Building the Bloomberg Interest Rate Curve – Definitions and Methodology.

Abstract

The goal of this document is to describe the process of interest rate (IR) curve construction and stripping in the Bloomberg terminal. We first introduce various types of rates used during curve stripping, then discuss the type of instruments commonly used in building the IR curves (cash rates, futures and IR swaps). Attention is given to a functional form of the curve (a.k.a. interpolation methods) and algorithms for building the curve under these different interpolation methods (i.e. curve *stripping*). We also cover various types of IR curve stripping including both single-currency and cross-currency curve stripping.

1. Interest Rate Curve – Definition

The IR Curve is an object which allows one to calculate a discount factor for every date in the future, thus providing us with the risk-free present value (PV) of a unit of currency (say, $\$1$) paying on that particular future date. It is widely used to calculate present values of a known set of payments (i.e. cash flows) for certain IR instruments. While in some situations one can construct an IR curve which takes in an additional discount (i.e. spread over risk free curve) due to risk of default of the counterparty, this document leaves the discussion of default or credit risk out. For the sake of simplicity, we will assume that the IR curves described in this document produce risk-free present values.

A second use for IR curves is to calculate projected forward rates between two dates ($d_1 \rightarrow d_2$) in the future. A typical example is to construct the payments of a 'floating leg' of an IR swap which pays, say, quarterly an amount of interest equal to 3-month LIBOR rate on a given notional. While the actual payments that will be made in the future are not known until we reach that point in time when LIBOR is fixed, the PV of this stream of payments is correct if we apply current projections of forward rates based on the known curve.

2. Types of Interest Rate

The definition of a *simple spot rate* r_s is expressed as:

$$DF(d_0, d) = \frac{1}{1 + r_s(t) \cdot \tau} \tag{1}$$

Here d_0 is the start date. Usually it is the settlement date of a financial instrument. d is some date in the future, $\tau = \tau(d_0, d)$ is the time interval between two dates (d_0, d) in years, and $DF(d_0, d)$ is the discount factor from the date d to start date d_0. The only undefined term in the above definition is the method to convert a pair of dates (d_0, d) into a time interval τ in years. This conversion method is formally called *Day Count Convention*. There is more than one way of doing this: e.g. both *ACT/360* and *ACT/365* are very widely accepted conventions in the financial markets. For *ACT/360*, we assume there are 360 days per year and $\tau(d_1, d_2) =$

$(d_2 - d_1)/360$. The operation $d_2 - d_1$ returns the actual number of calendar days between two dates. Similarity, for *ACT/365* $\tau(d_1, d_2) = (d_2 - d_1)/365$. There are other day count conventions such as ACT/ACT and 30U/360 which are outside the scope of this document.

The discount factor between two dates $DF(d_0, d)$ is fixed by the market, but day count convention is not, we can get different τ using different day count conventions, and therefore the corresponding rates will be different. Thus, when quoting the rate, it is important to note not only which rate it is (and frequency of compounding) but a day count convention as well. In this document every time we mention conversion of a pair of dates into a time interval t in years, it will be assumed that a day count convention is specified also.

A *simple forward rate* r_{sf} between two dates (d_1, d_2) in the future is defined as:

$$\frac{DF(d_0, d_2)}{DF(d_0, d_1)} = \frac{1}{1 + r_{sf}(t) \cdot \tau} \qquad (2)$$

where $\tau = \tau(d_1, d_2)$.

A *continuously compounded forward rate* $r_{cf}(\tau)$ between two dates (d_1, d_2) in the future is defined as:

$$DF(d_1, d_2) = exp\left[-\int_{T_1}^{T_2} r_{cf}(\tau) \cdot d\tau\right] \qquad (3)$$

where times T_1 and T_2 are equal to the time intervals between date d_0 now and dates d_1 and d_2 in future correspondingly: $T_1 = \tau(d_0, d_1)$ and $T_2 = \tau(d_0, d_2)$.

The *continuously compounded zero rate* $r_{cz}(t)$ is defined as:

$$DF(d_0, d) = exp[-r_{cz}(\tau) \cdot \tau] \qquad (4)$$

where time τ is the time interval between date d_0 now and date d in the future.

3. Building the Interest Rate Curve

The process of building of the IR curve (a.k.a. *curve stripping*) is a process of creating a curve object which would correctly price a set of N given instruments, e.g. produces correct discount factors and forward rates implied by these instruments.

Let us consider various types of IR instruments that can be used to build a typical vanilla swap curve (e.g. S201) in Bloomberg. They typically belong to 3 groups:

1) Cash or Deposit Rates in the short end
2) Interest Rate Futures or Forward Rate Agreements (FRAs) in the middle segment
3) Interest Rate Swaps in the long end

The user may choose the set of instruments to be applied in curve construction by typing **{SWDF <GO>}** in the Bloomberg terminal, choosing the right country/currency, selecting the source 8 (Bloomberg Recommended), saving their choice (**1 <GO>**) and clicking on the curve of interest. If the user knows the curve number, they can directly type **{SWDF ## <GO>}** in the terminal. A screen like the below should appear:

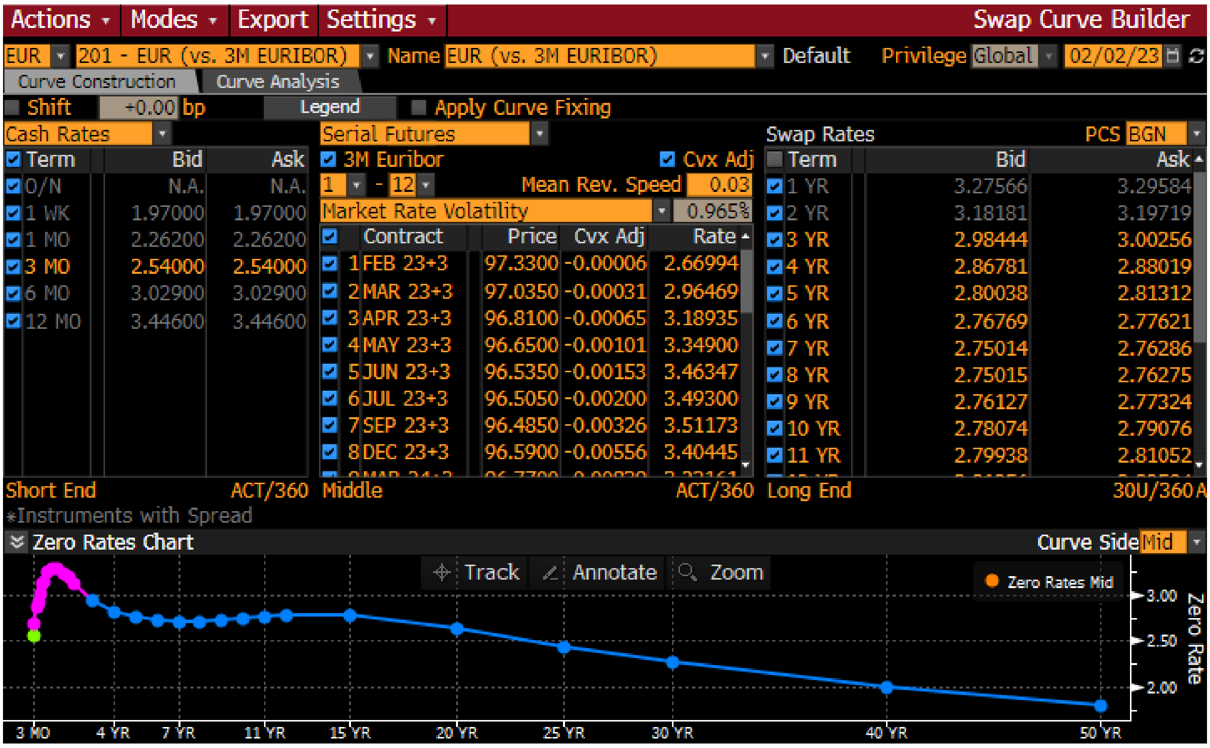


Fig. 1a. Example of Bloomberg screen to choose a set of instruments for building EUR IR curve S201.

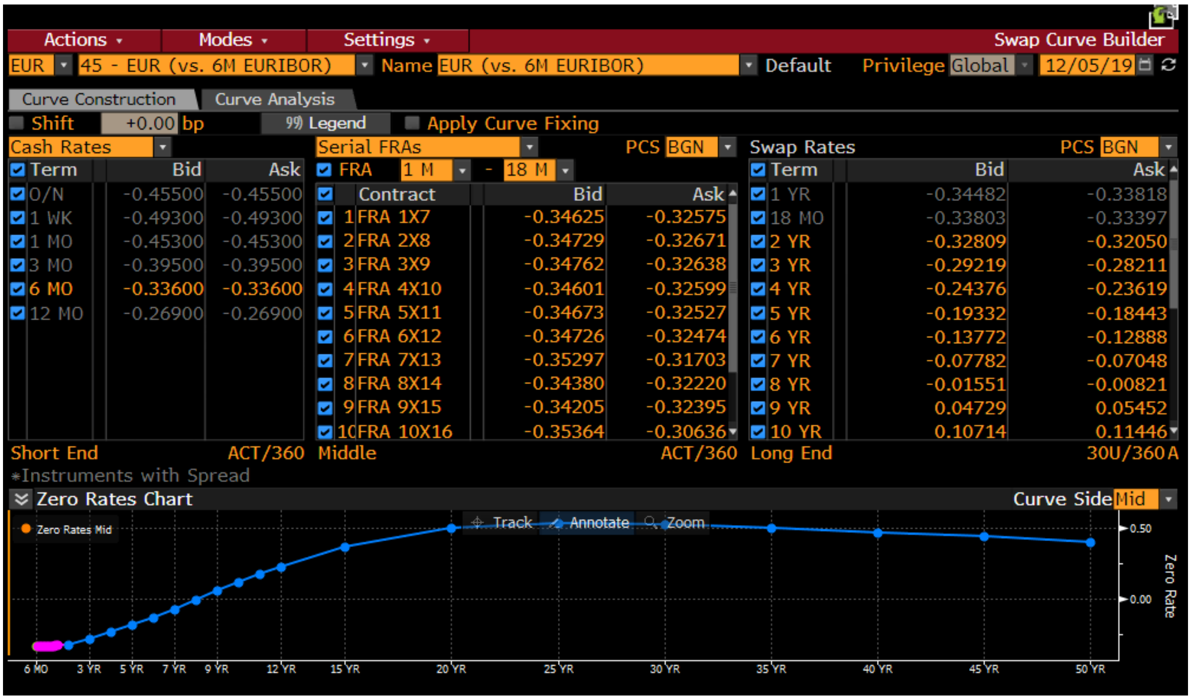


Fig. 1b. Example of Bloomberg screen showing instruments in EUR serial FRA IR curve S45.

The first group usually covers the short-end part of the curve; the cash/deposit securities are typically available up to 1 year tenor, but they are most commonly used for curve construction up to 3 or 6 months. Bloomberg recommends only including a single cash instrument with tenor equal to that of the curve being built. This is why in the example screen shot of S45 above only 6Mo has been selected.

The second group picks up where the first group ends. Futures strips will overlap with swaps, so the decision of where to switch from futures to swap rates is up to the user. As shown in the Fig.1a, the switch happens at 2 years for the default Bloomberg EUR curve S201. One may only use IR futures (contiguous or serial) on EUR curve S201, but for other curves, for instance, EUR curve S45, the user can choose FRAs, as shown in Fig. 1b.

The third group, swap rates, covers the time interval from where the user decides to end the use of futures/FRAs up to and including the final instrument used in the curve.

4. Interest Rate Instrument Pricing

Now let us consider each of these instruments in more detail, and understand what kind of constraint or mathematical equation they can provide to the IR curve.

1) Cash instruments:

Cash instruments are very straightforward. We are given a market quote for a given maturity. The correspondent equation is:

$$[1 + R \cdot \tau(t_s, t_e)] \cdot DF(t_e) = 1 \cdot DF(t_s) \qquad (5a)$$

Here R is the market quote for the cash instrument, t_s is accrual start time (usually the settlement time) and t_e is accrual end time (usually the maturity time) of this cash instrument, and $\tau(t_s, t_e)$ is the day count fraction for the rate. Thus $[1 + R * \tau(t_s, t_e)]$ is the total amount of money to be paid at payment time (usually the maturity time t_e). $DF(t)$ is the discount factor to be solved for at time t. Therefore, the left term in the above equation is the present value of the future payment and it must be equal to the right term which is the present value of the money we will pay for the deposit at the time t_s. Otherwise, there will be arbitrage which is not allowed.

