Consider the CRRA utility function u(w). Assume w ~ N(mu, signa^2). I want to take a second order Taylor series approximation of the function around expected utility.

To take a second-order Taylor series approximation of the CRRA utility function around the expected utility, we can follow these steps:

1. **CRRA Utility Function**: The CRRA (Constant Relative Risk Aversion) utility function is generally given by:

$$u(w) = \frac{w^{1-\gamma}}{1-\gamma}$$

   where w is wealth and γ is the coefficient of relative risk aversion. For $\gamma = 1$, the utility function becomes logarithmic: $u(w) = \log(w)$.

2. **Expected Wealth**: Let's denote the expected value of wealth w as μ (i.e., $\mathbb{E}[w] = \mu$).

3. **Taylor Series Expansion**: To approximate $u(w)$ around the expected wealth μ, we expand $u(w)$ in a Taylor series around μ:

$$u(w) \approx u(\mu) + u'(\mu)(w-\mu) + \frac{1}{2}u''(\mu)(w-\mu)^2$$

   Here, $u'(\mu)$ and $u''(\mu)$ are the first and second derivatives of the utility function with respect to wealth evaluated at μ.

4. **First Derivative** $u'(w)$:

$$u'(w) = \frac{d}{dw}\left(\frac{w^{1-\gamma}}{1-\gamma}\right) = w^{-\gamma}$$

   Evaluated at $w = \mu$:

$$u'(\mu) = \mu^{-\gamma}$$

5. **Second Derivative** $u''(w)$:

$$u''(w) = \frac{d}{dw}\left(w^{-\gamma}\right) = -\gamma w^{-\gamma-1}$$