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Arc routing problems: A review of the past, present, and future

Ángel Corberán[1] | Richard Eglese[2] | Geir Hasle[3] | Isaac Plana[4] | José María Sanchis[5]

[1]Departamento de Estadística e Investigación Operativa, Universidad de Valencia, Valencia, Spain
[2]Department of Management Science, Lancaster University, Lancashire, UK
[3]Department of Mathematics and Cybernetics, SINTEF Digital, Oslo, Norway
[4]Departamento de Matemáticas para la Economía y la Empresa, Universidad de Valencia, Valencia, Spain
[5]Departamento de Matemática Aplicada, Universidad Politécnica de Valencia, Valencia, Spain

Correspondence
Ángel Corberán, Departamento de Estadística e Investigación Operativa, Universidad de Valencia, Valencia, Spain.
Email: angel.corberan@uv.es

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Abstract

Arc routing problems (ARPs) are defined and introduced. Following a brief history of developments in this area of research, different types of ARPs are described that are currently relevant for study. In addition, particular features of ARPs that are important from a theoretical or practical point of view are discussed. A section on applications describes some of the changes that have occurred from early applications of ARP models to the present day and points the way to emerging topics for study. A final section provides information on libraries and instance repositories for ARPs. The review concludes with some perspectives on future research developments and opportunities for emerging applications.

KEYWORDS

applications, arc routing, future, history, state-of-the-art, vehicle routing

1 | INTRODUCTION

Routing problems have attracted the attention of many researchers and practitioners during the last 60 years because of their big economic impact and the mathematical challenges involved in their study and solution. Routing problems can be classified into node routing problems, in which customers can be represented by nodes in a network, and arc routing problems (ARPs), in which the service is performed on the arcs or edges of a network.

Although the research on routing problems has traditionally been focused more on node routing problems, the literature on ARPs is growing every day, and the number and efficiency of algorithms designed for these problems have increased considerably in recent years. Moreover, changes in the business of transportation logistics and technological innovations require new problems to be defined and studied, as well as incorporating new features to already existing problems.

Some examples of technological advances that are relevant include new types of vehicles, radio frequency identification (RFID) technology, or availability of real-time data, such as geolocalization, traffic flows, and communication between customers and drivers. Much early research concentrated on single objective problems where the objective was to minimize costs. There have already been developments to examine other objectives, such as the maximization of profit or to minimize emissions giving rise to greenhouse gas effects. Some approaches attempt to deal with two or more objectives using a multiobjective approach.

This paper aims to give an updated vision of the current state of the art of ARPs and tries to foresee what will be the most relevant topics and research lines in this area in the next years.

Dedicated to the memory of Nicos Christofides (1942-2019).

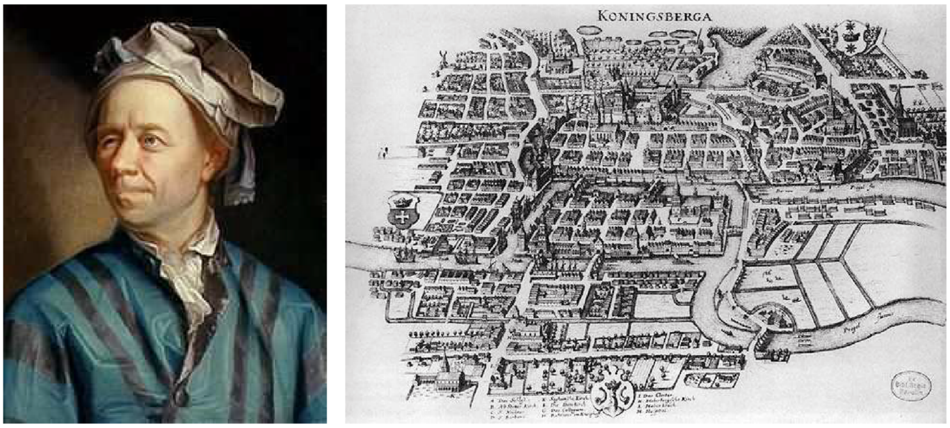

FIGURE 1 Euler and a map of the city of Königsberg [Color figure can be viewed at wileyonlinelibrary.com]

Over 230 references are cited to enable those interested to follow-up particular lines of research and details of existing methods. Of these references, over 23 of them are to articles published in *Networks*, illustrating the important role that this journal has played in the development of research into ARPs.

