实验八 曲线拟合

实验目的

1. 掌握曲线拟合的最小二乘法原理
2. 理解超定方程组的最小二乘法原理
3. 通过联系掌握实现最小二乘法曲线拟合的编程技巧

实验环境

1. 计算机
2. MATLAB 集成环境

实验内容与代码

某车间计划加工一批飞机零部件，为了规定工时定额，需要确定加工零件所花费的时间，为此进行了 **10** 次实验，收集数据如下：

$$\left[\begin{bmatrix} x \\ y \end{bmatrix} \quad \begin{bmatrix} 10 & 20 & 30 & 40 & 50 & 60 & 70 & 80 & 90 & 100 \\ 62 & 68 & 75 & 81 & 89 & 95 & 102 & 108 & 115 & 122 \end{bmatrix}\right]$$

1. 画出散点图
2. 用最小二乘法拟合直线 $y = a \cdot x + b$

```
clc;clear;
x = [10 20 30 40 50 60 70 80 90 100]
```

x = 1×10

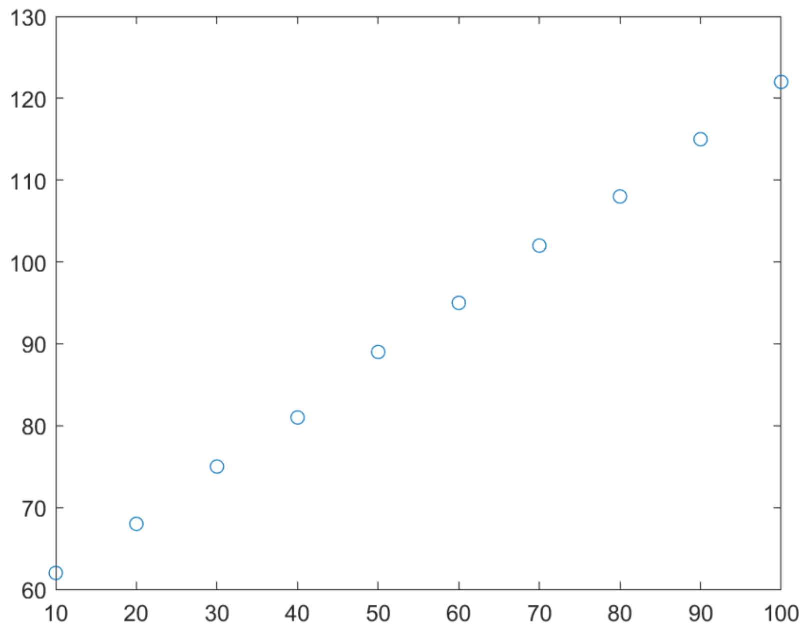
10	20	30	40	50	60	70	80	90	100

```
y = [62 68 75 81 89 95 102 108 115 122]
```

y = 1×10

62	68	75	81	89	95	102	108	115	122

```
plot(x, y, 'o')
```




```
p = polyFit(x, y, 1)
```

```
p = 1×2
     0.6685    54.9333
```

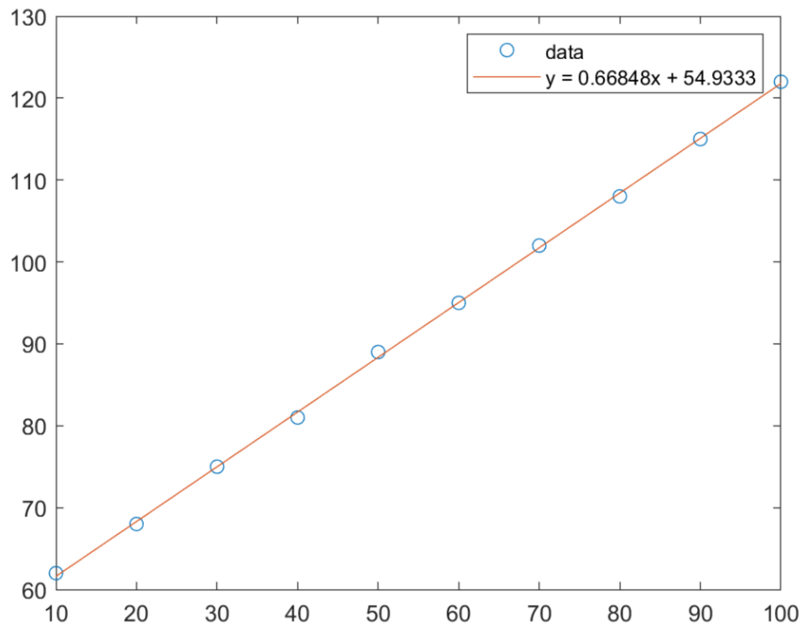
```
res = "y = " + p(1) + "x + " + p(2)
```

```
res = "y = 0.66848x + 54.9333"
```

```
f = polyval(p,x)
```

```
f = 1×10
   61.6182   68.3030   74.9879   81.6727   88.3576   95.0424  101.7273  ⋯
```

```
plot(x, y, 'o', x, f, '-')
legend({'data',res})
```