2) Forward and Future instruments:

For a forward instrument, the equation based on the no arbitrage principal is very similar to that of a cash instrument. It can be readily expressed as:

$$[1 + R \cdot \tau(t_1, t_2)] \cdot DF(t_2) - DF(t_1) = 0 \qquad (5b)$$

here R is the market quote for the forward instrument, t_1 is the start time and t_2 is the maturity time. $\tau(t_1, t_2)$ is the day count fraction for the rate. Thus $[1 + R * \tau(t_1, t_2)]$ is the total amount of money to be paid at maturity time t_2. Here we can see there are two discount factors (at t_1 and t_2) that come into play in this equation.

As to the future instrument, the future rate can be converted into an IR forward rate using a so-called convexity adjustment as discussed in [3]. This conversion is based on a Vasicek model with given volatility σ and mean reversion speed a. The user can specify these two parameters in the middle segment on the screen shown in Fig. 1a.

3) Swap Instruments:

An IR Swap is an instrument which exchanges a stream of payments (fixed rate vs floating rate), on some pre-defined notional principal N, between two counterparties.

For the fixed leg, the payments are calculated using a fixed rate K defined at the inception of the swap. The present value of this leg can be expressed as:

$$PV(fixed\ leg) = \sum_{i=1}^{n} [N \cdot K \cdot \tau(i) \cdot DF(t_i)] + N \cdot DF(t_n)$$

here $\tau(i)$ is the day count fraction that determines the i^{th} payment amount and $DF(t_i)$ is the discount factor at i^{th} payment time t_i. If the principal needs to be repaid at maturity, this introduces an extra $N \cdot DF(t_n)$ term in the above PV expression.

For the floating leg, the payment amount is not determined from a known fixed rate, but relies on projected forward rates based on the forward curve. Without loss of generality, the present value can be readily expressed as:

$$PV(float\ leg) = \sum_{i=1}^{n} [N \cdot R(i) \cdot \hat{\tau}(i) \cdot DF(t_i)] + N \cdot DF(t_n)$$

Here $R(i)$ is an effective rate to cover i^{th} payment on the floating leg, and $N \cdot R(i) \cdot \hat{\tau}(i)$ is the expected amount of money paid for i^{th} period.

For a typical vanilla swap, the payments in the floating leg are calculated based on a (L)IBOR rate at the accrual start time of the correspondent time interval. These rates change as market instrument rates change, thus the name *floating leg*.

Under the no-arbitrage rule the equation for a swap, in general, can be expressed as:

$$PV(leg1) - PV(leg2) = 0$$

Different type of swaps (vanilla swaps, single currency basis swaps, cross currency basis swaps) will have different ways to compute present value. We will cover this in more detail in section 7.

5. Interpolation Methods

One can define curve stripping as a process to find a discount factor function $DF(t)$ for all $t \geq 0$, such that it prices all the instruments back to a set of market quotations. In other words, the solution must satisfy a system of equations based on the instruments used in the curve.

If there are N number of instruments on the curve, to ensure that we have a unique solution, the discount factor function $DF(t)$ should have also N degrees of freedom. Therefore, $DF(t)$ is typically defined as a parametric function with N independent parameters:

$$DF(t) = DF(t, p_1, p_2, \dots p_N).$$

One way to achieve this is to break $DF(t)$ into N time intervals with the i^{th} interval covering period $[t_{i-1}, t_i]$. Here, t_i is the maturity time of the i^{th} instrument on the curve and $t_0 = 0$ is just the lower boundary of the discount factor function. The function will have N independent parameters, one for each segment. There are typically two methods to solve for this system to determine $DF(t)$:

1) *Bootstrapping* method, where each time interval on the curve has exactly one independent degree of freedom and it does not affect previous time intervals. In this case the curve is built by adjusting one piece at a time while moving from shorter maturities to longer maturities.

2) *Global* method, where all or at least some degrees of freedom of the curve affect its overall shape, and therefore one needs to solve a general system of N non-linear equations with N unknown variables at the same time.

Currently the Bloomberg terminal allows the user to choose one of the 4 functional forms for the IR curve (a.k.a. *interpolation methods*). By typing **{SWDF DFLT <GO>}**, one will see the following screen:

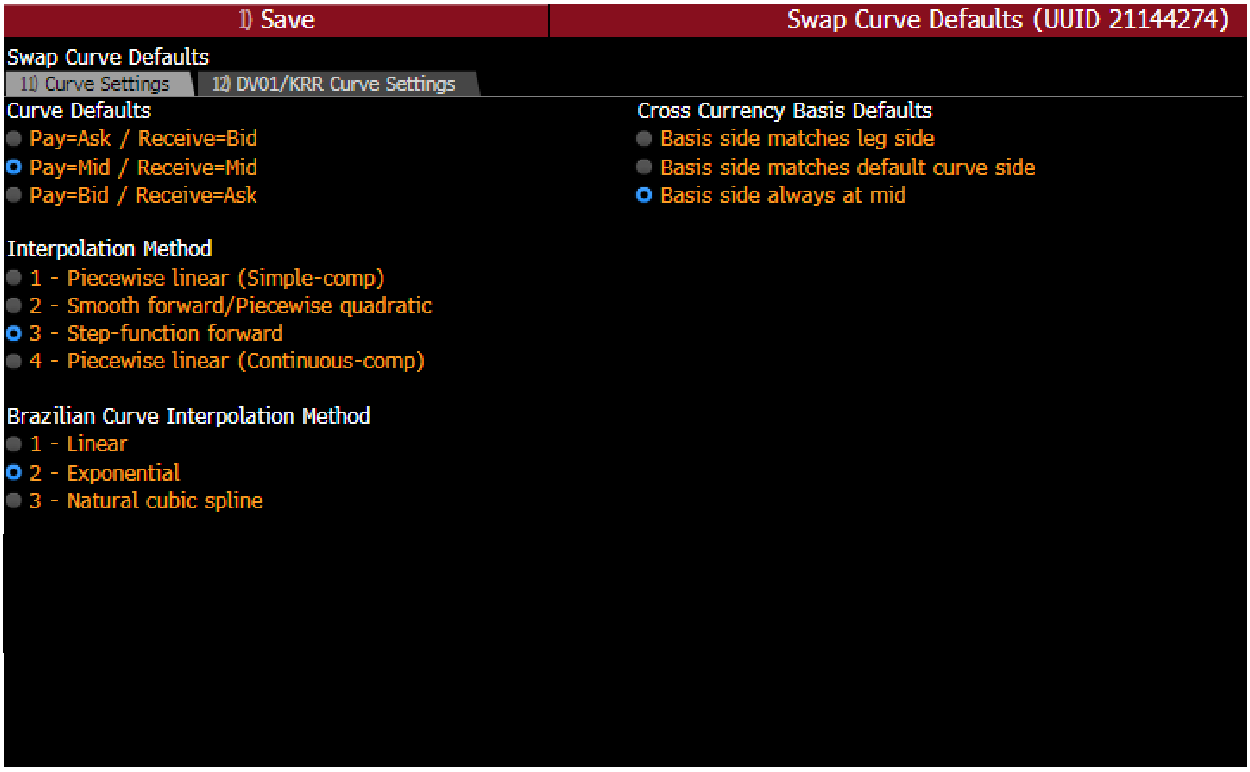


Fig. 2: Screen allowing user to choose curve interpolation method

On this screen one can choose 1 through 4 under the “Interpolation Method” section:

1) Piecewise linear
2) Smooth forward/Piecewise quadratic
3) Step-function forward
4) Piecewise linear

Interpolation method 1 - the simple zero rate r_{sz} defined by the formula (1) is a piecewise linear continuous function. A bootstrapping method is used to solve for all the simple zero rates at instrument maturities. The simple zero rate before the first instrument maturity is equal to the solved value from the first instrument. Similarly, in the extrapolated region after the last instrument maturity, the simple zero rate is equal to that of the last instrument. Thus, we have:

$$DF(t) = \frac{1}{1 + r_s(t_1) \cdot t} \qquad when\ \ t \in [0,\ t_1]$$

$$DF(t) = \frac{1}{1 + r_s(t_M) \cdot t} \qquad when\ \ t \in [t_M, \infty]$$

Here t_1 and t_M are the maturity time for the first instrument and last instrument respectively, and $r_s(t)$ is simple zero rate at time t. All dates are converted to time using ACT/360 convention. Thus, it produces a unique discount factor function $DF(t)$ that covers all the time $t \geq 0$ for this curve. Fig.3a shows an example of stripping results using interpolation method 1 for EUR curve S201.

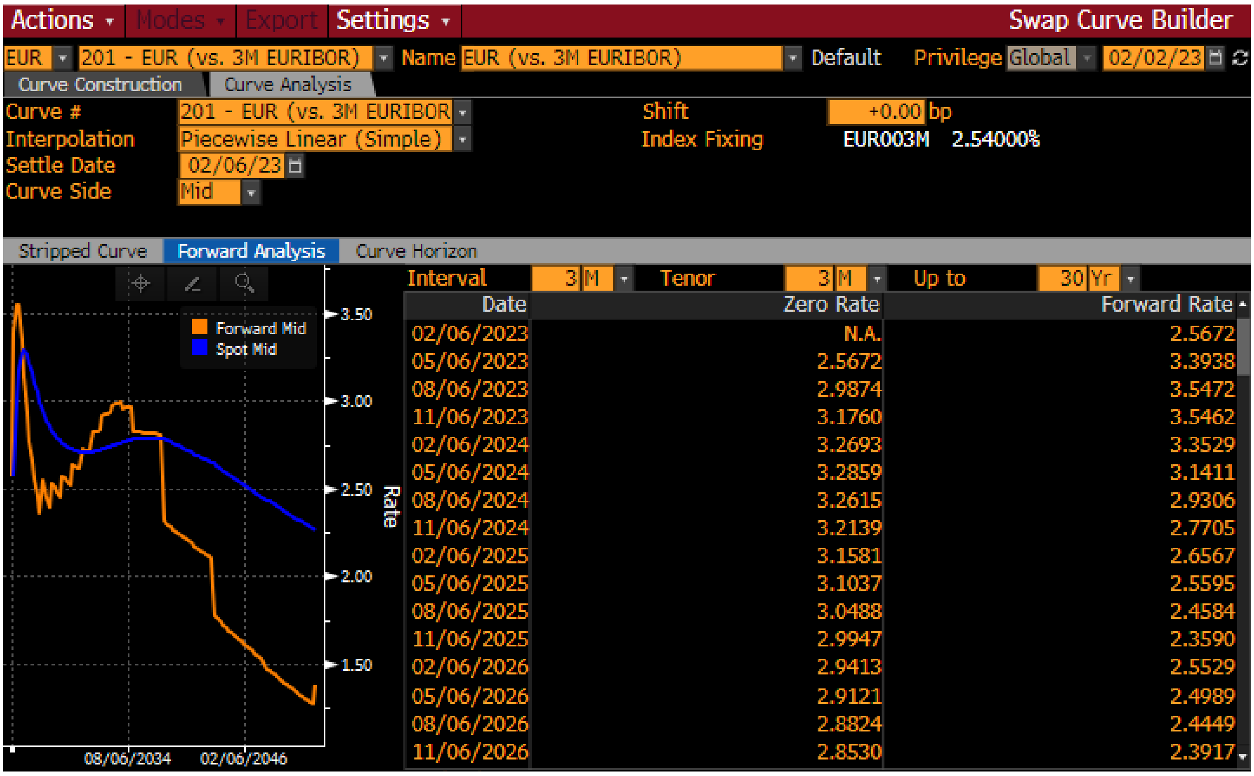


Fig. 3a: Spot rate (blue) and forward rate (orange) graphs for EUR curve with interpolation method 1.

Interpolation method 2 - smooth forward/piecewise quadratic continuously-compounded forward rate. The forward rate r_{cf} defined by formula (3) is piecewise-quadratic. The neighboring sections of the forward curve are connected in such a way that the first order derivative is continuous, which is reflected in name 'smooth'. In the extrapolated region after the last instrument maturity, the instantaneous forward rate is equal to that of the last instrument. Thus, we have:

$$DF(t) = exp[-L(t)] \qquad when\ t \in [0,\ t_1]$$

$$DF(t) = exp[-L(t)] = DF(t_M) * exp[-Fwd(t_M) \cdot (t - t_M)] \qquad when\ t \in [t_M, \infty]$$

Here $L(t)$ is a piecewise quadratic polynomial function and is determined after stripping (Refer to Appendix 1), t_M is the maturity time of the last instrument and $Fwd(t_M)$ is the instantaneous forward rate right at t_M. All dates are converted to time t using ACT/360 convention. Fig.3b shows an example of stripping results using interpolation method 2 for EUR curve S201. To solve such a system, the aforementioned global method is required. For details of the functional form of the curve see Appendix 1.

