

## Supplementary Material for Chapter 5

### Tables

Rank	Perfect Test	90% Sensitive & 90% Specific RDT			
	All Noise	1x Poisson Noise	7x Poisson Noise	1x Dynamical Noise	7x Dynamical Noise
1	Variance (0.62)	Variance (0.61)	Variance (0.6)	Variance (0.66)	Index of dispersion (0.47)
2	Index of dispersion (0.58)	Index of dispersion (0.59)	Index of dispersion (0.6)	Autocovariance (0.63)	Autocorrelation (0.45)
3	Autocovariance (0.58)	Autocovariance (0.55)	Coefficient of variation (0.59)	Index of dispersion (0.57)	Coefficient of variation (0.45)
4	Autocorrelation (0.38)	Coefficient of variation (0.51)	Autocovariance (0.51)	Mean (0.48)	Autocovariance (0.39)
5	Mean (0.38)	Autocorrelation (0.41)	Mean (0.37)	Autocorrelation (0.42)	Variance (0.38)
6	Coefficient of variation (0.15)	Mean (0.35)	Autocorrelation (0.36)	Coefficient of variation (0.12)	Skewness (0.11)
7	Skewness (0.06)	Skewness (0.14)	Skewness (0.1)	Skewness (-0.05)	Kurtosis (-0.19)
8	Kurtosis (-0.02)	Kurtosis (0.01)	Kurtosis (0.02)	Kurtosis (-0.11)	Mean (-0.21)

*Table 1: The ranking and mean value of Kendall's Tau computed on the subset of the emergent time series after the burn-in period, for a perfect test and an RDT with 90% sensitivity and 90% specificity, under high and low Poisson and dynamical noise systems*

Rank	Perfect Test	90% Sensitive & 90% Specific RDT			
	All Noise	1x Poisson Noise	7x Poisson Noise	1x Dynamical Noise	7x Dynamical Noise
1	Autocovariance (0.7)	Autocovariance (0.73)	Autocovariance (0.72)	Autocovariance (0.66)	Mean (0.55)
2	Variance (0.7)	Variance (0.71)	Mean (0.7)	Variance (0.64)	Variance (0.54)
3	Mean (0.68)	Mean (0.7)	Variance (0.68)	Mean (0.63)	Autocovariance (0.53)
4	Index of dispersion (0.63)	Index of dispersion (0.67)	Index of dispersion (0.68)	Index of dispersion (0.59)	Skewness (0.51)
5	Autocorrelation (0.62)	Autocorrelation (0.67)	Coefficient of variation (0.67)	Autocorrelation (0.57)	Kurtosis (0.51)
6	Skewness (0.6)	Coefficient of variation (0.6)	Autocorrelation (0.66)	Skewness (0.56)	Index of dispersion (0.5)
7	Kurtosis (0.53)	Skewness (0.58)	Skewness (0.6)	Kurtosis (0.51)	Autocorrelation (0.49)
8	Coefficient of variation (0.39)	Kurtosis (0.45)	Kurtosis (0.57)	Coefficient of variation (0.45)	Coefficient of variation (0.48)

*Table 2: The ranking of AUC computed on the subset of the emergent time series after the burn-in period, for a perfect test and an RDT with 90% sensitivity and 90% specificity, under high and low Poisson and dynamical noise systems*

Rank	Perfect Test		90% Sensitive & 90% Specific RDT			
	All Noise -  AUC-0.5	All Noise - Accuracy	1x Poisson Noise	7x Poisson Noise	1x Dynamical Noise	7x Dynamical Noise
1	Autocovariance (0.2)	Mean (0.72)	Mean (0.73)	Variance (0.73)	Variance (0.68)	Mean (0.6)
2	Variance (0.2)	Variance (0.72)	Variance (0.7)	Coefficient of variation (0.72)	Mean (0.66)	Skewness (0.57)
3	Mean (0.18)	Autocovariance (0.7)	Autocovariance (0.7)	Mean (0.72)	Autocovariance (0.65)	Kurtosis (0.55)
4	Index of dispersion (0.13)	Index of dispersion (0.63)	Index of dispersion (0.69)	Index of dispersion (0.72)	Skewness (0.6)	Autocorrelation (0.54)
5	Autocorrelation (0.12)	Autocorrelation (0.62)	Autocorrelation (0.67)	Autocovariance (0.71)	Index of dispersion (0.6)	Autocovariance (0.52)
6	Coefficient of variation (0.11)	Skewness (0.6)	Coefficient of variation (0.66)	Autocorrelation (0.66)	Kurtosis (0.57)	Coefficient of variation (0.52)
7	Skewness (0.1)	Kurtosis (0.58)	Skewness (0.62)	Skewness (0.66)	Autocorrelation (0.55)	Variance (0.52)
8	Kurtosis (0.03)	Coefficient of variation (0.5)	Kurtosis (0.56)	Kurtosis (0.57)	Coefficient of variation (0.51)	Index of dispersion (0.51)

Table 3: The ranking and  $|AUC - 0.5|$  for each metric computed on the emergent time series with a perfect test, and the alert accuracy with an RDT. The values are computed on the full time series, and the subset from after the completion of the burn-in period, with a perfect test

