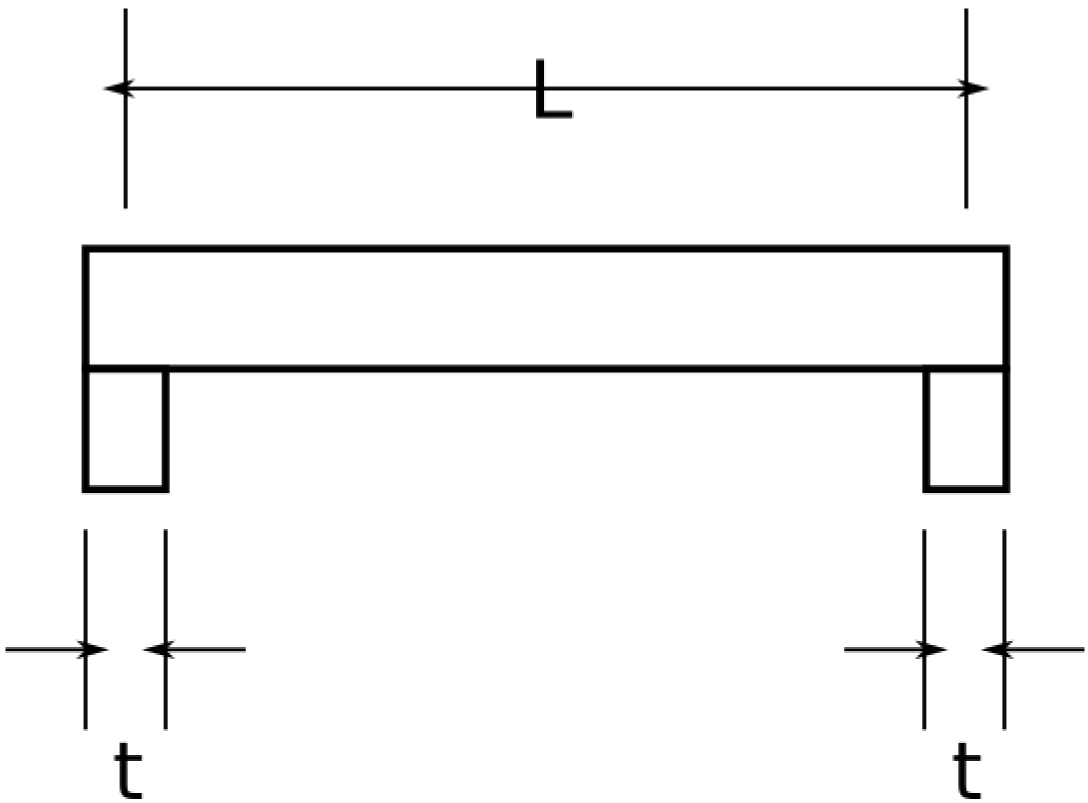
[2]:
```
%load_ext watermark
%watermark -i -m -v -iv
```

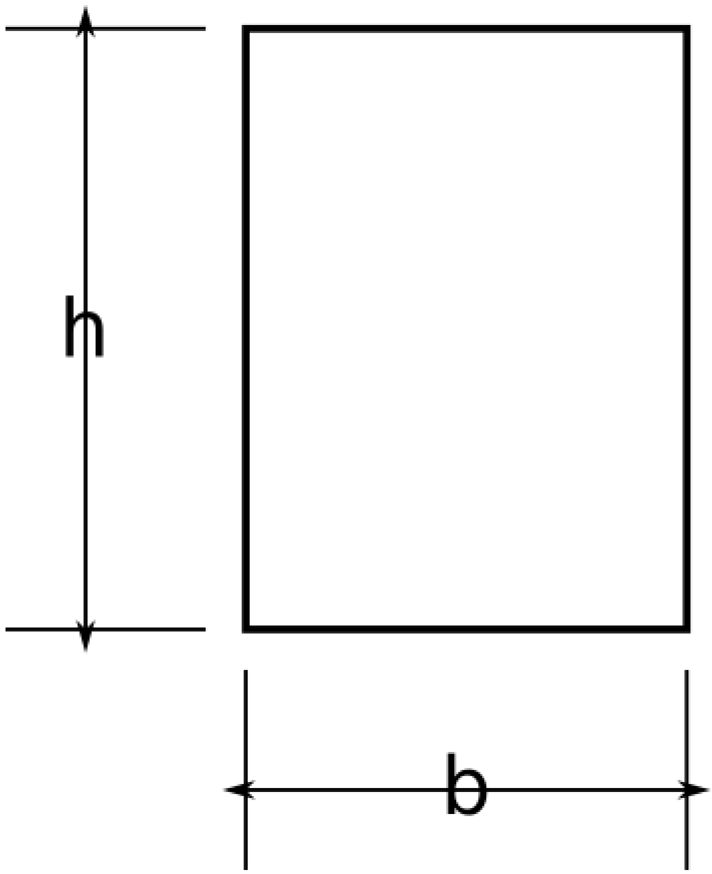
```
Python implementation: CPython
Python version       : 3.11.2
IPython version      : 8.14.0

Compiler    : GCC 12.2.0
OS          : Linux
```

```
Release     : 6.1.0-10-amd64
Machine     : x86_64
Processor   :
CPU cores   : 2
Architecture: 64bit

pint_pandas: 0.4
pint       : 0.19.2
matplotlib : 3.6.3
numpy      : 1.24.2
pandas     : 1.5.3
```