The contents of this paper are structured as follows. Section 2 gives a general view of the history of ARPs. The areas that we think will be more relevant in the near future are presented in Section 3, while Section 4 deals with several features that are important from the theoretical and practical point of view and can appear in many ARPs. Section 5 describes the most important applications of ARPs nowadays, as well as potential new applications, such as 3D printers, that will need to be studied from an arc routing perspective and tackled using arc routing techniques. Section 6 describes some of the main libraries and instance repositories related to ARPs. Finally some conclusions and perspectives are briefly presented in Section 7.

2 | HISTORY AND CURRENT STATE OF KNOWLEDGE

The origins of arc routing can be traced back to the 18th century when the Swiss mathematician Leonhard Euler studied the Königsberg bridges problem. This was based on the layout of seven bridges over branches of the river Pregel, as it flowed through Könisberg (now known as Kaliningrad in Russia) connecting Kneiphof Island and different areas of land. The problem concerned whether it was possible for anybody to find a closed walk that traversed each of the seven bridges, without traversing any bridge more than once. Euler developed a graph representation of the problem and was able to show that it was impossible to find such a path and generalized his result for other graphs. A history of the bridges of Könisberg can be found in Gribkovskaia et al. [108], while an account of Euler's achievements is provided in Assad [21] (Figure 1).

Euler showed that for a closed walk in an undirected connected graph to exist, all vertices must be of even degree. In this case, the graph is now called Eulerian.

However Euler was not concerned with the distance traveled in any path. It was not until the 1960s that a Chinese mathematician, Meigu Guan (Mei-Ko Kwan) introduced what has now become known as the Chinese postman problem (CPP). Guan [109] studied the plan for a mailman's route. The mailman has a set of street segments assigned to him and the problem is to find the shortest walking distance for the mailman from the post office, to visit each assigned street segment and return to the post office. The CPP can be expressed as the problem of finding a subset of edges with minimum total distance on an undirected connected graph which when added to the original graph produces an Eulerian graph.

The paper by Edmonds and Johnson [81] provided a polynomial algorithm which solves the CPP to optimality. It is based on a procedure for optimally matching the odd vertices of a graph that was developed in Edmonds [80].

If the underlying graph is directed, then Edmonds and Johnson [81] also showed that an optimal solution can be found efficiently by using the transportation algorithm to decide which arcs must be added to obtain a closed walk (tour) of minimum distance that covers each arc in the graph. This was also proposed at about the same time by Orloff [177] and Beltrami and Bodin [28].

Although the undirected and the directed CPP can be solved to optimality in polynomial time, many variations of ARPs are NP-hard so that the size of problem that can be solved to optimality is limited. For example, the mixed CPP, where the underlying graph contains both undirected edges and directed arcs, is NP-hard, as shown by Papadimitriou [183]. Norbert and Picard [173] present an optimal algorithm for the mixed CPP which includes a polynomial-time procedure to identify a violated balanced-set inequality in a mixed graph if one exists.

Another important variation is the rural postman problem (RPP), where some arcs or edges may require service, but others do not, though they are available for traversal in the solution. The objective of the RPP is to find a minimum cost tour that includes all the required arcs and edges. The RPP was introduced by Orloff [177] and was shown to be NP-hard by Lenstra and Rinnooy Kan [138].

