Now assume that the moment constraints are noisy

When the moment constraints are noisy, the principle of maximum entropy still applies, but we need to account for the uncertainty in the constraints. This situation introduces a layer of complexity because the exact moment constraints (e.g., expectations of outcomes) are not perfectly known; instead, we have estimates that are subject to error or noise.

1. Noisy Moment Constraints

Let's denote the noisy moment constraints as follows:

$$\mathbb{E}[Y_{it} \mid D_i, T_t] \approx \overline{Y}_{D_i,T_t} + \epsilon_{D_i,T_t}$$

Where ϵ_{D_i,T_t} represents the noise in the moment constraint for each group and time period. These noise terms are generally assumed to be mean-zero and could have a known variance.

2. Modified Entropy Maximization Problem

Given noisy constraints, the maximum entropy problem can be reformulated. Instead of enforcing the constraints exactly, we introduce a penalty term in the objective function that accounts for the deviations due to noise.

The new objective function to maximize is:

$$\max_P \quad H(P) - \sum_{D_i,T_t} \frac{\lambda_{D_i,T_t}}{2\sigma^2_{D_i,T_t}} \left(\mathbb{E}_P[Y_{it} \mid D_i, T_t] - (\overline{Y}_{D_i,T_t} + \epsilon_{D_i,T_t})\right)^2$$

Here:

- $H(P)$ is the entropy term as before.
- λ_{D_i,T_t} are Lagrange multipliers.
- $\sigma^2_{D_i,T_t}$ represents the variance of the noise in the moment constraints.

3. Solving the Modified Problem