

### 1. Second Derivative $u''(w)$ :

$$u''(w) = \frac{d}{dw} (w^{-\gamma}) = -\gamma w^{-\gamma-1}$$

Evaluated at  $w = \mu$ :

$$u''(\mu) = -\gamma \mu^{-\gamma-1}$$

### 2. Taylor Series Approximation:

Substituting these derivatives into the Taylor series expansion:

$$u(w) \approx \frac{\mu^{1-\gamma}}{1-\gamma} + \mu^{-\gamma}(w - \mu) + \frac{1}{2}(-\gamma \mu^{-\gamma-1})(w - \mu)^2$$

### 3. Expected Utility:

The expected utility, considering  $w$  is normally distributed  $w \sim N(\mu, \sigma^2)$ , is:

$$\mathbb{E}[u(w)] \approx \frac{\mu^{1-\gamma}}{1-\gamma} + \mu^{-\gamma} \mathbb{E}[w - \mu] + \frac{1}{2}(-\gamma \mu^{-\gamma-1}) \mathbb{E}[(w - \mu)^2]$$

Since  $\mathbb{E}[w - \mu] = 0$  and  $\mathbb{E}[(w - \mu)^2] = \sigma^2$ , this simplifies to:

$$\mathbb{E}[u(w)] \approx \frac{\mu^{1-\gamma}}{1-\gamma} - \frac{1}{2} \gamma \mu^{-\gamma-1} \sigma^2$$

Thus, the second-order Taylor series approximation of the expected CRRA utility function is:

$$\mathbb{E}[u(w)] \approx \frac{\mu^{1-\gamma}}{1-\gamma} - \frac{1}{2} \gamma \mu^{-\gamma-1} \sigma^2$$

This result shows how the expected utility is reduced by the variability (risk) in wealth  $\sigma^2$ , with the degree of risk aversion  $\gamma$  playing a crucial role in this reduction.