## Plots

### AUC Magnitude Heatmaps

*After 5yr Burn in*

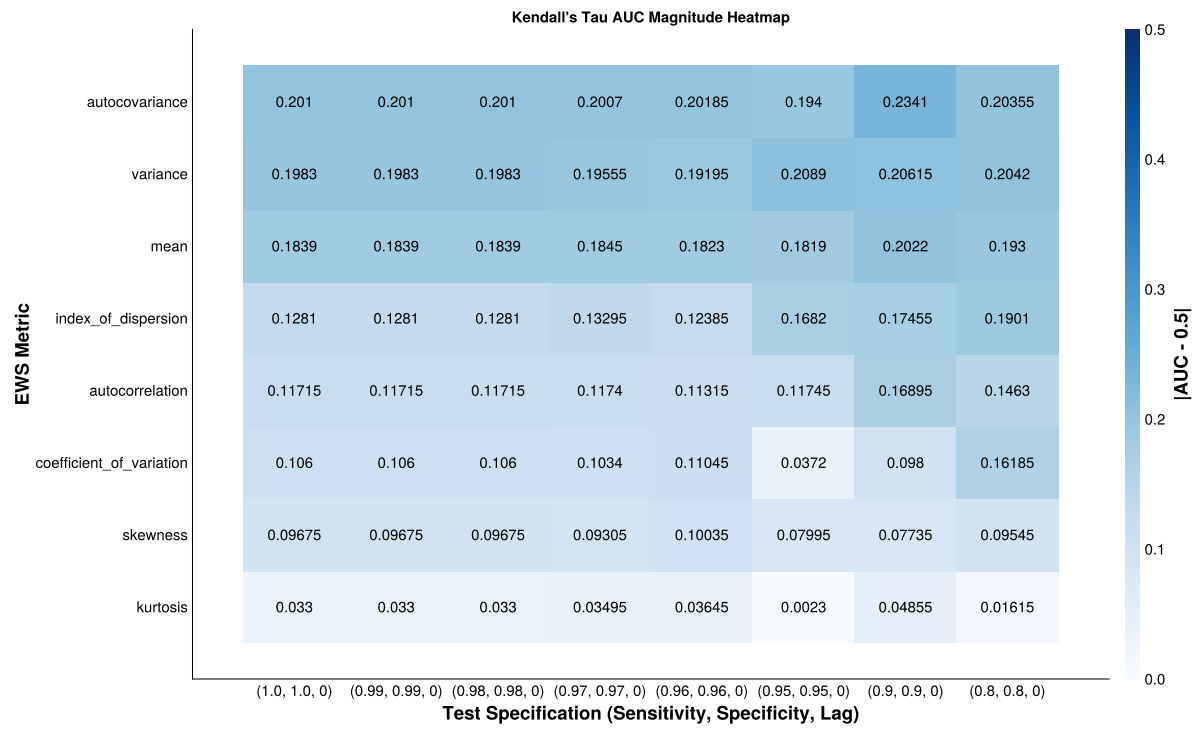


Figure 1: Poisson noise, 1x

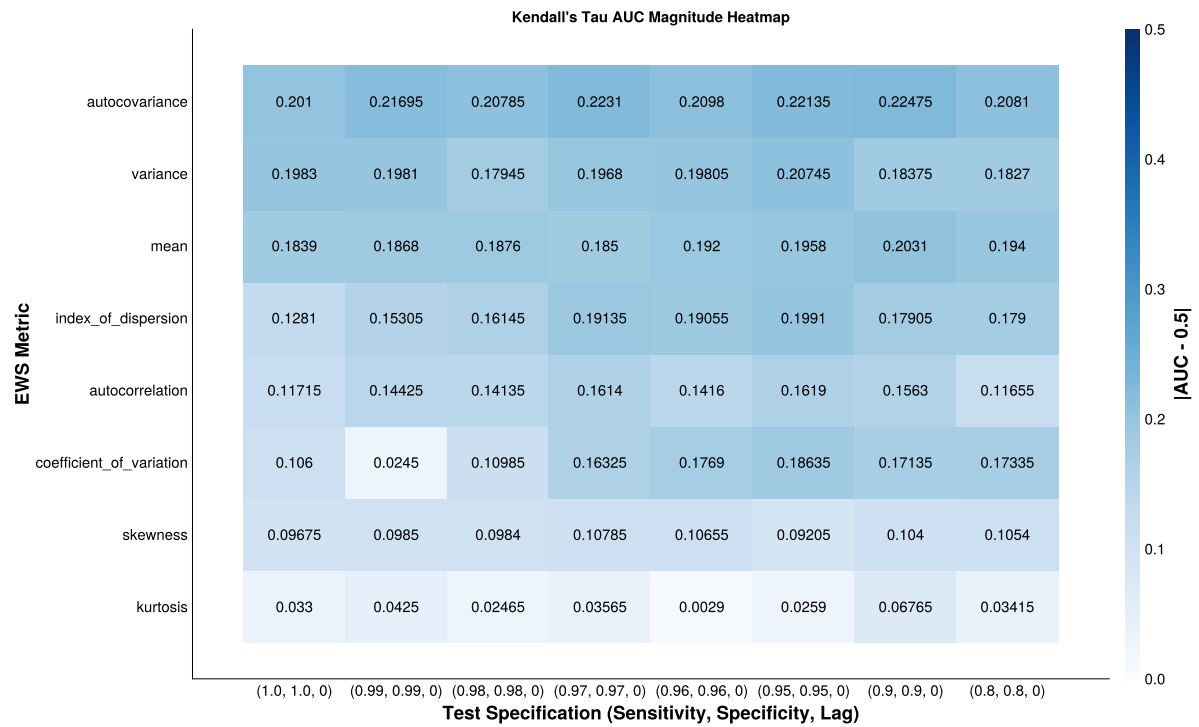


Figure 2: Poisson noise, 7x

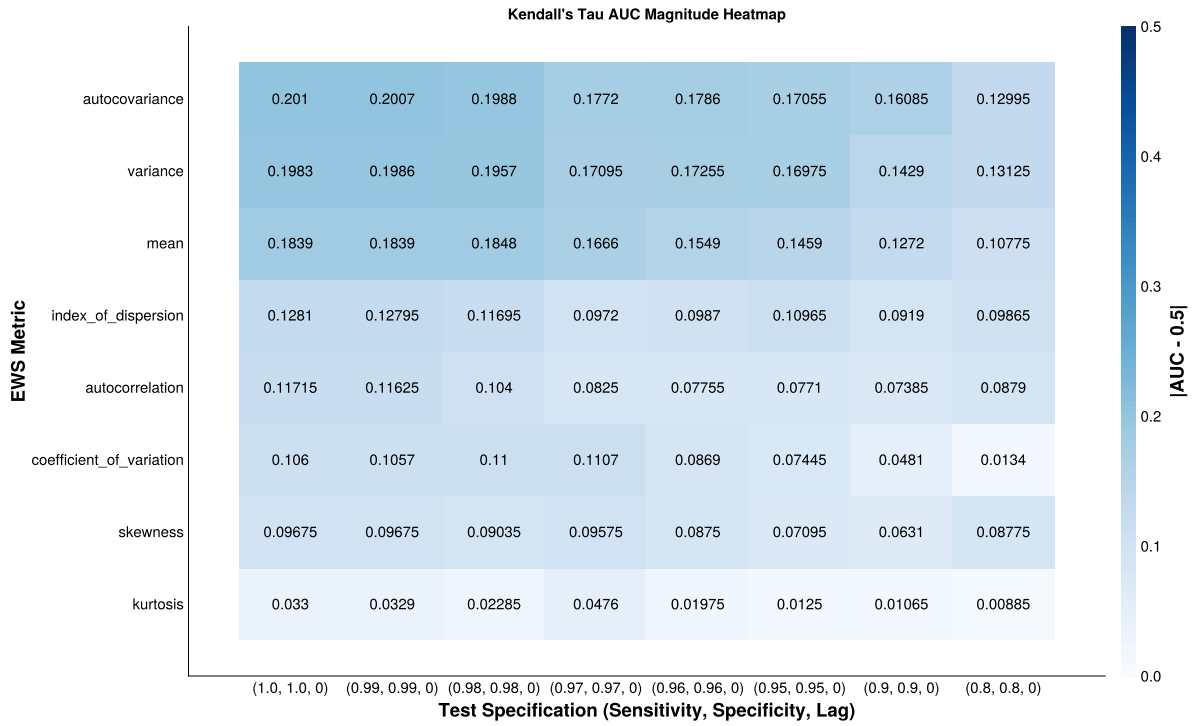


Figure 3: Dynamical noise, 1x

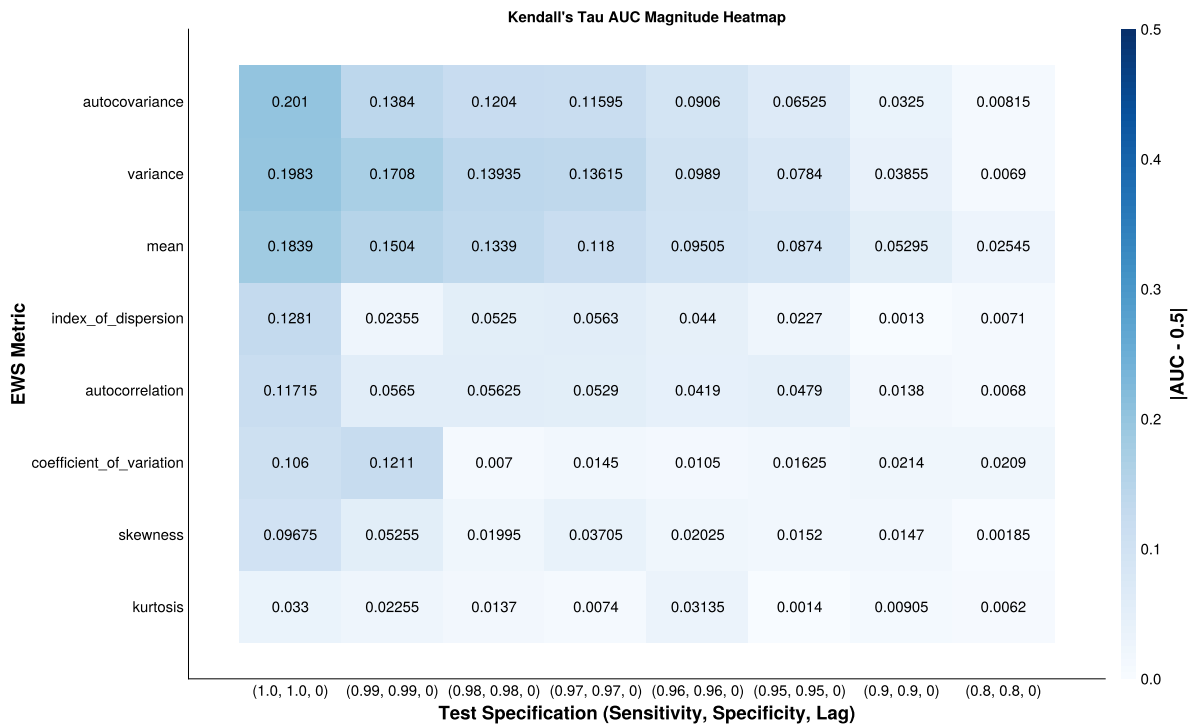


Figure 4: Dynamical noise, 7x

## AUC Heatmaps

### After 5yr Burn in

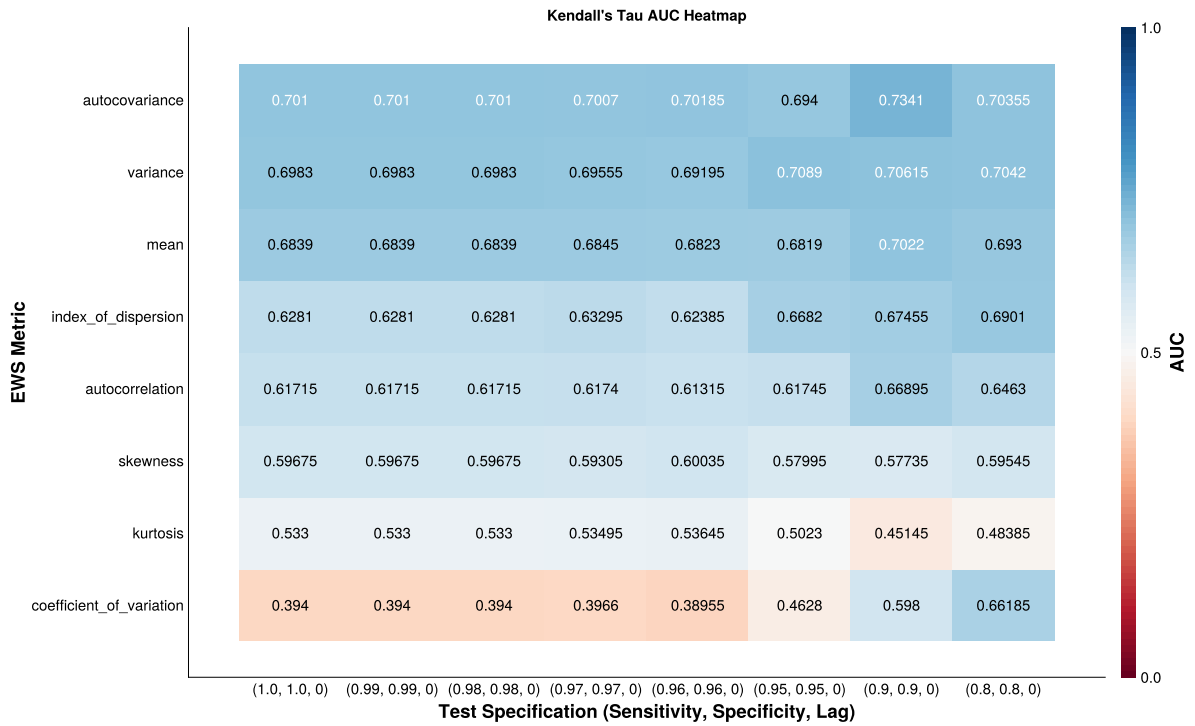