给定一组观察数据，试用最小二乘法拟合这组数据的多项式

$$\begin{bmatrix} x & 0.0 & 0.50 & 0.60 & 0.70 & 0.80 & 0.90 & 1.00 \\ f(x) & 1.0000 & 1.75 & 1.96 & 2.19 & 2.44 & 2.71 & 3.00 \end{bmatrix}$$

1. 画出拟合数据点的图形
2. 确定用几次的多项次拟合这组数据
3. 求 $f(x)$ 的最小二乘拟合函数

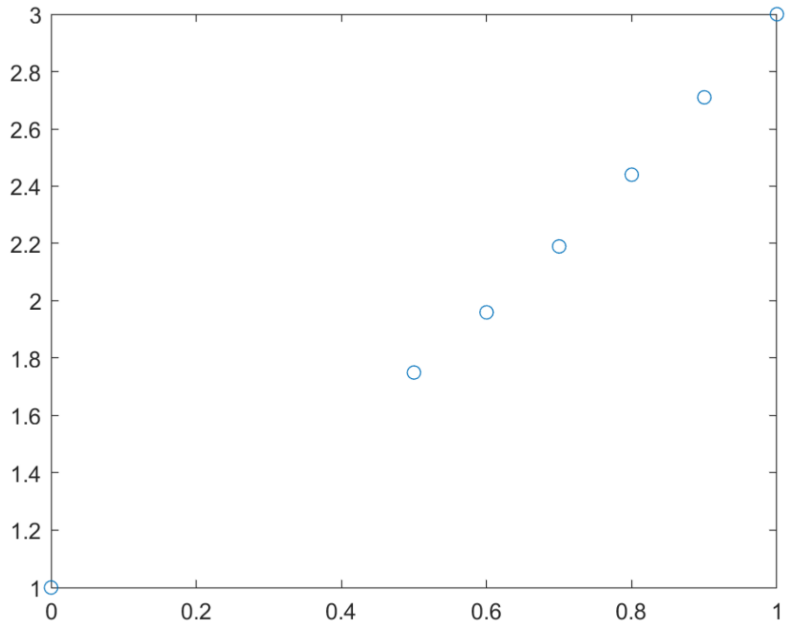
```
clc;clear;
x = [0 0.5 0.6 0.7 0.8 0.9 1]
```

```
x = 1×7
         0    0.5000    0.6000    0.7000    0.8000    0.9000    1.0000
```

```
y = [1 1.75 1.96 2.19 2.44 2.71 3]
```

```
y = 1×7
    1.0000    1.7500    1.9600    2.1900    2.4400    2.7100    3.0000
```

```
plot(x, y, 'o')
```




```
p2 = polyFit(x, y, 2)
```

```
p2 = 1×3
    1.0000    1.0000    1.0000
```

```
p3 = polyFit(x, y, 3)
```

```
p3 = 1×4
    0.0000    1.0000    1.0000    1.0000
```

```
p4 = polyFit(x, y, 4)
```

```
p4 = 1×5
    0.0000   -0.0000    1.0000    1.0000    1.0000
```

