- Output Primitives: Basic geometric structures (points, straight line segment, circles and other conic sections, quadric surfaces, spline curve and surfaces, polygon color areas, and character strings)
- These picture components are often defined in a continuous space.

- In order to draw the primitive objects, one has to first scan convert the object.
- Scan convert: Refers to be operation of finding out the location of pixels to the intensified and then setting the values of corresponding bits, in the graphic memory, to the desired intensity code.

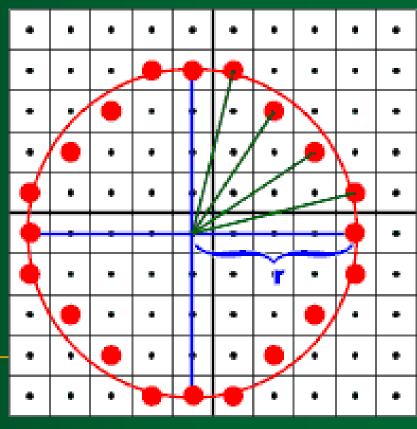
Each pixel on the display surface has a finite size depending on the screen resolution and hence a pixel *cannot* represent a single mathematical point.

The equation of a circle:

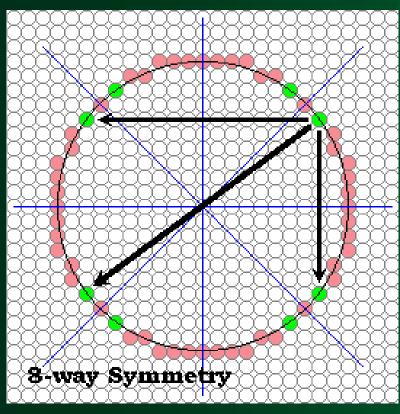
$$(x-x_0)^2 + (y-y_0)^2 = r^2$$

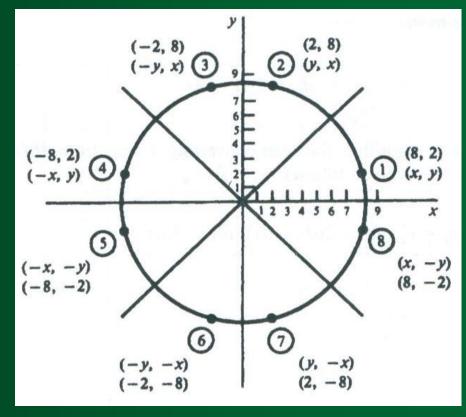
We could solve for y in terms of x

$$y = y_0 \pm \sqrt{r^2 - (x - x_0)^2}$$



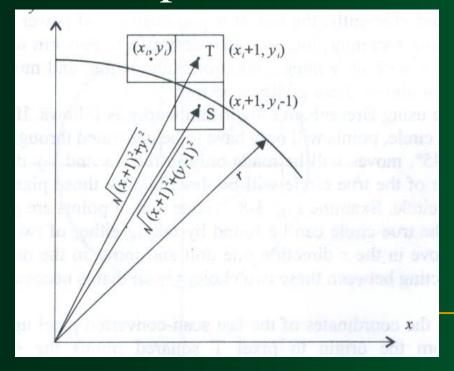
 Computation can be reduced by considering the symmetry of circles

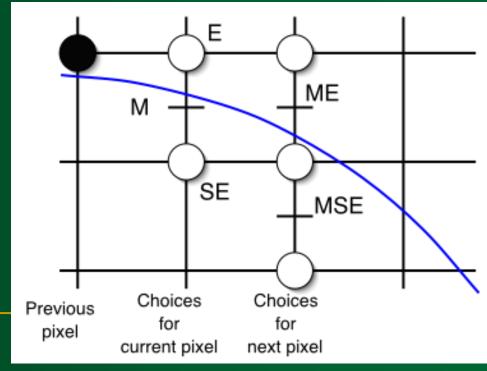




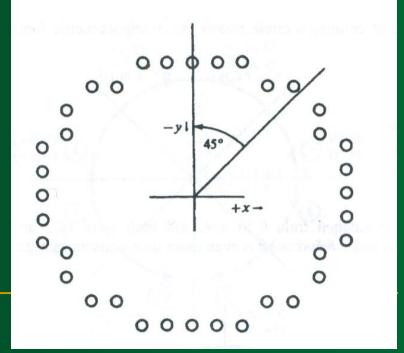
Midpoint Circle Algorithm

As in the line algorithm, we sample at unit intervals and determine the closet pixel position to the circle path at each step.