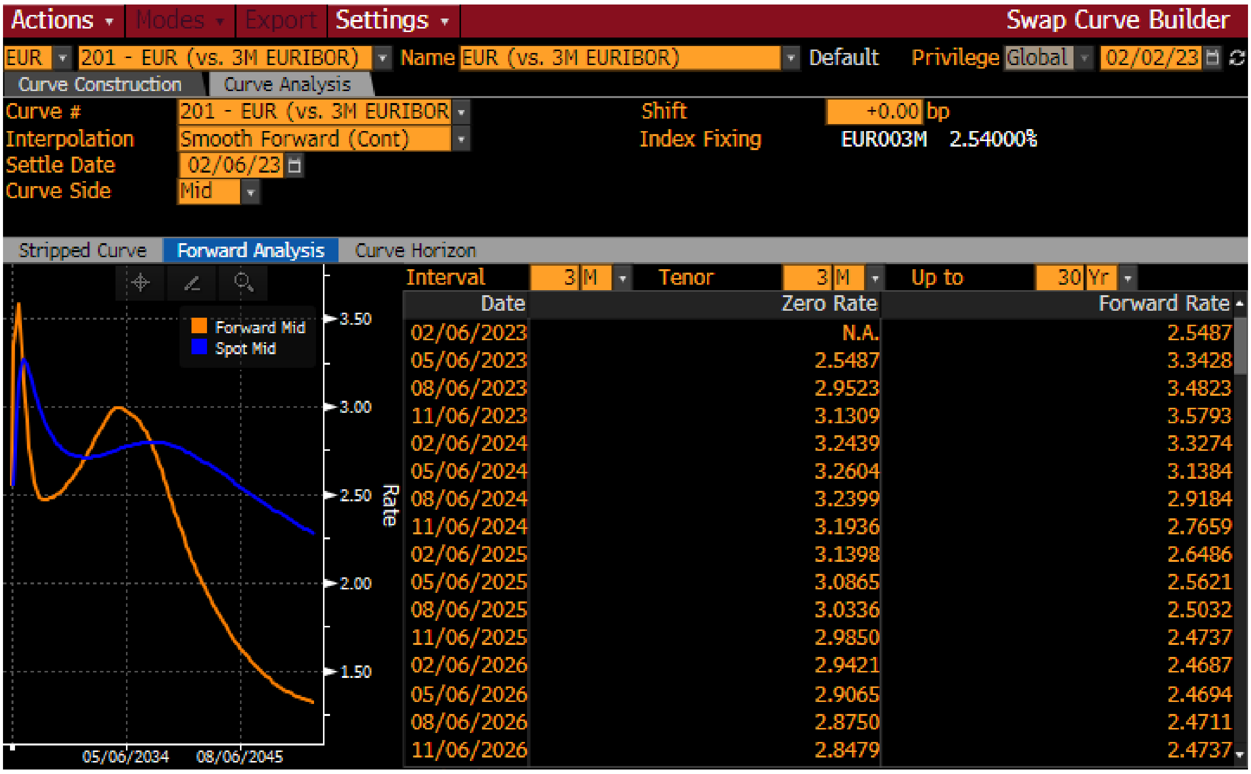


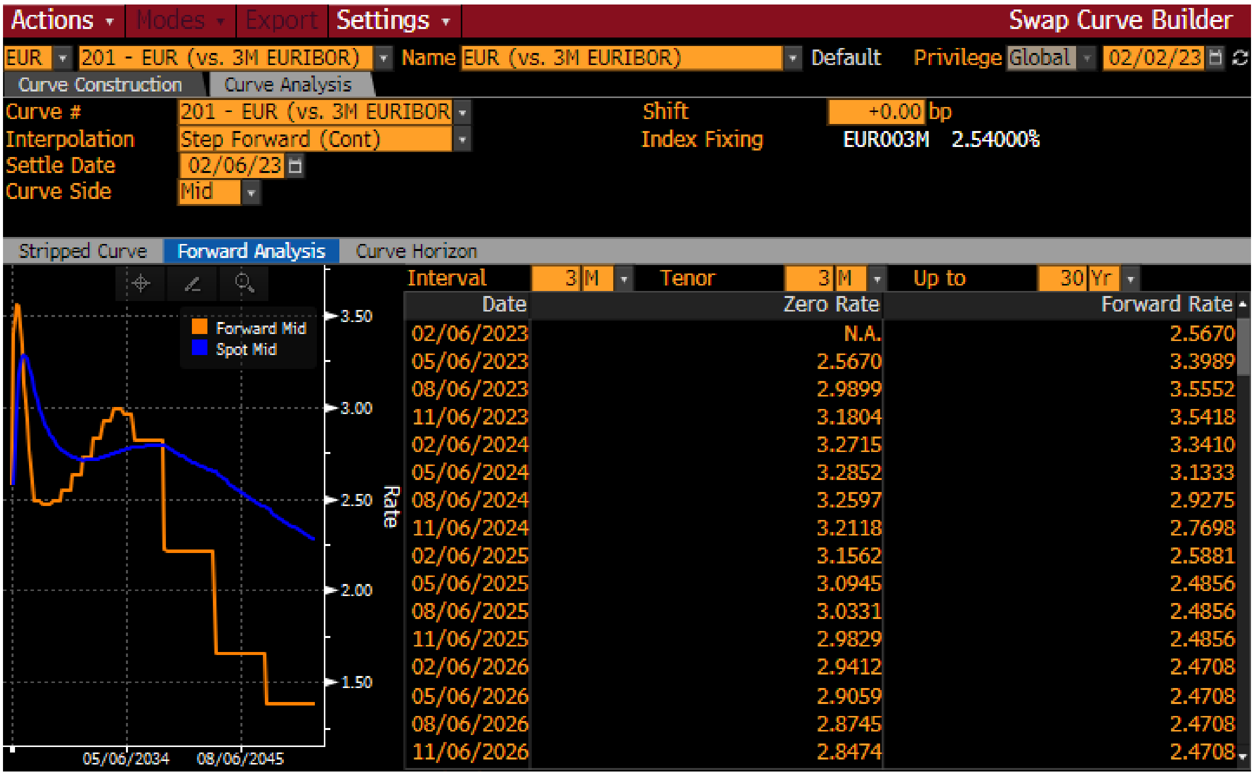
Fig. 3b: Spot rate (blue) and forward rate (orange) graphs for EUR curve with interpolation method 2.

Interpolation method 3 - step-function forward continuously-compounded (CC) forward rate. The forward rate r_{cf} defined by the formula (3) is piecewise constant. A bootstrapping method is used to solve for all the CC forward rates between maturities of neighboring instruments. The CC forward rate before the first instrument maturity is equal to the solved value from the first instrument. Similarly, in the extrapolated region after the last instrument maturity, the CC forward rate is equal to that of the last instrument. Thus, we have:

$$DF(t) = exp[-ccFwd(0, t_1) \cdot t] \qquad when\ t \in [0,\ t_1]$$

$$DF(t) = DF(t_M) \cdot exp[-ccFwd(t_{M-1}, t_M) \cdot (t - t_M)] \qquad when\ t \in [t_M, \infty]$$

Here t_1 is the maturity time of the first instrument and $ccFwd(0, t_1)$ is the CC forward rate at t_1. Similarly, t_M is the maturity time of the last instrument and $ccFwd(t_{M-1}, t_M)$ is the CC forward rate between the last two instruments ($t_{M-1} \rightarrow t_M$). All dates are converted to time t using ACT/360 convention. Thus, it produces a unique discount factor function $DF(t)$ that covers all the time $t \geq 0$ for this curve. Fig.3c shows an example of stripping results using interpolation method 3 for EUR curve S201.



Date	Zero Rate	Forward Rate
02/06/2023	N.A.	2.5670
05/06/2023	2.5670	3.3989
08/06/2023	2.9899	3.5552
11/06/2023	3.1804	3.5418
02/06/2024	3.2715	3.3410
05/06/2024	3.2852	3.1333
08/06/2024	3.2597	2.9275
11/06/2024	3.2118	2.7698
02/06/2025	3.1562	2.5881
05/06/2025	3.0945	2.4856
08/06/2025	3.0331	2.4856
11/06/2025	2.9829	2.4856
02/06/2026	2.9412	2.4708
05/06/2026	2.9059	2.4708
08/06/2026	2.8745	2.4708
11/06/2026	2.8474	2.4708

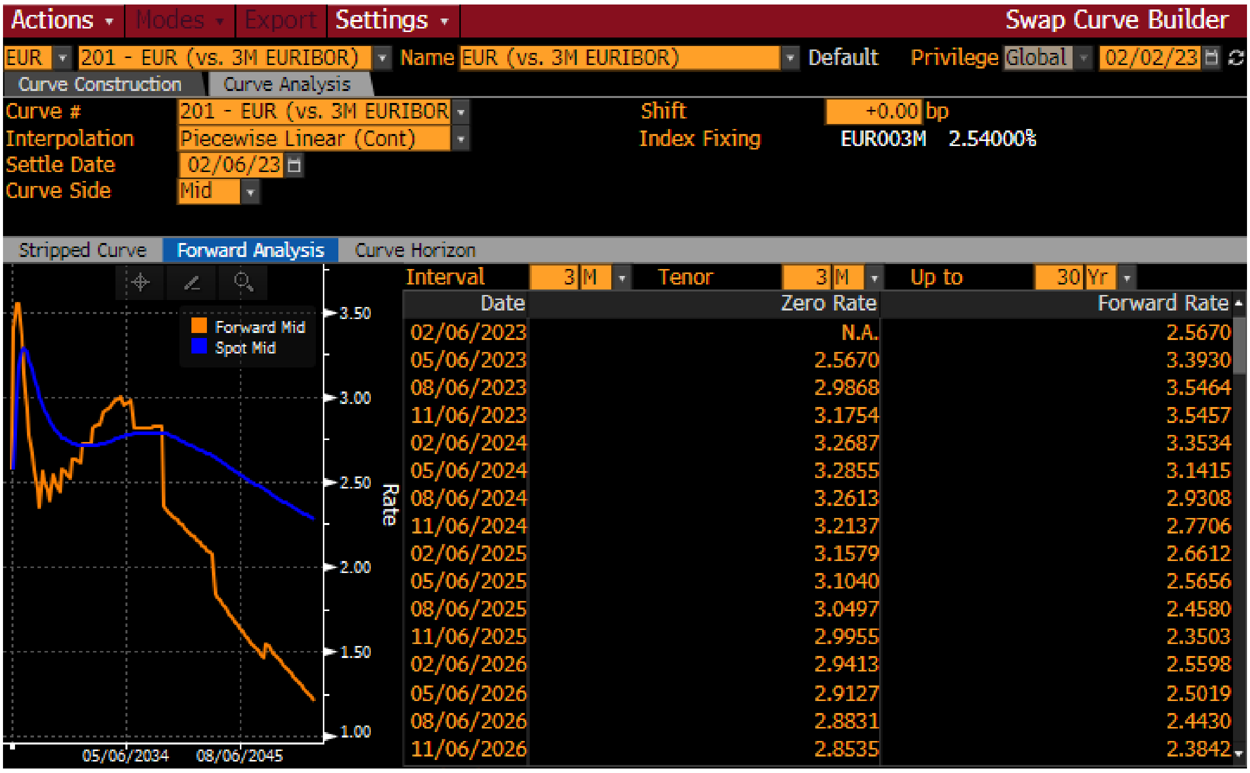
Fig. 3c: Spot rate (blue) and forward rate (orange) graphs for EUR curve with interpolation method 3.

Interpolation method 4 - piecewise linear continuously-compounded zero rate. The zero rate r_{sz} as defined by the formula (4) is piecewise linear continuous function. A bootstrapping method is used to solve for all the CC zero rates between maturities of neighboring instruments. The CC zero rate before the first instrument maturity is equal to the solved value from the first instrument. Similarly, in the extrapolated region after the last instrument maturity, the CC zero rate is equal to that of the last instrument, Thus, we have:

$$DF(t) = exp[-ccZero(t_1) \cdot t] \qquad when\, t \in [0,\ t_1]$$

$$DF(t) = exp[-ccZero(t_M) \cdot t] \qquad when\, t \in [t_M, \infty]$$

Here t_1 is the maturity time of the first instrument, t_M is the maturity time of the last instrument, and $ccZero(t)$ is the CC zero rate right at time t. All dates are converted to time using ACT/360 convention. Thus, it produces a unique discount factor function $DF(t)$ that covers all the time $t \geq 0$ for this curve. Fig.3d shows an example of stripping results using interpolation method 4 for USD curve S23.