3 Inputs

3.1 Geometry





[3]:
```
L = Q_(8, 'm')
h = Q_(500, 'mm')
b = Q_(400, 'mm')
```

3.2 Materials

3.2.1 Concrete

[4]:
```
fprime_c = Q_(35, 'MPa')
 _c = Q_(24, 'kN/m^3')
 _c = Q_(2300, 'kg/m^3')
```

3.2.2 Reinforcing

[5]:
```
f_y = Q_(400, 'MPa')
E_s = Q_(200_000, 'MPa')
bar_size_long = '20M'
bar_num_long = 3
bar_size_trans = '15M'
s_trans = Q_(200, 'mm')
cover = Q_(50, 'mm')
```

3.3 Loads

Live load applied as a point load at midspan

[6]:
```
P_LL = Q_(5, 'kN')
```

4 Basic Properties

4.1 Geometric

[7]:
```
I = 1/12 * h**3 * b ; I.to('mm^4')
print(f'Moment of inertia, {I=}')
```

```
Moment of inertia, I=<Quantity(4.16666667e+09, 'millimeter ** 4')>
```

4.2 Material

4.2.1 Concrete

[8]:
```
fprime_c.ito('MPa')
E_c = (3300 * sqrt(fprime_c.magnitude) + 6900 ) * (  _c / Q_(2300, 'kg/m^3') )␣
↪* Q_(1, 'MPa') # S6:19 §8.4.1.5.3
f_cr = 0.4 * sqrt(fprime_c.magnitude) * Q_(1, 'MPa') # S6:19 §8.4.1.8.1
M_cr = f_cr * I / ( h / 2 )
M_cr.ito('kN*m') ; print(f'Cracking moment, {M_cr=}')
 _1 = max(0.85 - 0.0015 * fprime_c.magnitude, 0.67)
 _1 = max(0.97 - 0.0025 * fprime_c.magnitude, 0.67)
n = E_s / E_c
print(f'{ _1=} S6:19 §8.8.3 ')
print(f'{ _1=} S6:19 §8.8.3 ')
# print(f'{n=} S6:19 8.6 ')
```

```
Cracking moment, M_cr=<Quantity(39.4405319, 'kilonewton * meter')>
 _1=0.7975 S6:19 §8.8.3
 _1=0.8825 S6:19 §8.8.3
```

4.2.2 Reinforcing

[9]:
```
reinforcing = pd.DataFrame(
{
    "Bar": pd.Series(['10M', '15M', '20M', '25M', '30M', '35M', '45M', '55M']),
    "d_b": pd.Series([11.3, 16.0, 19.5, 25.2, 29.9, 35.7, 43.7, 56.4],␣
↪dtype="pint[mm]"),
    "A_b": pd.Series([100, 200, 300, 500, 700, 1000, 1500, 2500],␣
↪dtype="pint[mm^2]")
})

print(f'Reinforcing bar properties \n {reinforcing}')
```

```
#print(reinforcing.at[2, 'A_b'])
#print(reinforcing.loc[reinforcing['Bar'] == '20M', 'A_b'].item())
#print(reinforcing.loc['Bar' == '20M])

A_s = bar_num_long * reinforcing.loc[reinforcing['Bar'] == bar_size_long,␣
  ↪'A_b'].item()
print(f'{A_s=}')

d_b = reinforcing.loc[reinforcing['Bar'] == bar_size_long, 'd_b'].item()
print(f'{d_b=}')

d_b_trans = reinforcing.loc[reinforcing['Bar'] == bar_size_trans, 'd_b'].item()
print(f'{d_b_trans=}')

d = h - cover - d_b_trans - d_b / 2
```

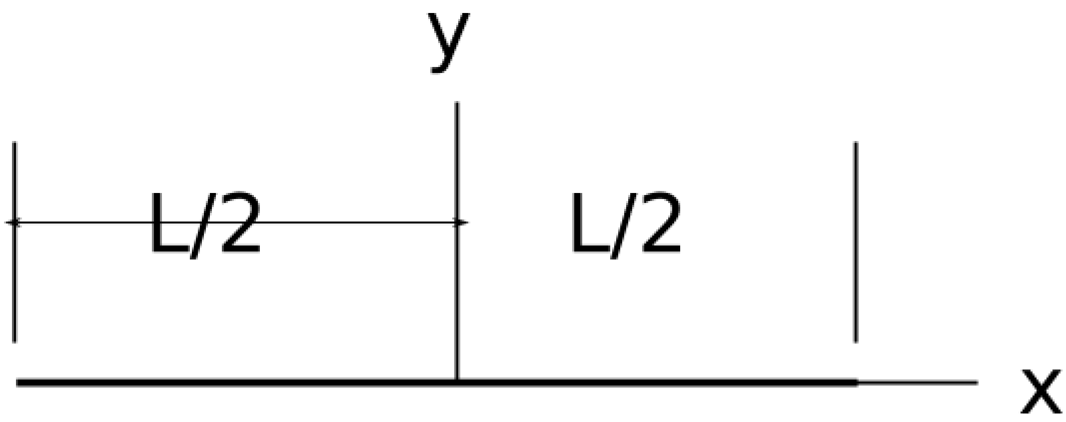
```
Reinforcing bar properties
    Bar   d_b   A_b
0   10M  11.3   100
1   15M  16.0   200
2   20M  19.5   300
3   25M  25.2   500
4   30M  29.9   700
5   35M  35.7  1000
6   45M  43.7  1500
7   55M  56.4  2500
A_s=<Quantity(900, 'millimeter ** 2')>
d_b=<Quantity(19.5, 'millimeter')>
d_b_trans=<Quantity(16.0, 'millimeter')>
```