In 1981, the capacitated arc routing problem (CARP) was introduced by Golden and Wong [104] where each required edge or arc in a RPP is associated with a demand. In the CARP, a set of routes is to be found for a set of vehicles with limited capacities based at a depot, such that the total demand serviced on each route does not exceed the vehicle capacity and the total cost is minimized. Many exact and heuristic algorithms have been proposed for the CARP since its introduction. Among them, the recent work by Pecin and Uchoa [184] analyzes the best existing exact algorithms for the CARP and proposes a new branch-and-cut-and-price algorithm that solves almost all instances from the classical benchmark CARP sets.

Since the 1980s, more ARPs have been studied, where the extensions have been designed to model practical applications more closely or to try to optimize other objectives. In general, these all represent challenging optimization problems where finding good solutions to problems of the size found in practice is not easy. Some of these extensions will be discussed in the following sections, but introductions into the current state of knowledge can be found in Corberán and Prins [63], Corberán and Laporte [58], and Mourão and Pinto [167].

Examples of these extensions are: the windy RRP, where the cost of traversing an edge depends on the direction of travel [162], the addition of time windows or time deadlines for the completion of service of sets of required arcs or edges [36], an objective relating to maximizing profit [153] and many others.

As different algorithms and approaches have been suggested for ARPs, one general issue has been whether it is better to model the problem as an ARP and to take advantage of the structure of the graph representing a road network where the degree of each vertex representing a junction is usually a small number, or whether it is better to convert the original ARP to a node routing problem with the addition of extra nodes and/or constraints, and then use a suitable node routing algorithm that has already been developed. The first approach is used to exploit sparcity by Letchford and Oukil [140]. An example of the second approach is given by Longo et al. [145].

3 | CURRENT STATE OF KNOWLEDGE AND FUTURE DEVELOPMENTS IN RELEVANT PROBLEMS

In this section, we summarize the state of the art of ARPs that we consider that will be relevant in the near future, namely periodic ARPs (PARPs), ARPs with profits, close enough ARPs, location ARPs, and ARPs with drones. Their importance can be due to their intrinsical difficulty that requires sophisticated mathematical methods to deal with them and their extensions, or to their applications in fields that we think will be the focus of great attention in the future.

3.1 | Periodic Arc Routing Problems

Some real applications may require the repetition of a service in some periods of time (e.g., days) on a time horizon (e.g., weeks or months). If the problem to solve at each period is modeled as an RPP or a CARP, for example, an instance of that problem has to be solved at each period. Since often not all the customers require service every day, the instances corresponding to different periods are different. PARPs arise when the requirement of service for some customers is not associated with some fixed periods but is defined as a given periodicity, or a given number of periods over the time horizon. The aim is to design a set of routes for each period satisfying the service requirements during the time horizon, while minimizing the total cost.

As far as we know, the most general way to define periodicity is the one proposed in Monroy et al. [163] and Benavent et al. [30] for the periodic CARP (PCARP) and periodic RPP (PRPP), respectively, with irregular services. In the PRPP with irregular services (PRPP-IS), for example, each customer (link) is associated with a partition of the set of periods. This partition is formed by a family of subsets of periods and each one of these subsets has associated a frequency (maybe zero) that indicates the number of times the link must be serviced during the days of the subset. For example, for a time horizon defined by the 7 days of a week, a customer could have the partition defined by the three subsets {*Monday*, *Tuesday*, *Wednesday*, *Thursday*}, {*Friday*, *Saturday*}, and {*Sunday*}, with corresponding frequencies 2, 1, and 0. This means that the customer must be serviced twice between Monday and Thursday, once either on Friday or on Saturday, and never on Sunday.

Note that most of the ARPs described in later sections, such as ARPs with profits, close-enough ARPs, ARPs with drones, time-dependent, or hierarchical ARPs, may have to be solved with a certain periodicity.

At first sight, PARPs may be considered as the repetition of a given ARP in each period of the time horizon. Nevertheless, decomposing the problem into an ARP per period and then solving each one of these problems separately does not yield the "global" optimal solution. As the PARP consists of finding this "global" optimal solution over the time horizon, it is significantly more difficult than the corresponding ARP and, hence, it is typically NP-hard.