Date	Zero Rate	Forward Rate
02/06/2023	N.A.	2.5670
05/06/2023	2.5670	3.3930
08/06/2023	2.9868	3.5464
11/06/2023	3.1754	3.5457
02/06/2024	3.2687	3.3534
05/06/2024	3.2855	3.1415
08/06/2024	3.2613	2.9308
11/06/2024	3.2137	2.7706
02/06/2025	3.1579	2.6612
05/06/2025	3.1040	2.5656
08/06/2025	3.0497	2.4580
11/06/2025	2.9955	2.3503
02/06/2026	2.9413	2.5598
05/06/2026	2.9127	2.5019
08/06/2026	2.8831	2.4430
11/06/2026	2.8535	2.3842

Fig. 3d: Spot rate (blue) and forward rate (orange) graphs for EUR curve with interpolation method 4.

The three functional forms – 1, 3, and 4 – have one internal parameter per piece, allowing one to modify the behavior of one piece at a time while not changing the shape of the curve in previous pieces. This makes it possible to build curves of these types using the bootstrap method, as mentioned above. It means that we build the curve from left to right, one piece at a time by solving one equation at a time – we thus reduce the problem of solving a system of N equations with N variables to N consecutive solutions of one equation with one variable.

In case of a smooth curve (interpolation method 2) the situation is different. The curve is defined in such a way that changes in each internal parameter or degree of freedom affects the shape of the whole curve, and therefore requires a solution of a non-linear system of N equations with N variables. The functional form of this curve is described in Appendix 1.

We also support 3 different interpolation methods that only apply to Brazilian (BRL) Curves:

1) Linear
2) Exponential
3) Natural cubic spline

In general, BRL spot curve is composed of a series of annually compounded zero rates that mature at different times, i.e., $[\ time[i],\ zeroRate[i]\ ]$. At any given time t, we can interpolate a zero rate, based on one of the above 3 methods, and convert to discount factor.

BRL Interpolation method 1 – Linear: the final zero rate is linearly interpolated based on neighboring points. The zero rate before the first instrument maturity is determined by the slope from the first two instruments. Similarly, in the extrapolated region after the last instrument maturity, the zero rate is determined by the slope from the last two instruments. Thus, we have:

$$slope_i = \frac{r_{i+1} - r_i}{t_{i+1} - t_i} \qquad where\ \ i = 1, \ldots., M - 1$$

$$interpRate(t) = \ r_1 + slope_1 * (t - t_1) \qquad when\ \ t \in [0,\ \ t_1]$$

$$interpRate(t) = \ r_i + slope_i * (t - t_i) \qquad when\ \ t \in [t_i,\ \ t_{i+1}]$$

$$interpRate(t) = \ r_M + slope_{M-1} * (t - t_M) \quad when\ \ t \in [t_M, \infty]$$

Here t_1 and t_M are the maturity time for the first instrument and last instrument respectively, and r_i is the zero rate at i^{th} point. Fig.3e shows an example of stripping results using BRL interpolation method 1 for BRL curve S89.

Fig. 3e: Spot rate graphs for BRL curve S89 with BRL interpolation method 1.

BRL Interpolation method 2 – Exponential: the final zero rate is interpolated based on neighboring points by keeping the continuous compounded forward (cc-forward) rate constant in this region. The zero rate before the first instrument maturity is determined by the cc-forward rate from the first two instruments. Similarly, in the extrapolated region after the last instrument maturity, the zero rate is determined by the cc-forward rate from the last two instruments. Thus, we have:

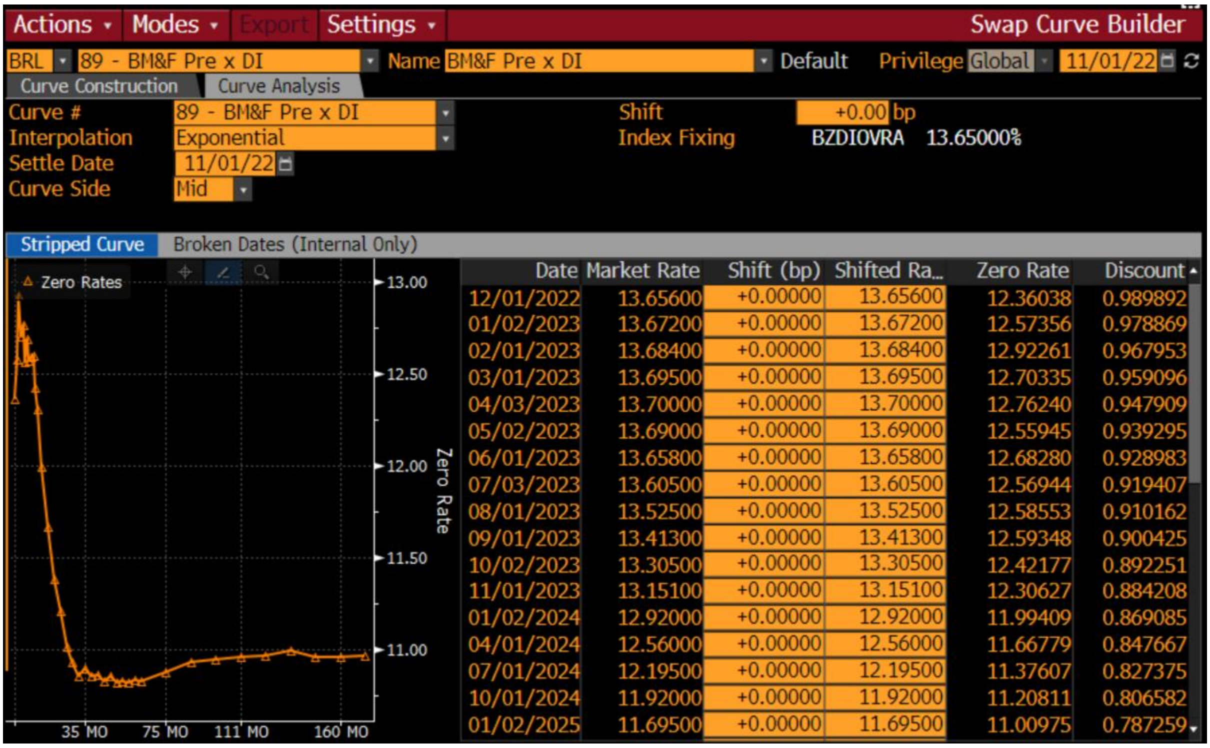
$$ccFwd_i = \frac{\ln\left(\frac{r_{i+1}}{r_i}\right)}{t_{i+1} - t_i} \quad where\ \ i = 1, \ldots., M-1$$

$$interpRate(t) = r_1 * \exp[ccFwd_1 * (t - t_1)] \qquad when\ \ t \in [0,\ t_1]$$

$$interpRate(t) = r_i * \exp[ccFwd_i * (t - t_i)] \qquad when\ \ t \in [t_i,\ t_{i+1}]$$

$$interpRate(t) = r_M * \exp[ccFwd_{M-1} * (t - t_M)] \quad when\ \ t \in [t_M, \infty]$$

Here t_1 and t_M are the maturity time for the first instrument and last instrument respectively, and r_i is the zero rate at i^{th} point. Fig.3f shows an example of stripping results using BRL interpolation method 2 for BRL curve S89.



Date	Market Rate	Shift (bp)	Shifted Ra...	Zero Rate	Discount
12/01/2022	13.65600	+0.00000	13.65600	12.36038	0.989892
01/02/2023	13.67200	+0.00000	13.67200	12.57356	0.978869
02/01/2023	13.68400	+0.00000	13.68400	12.92261	0.967953
03/01/2023	13.69500	+0.00000	13.69500	12.70335	0.959096
04/03/2023	13.70000	+0.00000	13.70000	12.76240	0.947909
05/02/2023	13.69000	+0.00000	13.69000	12.55945	0.939295
06/01/2023	13.65800	+0.00000	13.65800	12.68280	0.928983
07/03/2023	13.60500	+0.00000	13.60500	12.56944	0.919407
08/01/2023	13.52500	+0.00000	13.52500	12.58553	0.910162
09/01/2023	13.41300	+0.00000	13.41300	12.59348	0.900425
10/02/2023	13.30500	+0.00000	13.30500	12.42177	0.892251
11/01/2023	13.15100	+0.00000	13.15100	12.30627	0.884208
01/02/2024	12.92000	+0.00000	12.92000	11.99409	0.869085
04/01/2024	12.56000	+0.00000	12.56000	11.66779	0.847667
07/01/2024	12.19500	+0.00000	12.19500	11.37607	0.827375
10/01/2024	11.92000	+0.00000	11.92000	11.20811	0.806582
01/02/2025	11.69500	+0.00000	11.69500	11.00975	0.787259

Fig. 3f: Spot rate graphs for BRL curve S89 with BRL interpolation method 2.

BRL Interpolation method 3 – Natural Cubic Spline: the final zero rate is interpolated based on a series of piecewise cubic polynomials that is twice continuously differentiable at any time. The zero rate before the first instrument maturity is determined by a cubic polynomial that covers the first two instruments. Similarly, in the extrapolated region after the last instrument maturity, the zero rate is determined by a cubic polynomial that covers the last two instruments. Thus, we have:

$$cubicPoly(t,i) = \frac{r_i(t-t_{i+1}) - r_{i+1}(t-t_i)}{t_i - t_{i+1}} + \frac{k_i}{6}\left[\frac{(t-t_{i+1})^3}{t_i - t_{i+1}} - (t-t_{i+1})(t_i - t_{i+1})\right]$$

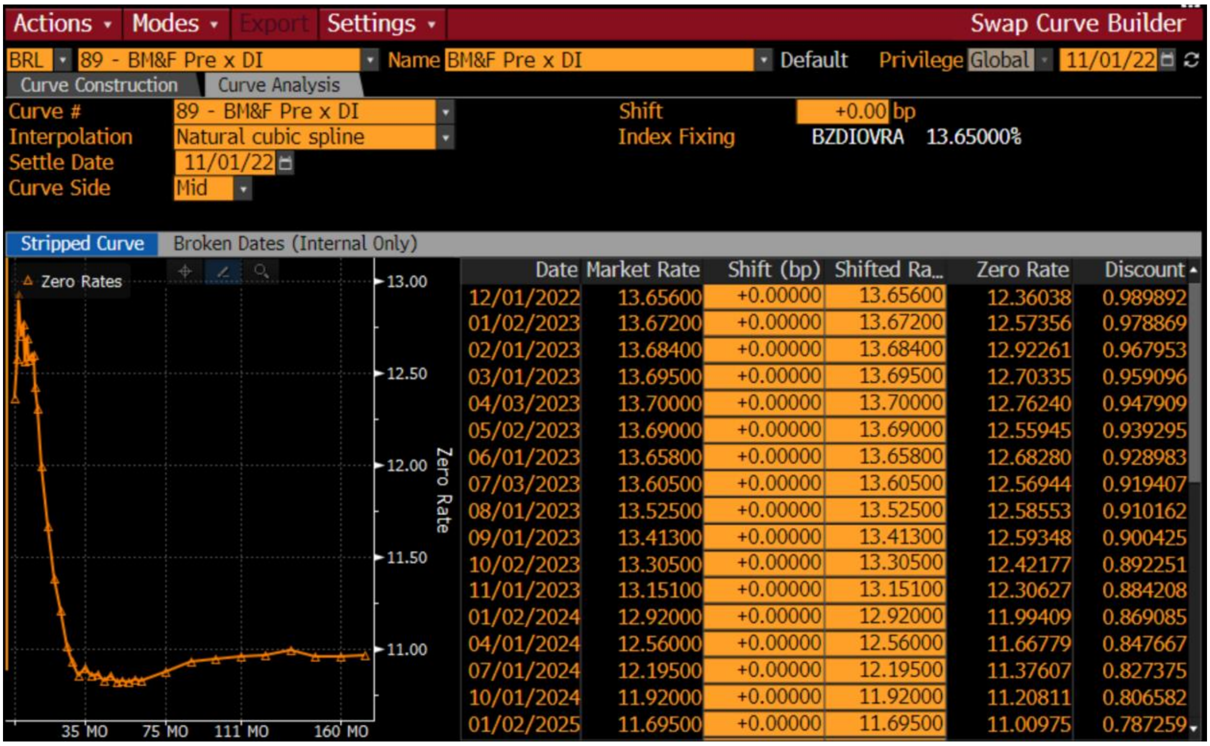
$$-\frac{k_{i+1}}{6}\left[\frac{(t-t_i)^3}{t_i - t_{i+1}} - (t-t_i)(t_i - t_{i+1})\right] \quad where\ \ i = 1, \ldots, M-1$$

$$interpRate(t) = cubicPoly(t,1) \qquad when\ \ t \in [0,\ t_1]$$

$$interpRate(t) = cubicPoly(t,i) \qquad when\ \ t \in [t_i,\ t_{i+1}]$$

$$interpRate(t) = cubicPoly(t,M-1) \quad when\ \ t \in [t_M, \infty]$$

Here t_1 and t_M are the maturity time for the first instrument and last instrument respectively, and r_i is the zero rate at i^{th} point. k_i is the second order derivative at t_i and this is determined from solving a series of continuity equations at the boundaries (see Appendix 3). Fig.3g shows an example of stripping results using BRL interpolation method 3 for BRL curve S89.



