

TODO

- Main Body
- Update/Verify tables
 - Add GREXIT Analysis
 - Calculate Roll and Amihud Measures for GREXIT
- Appendices
- Appendix: Reanalysis w/ outliers included
 - Appendix: Sensitivity to CARA utility assumptions
 - Appendix: Alternative estimators for D'D
 - Appendix: Alternative estimator for volatility

Optimal Order Qty under CRRA utility

The final wealth of the liquidity trader is given by:

$$\begin{aligned}w(Q) &= Q \cdot p_b + (L-Q) \cdot (1+z) = Q \cdot p_b + L - Q + (L-Q)z \\ &= Q(p_b - 1) + L + (L-Q)z\end{aligned}$$

Let the trader's utility fn over wealth be:

$$u(w) = \begin{cases} \frac{w^{1-\gamma} - 1}{1-\gamma}, & \gamma \neq 1 \\ \ln w, & \gamma = 1 \end{cases}$$

Case 1: $\gamma \neq 1$

Let $\mu_w = E[w(Q)] = Q \cdot (p_b - 1) + L$ be the expected wealth. A second-order Taylor-series approximation of the utility fn around μ_w is given by

$$\begin{aligned}u'(w) &= w^{-\gamma} \\ u''(w) &= -\gamma w^{-\gamma-1}\end{aligned}$$

Aside: Measure of relative risk aversion

$$RRA = -\frac{w u''}{u'} =$$

$$u(w) \Big|_{w=\mu_w} \approx u(\mu_w) + (w-\mu_w) u'(\mu_w) + \frac{1}{2} (w-\mu_w)^2 u''(\mu_w) \quad \rightarrow \quad \left[-\frac{\gamma w^{-\gamma-1} \cdot w}{w^{-\gamma}} \right]$$

1

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
 $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

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100

$$\begin{aligned}
 u(w) \Big|_{w=\mu_w} &\approx u(\mu_w) + (w-\mu_w)u'(\mu_w) + \frac{1}{2}(w-\mu_w)^2 u''(\mu_w) \\
 E[u(w)] &\approx u(\mu_w) + \frac{1}{2}(w-\mu_w)^2 u''(\mu_w)
 \end{aligned}
 \left. \begin{array}{l} \text{Aside: RRA measure} \\ RRA = - \left[\frac{-\gamma w^{-\gamma-1} \cdot w}{w^{-\gamma}} \right] \\ RRA = \gamma \text{ constant} \end{array} \right\}$$

where I have used the fact that $E[w - \mu_w] = 0$
 Note that $E(w - \mu_w)^2 = (L - Q)^2 \sigma^2$

$$\therefore E[u(w)] \Big|_{w=\mu_w} \approx \frac{\mu_w(Q)^{1-\gamma} - 1}{1-\gamma} - \frac{\gamma}{2} \mu_w(Q)^{-\gamma-1} (L-Q)^2 \sigma^2$$

where I write $\mu_w = \mu_w(Q)$ to highlight that the expected wealth is a fn. of the order qty.

To find the optimal Q , take the derivative of EU and set it to 0:

Foc:

$$0 = \frac{d}{dQ} E[u(w(Q))] = \frac{d}{dQ} \left[\frac{\mu_w(Q)^{1-\gamma} - 1}{1-\gamma} - \frac{\gamma}{2} \mu_w(Q)^{-\gamma-1} (L-Q)^2 \sigma^2 \right]$$

$$\text{Note: } w(Q) = Q(p_b - 1) + L + (L-Q)\epsilon \rightarrow E[w(Q)] = \mu_w = Q(p_b - 1) + L$$

$$\frac{d \mu_w(Q)}{dQ} = \frac{d}{dQ} [Q(p_b - 1) + L] = p_b - 1$$

$$\frac{d}{dQ} [E[u(w(Q))]] = \frac{(1-\gamma) \mu_w(Q)^{-\gamma} (p_b - 1)}{1-\gamma} - \frac{\gamma(-\gamma-1) \mu_w(Q)^{-\gamma-2} (p_b - 1) (L-Q)^2 \sigma^2}{2}$$

$$- \gamma \cdot \mu_w(Q)^{-\gamma-1} (L-Q)(-1) \sigma^2 = 0$$

$$\text{which simplifies to: } \mu_w(Q)^{-\gamma} (p_b - 1) + \frac{\gamma(L-Q)\sigma^2}{\mu_w(Q)^{\gamma+1}} = 0$$