Figure 5: Poisson noise, 1x



Figure 6: Poisson noise, 7x

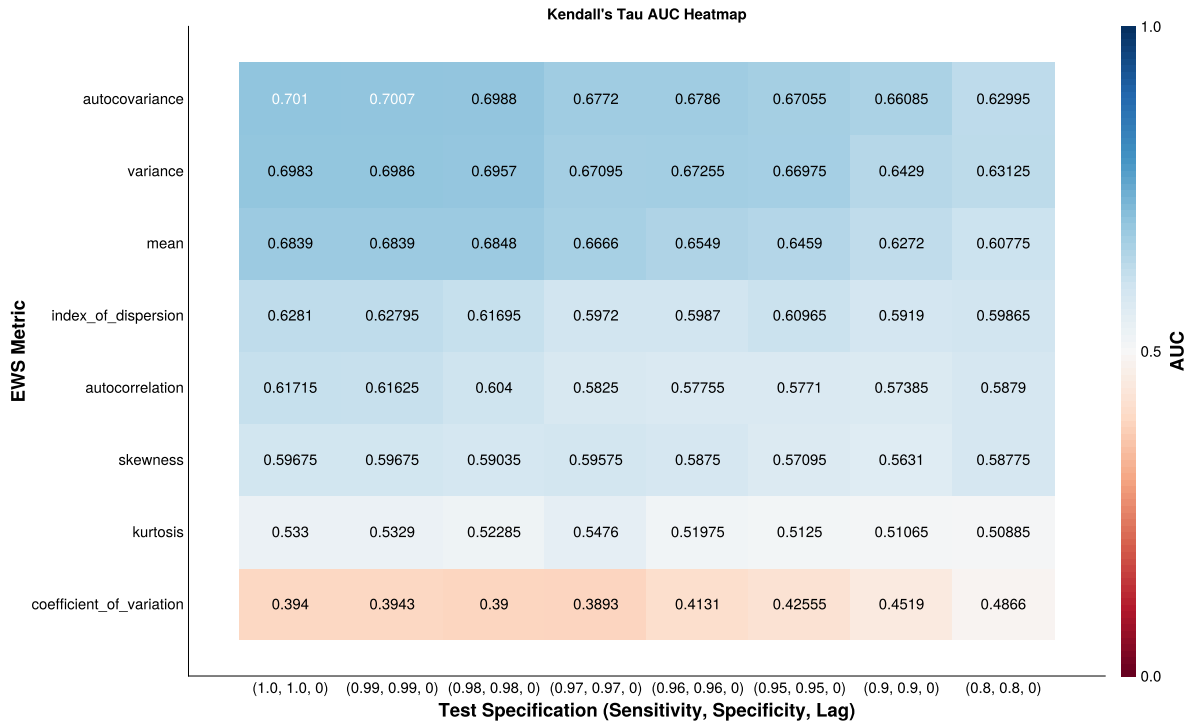


Figure 7: Dynamical noise, 1x

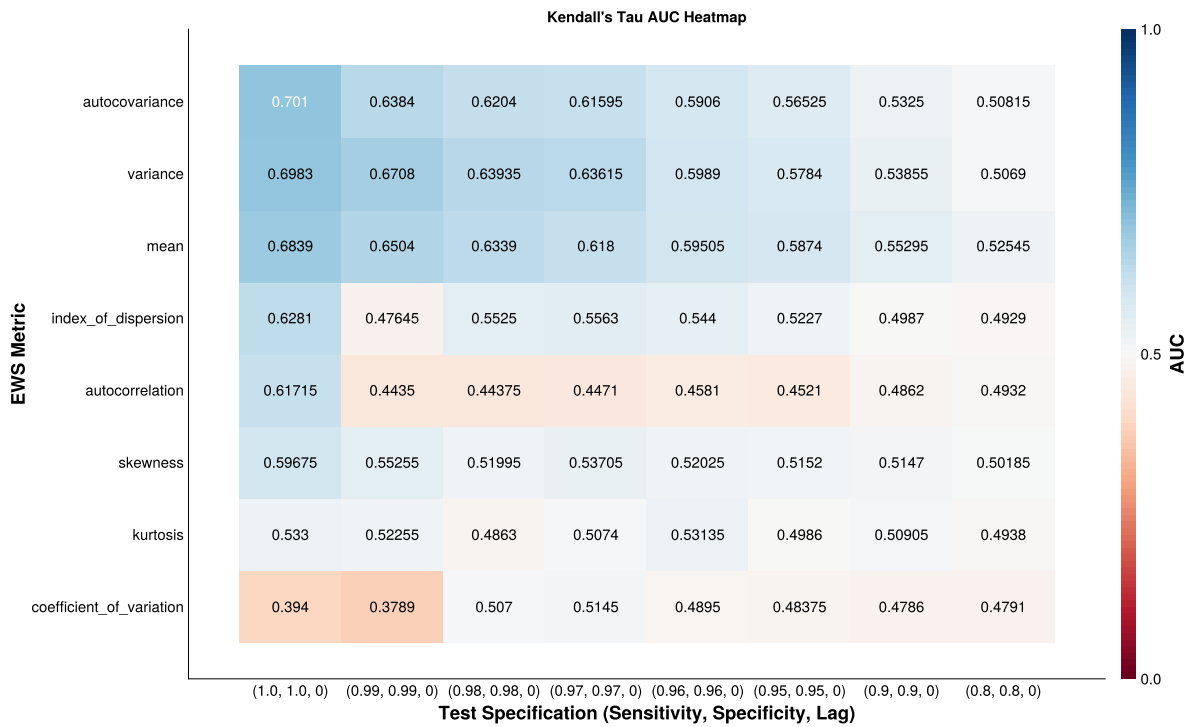


Figure 8: Dynamical noise, 7x

## Optimal Threshold Accuracies

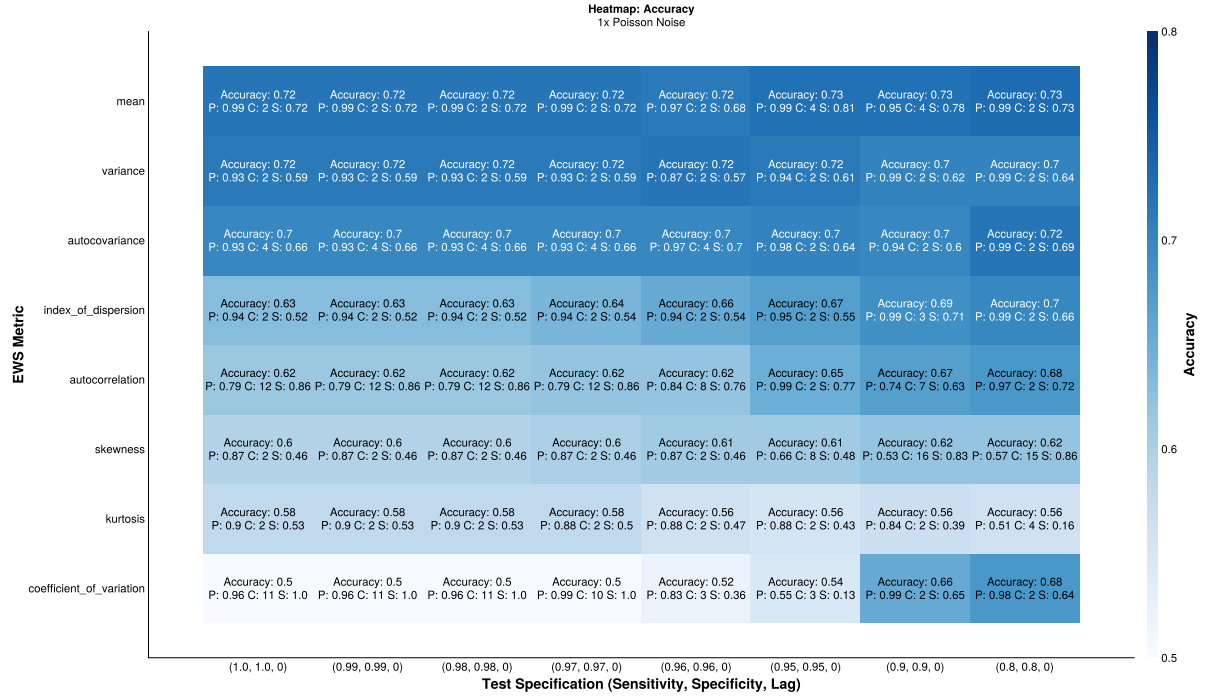


Figure 9: The maximal alert accuracy under 1x Poisson noise. P) refers to the long-running percentile threshold to return a flag, and C) the number of consecutive flags to trigger