Date	Market Rate	Shift (bp)	Shifted Ra...	Zero Rate	Discount
12/01/2022	13.65600	+0.00000	13.65600	12.36038	0.989892
01/02/2023	13.67200	+0.00000	13.67200	12.57356	0.978869
02/01/2023	13.68400	+0.00000	13.68400	12.92261	0.967953
03/01/2023	13.69500	+0.00000	13.69500	12.70335	0.959096
04/03/2023	13.70000	+0.00000	13.70000	12.76240	0.947909
05/02/2023	13.69000	+0.00000	13.69000	12.55945	0.939295
06/01/2023	13.65800	+0.00000	13.65800	12.68280	0.928983
07/03/2023	13.60500	+0.00000	13.60500	12.56944	0.919407
08/01/2023	13.52500	+0.00000	13.52500	12.58553	0.910162
09/01/2023	13.41300	+0.00000	13.41300	12.59348	0.900425
10/02/2023	13.30500	+0.00000	13.30500	12.42177	0.892251
11/01/2023	13.15100	+0.00000	13.15100	12.30627	0.884208
01/02/2024	12.92000	+0.00000	12.92000	11.99409	0.869085
04/01/2024	12.56000	+0.00000	12.56000	11.66779	0.847667
07/01/2024	12.19500	+0.00000	12.19500	11.37607	0.827375
10/01/2024	11.92000	+0.00000	11.92000	11.20811	0.806582
01/02/2025	11.69500	+0.00000	11.69500	11.00975	0.787259

Fig. 3g: Spot rate graphs for BRL curve S89 with BRL interpolation method 3.

IR Curve stripping

We detail the stripping process for various types of IR curve in this section. Please note that all the unknown variables (i.e. to be solved for) are marked in red in the formulas below.

6.1 Vanilla Swap Curve

A vanilla swap instrument comprises a fixed and floating leg with associated market quote being the rate on the fixed leg. Figure 6a shows a standard 5 year vanilla swap instrument on SAR 3Mo SAIBOR curve S166. For the sake of simplicity, we still start with a currency where the market (generally) and Bloomberg still assume single curve stripping. In Fig.6a, the fixed leg pays 4.1935% interest every year, while the floating leg pays a floating 3Mo SAIBOR rate every 3 months.

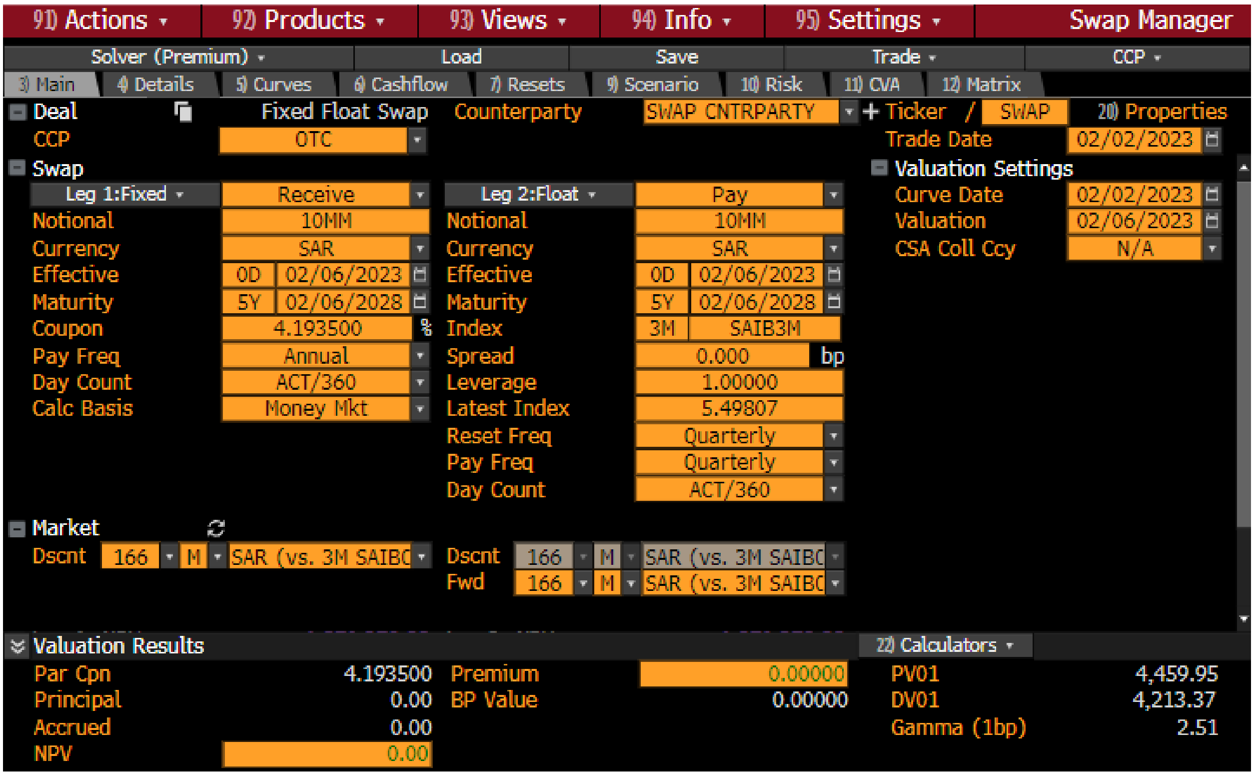


Fig. 6a. Standard 5Y swap instrument on curve S166

The present value of both sides can be expressed as the summation of a series of cash flows in the future and discounted back to the start time:

$$PV(fixed) = \sum_i [N \cdot K \cdot \tau(i) \cdot df(t_i, S166)] + N \cdot df(t_M, S166)$$

$$PV(float) = \sum_j [N \cdot R(j, S166) \cdot \hat{\tau}(j) \cdot df\left(t_j, 166\right)] + N \cdot df(t_M, S166)$$

On the fixed leg, all the unknowns are discount factors from S166, which is the curve to be stripped. On the floating leg, the first effective rate is the quoted index instrument (e.g. SAIB3M Index for S166) and all the remaining ones are the projected forward rates from S166, which in turn can be directly calculated based on the discount factors from S166. By solving $PV(fixed) = PV(float)$, we are able to determine a series of discount factors and thus strip S166.

6.2 OIS-Discounting/Dual-Curve Stripping

OIS discounting means discounting the expected cash flows of a derivative using a nearly risk free curve such as an overnight index swap (OIS) curve. OIS-discounted IR curves are built using a dual-curve (DC) stripping technique.

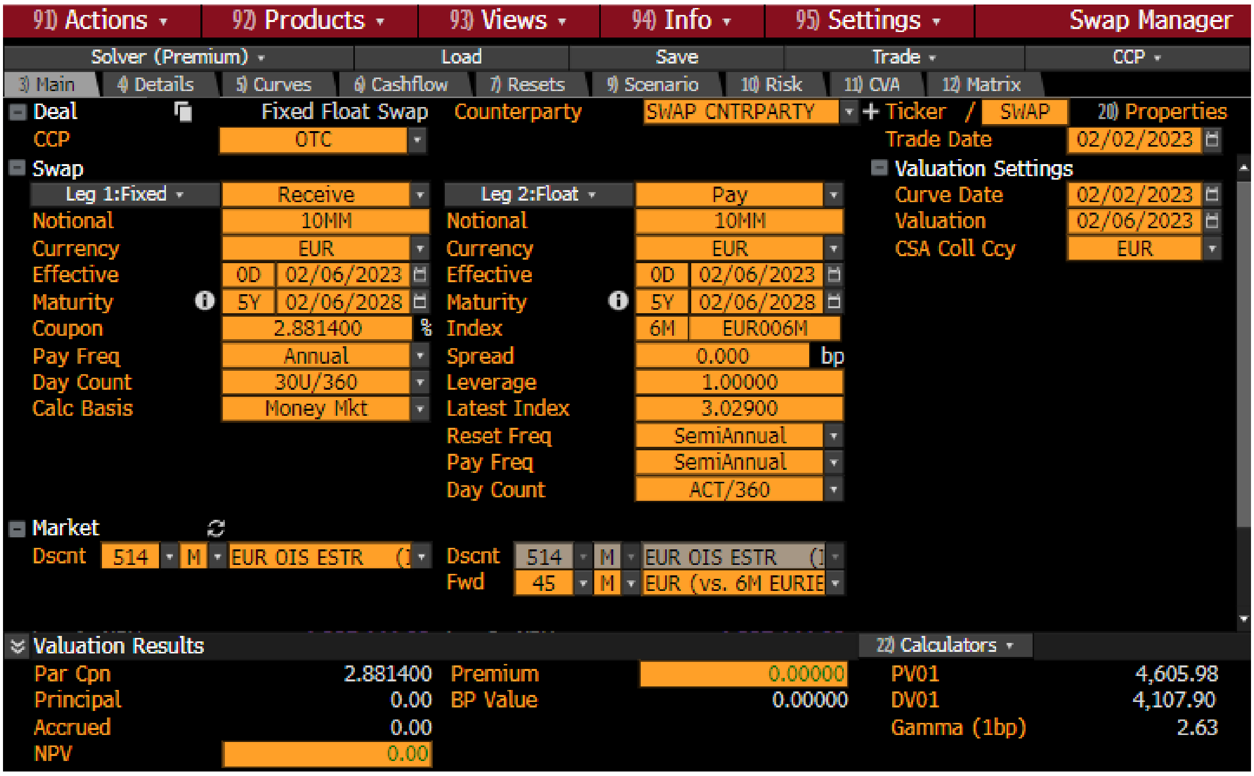


Fig. 6b. Standard 5Y swap instrument on curve S45 with OIS-discounting.

With OIS-discounting, swap rates are calculated using a different formula to its single-curve counterpart because a different curve is used to discount cash flows. Thus, the present value of each leg can be expressed as:

$$PV(fixed) = \sum_i [N \cdot K \cdot \tau(i) \cdot df(t_i, S514)] + N \cdot df(t_M, S514)$$

$$PV(float) = \sum_j \left[N \cdot R(j, S45) \cdot \hat{\tau}(j) \cdot df\left(t_j, S514\right)\right] + N \cdot df(t_M, S514)$$

Since OIS curve S514 has already been stripped before stripping S45 all the discount factors from S514 are known here. The only unknown value in the above two PV expressions are the payment values on the floating leg, which can be calculated based on the discount factors from S45. Therefore, $PV(fixed)$ is actually a constant and all the unknown variables are on the floating leg. The stripping process is to find proper discount factors for S45 to make the above two present values equal.

6.3 Single Currency Basis Curve

Single currency basis swaps (also known as tenor basis swaps) are financial instruments that exchange a series of floating rate payments. Both sides reference index instruments from the same currency but with different tenors. Fig. 6c displays a standard tenor basis swap on curve S485: one floating leg references a 3Month index (NFIX3FRA Index) while the other references a 6Month index (NFIX6FRA Index). Tenor basis swaps are typically quoted as a spread applied on one leg. For example, Fig. 6c shows a spread of 5bp needs to be applied to the 3Month leg in order to make its value equal to the 6Month leg.

Fig. 6c. Standard 5Y tenor basis swap instrument on curve S485.