4.3 Loads

Self-weight

[10]:
```
w_SW =  _c * b * h
```

5 Analysis



TODO Move x = 0 to left support

5.1 Load patterns

5.1.1 UDL on entire span

Add image

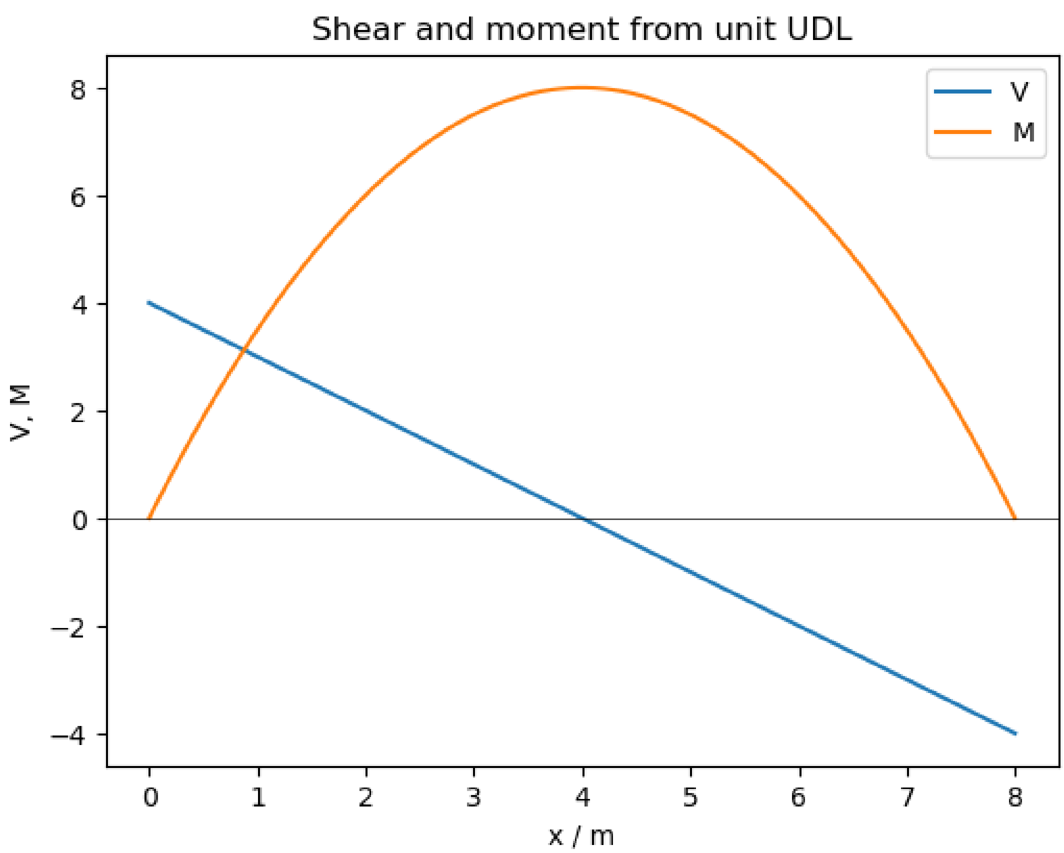
[11]:
```python
def V_1(x, w):
    """Return the shear at point x when loaded uniformly by UDL w."""
    Vee =  w * ( L / 2 - x )
    return Vee

def M_1(x, w):
    """Return the bending moment at point x when loaded uniformly by UDL w."""
    Emm = w * x / 2 * ( L - x )
    return Emm

def Δ_1(x, w):
    """Return the deflection down at point x when loaded uniformly by UDL w."""
    Delta = w * x / ( 24 * E_c * I ) * ( L**3 - 2 * L * x**2 + x**3 )
    return Delta
```

[12]:
```python
x1 = np.linspace(0, L.magnitude, 100)
x11 = x1 * L / L.magnitude
```

[13]:
```python
fig, ax = plt.subplots()

ax.plot(x11, V_1(x11, Q_(1, 'kN/m')).magnitude, label='V')
ax.plot(x11, M_1(x11, Q_(1, 'kN/m')).magnitude, label='M')
plt.axhline(y = 0, color = 'k', lw = 0.5)

plt.xlabel('x / m')
ax.xaxis.set_units(ureg['m'])
```

```
plt.ylabel('V, M')
ax.legend()

plt.title('Shear and moment from unit UDL')
plt.savefig('ein-images/shear-moment-1.png')
plt.show
```