PARPs appear naturally to deal with repetitive actions such as urban garbage collection, snow plowing, and so on. In fact, some authors [130] claim that periodicity fits reality better. In urban garbage collection, it is necessary to collect the garbage periodically and streets with different rates of garbage accumulation may need to be serviced with different frequencies. In the inspection of power lines, water and gas underground pipelines or road networks, the frequencies of service in the required links may be regularly planned. In the inspection of road networks, periodicity may also appear because the traffic conditions change during the week considerably. In other applications such as collecting or delivering cash, personal protection, patrolling areas, and so on, periodicity may be implied for security reasons.

In what follows, we present some variants and extensions of the PARP. Regarding the number of vehicles, we can consider PARPs with only one vehicle per period or with several vehicles. Each vehicle may have a limited capacity and a maximum travel distance (or time). The vehicle fleet can be homogeneous or heterogeneous and time windows may arise, for example, when it is not recommended to perform a service such as waste collection at rush hour [219].

Biobjective PARPs, minimizing the total routing cost and the number of vehicles, are studied in Mei et al. [159] and Zhang et al. [233], whereas Fröhlich and Dörner [96] consider costs and security as the two objectives to optimize. The security of a solution is associated with route inconsistency, which measures how often links are used within given periods and whether the sequences in which the services take place have similar subsequences.

Riquelme-Rodriguez et al. [196,197] describe the PARP with inventory constraints. Arcs in the network represent customers consuming a commodity (water in this case) over time. When the vehicle arrives, it services the arc by replenishing the commodity up to an inventory level. Huang and Lin [123] generalize this problem with the inclusion of some refill nodes in the network where vehicles make a pit stop to recover their capacity.

Monroy et al. [163] study the PCARP with irregular services (PCARP-IS), where roads are serviced a given number of times in subperiods along the time horizon and according to a hierarchy of link classes. The authors claim that this model is a better fit to applications such as road maintenance and street security surveillance.

As already mentioned, the PARP is much more difficult than the ARP associated with each time period. Still, several exact procedures based on mathematical formulations have been proposed. Benavent et al. [30] present a mathematical model and a branch-and-cut algorithm for the PRPP-IS. Other mathematical formulations and exact procedures are proposed in Monroy et al. [163], for the PCARP-IS, Riquelme-Rodriguez et al. [197] for the PCARP with inventory constraints, Laganà et al. [135] for the periodic multi-vehicle ARP, and Thomaz et al. [219] for the PARP with time windows.

More useful in real applications, as they can address larger instances, are the heuristic algorithms. Mei et al. [159] and Zhang et al. [233] propose two memetic algorithms for the PCARP, Monroy et al. [163] present a cluster-first route-second heuristic for the PCARP-IS and in [196] an adaptive large neighborhood search is proposed for the PCARP with inventory constraints. As in other applications, transformations of the studied ARP into a node routing problem have also been proposed in Huang and Lin [123] and Fröhlich and Dörner [96].

In the near future there will surely be advances in the solution of classic ARPs such as the RPP, the general routing problem (GRP), and the CARP. These methods may be applied to their corresponding periodic versions, either to optimally solve larger instances or, at least, to obtain good lower bounds that can be used to test the quality of the solutions provided by heuristic algorithms. Many periodic versions of well-known ARPs will surely be studied in the near future and applied to real-life applications that cannot be addressed nowadays due to their complexity and the lack of methods that guarantee a solution of good enough quality. The difficulty of these problems will make the use of tailored heuristic algorithms necessary.

3.2 | ARPs with profits

In some applications the number of links to service is so large that it is not possible to service them all in a day because vehicles or time, for example, are not sufficient. Therefore, the goal is not to traverse all the required links (those requiring a service) but only the most "valuable" of them. If a link is serviced, the company obtains a profit, and the more links are serviced, the greater is the profit obtained. Frequently, the company has a set of links that must be serviced and there is a large set of potential customers that may be chosen for service only if it is worthwhile. A penalty cost may also be considered when a customer is not serviced [19].