1. The first part of the paper is devoted to a general discussion of the problem.

2. In the second part we shall consider the case of a homogeneous medium.

3. The third part is devoted to the study of the properties of the solutions.

4. In the fourth part we shall discuss the question of the stability of the solutions.

5. The fifth part is devoted to the study of the asymptotic behavior of the solutions.

6. In the sixth part we shall consider the case of a non-homogeneous medium.

7. The seventh part is devoted to the study of the properties of the solutions.

8. In the eighth part we shall discuss the question of the stability of the solutions.

9. The ninth part is devoted to the study of the asymptotic behavior of the solutions.

10. In the tenth part we shall consider the case of a non-homogeneous medium.

11. The eleventh part is devoted to the study of the properties of the solutions.

Solving this for Q one obtains:

~~Aside RPA measure~~

$$Q = L + \frac{(p_b - 1) \mu_w(Q)^{2\gamma+1}}{\gamma \sigma^2}$$

Note that this expression is similar to the optimal order quantity under CARA (previously derived):

$$Q_{\text{CARA}} = \frac{1}{\alpha \sigma^2} (p_b^{\text{CARA}} - 1) + L \quad \text{w/ an additional}$$

$\mu_w(Q)^{2\gamma+1}$ term in the numerator of the fraction showing order qty. is sensitive to the (expected) level of wealth of the trader.

The optimal bid price for dealer 1

I now derive the optimal bid price for dealer 1 (Note: the problem is symmetric so dealer 2 will have the same solution).

Dealer 1 receives random inventory pos'n $I_1^1 \sim U(-R, R]$. If s/he has the best bid price and serves order qty Q , his/her wealth at the end of the game is:

$$W = (I_1^1 + Q)(1+z) - p_b^1 Q$$

If s/he does not receive the order flow, s/he will have wealth

$$W^{NT} = I_1^1 (1+z)$$

At some price $p_{b,r}^1$ the dealer will be indifferent between trading and not trading. This happens when $u(W^{NT}) = u(W)$.

Monday, April 1st 1912

1912

April 1st 1912

Left at 10:00 AM for the city and arrived at 12:30 PM. Found everything in a state of confusion.

Visited the city and saw many interesting things. The people were very friendly and the food was excellent.

Spent the afternoon at the park and saw many beautiful flowers. The children were playing happily and the old people were sitting on the benches.

Went to the city and saw many interesting things. The people were very friendly and the food was excellent.

Spent the evening at the city and saw many beautiful things. The people were very friendly and the food was excellent.

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Assume dealer 1's utility is:

$$u(w) = \begin{cases} \frac{w^{1-\gamma} - 1}{1-\gamma}, & \gamma \neq 1 \\ \ln w, & \gamma = 1 \end{cases}$$

Case 1: $\gamma \neq 1$

Let $\mu_w(p_{b,r}^1)$ ~~be the~~ $= \mathbb{E}[(I_z^1 + Q)(1+z) - p_{b,r}^1 Q]$
 $= I_z^1 + Q - p_{b,r}^1 Q = I_z^1 + Q(1 - p_{b,r}^1)$

(Note: I_z^1 has been realized and is a constant)

The 2nd-order Taylor-series approx of $u(w)$ about μ_w is:

$$u(w) \Big|_{w=\mu_w} \approx u(\mu_w) + (w - \mu_w)u'(\mu_w) + \frac{1}{2}(w - \mu_w)^2 u''(\mu_w)$$