and alert, that in combination produce the maximal accuracy. S) refers to the specificity of the alert system

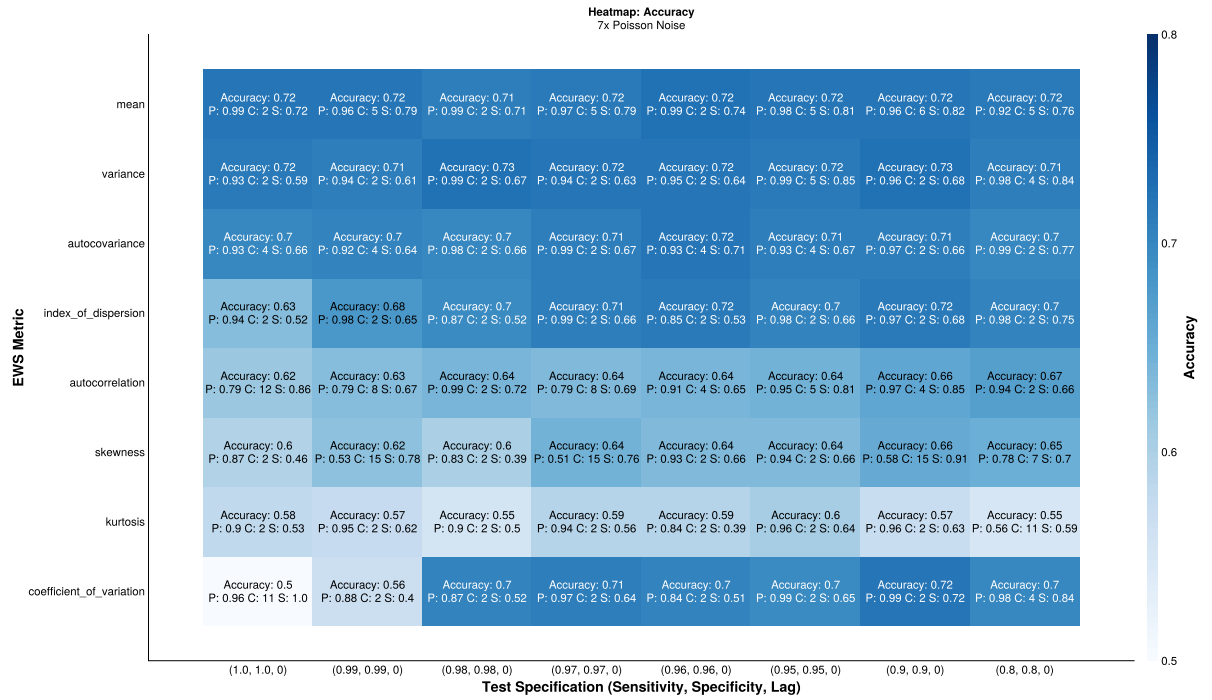


Figure 10: The maximal alert accuracy under 7x Poisson noise. P) refers to the long-running percentile threshold to return a flag, and C) the number of consecutive flags to trigger and alert, that in combination produce the maximal accuracy. S) refers to the specificity of the alert system