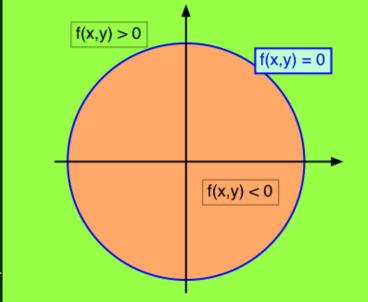


- Points are generated from 90° to 45°, moves will be made only in the +x and -y direction.
- positive x direction over this octant and use
 a decision parameter



We define a circle function:

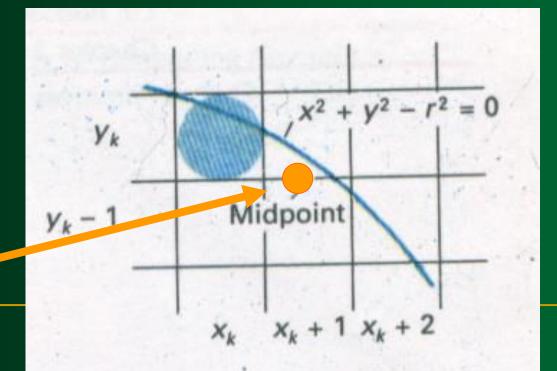
$$f_{circle}(x, y) = x^{2} + y^{2} - r^{2} \begin{cases} < 0 \\ = 0 \\ > 0 \end{cases}$$



Midpoint

$$(x_i + 1, y_i - \frac{1}{2})$$

 Consider the coordinates of the point halfway between pixel T and pixel S

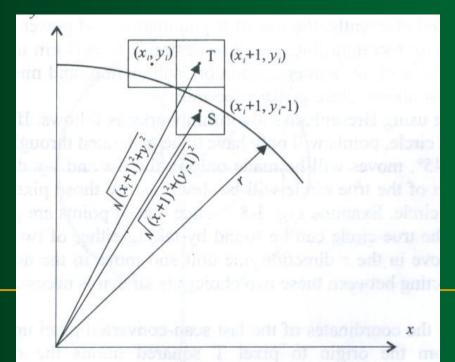


Midpoint

We use it to define a decision parameter

$$p_i = f(x_i + 1, y - \frac{1}{2}) = (x_i + 1)^2 + (y_i - \frac{1}{2})^2 - r^2$$

- If $p_i < 0$ is negative, the midpoint is inside the pixel, and we choose pixel T.
- If $p_i \ge 0$ we choose pixel S.



Parameter for the next step is:

$$p_{i+1} = (x_{i+1} + 1)^2 + (y_{i+1} - \frac{1}{2}) - r^2$$

since

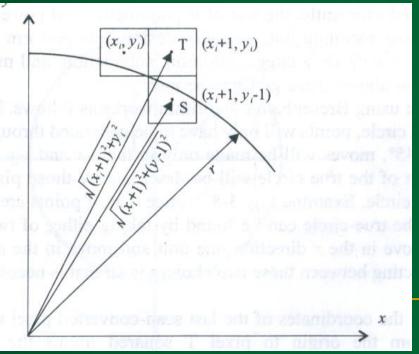
$$x_{i+1} = x_i + 1$$

$$p_{i+1} - p_i = [(x_i + 1) + 1]^2 - (x_i + 1)^2 + (y_{i+1} - \frac{1}{2})^2 - (y_i - \frac{1}{2})^2$$
$$p_{i+1} = p_i + 2(x_i + 1) + 1 + (y_{i+1}^2 - y_i^2) - (y_{i+1} - y_i)$$

If T is chosen ($p_i < 0$) we have: $y_{i+1} = y_i$

If pixel S is chosen ($p_i \ge 0$) we

have $y_{i+1} = y_i - 1$



In terms of (x_{i+1}, y_{i+1})

$$p_{i+1} = \begin{cases} p_i + 2x_{i+1} + 1 & \text{if } p_i < 0\\ p_i + 2(x_{i+1} - y_{i+1}) + 1 & \text{if } p_i \ge 0 \end{cases}$$

Initial value for the decision parameter using the original function of (0,r(

$$p_{i+1} = (x_{i+1} + 1)^2 + (y_{i+1} - \frac{1}{2})^2 - r^2$$

$$p_0 = (0+1)^2 + (r - \frac{1}{2})^2 - r^2 = \frac{5}{4} - r$$

When r is an integer we can simply set

$$p_0 = 1 - r$$

- **Example:** A circle radius r=10

x=0 to x=y
$$P_0=1-r=-9$$

$$p_{i+1} = \begin{cases} p_i + 2x_{i+1} + 1 & \text{if } p_i < 0 \\ p_i + 2(x_{i+1} - y_{i+1}) + 1 & \text{if } p_i < 0 \end{cases}$$

$$p_0 = 1 - r$$

Exercises