[13]: <function matplotlib.pyplot.show(close=None, block=None)>



5.1.2 Point load at midspan

Add image

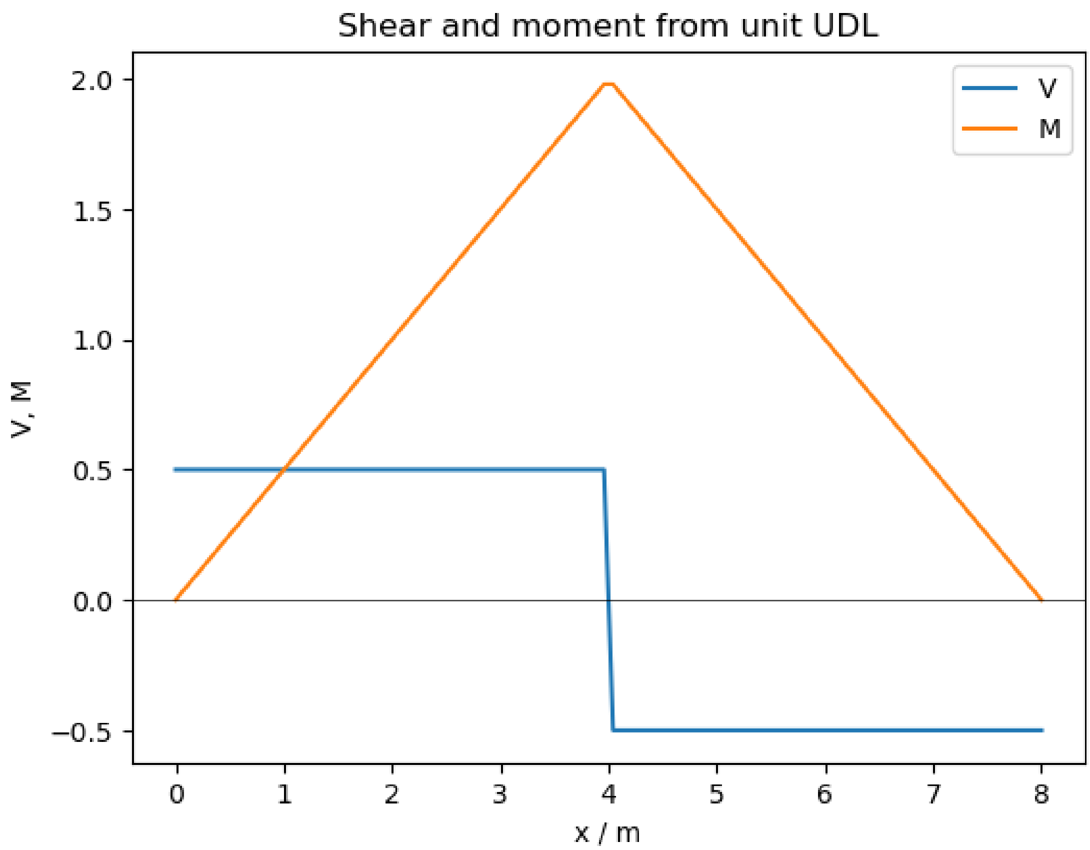
[14]:
```
def V_2(x, P):
    """Return the shear at point x when loaded by point load P at midspan."""
    if isinstance(x.magnitude, np.ndarray):
        Vee = np.where( x < L / 2, P / 2, -P / 2 )
    else:
        if x < L / 2:
            Vee = P / 2
```

```python
        elif x > L / 2:
            Vee = -P / 2
    return Vee

def M_2(x, P):
    """Return the bending moment at point x when loaded by point load P at midspan."""
    if isinstance(x.magnitude, np.ndarray):
        Emm = np.where( x < L / 2, P / 2 * x, P / 2 * ( L - x ) )
    else:
        if x < L / 2:
            Emm = P / 2 * x
        elif x > L / 2:
            Emm = P / 2 * ( L - x )
    return Emm

def Δ_2(x, P):
    """Return the deflection down at point x when loaded by point load P at midspan."""
    if isinstance(x.magnitude, np.ndarray):
        Delta = np.where( x < L / 2, P * x / ( 48 * E_c * I ) * ( 3 * L**2 - 4 * x**2 ),
                                    P * ( L - x ) / ( 48 * E_c * I ) * ( 3 * L**2 - 4 * (L - x )**2 ) )
    else:
        if x < L / 2:
            Delta = P * x / ( 48 * E_c * I ) * ( 3 * L**2 - 4 * x**2 )
        elif x > L / 2:
            Delta = P * ( L - x ) / ( 48 * E_c * I ) * ( 3 * L**2 - 4 * (L - x)**2 )
    return Delta
```

[15]:

```python
fig, ax = plt.subplots()

ax.plot(x11, V_2(x11, Q_(1, 'kN')).magnitude, label='V')
ax.plot(x11, M_2(x11, Q_(1, 'kN')).magnitude, label='M')
plt.axhline(y = 0, color = 'k', lw = 0.5)

plt.xlabel('x / m')
ax.xaxis.set_units(ureg['m'])
plt.ylabel('V, M')
ax.legend()

plt.title('Shear and moment from unit midspan point load')
plt.savefig('ein-images/shear-moment-2.png')
plt.show
```

[15]: <function matplotlib.pyplot.show(close=None, block=None)>



5.2 Applied loads

5.2.1 Self-weight

[16]:
```
def V_SW(x):
    """Return the shear at point x from self-weight."""
    return V_1(x, w_SW)

def M_SW(x):
    """Return the shear at point x from self-weight."""
    return M_1(x, w_SW)
```