The present value of each leg can be expressed as the summation of a series of discounted cash flows:

$$PV(3Mo\ leg) = \sum_i [N \cdot (R(i, S15) + spread) \cdot \hat{\tau}(i) \cdot df(t_i, S198)] + N \cdot df(t_M, S198)$$

$$PV(6Mo\ leg) = \sum_j [N \cdot R(j, S485) \cdot \hat{\tau}(j) \cdot df\big(t_j, S198\big)] + N \cdot df(t_M, S198)$$

Here, $R(i)$ is the effective rate to cover the i^{th} payment period. Usually it is just the simple forward rate of the same period. However, on the 3Mo leg here, the floating rates are reset quarterly while the payment happens semiannually. Thus, $R(i)$ is a compounded rate (6Mo) based on two different reset rates (3Mo).

Please note that both curve S15 and S198 are stripped before stripping curve S485, thus the PV on the 3Mo leg can be readily calculated and becomes a constant. While on the 6Mo leg, the effective rates from S485 are directly linked to the unknown discount factors from S485, which is similar to the above OIS discounting case. Finally, solving $PV(3Mo\ leg) = PV(6Mo\ leg)$ can determine a series of discount factors and finalize the $DF(t)$ for S485.

As most of the liquidity in a given currency resides in the standard tenor IR swaps (NFIX3FRA for NZD), we often find we have less IR swap market data to define different tenor IBOR curves. If one directly constructs a non-standard tenor IBOR curve using non-standard tenor

IR swaps with limited liquidity, then the fact that the spread between the curves is often relatively stable is not enforced, and can be violated if left unconstrained. An alternative approach to build the non-standard tenor IBOR curves is to define the parameters of the calibration in terms of spreads over the existing curve. Bloomberg implements a so-called **step forward spread interpolation method**. This interpolation method is not configurable and the details of are coved in the Appendix 2.

6.4 Building Cross Currency Curves using FX Forwards

Cross currency curves are divided into two categories: Cross Currency Fixed vs. Floating Swap (CCS) curves and Cross Currency Basis (FX Basis) curves. The instruments we use to build the long end maturities of both types of curves are either Cross Currency Fixed vs. Floating swaps (in the case of CCS curves) or Cross Currency Basis swaps (in the case of FX Basis curves). For tenors at or above some threshold tenor, there are liquid markets for both types of swaps. However, for tenors below that threshold tenor, cross currency swaps are not as liquid. For this region, we use FX Forwards as the par instruments with which to bootstrap the short end of the curve.

Take, as an example, curve S92, the EUR-USD FX Basis curve. The current default Bloomberg curve for S92 is *EUR vs. USD Basis with FX Forwards*. The main ICVS page for this curve is:

Term	Ticker	FX (pts) Mid	FX (outright) Mid	Basis Mid
3 MO	EUR3M	54.56	1.074911	-19.2725
4 MO	EUR4M	72.11	1.076666	-22.1757
5 MO	EUR5M	89.33	1.078388	-22.2362
6 MO	EUR6M	103.92	1.079847	-22.2831
9 MO	EUR9M	154.99	1.084954	-33.7205
12 MO	EUR12M	193.25	1.088780	-32.1228
15 MO	EUR15M	224.80	1.091935	-34.7619
18 MO	EUR18M	251.25	1.094580	-36.7023
2 YR	EUR2Y	297.27	1.099183	-31.0000
3 YR	EUR3Y	390.40	1.108495	-30.3750
4 YR	EUR4Y	490.34	1.118489	-30.0000
5 YR	EUR5Y	592.50	1.128705	-29.7500
3 MO	EUX0QQC	54.56	1.074911	-19.2500
6 MO	EUX0QQF	103.92	1.079847	-22.5000
9 MO	EUX0QQI	154.99	1.084954	-33.8750
1 YR	EUX0QQ1	193.25	1.088781	-32.1250
2 YR	EUX0QQ2	297.95	1.099251	-31.0000
3 YR	EUX0QQ3	388.77	1.108332	-30.3750

The instruments on and after the 2Year tenor are all basis swaps. The orange-highlighted instruments on or below 12Months are FX Forwards. The instruments in grey are additional longer dated FX Forwards as well as short-dated basis swaps. They are not used to build the curve but are provided in case users want to customize their own versions of the curve.

When building curve S92 using FX Forwards, we use the *Covered Interest Rate Parity* relationship between FX rates and Interest rates. This relationship is:

$$\boldsymbol{FXFwd(t) = FXSpot * \frac{df(t_{FXS}, S92)}{df(t_{FXS}, S490)} * \frac{df(t, S490)}{df(t, S92)}}$$

where FXSpot is the USD-EUR spot exchange rate, FXFwd(t) is USD-EUR FX Forward rate at maturity t, and t_{FXS} is the FX settlement time. When we begin building the curve, all the terms in the above equation are known except for $df(t_{FXS},S92)$ and $df(t,S92)$. When the first FX forward instrument is added, the curve builder solves for both $df(t_{FXS},S92)$ and $df(t,S92)$. Once $df(t_{FXS},S92)$ is known and stored, adding subsequent maturities, t, completely determine $df(t,S92)$.

For all Cross Currency curves (CCS and FX Basis), we transform the FX Forwards instruments into swap instruments when the curve is loaded. The swap instruments (either FX Basis swaps or CC swaps) are chosen to produce the same discount factors at the FX Forward maturities as the original FX Forwards. The reason we do this is so that a curve does not show FX risk when pricing short-dated instruments. The idea is that curves should produce risk only to interest rate and basis instruments, and not to FX instruments.

You can see the transformed instruments in the ICVS snapshot above. In the FX forwards section, the last column shows the basis spread of the FX Basis swap that produces the same discount factor at the corresponding maturity as the FX forward, for that tenor.

6.5 Cross Currency Basis Curve Stripping (Non Mark-to-Market)

A cross currency (XCCY) basis swap is a financial instrument where each counterparty exchanges payments in a different currency.

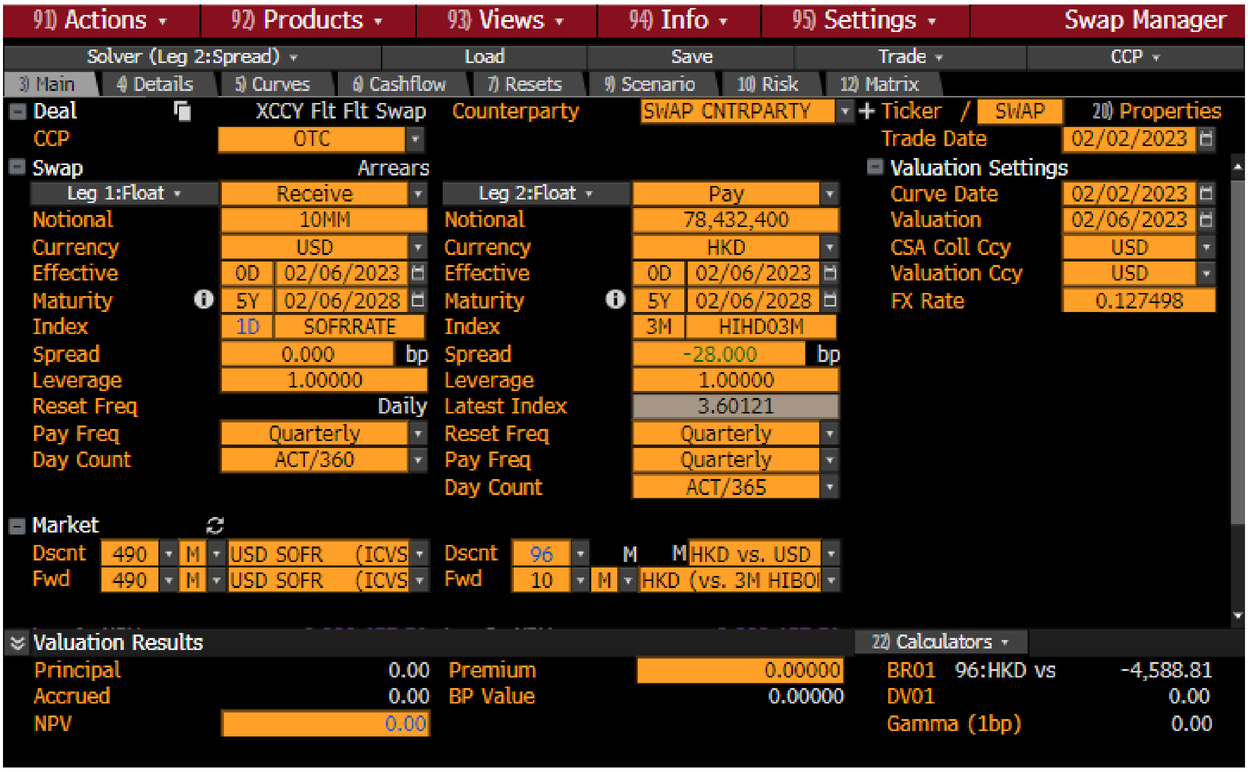


Fig. 6d. 5Y HKD/USD XCCY basis swap instrument on curve S96.

Please note that both legs are floating legs, the market quote (-28bps) for this swap is a spread applied to the HKD leg and there is no spread on the USD leg.

Theoretically, an XCCY basis swap should have a zero spread. However, in practice, due to supply and demand, spreads are generally non-zero. In the above example, the market is charging -28bp spread on the HKD leg for the right to swap floating payments into USD. Under the no-arbitrage principal, this market quote will make the present value of the HKD floating leg equal to the present value of the floating leg on the USD side. This forms the foundation to strip this curve.

It is straightforward to calculate the present value of the USD leg as:

$$PV(USD) = \sum_i [N_{USD} \cdot R(i, S490) \cdot \hat{\tau}(i) \cdot df(t_i, S490)] + N_{USD} \cdot df(t_M, S490)$$

Here N_{USD} is the principal on the USD leg and it is held constant throughout the life of the swap. Each payment can be calculated using the projected forward rate based on the USD forward curve (S490) and then discounted by the discount factor from the USD discount curve (S490). Finally, there is an extra term representing the repayment of principal at maturity.

On the HKD leg, the existence of a basis spread will change the way we compute the payment amount:

$$PV(HKD) = \sum_j \left[N_{HKD} \cdot [R(j, S10) + spread] \cdot \hat{\tau}(j) \cdot df\left(t_j, S96\right)\right] + N_{HKD} \cdot df(t_M, S96)$$

As can be seen, the spread is added to the projected forward rate based on the 3Mo HIBOR curve (S10) to determine the actual payment amounts.

During stripping, the mathematical equation representing this swap is simply $PV(USD) = PV(HKD)$. Please note that S10 and S490 are already stripped, so all their derived values (forward rates / discount factors) are already determined. Principals on the USD/HKD legs are also determined at the inception of the swap. Thus $PV(USD)$ is actually a constant and the only unknowns are the discount factors from S96, which can be determined by solving the above equation.

Please note that the initial FX spot rate, i.e. the ratio $\frac{N_{HKD}}{N_{USD}}$ does not impact the stripping results, because when setting equation $PV(USD) = PV(HKD)$, we are comparing the value of money, not the absolute number in each currency. This leads to the initial FX spot rate being cancelled on both sides.

6.6 Cross Currency Basis Curve Stripping (Mark-to-Market)

Mark-to-market (MTM) cross currency basis swaps are similar to the non-MTM XCCY basis swaps introduced above, but the MTM type is much more widely traded in financial markets, especially for major currencies. MTM XCCY basis swaps differ in that the principal on the leg paying zero spread (generally the USD leg) is adjusted at the start of every payment period based on the spot foreign exchange (FX) rate. The difference between the principal used for the previous period and the next is paid or received. Here, the principal for the other currency (the leg with basis spread applied) is kept constant throughout the contract.

Basis swaps with this MTM feature are commonly referred to as resettable basis swaps, and an example can be viewed in the Bloomberg terminal by entering **{SWPM -FLFL USD EUR -MTM<GO>}**. In Fig.6e, please note the USD leg has a "*Notional Reset by FX" label at the top to indicate the mark-to-market notional resetting feature has been enabled for this leg of the swap. Also, in Fig 6f, we can see that the notional principal changes at every payment time on the USD leg.

Fig. 6e. 5Y MTM cross currency EUR/USD basis swap instrument on curve S92.

