Spatial Interpolation Notes

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1 Introduction

Information from ARCGIS unless listed otherwise

Kriging is a way to interpolate data and estimate a surface for geospatial data. There are two main ways of doing this:

- Inverse distance weighting (IDW) and spline methods
- Kriging

IDW are deterministic interpolation methods as they are directly based on the surrounding values or smoothed formulas. Kriging is different as it uses autocorrelation and takes position into account in the statistical models. Kriging uses a certain number of neighbouring points, or all points within a specified radius (cf kNN).
The general formula for IDW and kriging is:

$$\hat{Z}(s_0) = \sum_{i=1}^{N} \lambda_i Z(s_i)$$

where: $Z(s_i)$ = the measured value at the ith location
λ_i = an unknown weight for the measured value at the ith location
s_0 = the prediction location
N = the number of measured values

The difference between IDW and kriging is that in IDW, λ_i only depends on distance to prediction location. In kriging λ_i also depends on autocorrelation i.e. spatial relationship between prediction locations.

1.1 Assumptions in kriging

Information for subsection from Columbia

For kriging to be used, there are a number of assumptions/conditions to be met. These conditions can be checked in exploratory data analysis.

1. The data should be stationary
   - Means that the joint probability distribution does not vary across the study space, so the same parameters (e.g. mean, range and sill etc) are valid across the space
   - Means one variogram is valid across the space
2. Assumption of isotropy
   - Uniformity in all directions (semivariance identical in all directions)

2 Creating a prediction map with kriging

There are two steps:

1. Create the variograms and covariance functions to estimate the spatial autocorrelation values that depend on the model of autocorrelation (fitting a model).
2. Predict the unknown values

2.1 Variography (spatial modelling/structural analysis)

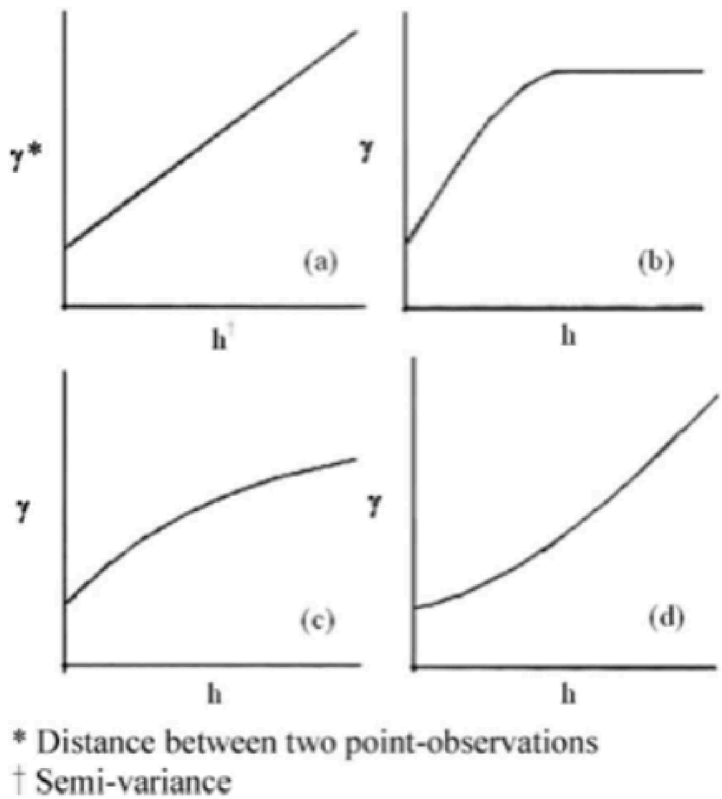
There are often too many pairs of spatial points to calculate and plot the distance for each pair. Instead, spatial distances are put into lag bins i.e. all points in the range $40m < x \leq 50m$ of point A, and calculate the semivariance. The semivariance is equal to half the variance of the differences between all possible points spaced a constant distance apart.

$$\gamma(h) = \frac{1}{2}\left[z(x_i) - z(x_j)\right]^2$$

Plotting the distance vs semivariance produces an empirical semivariogram. Closer items should be more similar, therefore lower semivariance. The opposite is true for further points.
A model is fit to the empirical semivariogram (cf regression). Different types of models can be fit the the semivariogram:

- Spherical (most common)
- Circular
- Exponential
- Gaussian
- Linear

Figure 1: Different types of models used in spatial modelling (Poilou 2008). a) Linear semi-variogram; (b) spherical semi-variogram; (c) exponential semi-variogram; and (d) power semi-variogram



There are a number of key points on the figures:

- Range
  - The Range is the point at which the semivariance first levels off
  - Items within the range are autocorrelated (distance matters)
  - Items outside the range are not autocorrelated (distance no longer changes the semivariance)
- Sill

    - The Sill is the height at which the semivariance levels off to

- Nugget
    - The minimum value of semivariance ($\gamma\,(h = 0)$)
    - Theoretically there is no semivariance when $h = 0$, but in reality it is present due to measurement error or spatial sources of variation at distances smaller than the sample interval (or both)

- Partial Sill
    - Amount of semivariance between Sill and the Nugget

2.2 Predictions

Now a model has been fit to the semivariance and autocorrelation can be observed, predictions can be made within the domain. Kriging differs from IWD as it uses the semivariogram to calculate the weights. There are two methods used in kriging:

1. Ordinary kriging
    - Assumes the constant mean is unknown
2. Universal kriging
    - Assumes there's a prevailing trend
    - Trend is modelled with polynomial function, and subtracted from observed
    - Semivariogram is modelled on the residuals to produce autocorrelations