To illustrate the increasing applicability of the ARPs with profits, we comment on two real-life applications related to leisure and tourism. The first one [213] deals with the design of cycling tours formed by selecting some arcs representing sections of routes. Each arc has an associated score (profit) and the problem is to maximize the total collected score within the defined characteristics of the tour. The second application [99] describes the route planning for tourists who are visiting a destination with several points (nodes) and panoramic views (street segments) of interest. The objective is to design walks that maximize the score collected with a duration below a certain threshold and so that the nodes and edges are visited within its time window.

In ARPs with profits three nonnegative values may be associated with the traversal of each link: a traveling cost, a demand, and a traveling time. In addition, a deadheading cost and deadheading time may be associated with the traversal of a link without servicing it. There is a set of (profitable) links that may be serviced or not, and a nonnegative profit associated with the service of any profitable link. Moreover, there may be a set of links that must be serviced. A single vehicle or fleet vehicles are available at the depot and each vehicle has a given capacity and a limit on the traveling time of its route. In street cleaning, snow plowing, snow salting, or security patrolling, for example, the service of a link can be done several times and with different costs and profits. These are problems with multiple services [61,86,207] where the aim is to decide how many times each link is serviced to maximize the benefit.

Beyond the type of underlying graph (undirected, directed, mixed, or windy), if the problem is a single or a multivehicle problem, and so on, the classification of ARPs with profits depends on the objective function pursued [20,167]:

- Profitable ARPs (P-ARPs), in which the objective is to maximize the net profit, given as the difference between the collected profit and the traversal cost.
- Orienteering ARPs (OARPs), where the objective is to maximize the total collected profit within a limited traveling time for the vehicles.
- Minimum bound collected profit ARPs (MBPARPs), in which the objective is to minimize the total travel cost while collecting a minimum profit threshold. These problems are also known as prize-collecting ARPs.

Among the variants and extensions of ARPs with profits we can mention the windy clustered P-ARP [57], in which the required links are divided into clusters and, if a required link is serviced, all the links in the same cluster must also be serviced. Arbib et al. [12] are the first exploring the integration of facility location issues in a P-ARP. The authors consider that the selection of a profitable link implies the selection of the two facilities located at the end-points of the link, with a fixed installation cost. Fernández et al. [88] introduce the collaboration uncapacitated ARP, where there is a set of carriers and a set of link tasks, each assigned to a carrier, which can be performed by a collaboration of carriers to maximize the total profit.

ARPs with profits are not necessarily harder than their associated ARPs with a fixed set of customers, and several exact procedures based on mathematical formulations have been proposed for their solution.

For single-vehicle problems, Benavent et al. [29] propose flow-based models evaluated by CPLEX for several ARPs with profits on mixed graphs: OARP, P-ARP, and MBPARP, solving to optimality instances with up to 426 nodes and 1012 links, in short computational times. Archetti et al. [16] use large families of facet-inducing inequalities in a branch-and-cut algorithm with the help of a heuristic for the OARP, and many instances with 100-2000 nodes and 7000-14 000 arcs were solved within a few minutes. Colombi and Mansini [55] propose inequalities that are added to an existing model of Archetti et al. [19] for the P-ARP, while Ávila et al. [22] have developed a branch and cut for the windy P-ARP that optimally solves instances with up to 1500 nodes. For the P-ARP with multiple services, Corberán et al. [61] propose a formulation and a branch-and-cut algorithm that solves instances with up to 1000 nodes and 3000 edges. Arbib et al. [12] present an ILP model for the profitable location ARP and a branch-and-cut procedure to solve instances with up to 102 nodes and 200 arcs.

