Problem Set 2

Problem 1

$p_{x,y}(\text{k, l}) = c(k^2 + l^2)$

k : {-1, 0, 1, 3}

l: {-1, 2, 3}

$\sum_k \sum_l p_{x,y} = 1$

89c = 1

c = 1/89

Checking my work

```
pxy = c (k^2 + l^2)
soln = Solve[Sum[Sum[pxy, {k, {-1, 0, 1, 3}}], {l, {-1, 2, 3}}] == 1, c]
pxy /. soln /. {k → 3, l → 3}
```

Part II

$p_{x,y}(\text{k, l}) = \frac{1}{89}(k^2 + l^2)$

(y, x)	-1	0	1	3
-1	$\frac{2}{89}$	$\frac{1}{89}$	$\frac{2}{89}$	$\frac{10}{89}$
2	$\frac{5}{89}$	$\frac{4}{89}$	$\frac{5}{89}$	$\frac{13}{89}$
3	$\mathbf{\frac{10}{89}}$	$\mathbf{\frac{9}{89}}$	$\mathbf{\frac{10}{89}}$	$\frac{18}{89}$

$P(X \leq 1, Y > 2) = \frac{29}{89}$

Problem 2

In[7]:=
```
pxy = { 24 x y   0 < x < 1 && 0 < y < 1 && x + y < 1
      { 0        True
```

$$\int_{-\infty}^{\infty}\int_{-\infty}^{(1/2)-x} pxy\, dy\, dx$$

```
N[%]
```

Out[7]= $\begin{cases} 24\,x\,y & 0 < x < 1 \,\&\&\, 0 < y < 1 \,\&\&\, x + y < 1 \\ 0 & \text{True} \end{cases}$

Out[8]= $\frac{1}{16}$

Out[9]= 0.0625

Problem 3

```
cxy = { (1 - Exp[-x^2]) (1 - Exp[-y^2])   x ≥ 0 && y ≥ 0
      { 0                                 True
```

$pxy = \partial_y \partial_x cxy$

$\begin{cases} \left(1 - e^{-x^2}\right)\left(1 - e^{-y^2}\right) & x \geq 0 \,\&\&\, y \geq 0 \\ 0 & \text{True} \end{cases}$

$\begin{cases} 4\,e^{-x^2}\,x\,y - 4\,e^{-x^2-y^2}\left(-1 + e^{y^2}\right)x\,y & x \geq 0 \,\&\&\, y \geq 0 \\ 0 & \text{True} \end{cases}$

Problem 4

In[34]:=
```
cxy = { 1 - Exp[-x] - Exp[-y] + Exp[-x - y]   x ≥ 0 && y ≥ 0
      { 0                                     True
```

$pxy = \partial_y \partial_x cxy$

$$\left(\int_{3}^{\infty}\int_{-\infty}^{y-3} pxy\, dx\, dy\right) \text{ // FullSimplify}$$

```
N[%]
```

Out[34]= $\begin{cases} 1 - e^{-x} + e^{-x-y} - e^{-y} & x \geq 0 \,\&\&\, y \geq 0 \\ 0 & \text{True} \end{cases}$

Out[35]= $\begin{cases} e^{-x} - e^{-x-y}\left(-1 + e^{y}\right) & x \geq 0 \,\&\&\, y \geq 0 \\ 0 & \text{True} \end{cases}$

Out[36]= $\frac{1}{2\,e^{3}}$

Out[37]= 0.0248935

Problem 5

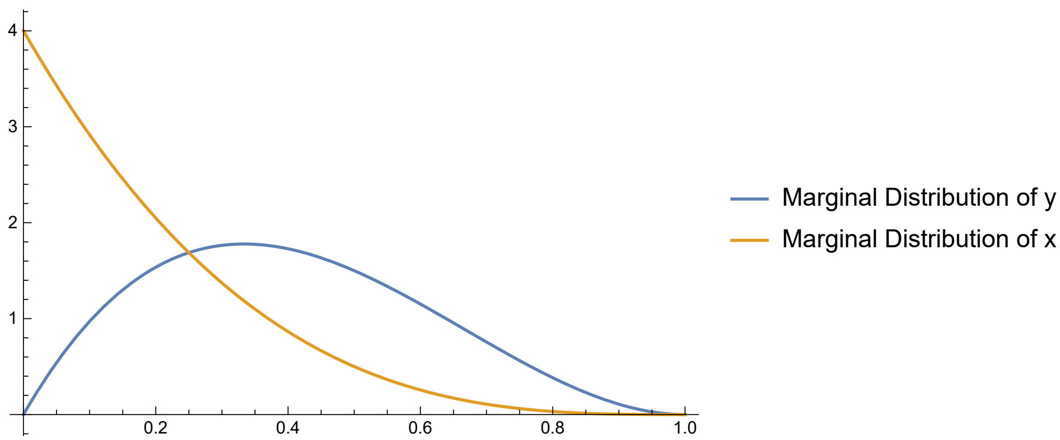
$$\mathbf{pxy} = \begin{cases} 24\,y\,(1-x-y) & x > 0\ \&\&\ y > 0\ \&\&\ x + y < 1 \\ 0 & \text{True} \end{cases}$$