5.2.2 Live point load

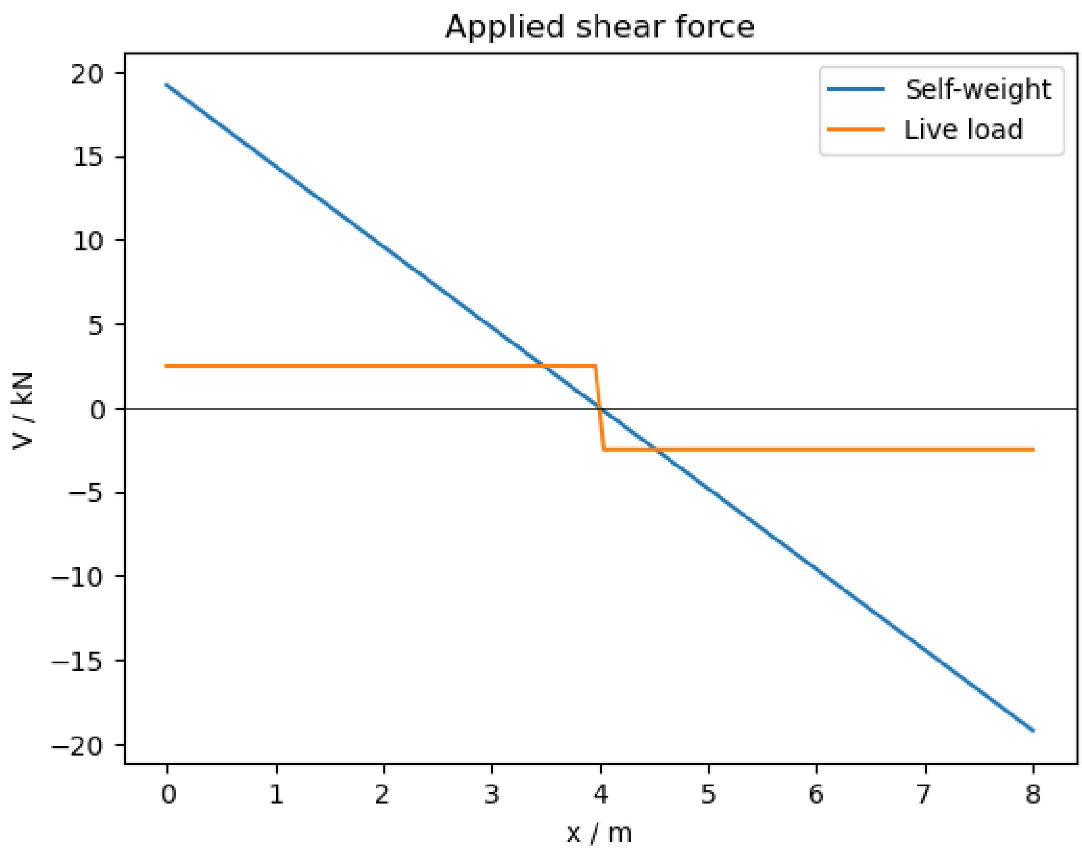
[17]:
```
def V_LL(x):
    """Return the shear at point x from point live load."""
    return V_2(x, P_LL)
```

```
def M_LL(x):
    """Return the shear at point x from point live load."""
    return M_2(x, P_LL)
```

5.3 Load effects

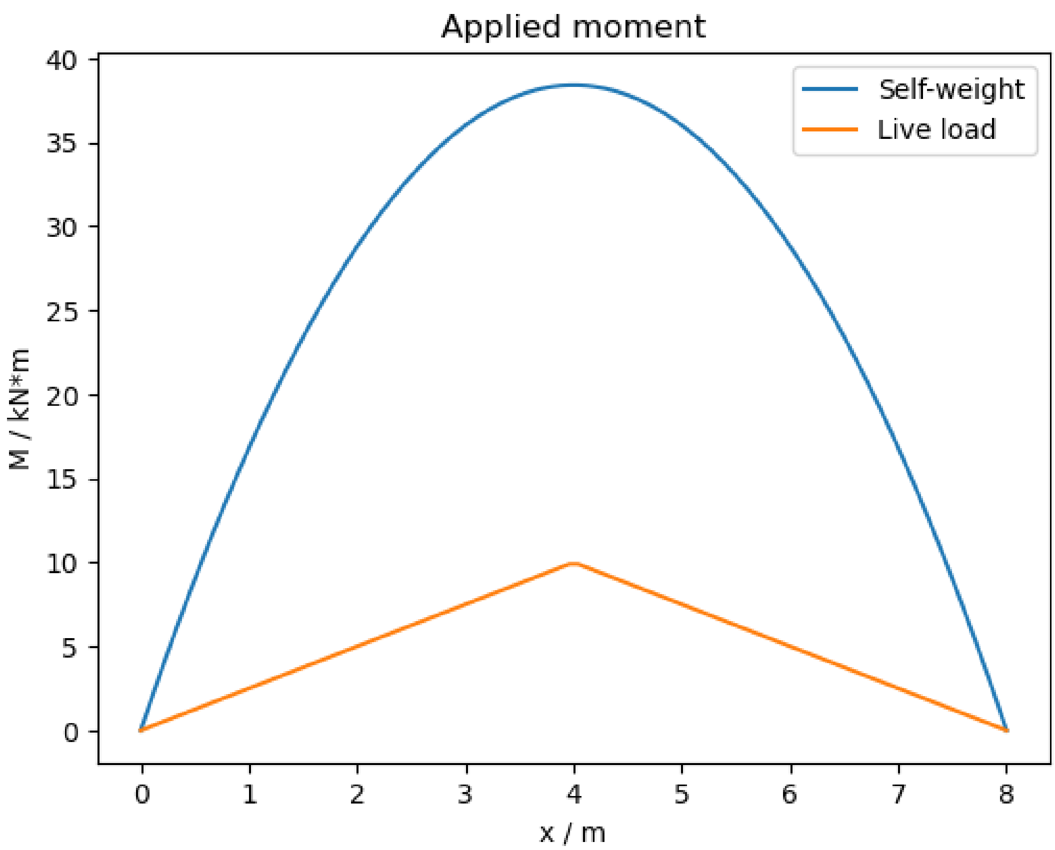
[18]:
```
fig, ax = plt.subplots()

ax.plot(x11, V_SW(x11), label='Self-weight')
ax.plot(x11, V_LL(x11), label='Live load')
plt.axhline(y = 0, color = 'k', lw = 0.5)

plt.xlabel('x / m')
ax.xaxis.set_units(ureg['m'])
plt.ylabel('V / kN')
ax.yaxis.set_units(ureg['kN'])
ax.legend()

plt.title('Applied shear force')
plt.savefig('ein-images/shear-1.png')
plt.show
```