Taking expectations:

$$\mathbb{E}[u(w)] \approx \mathbb{E}[u(\mu_w)] + \frac{1}{2} u''(\mu_w) (w - \mu_w)^2$$

where: I have used $\mathbb{E}[w - \mu_w] = 0$. Note that $\mathbb{E}[(w - \mu_w)^2] = (I_z^1 + Q)^2 \sigma^2$. $\therefore \mathbb{E}[u(w)] \approx \left[I_z^1 + Q(1 - p_{b,r}^1) \right]^{1-\gamma} - 1$
 $\dots + \frac{1}{2} (I_z^1 + Q)^2 \sigma^2 \cdot \frac{-\gamma (I_z^1 + Q(1 - p_{b,r}^1))^{-\gamma-1}}{1-\gamma}$

The (expected) utility from no trade is:

$$\mathbb{E}[u(w^{NT})] = \mathbb{E}\left[\frac{I_z^1 (1+z)^{1-\gamma} - 1}{1-\gamma} \right]$$

Taking the Taylor-series approx. around $\mu_w(p_{b,r}^1)$:

$$u(w^{NT}) \Big|_{w=\mu_w} \approx u(\mu_w) + (w^{NT} - \mu_w)u'(\mu_w) + \frac{1}{2} u''(\mu_w) (w^{NT} - \mu_w)^2$$

Taking expectations: $\mathbb{E}[u(w^{NT})] \approx \frac{I_z^1 + Q(1 - p_{b,r}^1)^{1-\gamma} - 1}{1-\gamma} + \frac{1}{2} \frac{I_z^1^2 \sigma^2}{1-\gamma}$

1. The first part of the paper is devoted to a discussion of the general principles of the theory of the structure of the atom.

2. The second part is devoted to a discussion of the experimental results obtained in the study of the structure of the atom.

3. The third part is devoted to a discussion of the theoretical results obtained in the study of the structure of the atom.

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12. The twelfth part is devoted to a discussion of the experimental results obtained in the study of the structure of the atom.

$$\dots - \gamma \mu_w^{1-\gamma-1}$$

$$E[u(w^{NT})] \approx \frac{\mu_w(p_{b,r}^1)^{1-\gamma} - 1}{1-\gamma} + \frac{\gamma}{2} \sigma^2 \mu_w(p_{b,r}^1)^{-\gamma-1}$$

Dealer 1
The ~~trader~~ is indifferent when:

$$E[u(w^{NT})] = E[u(w)]$$

$$\frac{[\mu_w(p_{b,r}^1)]^{1-\gamma} - 1}{1-\gamma} = \frac{\gamma}{2} \sigma^2 [\mu_w(p_{b,r}^1)]^{-\gamma-1}$$

$$= \frac{[\mu_w(p_{b,r}^1)]^{1-\gamma} - 1}{1-\gamma} = \frac{\gamma}{2} (I^1 + Q)^2 \mu_w(p_{b,r}^1)^{-\gamma-1}$$

$$I^{1^2} [\mu_w(p_{b,r}^1)]^{-\gamma-1} = (I^{1^2} + 2QI^1 + Q^2) [\mu_w(p_{b,r}^1)]^{-\gamma-1}$$

$$(I^1)^2 = (I^1)^2 + 2QI^1 + Q^2$$

$$(I^1)^2 - (I^1)^2 - 2QI^1 = 0$$

this has

Note that the reservation price is

$$p_{b,r}^1 \approx \frac{\gamma \sigma^2}{2} \mu_w(p_{r,b}^1)$$

w/o solving explicitly

I argue that under ^{perfect} competition, the dealer's markup (not shown) will tend to zero and prices will tend to the reservation price.

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2. The second part is devoted to a detailed analysis of the results.

3. The third part is devoted to a discussion of the conclusions.

4. The fourth part is devoted to a discussion of the future work.

5. The fifth part is devoted to a discussion of the references.

6. The sixth part is devoted to a discussion of the appendix.

7. The seventh part is devoted to a discussion of the bibliography.

8. The eighth part is devoted to a discussion of the index.

9. The ninth part is devoted to a discussion of the table of contents.

10. The tenth part is devoted to a discussion of the list of figures.

11. The eleventh part is devoted to a discussion of the list of tables.