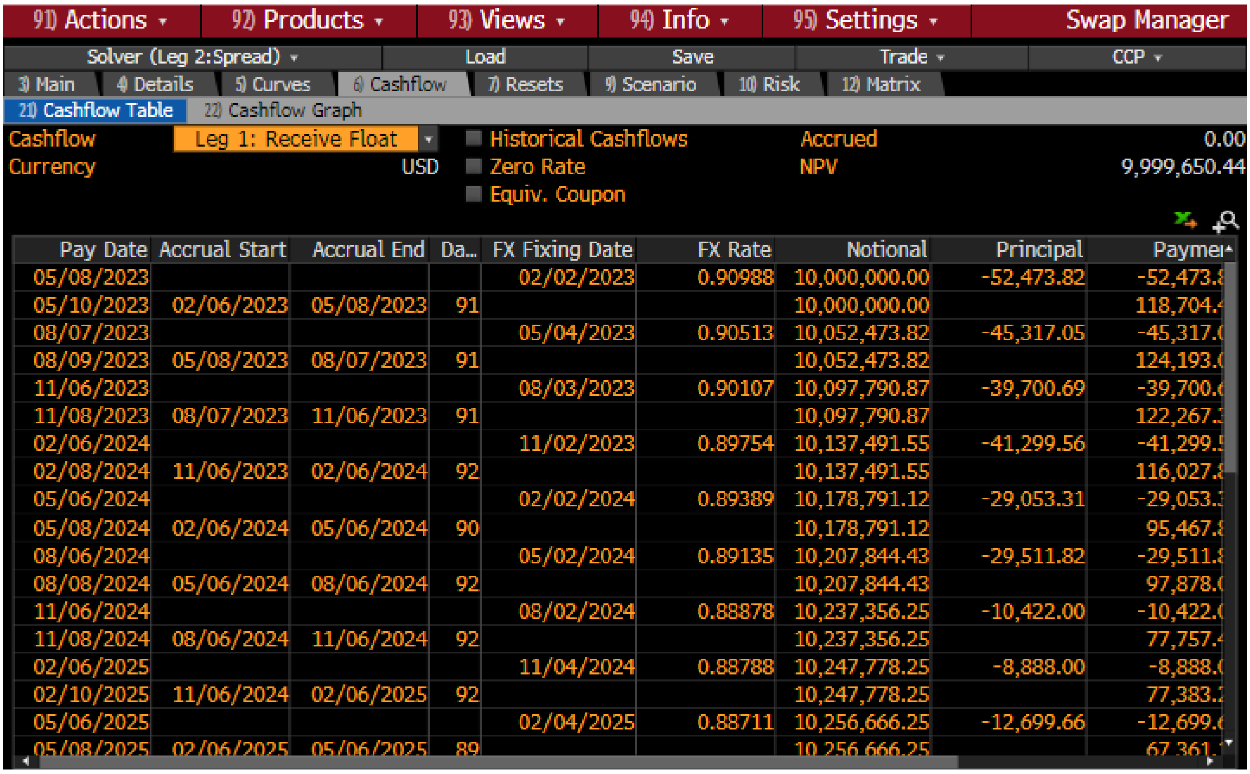


Fig. 6f. Example of cash flows of a 5Y MTM cross currency EUR/USD basis swap instrument on curve S92.

Similar to the non-MTM XCCY basis curve stripping, the key to strip the MTM type of XCCY basis curve is to determine the discount factors that can make the net present value of the swap zero in order to satisfy the no-arbitrage principal.

Let's demonstrate this process using curve S92 (EUR/USD) as an example. As can be seen in Fig. 6e, the forward curve on USD side is S490 and it is S514 on EUR side, while the discounting curve on USD side is S490 and it is S92 on EUR side.

At the i^{th} payment time for each side, let's assume $Notional(i)$ is the principal for this period, $F(i)$ is the projected forward rate derived from curve S490 or S514, $df(i)$ is the corresponding discount factor and $spread$ is the basis spread quote applied (on EUR leg for curve S92). We also assume the final principals need to be exchanged at maturity.

On the EUR leg, the $Notional(i, EUR)$ is fixed and we can set its value to 1, i.e. $Notional(EUR) = 1$. Let's assume N is the total number of payments on this side. Also note that the i^{th}coupon payment is determined by projected forward rate and basis spread, we can write the present value of EUR leg as follows:

$$PV(EUR) = \sum_{i=1}^{N} [R(i, S514) + spread] \cdot \tau(i) \cdot df(t_i, S92) + df(t_N, S92)$$

On the USD leg, the principal needs to be adjusted to mark-to-market, thus the j^{th} payment has two components:

$$Coupon\ Payment\ CP(j) = \ Notional(j, USD) \cdot R(j, S490) \cdot \tau(j)$$

$$Principal\ Exchange\ PE(j) = Notional(j, USD) - Notional(j+1, USD)$$

Please note the final principal exchange pays off the entire principal amount, and $Notional(j, USD)$ becomes zero after the last payment.

Based on Interest Rate Parity and the assumption that $Notional(EUR) = 1$, we have:

$$Notional(j, USD) = Notional(USD) \cdot \frac{df\left(t_{j-1}^{FX}, EUR\right)}{df\left(t_{j-1}^{FX}, USD\right)} = Notional(USD) \cdot \frac{df\left(t_{j-1}^{FX}, S92\right)}{df\left(t_{j-1}^{FX}, S490\right)}$$

Here t_{j-1}^{FX}means the FX-fixing time for j^{th}payment, and it is usually a few days ahead of the accrual start time t_{j-1} for j^{th} period $\left[t_{j-1}, t_j\right]$. Also note that $Notional(1, USD) = Notional(USD)$ since there is no notional resetting for the 1^{st} payment, thus we can set $t_0^{FX} = 0$ to automatically satisfy this since $df(0) = \ 1$ always holds.

Since the notional on EUR-leg is fixed to be 1, $Notional(USD)$ is just initial FX rate (USD:EUR). Thus, we can write $PV(USD)$ as follows (M is the number of payments on this leg):

$$PV(USD) = \sum_{j=1}^{M} [CP(j) + PE(j)] \cdot df\left(t_j, S490\right)$$

$$= \sum_{j=1}^{M-1} [CP(j) + PE(j)] \cdot df\left(t_j, S490\right) + [CP(M) + Notional(M, USD)] \cdot df(t_M, S490)$$

$$= \sum_{j=1}^{M-1} \left[\frac{df(t_{j-1}^{FX}, S92)}{df(t_{j-1}^{FX}, S490)} \cdot fwdRate(j, S23) \cdot cov(j) + \frac{df(t_{j-1}^{FX}, S92)}{df(t_{j-1}^{FX}, S490)} - \frac{df(t_{j}^{FX}, S92)}{df(t_{j}^{FX}, S490)} \right] \cdot df(t_j, S490) + \left[\frac{df(t_{M-1}^{FX}, S92)}{df(t_{M-1}^{FX}, S490)} \cdot fwdRate(M, S490) \cdot cov(M) + \frac{df(t_{M-1}^{FX}, S92)}{df(t_{M-1}^{FX}, S490)} \right] \cdot df(t_M, S490)$$

Since curve S490 and S514 are already stripped, the only unknown variables are the discount factors from S92. By solving equation $PV(EUR) = PV(USD)$, we can determine the unique $DF(t)$ for S92.

6.7 Special Considerations for Curves with Serial FRAs

Incorporation of serial FRAs in curve building by itself does not require any special treatment from the stripping algorithm, in the sense that the stripper will succeed in producing a curve that matches all input instruments. However, converting FRAs to corresponding discount factors relies implicitly on the zero rates at the start dates of FRA instruments and these zero rates are often interpolated from the curve. For a curve with a few FRA instruments, the forward rates produced directly from stripping of the original curve can sometimes be quite bumpy, especially in the segment that covers the FRAs instruments.

In order to help solve this and make the stripping result smoother in the forward space, we adopt a special logic by adding a few so-called “implied cash rates” in the very front of any curve with serial FRAs before final stripping commences. These implied cash instruments are spaced monthly and cover the region before the first instrument on the curve. For example, EUR curve S45 usually starts with a 6Mo index instrument, and thus five extra cash instruments at 1Mo, 2Mo,…, 5Mo will be added to the front. The rates on these cash instruments are derived or implied based on the original curve S45.

In brief, we first partially strip the curve up to the first swap instrument (2Y for S45) to get all the discount factors in this region. Then a so-called “anchor chain” is constructed with the following instruments: 6Mo (Cash) / 6M X 12M (FRA) / 12M X 18M (FRA) / 18M X 24M (FRA*). Please note the last 18M X 24M FRA instrument is a derived one based on the discount factors at 18Mo and 2Y, while the rest are just the original instruments on curve S45. Let’s move on to consider another chain: 5Mo (Cash*) / 5M X 11M (FRA) / 11M X 17M (FRA) / 17M X 23M (FRA*). Here the 17M X 23M FRA rate is linearly interpolated based on the 12M X 18M (FRA) / 18M X 24M (FRA) from the anchor chain, and we are able to reverse calculate the implied cash rate at 5Mo based on the discount factor at 23M. All the other implied cash points at 1 - 4Mo can be determined in a similar manner. This way, the actual stripping result with these implied cash instrument included can produce much smoother forward rates across the FRA segment.

Since these implied cash instruments do not exist on the curve, they cannot be seen in **{SWDF 45 <GO>}**. Some users are puzzled when comparing zero rates before 6Mo since the result won’t match the interpolation (extrapolation, more accurately) method chosen. This is a direct consequence of the implied cash rates.

6.8 Ultimate Forward Rate (UFR) Curve Stripping (EIOPA methodology)

Insurance companies, for example, write insurance policies that can imply liabilities that materially exceed the tenors for most routinely traded securities in the public markets. Life insurance liabilities can routinely exceed 60-70 years, and retirement participation agreements have actuarial requirements that mandate the use of longevity risk that can exceed 100 years. Thus, the European Insurance and Occupational Pensions Authority (EIOPA) has chosen a prescriptive approach to the accounting of forward rates beyond those visible in the market.

In brief, this methodology requires constructing a risk-free interest rate curve based on selected market instruments which are further adjusted for credit risk using credit adjustment spread (CAS). Then this curve is stripped in accordance with the Smith-Wilson interpolation method, which ensures that the long end forward rate will converge to the UFR value published by EIOPA, at a speed determined by the given convergence criteria. The detailed information regarding this methodology can be found on the official EIOPA website:

https://www.eiopa.europa.eu/tools-and-data/risk-free-interest-rate-term-structures_en

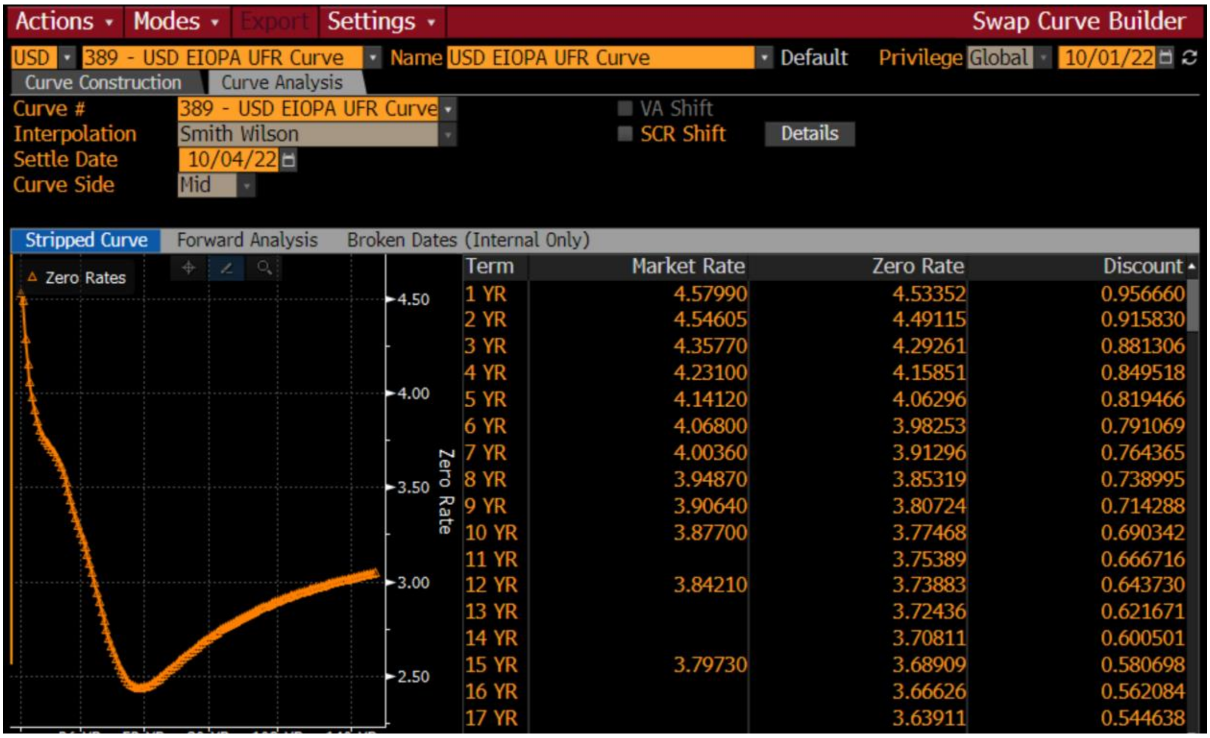


Fig. 6g. Example of the stripping results from USD UFR curve S389.