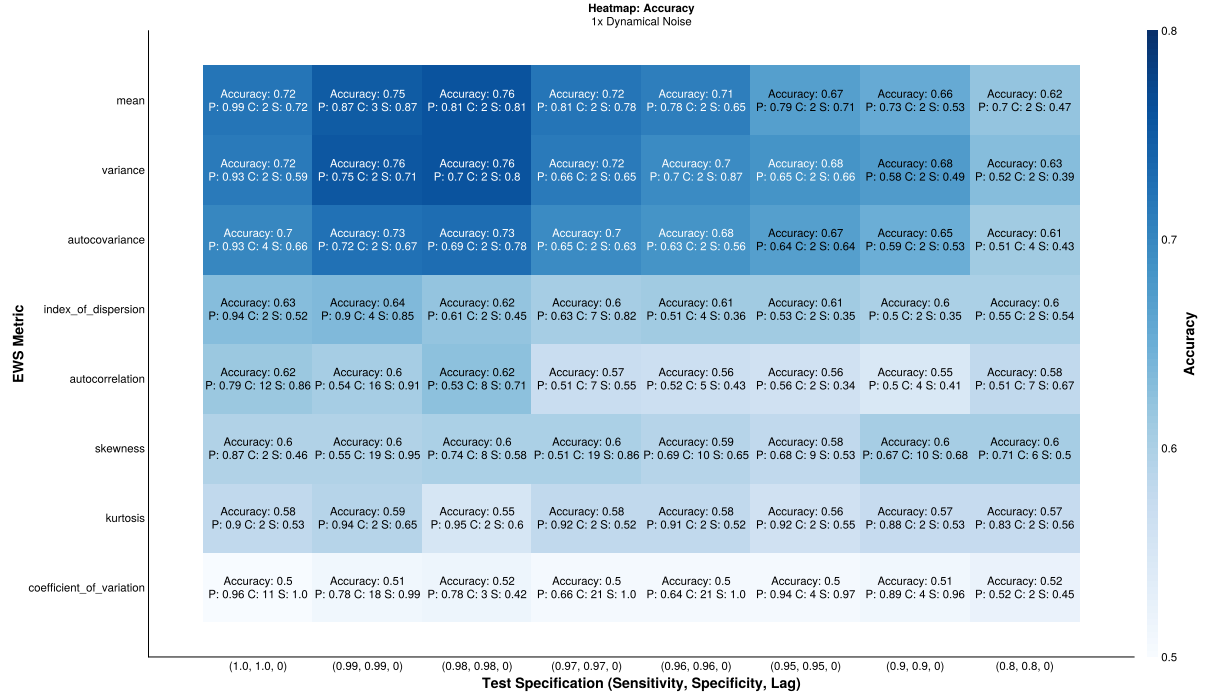


Figure 11: The maximal alert accuracy under 1x Dynamical noise. *P*) refers to the long-running percentile threshold to return a flag, and *C*) the number of consecutive flags to trigger and alert, that in combination produce the maximal accuracy. *S*) refers to the specificity of the alert system

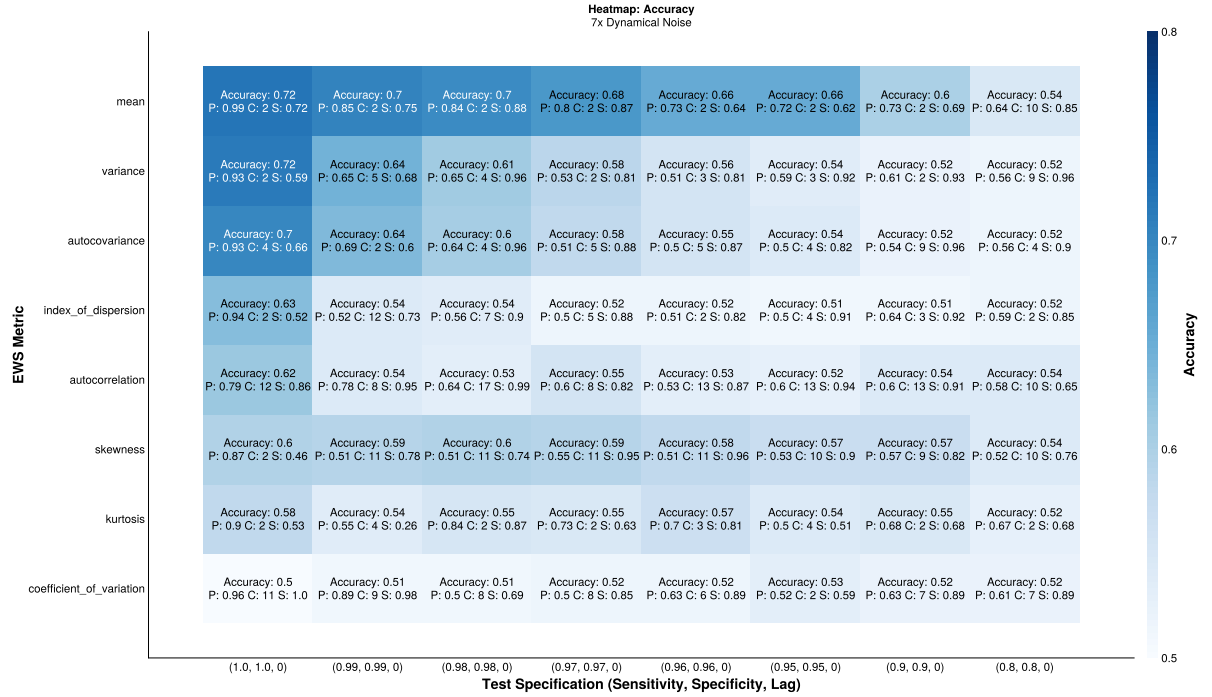


Figure 12: The maximal alert accuracy under 7x Dynamical noise. *P*) refers to the long-running percentile threshold to return a flag, and *C*) the number of consecutive flags to trigger and alert, that in combination produce the maximal accuracy. *S*) refers to the specificity of the alert system