Regarding problems with several vehicles, Archetti et al. [17] presented a branch-and-price algorithm for the undirected Team Orienteering ARP (TOARP) capable of optimally solving instances with up to 50 nodes, 97 profitable edges, and 2-4 vehicles. A directed TOARP is studied in Archetti et al. [14], who propose a branch-and-cut algorithm embedding families of valid and facet-inducing inequalities. Riera-Ledesma and Salazar-González [195] transform the same problem into a node routing problem and describe two exact algorithms, a branch and price and a branch and price and cut, to solve it. Riera-Ledesma and Salazar-González [195] report a kind of complementarity between branch-and-cut and column generation approaches, and point out that in instances with a tight time limit constraint column generation works better.

In Benavent et al. [29], aggregated compact flow-based models for the TOARP, the K-vehicles P-ARP and the K-vehicles MBPARP, are presented. The model for the TOARP was tested on the same set of instances as [14,17], obtaining slightly worse gap values. The procedures for the K-vehicles P-ARP and the K-vehicles MBPARP were tested on two sets of instances with up to 50 and 428 nodes, 156 and 1074 links, and 4-12 and 2-4 vehicles, respectively. As expected, as the number of vehicles increases, the results deteriorate. Archetti et al. [13] propose a two-phase exact algorithm for a profitable GRP, in which, in addition to profitable edges, there are also profitable nodes. First, an aggregated formulation is solved to optimality with a branch-and-cut algorithm. Then, the inequalities found in the first phase are used on a branch-and-cut algorithm for the complete disaggregate formulation.

Among the heuristic algorithms for ARPs with profits with a single vehicle we can mention the ant colony procedure for the windy P-ARP proposed by Schaeffer et al. [206] and the GRASP and path relinking heuristic of Aráoz et al. [10] for the undirected clustered P-RPP. Black et al. [34] propose two metaheuristics for the time-dependent P-ARP, obtaining good quality results. For the same problem, Yu and Lin [226] describe an iterated greedy heuristic that provides the best solutions for most of the instances presented in [34].

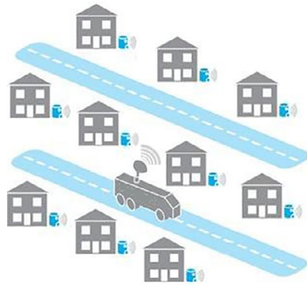

FIGURE 2 Close enough ARPs for meter reading [Color figure can be viewed at wileyonlinelibrary.com]

A number of metaheuristics have been proposed for multivehicle ARPs with profits. For the undirected TOARP, Archetti et al. [17] present a variable neighborhood search (VNS) and two tabu search methods, and Cura [66] proposes a bee colony approach. For the directed TOARP, the algorithm proposed in Archetti et al. [15] successfully combines the solution of integer models with a tabu search. For the multiple profitable arc routing (KP-ARP), Zachariadis and Kiranoudis [227] describe a local search metaheuristic for a biobjective problem. Instead of considering the net profit, they maximize the gross profit and minimize the travel time. For the multiple vehicle GRP with profits and time windows, Gavalas et al. [99] present a formulation, a preprocessing procedure, an iterated local search, and a simulated annealing algorithm. Euchi and Chabchoub [86] present two metaheuristics for the multiple P-ARP with multiple services (KPARPM), both based on an adaptive memory procedure but combined with two different methods: a tabu search and a VNS. They report small gaps, of up to 1%, over the best known solution values on instances with up to 140 nodes.

The study and solution of ARPs with profits will be increasingly useful in the future because their applications involve more and more customers, companies, and services, and have a larger global component. With the exception of multiservice problems, in which the same link can be traversed several times with a different cost and producing a different profit each time, it seems that ARPs with profits are very similar to classic ARPs in complexity and in the way they are solved. Hence, it is very likely that all the advances in the solution of the classic ARPs, that will surely occur in the future, will be directly applied to the solution of the ARPs with profits.

3.3 | Close enough (or generalized) ARPs

The term "close enough" in routing problems refers to situations in which the vehicle is not required to reach the exact point where the customer is located in order to perform its service, since the service can be performed from a certain distance and the vehicle only needs to pass "close enough" to the point where the customer is actually located (Figure 2).