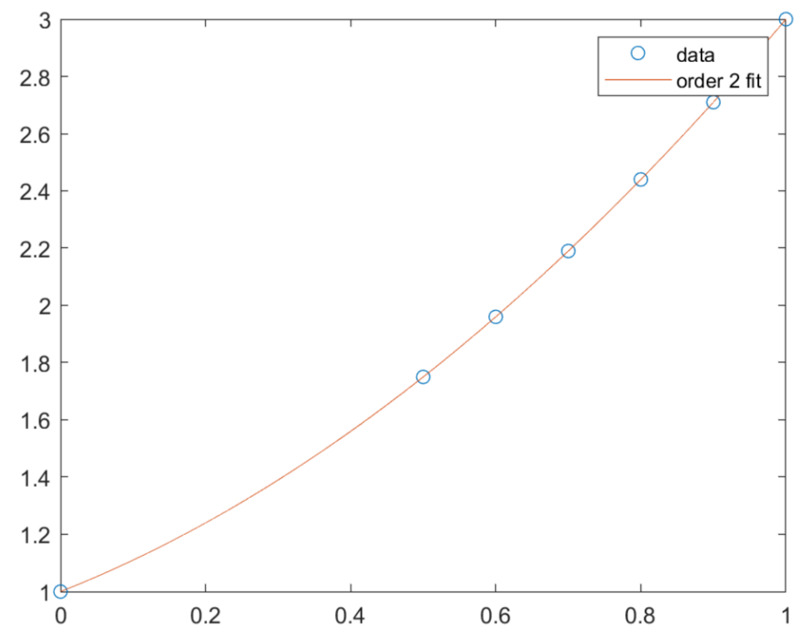
```
x1 = 0:0.01:1
```

```
x1 = 1×101
         0    0.0100    0.0200    0.0300    0.0400    0.0500    0.0600 ⋯
```

```
f = polyval(p2,x1)
```

```
f = 1×101
    1.0000    1.0101    1.0204    1.0309    1.0416    1.0525    1.0636 ⋯
```

```
plot(x, y, 'o', x1, f, '-')
legend({'data','order 2 fit'})
```



由上述结果可知：该函数从 2 次开始拟合结果相同

则可知，拟合函数为： $f(x) = x^2 + x + 1$

此实时脚本中使用的函数:

```
function [p] = polyFit(x,y,n)
    %POLYFIT Fit polynomial to data.
    %   P = POLYFIT(X,Y,N) finds the coefficients of a polynomial P(X) of
    %   degree N that fits the data Y best in a least-squares sense. P is a
    %   row vector of length N+1 containing the polynomial coefficients in
    %   descending powers, P(1)*X^N + P(2)*X^(N-1) +...+ P(N)*X + P(N+1).
    %
    %   Example: simple linear regression with polyfit
    %
    %     % Fit a polynomial p of degree 1 to the (x,y) data:
    %       x = 1:50;
    %       y = -0.3*x + 2*randn(1,50);
    %       p = polyFit(x,y,1);
    %
    %     % Evaluate the fitted polynomial p and plot:
    %       f = polyval(p,x);
    %       plot(x,y,'o',x,f,'-')
    %       legend('data','linear fit')
    %
    %   Class support for inputs X,Y:
    %      float: double, single
    %

    x = x(:);
    y = y(:);

    outputClass = superiorfloat(x,y);

    % Construct the Vandermonde matrix V = [x.^n ... x.^2 x ones(size(x))]
    V(:,n+1) = ones(length(x),1,class(x));
    for j = n:-1:1
        V(:,j) = x.*V(:,j+1);
    end

    % Convert y to the same class as V
    y1 = cast(full(y), class(V));

    % Solve least squares problem p = V\y to get polynomial coefficients p.
    [QRfactor, tau, perm, ~] =
matlab.internal.decomposition.builtin.qrFactor(V, -2);
    % use nonzero diagonal entries to determin rank for qrSolve.
    rV = sum(abs(getDiag(QRfactor)) ~= 0);
    % QR solve with rank = rV.
```

```
    p = matlab.internal.decomposition.builtin.qrSolve(QRfactor, tau, perm, y1,
rV);

    % Get correct output class
    p = cast(p, outputClass);

    p = p.'; % Polynomial coefficients are row vectors by convention.
end

function d = getDiag(X)
    % get diagonal entries of X.
    if isvector(X)
        if isempty(X)
            d = X(:);
        else
            d = X(1);
        end
    else
       d = diag(X);
       d = d(:); %handle diag([])
    end
end
```

实验小结

通过此次实验，掌握了曲线拟合的最小二乘法原理，理解了超定方程组的最小二乘法原理，并通过练习掌握了实现最小二乘法曲线拟合的编程技巧