[18]:
```
<function matplotlib.pyplot.show(close=None, block=None)>
```



[19]:
```
fig, ax = plt.subplots()

ax.plot(x11, M_SW(x11), label='Self-weight')
ax.plot(x11, M_LL(x11), label='Live load')

plt.xlabel('x / m')
ax.xaxis.set_units(ureg['m'])
plt.ylabel('M / kN*m')
ax.yaxis.set_units(ureg['kN*m'])
ax.legend()

plt.title('Applied moment')
plt.savefig('ein-images/moment-1.png')
plt.show
```

[19]:
```
<function matplotlib.pyplot.show(close=None, block=None)>
```



6 Load Combinations

6.1 Load factors

[20]:
```
_D1_max = 1.10                  # S6:19 Tab. 3.3
_D1_min = 0.95
_D2_max = 1.20
_D2_min = 0.90
_D3_max = 1.25
_D3_min = 0.80
_D_ULS9 = 1.35                  # S6:19 Tab. 3.1

_L_ULS1 = 1.70                  # S6:19 Tab. 3.2
_L_SLS1 = 0.90                  # S6:19 Tab. 3.1
```

6.2 Ultimate limit states

[21]:
```
def V_f_ULS1a(x):
    """Return shear at ULS1 with maximum load factors."""
    return  _D2_max * V_SW(x) +  _L_ULS1 * V_LL(x)

def M_f_ULS1a(x):
    """Return moment at ULS1 with maximum load factors."""
    return  _D2_max * M_SW(x) +  _L_ULS1 * M_LL(x)

def V_f_ULS9(x):
    """Return shear at ULS9 with maximum load factors."""
    return  _D_ULS9 * V_SW(x)

def M_f_ULS9(x):
    """Return moment at ULS9 with maximum load factors."""
    return  _D_ULS9 * M_SW(x)
```

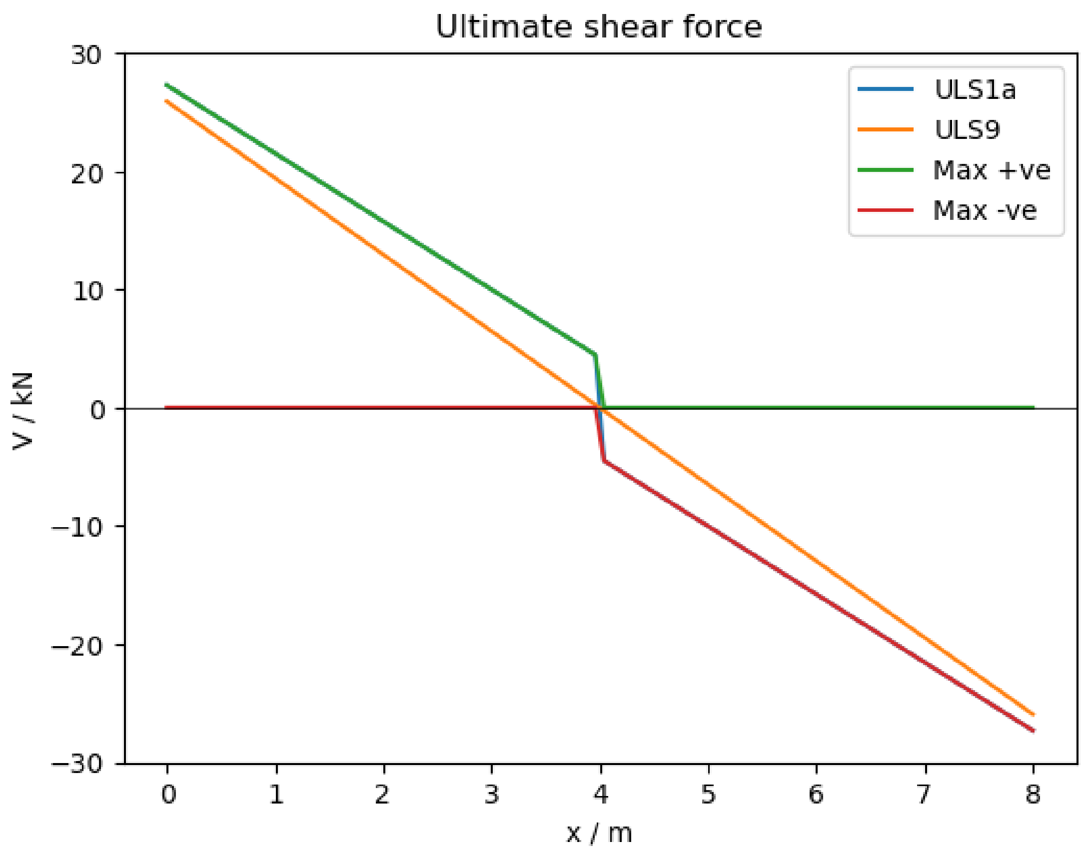
[22]:
```
def V_f_maxp(x):
    """Return greatest positive shear for all ULS combinations."""
    if isinstance(x.magnitude, np.ndarray):
        z = np.zeros(len(x)) * Q_(0, 'kN')
        Vf = V_f_ULS1a(x).to('kN'), V_f_ULS9(x).to('kN'), z
        return np.amax(Vf, axis=0) * Q_(1, 'kN')
    else:
        return max(V_f_ULS1a(x), V_f_ULS9(x), Q_(0, 'kN'))

def M_f_maxp(x):
    """Return greatest positive moment for all ULS combinations."""
    if isinstance(x.magnitude, np.ndarray):
        z = np.zeros(len(x)) * Q_(0, 'kN*m')
        Mf = M_f_ULS1a(x).to('kN*m'), M_f_ULS9(x).to('kN*m'), z
        return np.amax(Mf, axis=0)  * Q_(1, 'kN*m')
    else:
        return max(M_f_ULS1a(x), M_f_ULS9(x), Q_(0, 'kN*m'))

def V_f_maxn(x):
    """Return greatest negative shear for all ULS combinations."""
    if isinstance(x.magnitude, np.ndarray):
        z = np.zeros(len(x)) * Q_(0, 'kN')
        Vf = V_f_ULS1a(x).to('kN'), V_f_ULS9(x).to('kN'), z
        return np.amin(Vf, axis=0) * Q_(1, 'kN')
    else:
        return min(V_f_ULS1a(x), V_f_ULS9(x), Q_(0, 'kN'))

def M_f_maxn(x):
    """Return greatest negative moment for all ULS combinations."""
    if isinstance(x.magnitude, np.ndarray):
```

```
        z = np.zeros(len(x)) * Q_(0, 'kN*m')
        Mf = M_f_ULS1a(x).to('kN*m'), M_f_ULS9(x).to('kN*m'), z
        return np.amin(Mf, axis=0) *  Q_(1, 'kN*m')
    else:
        return min(M_f_ULS1a(x), M_f_ULS9(x), Q_(0, 'kN*m'))
```