This situation was first described in Gulczynski et al. [111] for a node routing problem, the close enough traveling salesman problem (CETSP). In the CETSP customers are located at points in the plane and the salesman must travel within a required radius r of each point to service the customer. Since then, this problem and some variants where the radius associated with each customer may vary, or the shape of the area around the customer is not a circle, have been studied by several authors: Dong et al. [72], Mennell [161], Shuttleworth et al. [210], Behdani and Smith [25], Coutinho et al. [65], and Carrabs et al. [44]. Also closely related is the covering tour problem studied by Gendreau et al. [100]. These articles deal with node routing problems as in most of them it is assumed that the salesman can move between any pair of points in the plane following a straight line whose cost is the Euclidean distance.

The first paper in which the concept "close enough" is related to an ARP is the one of Shuttleworth et al. [210]. In this paper, although the original problem is a CETSP, the proposed heuristic procedures have a phase that consists in the solution of a directed RPP. But it was Há et al. [114] who first considered that the vehicle is restricted to a road network and the customers are points that can be located on the network links or even outside the network. They called it the close enough arc routing problem (CEARP) because, to service a customer, the vehicle has to pass close enough to the customer while it travels through the network links. Since a customer c is a point on the plane, a customer is fully determined by the set of links H_c whose distance to the point is less than a given value. The goal of the CEARP is to find a shortest route, starting and ending at the depot, such that at least one link from each H_c is traversed. In [114], a formulation for the CEARP and a branch-and-cut algorithm producing very good computational results are proposed. Ávila et al. [23] present two new formulations, a polyhedral study and a branch-and-cut algorithm whose performance is compared with that of the algorithm described in Hà et al. [114]. In Cerrone et al. [47] a new flow-based formulation is given, as well as some techniques to reduce the size of the graph. The results obtained on one of the set of instances proposed in [113] using this new formulation improve those of Hà et al., but are slightly worse than those in [23]. A stochastic version of the CEARP has been studied by Renaud et al. [194]. Authors point out that the remote reading of a meter may fail and therefore there is an uncertainty in the collection of the data. They introduce the probability of reading a meter as a function of the distance of the customer from the vehicle route, and propose a mathematical formulation and a cutting-plane algorithm and several heuristics for its solution.

If in the CEARP we consider the links covering each customer as a subset of the network links, the CEARP is equivalent to the generalized RPP introduced and studied by Drexl in [73,74]. The CEARP is also related to the generalized arc routing problem defined in Araoz et al. [11], but both problems differ in that in the CEARP the subsets of links associated with the service of the customers do not need to be disjoint nor to induce a connected graph.

Only a few articles deal with the CEARP, or variants, with several vehicles. The first multi-vehicle version of a CEARP was the distance constrained CEARP (DC-CEARP) introduced by Ávila et al. [24] and also studied in Corberán et al. [60]. Several formulations and valid inequalities have been proposed in both papers, as well as exact solution methods producing good computational results. A matheuristic for the same problem is described in [59]. Bianchessi et al. [32] study the min-max version of the multi-vehicle CEARP (MM-CEARP). The aim of these problems is to find a set of K routes, starting and ending at the depot, such that at least one link from each H_c is traversed by at least one route. In the DC-CEARP, the length of each route must not exceed a given limit while the total distance is minimum. In the MM-CEARP, the objective is to minimize the length of longest route in order to find a set of balanced routes.

A natural application of CEARPs is on the design of routes for meter readers that use electronic devices with RFID technology, allowing utility meters to be read from a distance under a threshold. Eglese et al. [83] present a very interesting survey of meter reading applications, including the historical evolution of the associated problems and the proposed solution approaches. Other potential applications are mentioned in Araoz et al. [11] and are related to quality control for networks maintenance, where only a small subset of the links of a network has to be traversed.

