How about if the utility function in ln w?

If the utility function is $u(w) = \ln(w)$, this represents a logarithmic utility function, which corresponds to the special case of CRRA utility with $\gamma = 1$. Let's re-derive the optimal quantity Q under this utility function.

Step 1: Define the Final Wealth

The agent's final wealth after selling Q units of the risky asset at price p_b is:

$$w(Q) = Q \cdot p_b + (L - Q) \cdot (1 + z)$$

Step 2: Expected Utility

The expected utility function is:

$$\mathbb{E}[u(w(Q))] = \mathbb{E}[\ln(w(Q))]$$

Substituting for $w(Q)$:

$$\mathbb{E}[\ln(Q \cdot p_b + (L - Q) \cdot (1 + z))]$$

Step 3: Approximating the Utility Using Taylor Expansion

For small z, we can approximate $\ln(w(Q))$ using a first-order Taylor expansion around the expected value $\mu_w(Q) = Q \cdot p_b + (L - Q)$:

$$\ln(w(Q)) \approx \ln(\mu_w(Q)) + \frac{w(Q) - \mu_w(Q)}{\mu_w(Q)} - \frac{1}{2} \cdot \frac{(w(Q) - \mu_w(Q))^2}{\mu_w(Q)^2}$$

Taking the expectation:

$$\mathbb{E}[\ln(w(Q))] \approx \ln(\mu_w(Q)) - \frac{1}{2} \cdot \frac{\mathrm{Var}(w(Q))}{\mu_w(Q)^2}$$

Step 4: Variance of Wealth

The variance of $w(Q)$ is:

$$\mathrm{Var}(w(Q)) = \mathrm{Var}((L - Q) \cdot z) = (L - Q)^2 \cdot \sigma^2$$