[23]:
```
fig, ax = plt.subplots()

ax.plot(x11, V_f_ULS1a(x11), label='ULS1a')
ax.plot(x11, V_f_ULS9(x11), label='ULS9')
ax.plot(x11, V_f_maxp(x11), label='Max +ve')
ax.plot(x11, V_f_maxn(x11), label='Max -ve')
plt.axhline(y = Q_(0, 'kN'), color = 'k', lw = 0.5)

plt.xlabel('x / m')
ax.xaxis.set_units(ureg['m'])
plt.ylabel('V / kN')
ax.yaxis.set_units(ureg['kN'])
ax.legend()

plt.title('Ultimate shear force')
plt.savefig('ein-images/shear-ULS-1.png')
plt.show
```

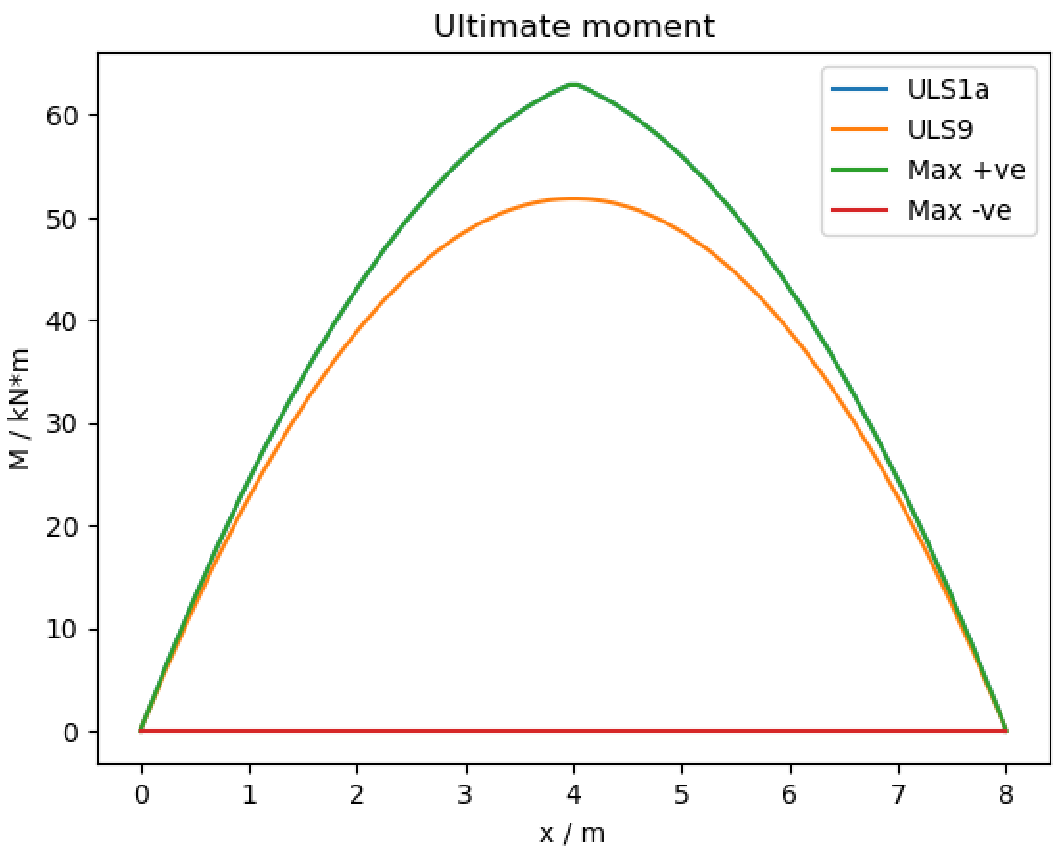
```
/usr/lib/python3/dist-packages/numpy/core/fromnumeric.py:86:
UnitStrippedWarning: The unit of the quantity is stripped when downcasting to
ndarray.
  return ufunc.reduce(obj, axis, dtype, out, **passkwargs)
```