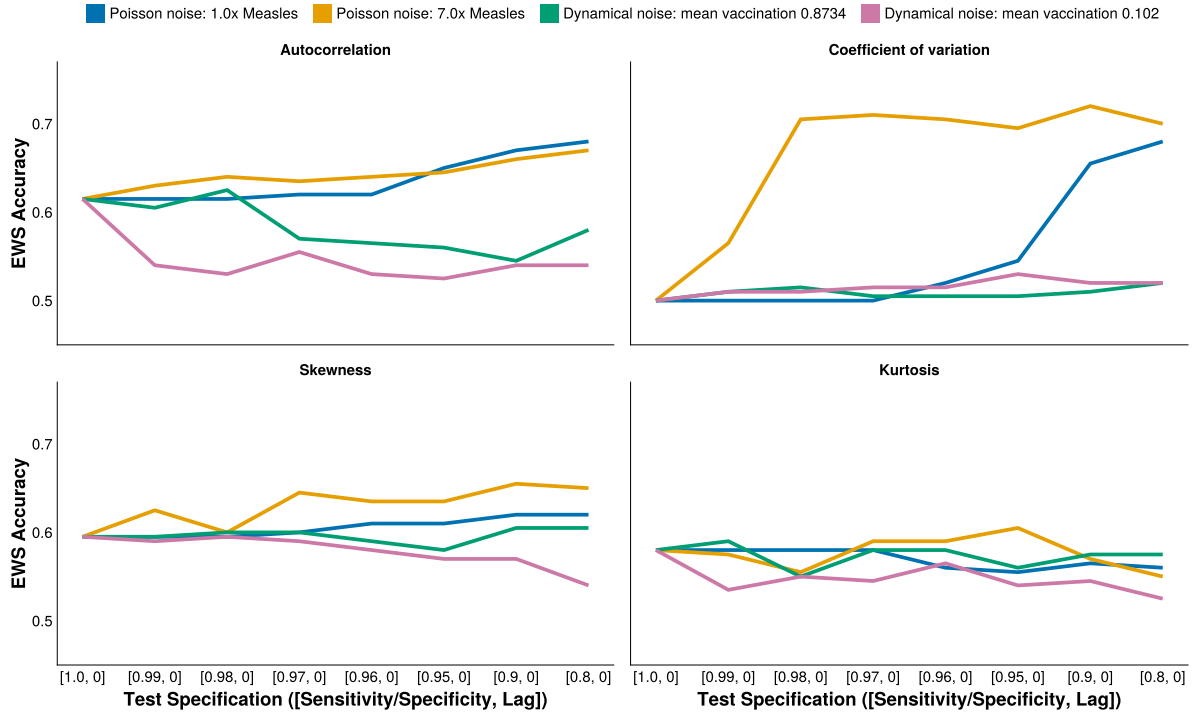


Figure 13: The change in alert accuracy for less correlated EWS metrics under increasing diagnostic uncertainty, and low and high levels of Poisson or dynamical noise

## Survival Analysis

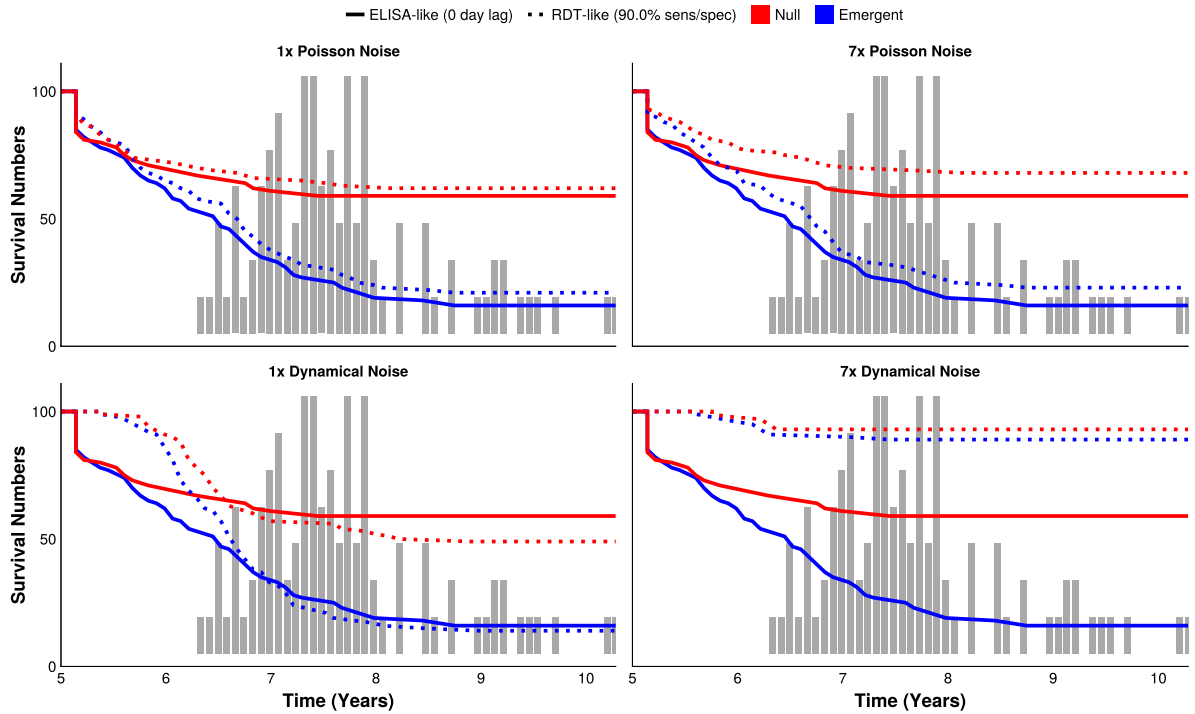


Figure 14: Survival curves for the variance EWS metric computed on emergent and null simulations, with a perfect test and an RDT equivalent with 90% sensitivity and specificity. The histogram depicts the times when the tipping point is reached ( $R_E = 1$ ) under the emergent simulation, right-truncating the curves.

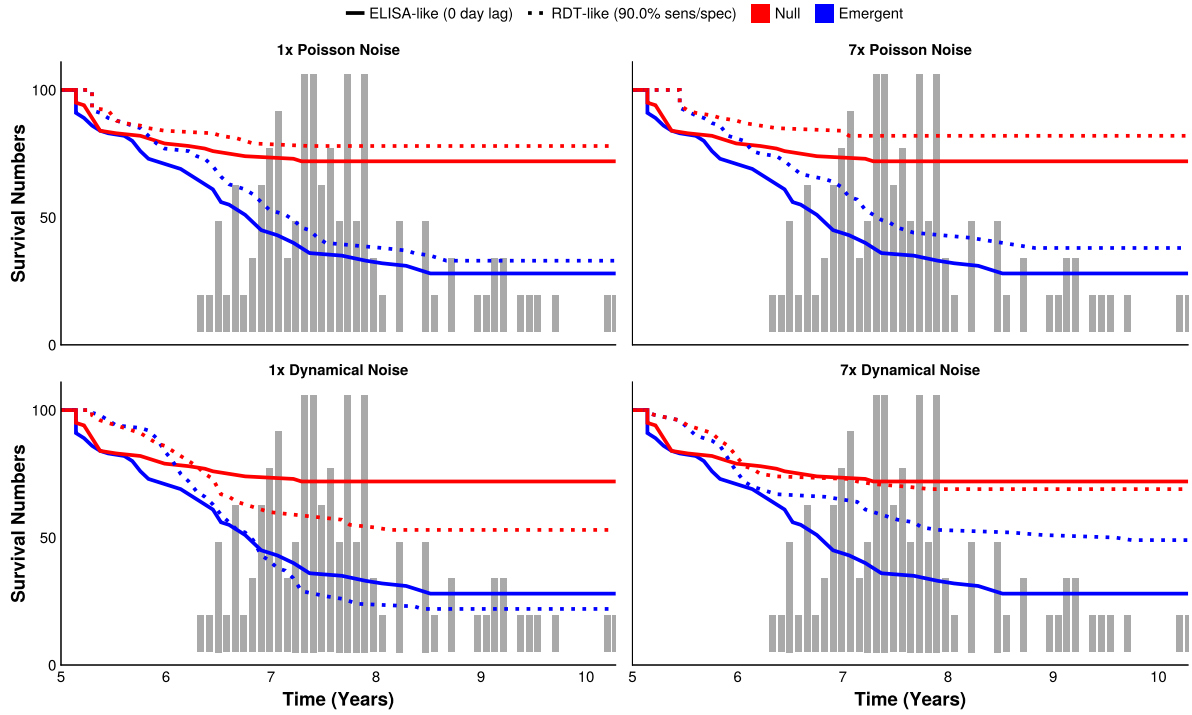


Figure 15: Survival curves for the mean EWS metric computed on emergent and null simulations, with a perfect test and an RDT equivalent with 90% sensitivity and specificity. The histogram depicts the times when the tipping point is reached ( $R_E = 1$ ) under the emergent simulation, right-truncating the curves.