$$\begin{cases} 24\,(1-x-y)\,y & x > 0\ \&\&\ y > 0\ \&\&\ x + y < 1 \\ 0 & \text{True} \end{cases}$$

$$\mathbf{mx} = \int_{-\infty}^{\infty} \mathbf{pxy}\, dx$$

$$\begin{cases} 12\,(y - 2\,y^2 + y^3) & 0 < y < 1 \\ 0 & \text{True} \end{cases}$$

$$\begin{cases} -4\,(-1+x)^3 & 0 < x < 1 \\ 0 & \text{True} \end{cases}$$



Problem 6

```
MomentGeneratingFunction[ProbabilityDistribution[2 × 3^-x, {x, 1, ∞}], t]
```

$$\mathbf{firstMoment} = \partial_t\left(-\frac{2\,e^t}{3\,t - \mathrm{Log}[27]}\right)$$

$$\mathbf{secondMoment} = \partial_t\left(\frac{6\,e^t}{(3\,t - \mathrm{Log}[27])^2} - \frac{2\,e^t}{3\,t - \mathrm{Log}[27]}\right)$$

$$-\frac{2\,e^t}{3\,t - \mathrm{Log}[27]}$$

$$\frac{6\,e^t}{(3\,t - \mathrm{Log}[27])^2} - \frac{2\,e^t}{3\,t - \mathrm{Log}[27]}$$

$$-\frac{36\,e^t}{(3\,t - \mathrm{Log}[27])^3} + \frac{12\,e^t}{(3\,t - \mathrm{Log}[27])^2} - \frac{2\,e^t}{3\,t - \mathrm{Log}[27]}$$

Problem 7

$Z = (X - 3)/4$

$Z = \frac{1}{4}x - \frac{3}{4}$

$M_x(t) = e^{3t+8t^2}$

$M_z(t) = e^{at}\, M_x(bt)$

$M_z(t) = e^{t/4}\, M_x(3t/4) = e^{t/4}\, e^{3(3t/4)+8(3t/4)^2} = e^{\frac{1}{2} t\,(5+9t)}$

$M_z'(t) = \frac{1}{2}\, e^{\frac{1}{2} t\,(5+9t)}\, (5 + 18t)$

$M_z'(0) = 5/2$

$M_z''(t) = \frac{1}{4}\, e^{\frac{1}{2} t\,(5+9t)}\, (61 + 36t\,(5+9t))$

$M_z''(0) = \frac{61}{4}$

$M_z''(0) - M_z'(0)^2 = \frac{61}{4} - (5/2)^2$

$M_z''(0) - M_z'(0)^2 = 9$

$\mu = \frac{5}{2},\ \sigma = 9$

```
Exp[t / 4] Exp[3 (3 t / 4) + 8 (3 t / 4) ^2]
D[%, t]
D[%, t] /. {t → 0} // FullSimplify
% - ((5 / 2) ^2)
```

$e^{\frac{5t}{2}+\frac{9t^2}{2}}$

$e^{\frac{5t}{2}+\frac{9t^2}{2}} \left(\frac{5}{2} + 9t\right)$

$\frac{61}{4}$

9

Problem 8

$f_x(x) = 2\,x\, e^{-x^2}\, 1_{[0,\infty)}(x)$

$Y = X^2$

$X = Y^{1/2} = g^{-1}(y)$

$\frac{\partial g^{-1}(y)}{\partial y} = \frac{1}{2}\, y^{\frac{-1}{2}}$

$f_y(y) = f_x(y^{1/2})\, \frac{1}{2}\, y^{\frac{-1}{2}} = 2\,(y^{1/2})\, e^{\left(-y^{\frac{1}{2}}\right)^2}\, \frac{1}{2}\, y^{\frac{-1}{2}} = e^{-y}$