[23]:
```
<function matplotlib.pyplot.show(close=None, block=None)>
```



[24]:
```
fig, ax = plt.subplots()

ax.plot(x11, M_f_ULS1a(x11), label='ULS1a')
ax.plot(x11, M_f_ULS9(x11), label='ULS9')
ax.plot(x11, M_f_maxp(x11), label='Max +ve')
ax.plot(x11, M_f_maxn(x11), label='Max -ve')

plt.xlabel('x / m')
ax.xaxis.set_units(ureg['m'])
plt.ylabel('M / kN*m')
ax.yaxis.set_units(ureg['kN*m'])
ax.legend()

plt.title('Ultimate moment')
plt.savefig('ein-images/moment-ULS-1.png')
plt.show
```

[24]:
```
<function matplotlib.pyplot.show(close=None, block=None)>
```



6.3 Fatigue limit states

Not applicable

6.4 Serviceability limit states

[25]:
```
def M_s(x):
    """Return moment at ULS1 with maximum load factors."""
    return M_SW(x) +  _ULS1_L * M_LL(x)
```

7 Ultimate Limit States Checks

7.1 Resistance factors

[26]:
```
_c = 0.75                      # S6:19 Tab. 8.1
_s = 0.90                      # Greek phi symbol
```

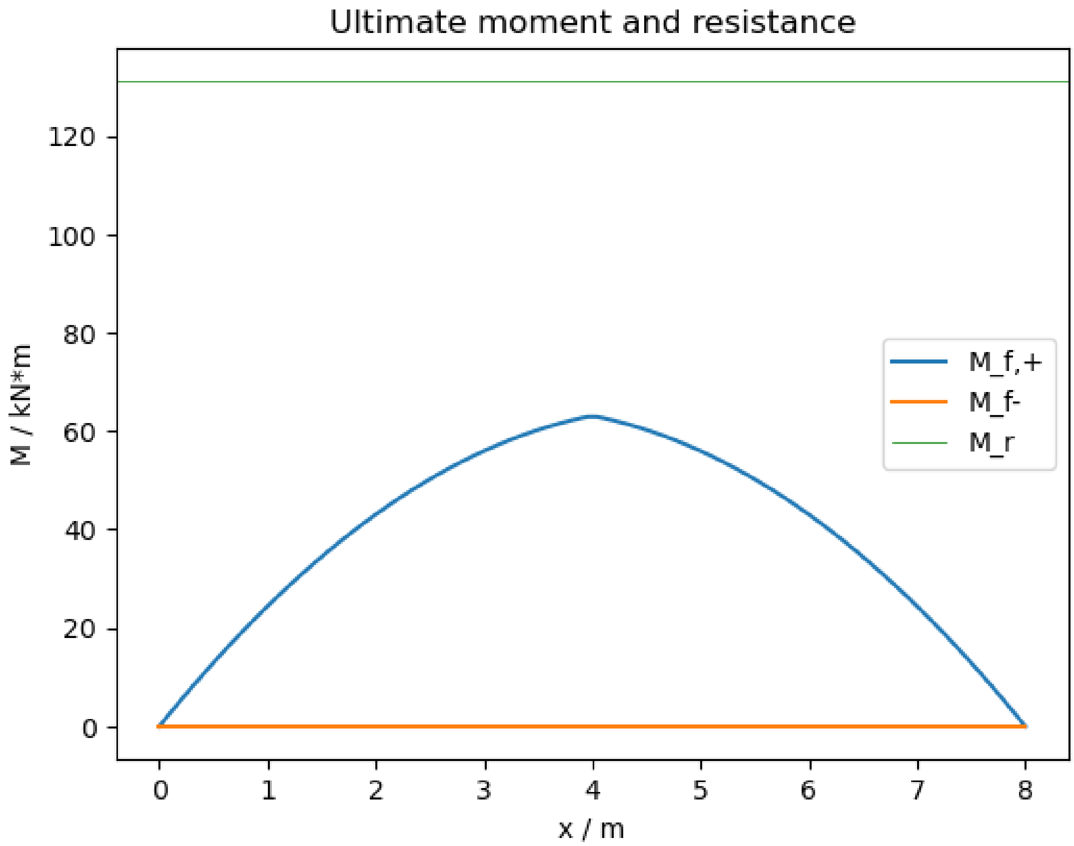
7.2 Bending moment

[27]:
```
a =  _s * A_s * f_y / (  _1 *  _c * fprime_c * b )
M_r =  _s * A_s * f_y * ( d - a / 2 )
M_r.ito('kN*m')
print(f'Moment resistance, {M_r=}')
```

```
Moment resistance, M_r=<Quantity(131.188841, 'kilonewton * meter')>
```

[28]:
```
fig, ax = plt.subplots()

ax.plot(x11, M_f_maxp(x11), label='M_f,+')
ax.plot(x11, M_f_maxn(x11), label='M_f-')
plt.axhline(y = M_r, color = 'g', lw = 0.5, label='M_r')

plt.xlabel('x / m')
ax.xaxis.set_units(ureg['m'])
plt.ylabel('M / kN*m')
ax.yaxis.set_units(ureg['kN*m'])
ax.legend()

plt.title('Ultimate moment and resistance')
plt.savefig('ein-images/Mf-Mr-1.png')
plt.show
```

[28]:
```
<function matplotlib.pyplot.show(close=None, block=None)>
```



7.3 Shear

[29]:
```
d_v = min
```

[30]:
```
 _x =

  =
```

```
  Cell In[30], line 1
     _x =
         ^
SyntaxError: invalid syntax
```

[ ]:
```
V_s =

V_c =

V_r = min
```

```
[ ]:
```

8 Serviceability Limit States Checks

8.1 Cracking

```
[ ]:
```

9 Other Checks

10 Conclusions