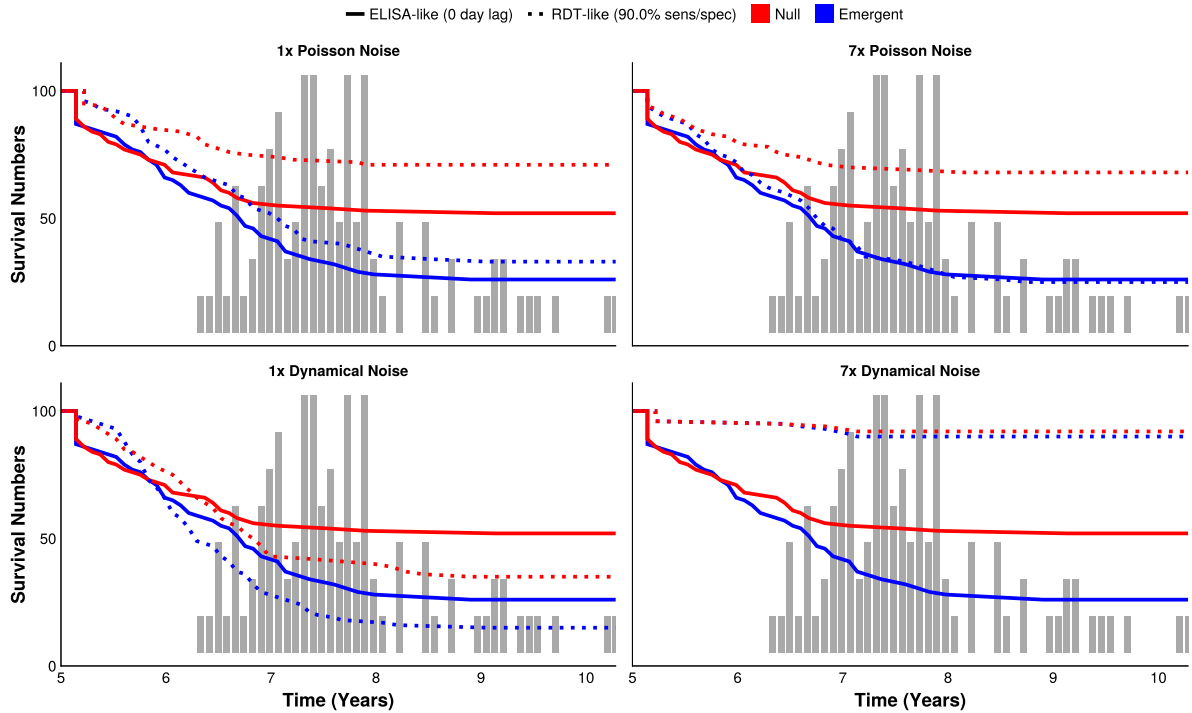


Figure 16: Survival curves for the index of dispersion EWS metric computed on emergent and null simulations, with a perfect test and an RDT equivalent with 90% sensitivity and specificity. The histogram depicts the times when the tipping point is reached ( $R_E = 1$ ) under the emergent simulation, right-truncating the curves.

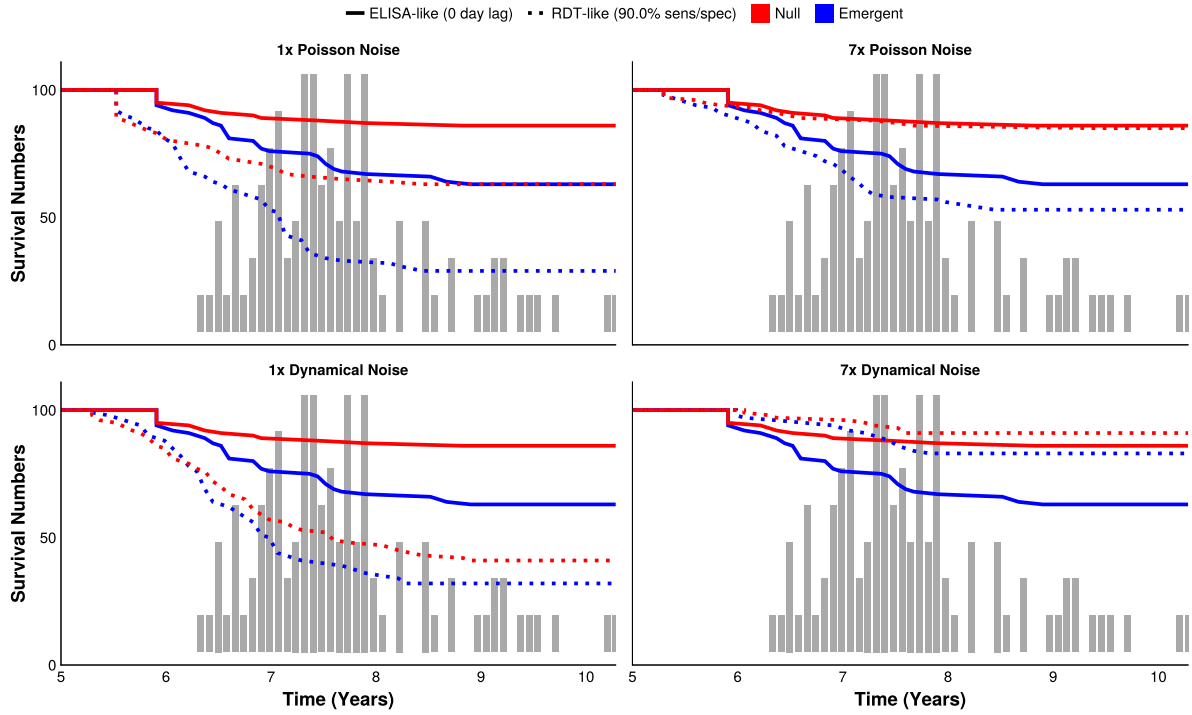


Figure 17: Survival curves for the autocorrelation EWS metric computed on emergent and null simulations, with a perfect test and an RDT equivalent with 90% sensitivity and specificity. The histogram depicts the times when the tipping point is reached ( $R_E = 1$ ) under the emergent simulation, right-truncating the curves.