Appendix 1: Functional Form of Smooth Curve (Piecewise Quadratic Forward Rate)

In this interpolation method, the functional form of the continuously compounded forward rate is a piecewise quadratic polynomial function $L(t)$, i.e.:

$$DF(t) = exp[-L(t)]$$

And L-polynomial is constructed as in its general form:

$$L(t) = (b \cdot t^2 + c \cdot t + d) + \frac{1}{6}\sum_{T_i < t} \lambda_i \cdot (t - T_i)^3 - \frac{1}{6} P \cdot t^3$$

Here T_i is the maturity of the i^{th} instrument that matures after t_c on the curve. The big summation is done for all the instruments at $i \in [1, \dots . N]$ such that $T_i < t$, and this term makes $L(t)$ a piecewise cubic polynomial function. At each time $= T_i$, a new cubic polynomial term is added in a manner that assures that first and second order derivatives of $L(t)$ are continuous. There are N lambdas together with four extra parameters (b, c, d and P) in the above formula, therefore we need four boundary conditions to reduce the number of freedoms back to N in the curve stripping.

Furthermore, in order to better support its application in multi-segment interpolation, i.e. mixed interpolation in different time intervals, we can extend the above discount factor function $DF(t)$ to allow a different interpolation method on the front end:

$$DF(t) = \begin{cases} df(t) & t \in [0, t_c] \\ exp[-L(t - t_c)] & t \in (t_c, +\infty] \end{cases}$$

where $df(t)$ is an arbitrary function that covers the region from zero to the joint time t_c. Based on the requirements, we can impose appropriate boundary conditions at time t_c to connect these two functions properly.

Let's consider the behavior of $DF(t)$ in the long end first, where the curve is extrapolated when $t \to +\infty$. We choose to hold the instantaneous forward rate $F(t)$ a constant beyond T_N and then

$$0 = \frac{dF(t)}{dt} = -\frac{d^2[\ln(DF(t))]}{dt^2} = \frac{d^2 L(t)}{dt^2}$$

This leads to two constraint equations:

$$\sum_{i=1}^{N} \lambda_i - P = 0 \qquad (eq. 1)$$

$$2 \cdot b - \sum_{i=1}^{N} \lambda_i \cdot T_i = 0 \qquad (eq. 2)$$

Let's move on to the short end side, where we need to satisfy the continuity condition at the joint point t_c. Please note this is the $t = 0$ point for $L(t)$ internally, and in this region L-polynomial is reduced to:

$$L(t) = b \cdot t^2 + c \cdot t + d - \frac{1}{6} P \cdot t^3$$

Let's consider two common boundary requirements here:

1) Continuity in $DF(t)$ only at t_c

Since $L(0) = d$, we can see that parameter d is pre-determined by the joint function $df(t)$ at t_c point. And there is one more freedom to eliminate, we usually set $b = 0$ for simplicity. It will not only make $b \cdot t^2$ term disappear, but also lead to a simpler form of the above eq.2. After renaming the unknown parameter c to λ_0, L-polynomial becomes:

$$L(t) = (\lambda_0 \cdot t + d) + \frac{1}{6}\sum_{T_i < t} \lambda_i \cdot (t - T_i)^3 - \frac{1}{6} P \cdot t^3$$

The ultimate unknown parameters to be solved are $[\lambda_0, \lambda_1, \ldots \lambda_{N-1}]$ and both λ_N and P can be derived from eq.1 and eq.2 above.

In the extreme case where $t_c = 0$ and $df(0) = 1$, i.e. no constraint at all, L-polynomial will reduce to its traditional form that covers the whole $t \geq 0$ region:

$$L(t) = \lambda_0 \cdot t + \frac{1}{6}\sum_{T_i < t} \lambda_i \cdot (t - T_i)^3 - \frac{1}{6} P \cdot t^3$$

2) Continuity in both $DF(t)$ and $DF'(t)$ at t_c

Since $L(0) = d$ and $L'(0) = c$, both parameter d and c are pre-determined by the joint function $df(t)$ at t_c point. The corresponding constraint equations are:

$$df(t_c) = \exp(-d) \qquad (eq.3)$$

$$df'(t_c) = df(t_c) \cdot (-c) \qquad (eq.4)$$

After renaming unknown parameter b to λ_0, L-polynomial becomes:

$$L(t) = (\lambda_0 \cdot t^2 + c \cdot t + d) + \frac{1}{6}\sum_{T_i < t} \lambda_i \cdot (t - T_i)^3 - \frac{1}{6} P \cdot t^3$$

Similarly, the ultimate unknown parameters to be solved are $[\lambda_0, \lambda_1, \ldots \lambda_{N-1}]$ and both λ_N and P can be derived from eq.1 and eq.2 above.

Overall, the curve is 'smooth' e.g. the forward rate has a continuous first order derivative, and it has N degrees of freedom. It is easy to see that change in any parameter λ_i where $i \in [0, \ldots. N-1]$ leads to changes in λ_N and P, thus affecting the value of the function everywhere. Therefore, to strip curve with this interpolation, one needs to solve a system of N non-linear equations with N variables. The Newton Raphson method is used, with an initial guess found using interpolation method 1 (linear simple zero rate).

Appendix 2: Step Forward Spread Interpolation Method

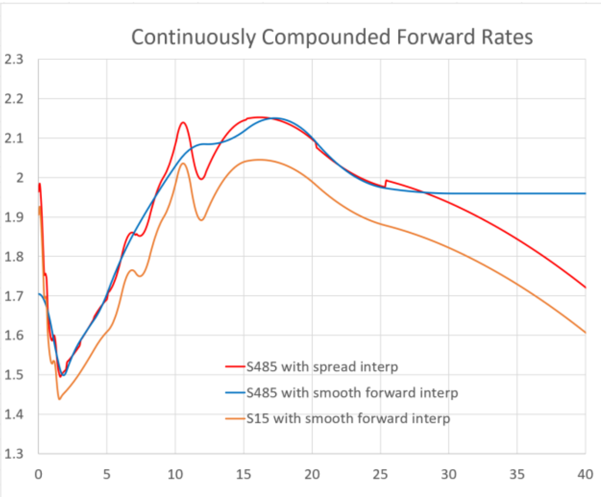
This interpolation takes in a so-called base curve and applies a constant continuous forward spread (for each segment) on top of the forward rate generated directly from base curve to get the final forward rate. So, the final continuously compounded forward rate can be expressed as following:

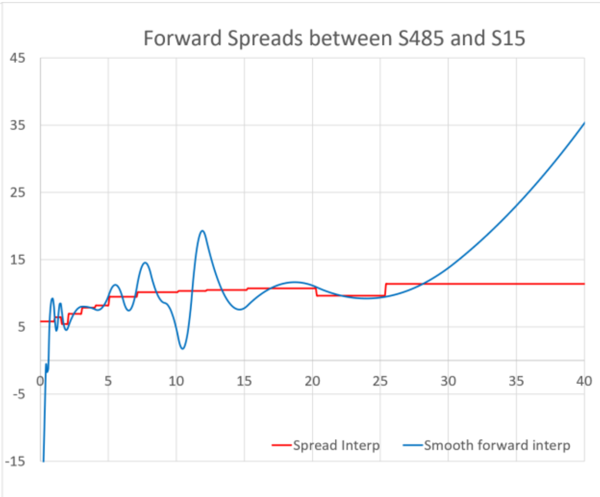
$$F(t_1, t_2) = fwdSpread + baseCCFwd(t_1, t_2)$$

Usually, t_1 is taken at the start time of this segment (t_s), which is the maturity time of the pervious instrument for most of the case. Thus, the discount factors at any time t within this segment can be readily calculated by:

$$DF(t) = DF(t_s) * \exp[-F(t_s, t) * (t - t_s)]$$

Since the base curve is given from outside and already stripped, the $baseCCFwd(t_1, t_2)$ can be directly computed at any period. The only unknown variables left are the above $fwdSpread(s)$, one for each segment. The stripping process will apply a bootstrapping solver to determine all of these values one by one under no-arbitrage principal. The biggest advantage of this interpolation method is to allow the user to capture the feature of the benchmark (base curve), and focus on the deviation from it. Below is a graph that plots the continuously compounded forward rates for NZD 6Mo basis curve S485 under spread interpolation (red) as a function of time. It also displays the same forward rates for S485 using smooth forward interpolation, and the underlying NZD 3Mo curve S15 for comparison. It is quite obvious that the stripping results based on the spread interpolation are tracking the underlying curve much better than that from the smooth forward interpolation. Furthermore, smooth forward interpolation completely loses track of the underlying curve especially in the extrapolation region (beyond 30 years), while the spread interpolation still tracks it very well by keeping a constant distance above S.





Appendix 3: Cubic Spline Interpolation

In this interpolation method, a series of piecewise cubic polynomials $f_i(x)$ are applied to the given set of points, i.e., $[x_i, y_i]$, to ensure smoothness up to the second order derivatives at any x.

Let's first assume the second order derivatives at boundary x_i is k_i. Since the second order derivative $f_i''(x)$ of a cubic polynomial $f_i(x)$ is a linear function of x, we can rewrite $f_i''(x)$ to the following form based on k_i and k_{i+1}:

$$f_i''(x) = k_i \frac{x - x_{i+1}}{x_i - x_{i+1}} + k_{i+1} \frac{x - x_i}{x_{i+1} - x_i} \qquad when\ x \in [x_i, x_{i+1}]$$

After integrating this function twice, we can get a general expression for the cubic polynomial $f_i(x)$ as:

$$f_i(x) = \frac{k_i}{6(x_i - x_{i+1})}(x - x_{i+1})^3 + \frac{k_{i+1}}{6(x_{i+1} - x_i)}(x - x_i)^3$$
$$+C_1(x - x_{i+1}) + C_0(x - x_i) \qquad when\ x \in [x_i, x_{i+1}]$$

Here C_1 and C_0 are two constants of integration. They can be further determined by using continuity constraints at boundaries: $f_i(x_i) = y_i$ and $f_i(x_{i+1}) = y_{i+1}$. This will finally transform $f_i(x)$ to:

$$f_i(x) = \frac{y_i(x - x_{i+1}) - y_{i+1}(x - x_i)}{x_i - x_{i+1}} + \frac{k_i}{6}\left[\frac{(x - x_{i+1})^3}{x_i - x_{i+1}} - (x - x_{i+1})(x_i - x_{i+1})\right]$$
$$- \frac{k_{i+1}}{6}\left[\frac{(x - x_i)^3}{x_i - x_{i+1}} - (x - x_i)(x_i - x_{i+1})\right] \quad when\ x \in [x_i, x_{i+1}] \qquad (eq. 1)$$

This is the most general form of cubic splines based on the unknown second order derivatives at the boundaries. Additionally, we still need to ensure smoothness of the first order derivatives at the boundary, i.e., $f_{i-1}'(x_i) = f_i'(x_i)$, and this forces all the k_i to satisfy:

$$k_{i-1}(x_{i-1} - x_i) + 2k_i(x_{i-1} - x_{i+1}) + k_{i+1}(x_i - x_{i+1}) = 6\left(\frac{y_{i-1} - y_i}{x_{i-1} - x_i} - \frac{y_i - y_{i+1}}{x_i - x_{i+1}}\right) \quad (eq. 2)$$

It is obvious to see from above equation that not all k_i are independent and we only have two extra degrees of freedom here. Normally, we will set the first and last k_i to be zero and the resulting $f_i(x)$ is called **natural cubic splines** which is also the BRL interpolation method 3 in our system. However, natural cubic spline is not the only valid result. By imposing different two extra constraints, we can have different final form of cubic splines $f_i(x)$ and they are all valid solutions to the given set of points.

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