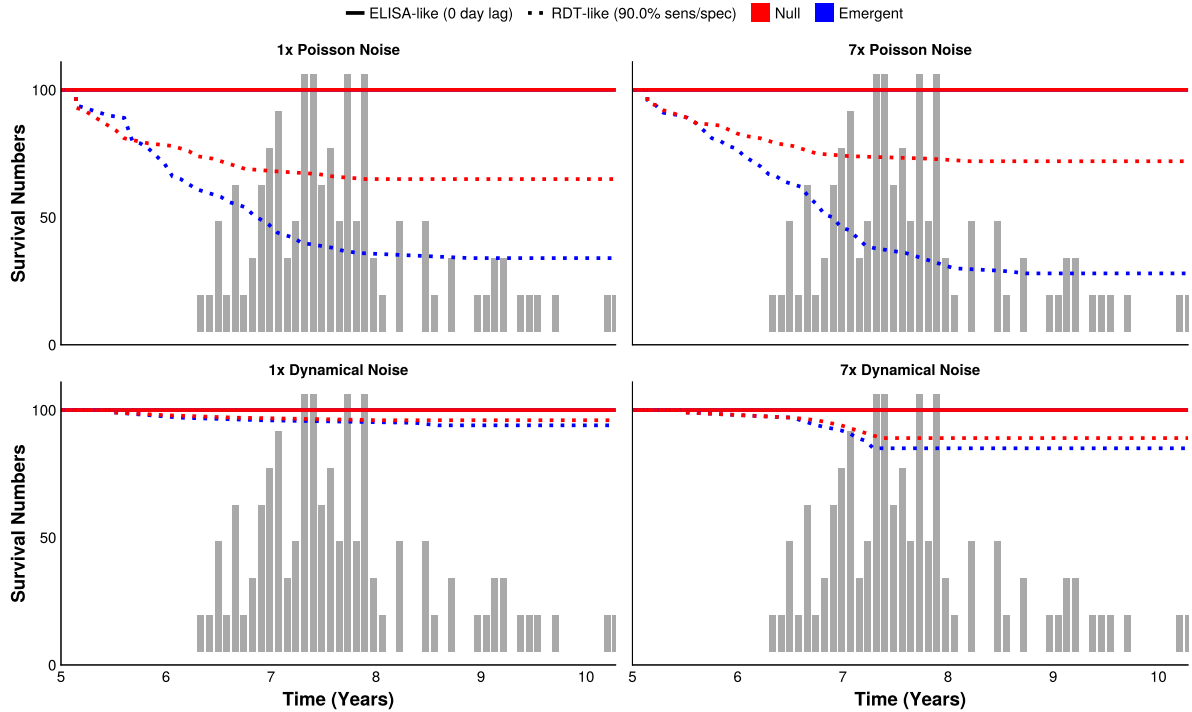


Figure 18: Survival curves for the coefficient of variation EWS metric computed on emergent and null simulations, with a perfect test and an RDT equivalent with 90% sensitivity and specificity. The histogram depicts the times when the tipping point is reached ( $R_E = 1$ ) under the emergent simulation, right-truncating the curves.

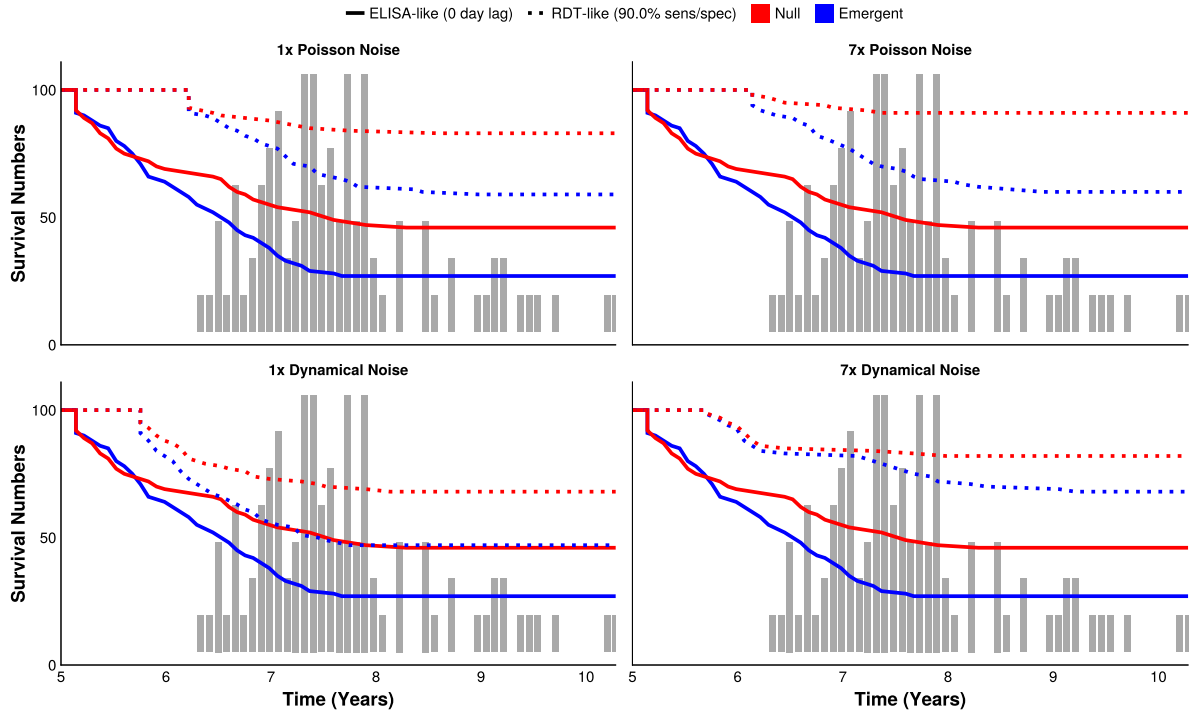


Figure 19: Survival curves for the skewness EWS metric computed on emergent and null simulations, with a perfect test and an RDT equivalent with 90% sensitivity and specificity. The histogram depicts the times when the tipping point is reached ( $R_E = 1$ ) under the emergent simulation, right-truncating the curves.

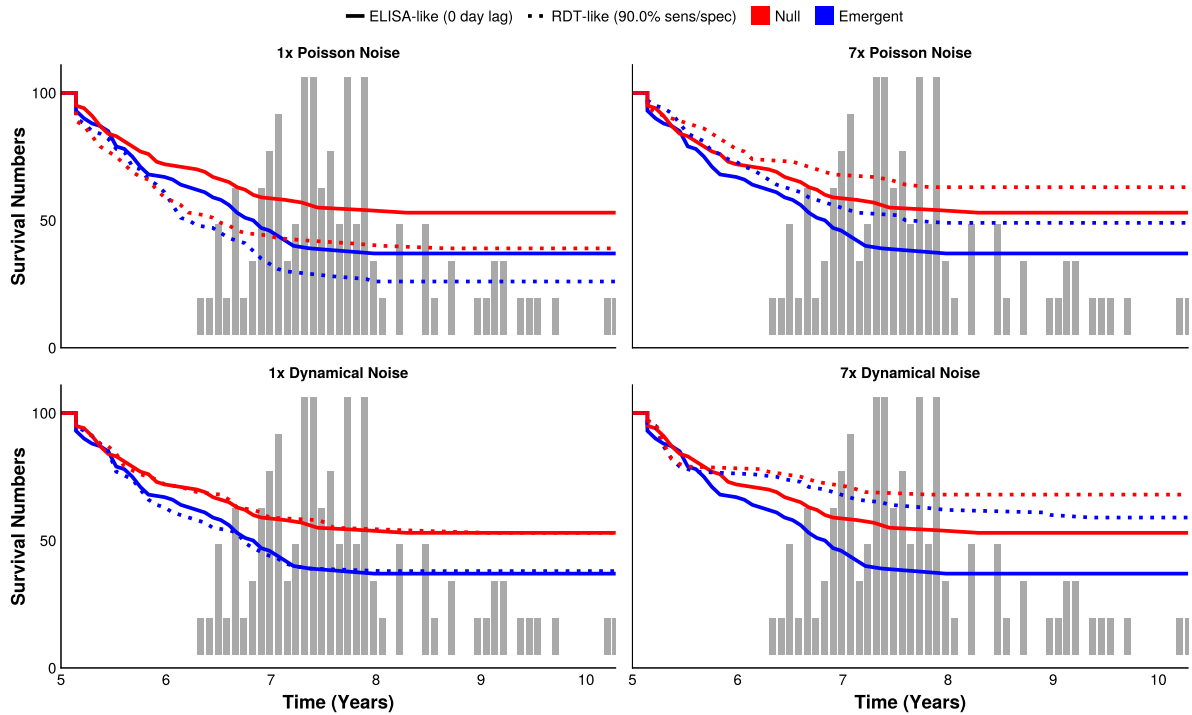


Figure 20: Survival curves for the kurtosis EWS metric computed on emergent and null simulations, with a perfect test and an RDT equivalent with 90% sensitivity and specificity. The histogram depicts the times when the tipping point is reached ( $R_E = 1$ ) under the emergent simulation, right-truncating the curves.