Recent real-life problems involving "close enough" features are related to routing with drones. In surveillance and maintenance tasks, for example, the drone does not need to pass exactly over the nodes or lines to be monitored. It is enough that the drone approaches the target less than a certain distance that depends on the technical characteristics of the device. As, in general, drones are not constrained to fly over the links but can fly freely between any two points in the plane, these applications would be modeled as node routing problems. However, as the use of drones becomes widespread, it is increasingly frequent for drone flights to be restricted to certain flight corridors. In these cases, the applications are modeled as ARPs. As an illustration, a combination of CEARP and CETSP is presented in [202] to model some real-world applications related to drone routing. The authors distinguish between the free flight zone and the constrained flight zone, where the flight is constrained to specific moving corridors. See Section 3.5 for more details about this interesting area offering a large number of research and applications opportunities for the future.

3.4 | Location arc routing problems

Location arc routing problems (LARPs) is the name for a family of problems involving decisions related to the location of facilities and the design of routes for vehicles based at these facilities (depots) that service some or all the links (arcs or edges) of a graph.

Like other ARPs, LARPs can be defined on undirected, directed, or mixed graphs. They can consider capacities for the facilities and fixed costs for opening a facility or an upper bound on the number of open facilities. Vehicles can perform one or several trips, can have the same characteristics or not, and there may be a fixed cost per vehicle used. Moreover, different objectives can be defined as, for example, to minimize the total cost, or the length of the longest route, or maximize the sum of collected profits associated with the service of the customers.

Since most location and ARPs are NP-hard, the combined problems are also NP-hard and more complex and difficult to solve that those problems separately. This may explain the small number of papers devoted to these problems and that most of them deal with the design and implementation of heuristic or metaheuristic algorithms for LARP variants or related problems. Among them are the papers by Ghiani et al. [101], Del-Pia and Filippi [68], Hashemi et al. [118], Borges-Lopes et al. [37], and Riquelme-Rodríguez et al. [198]. Recent articles summarizing the work done on LARPs are those by Albareda-Sambola [5] and Mourão and Pinto [167]. Although focused on location-node routing, the paper by Nagy and Salhi [172] is a very good discussion of many aspects related to this subject.

As far as we know, the most recent and successful exact methods for several classes of LARPs are proposed in Fernández et al. [90]. In that paper, the authors model and solve six problems defined on undirected graphs with different objective functions (minimizing the total routing cost or the makespan) and characteristics (capacities for the facilities or an upper bound on the number of open facilities). Although edge capacities are not considered in these models, some of them contain an upper bound on the number of edges that can be serviced from an open facility. For the six studied problems, Fernández et al. [90] present ILP formulations and branch-and-cut algorithms providing very good computational results.

Borges-Lopes et al. [37] applied LARP to the following LARP with capacities on facilities and vehicles. Their definition, the arc routing counterpart (defined on a directed graph) to the node routing version (defined on an undirected graph) given in

Prodhon and Prins [191], is as follows. Consider a directed graph, a subset of vertices representing the set of potential depot locations and a set of required arcs (customers). Each potential depot has associated an opening cost and a capacity, while there is a demand and a cost associated with the service of the required arcs. A fleet of vehicles with the same capacity is available, and there is a fixed cost per vehicle used. The problem consists of deciding which depots to open, assigning the required arcs to the depots, and building the routes for each depot in such a way that the total cost is minimized and: (i) the total demand of customers assigned to one depot does not exceed its capacity; (ii) each route begins and ends at the same depot; (iii) each vehicle performs at most one trip; (iv) each customer is served by a single vehicle; and (v) the total demand of customers serviced by a vehicle satisfies vehicle capacity. Since, as said before, there is no a single LARP but many problems involving location and routing decisions, we think it is better to consider the previous problem, defined on an undirected, directed or mixed graph, as a base capacitated LARP.

The above capacitated LARP, although "simple" if we think in terms of real-world applications, is a very difficult problem for which no exact method has been proposed so far. Even the existing approximate procedures do not consider all the characteristics and possible extensions of these problems. Note that, as pointed out in [5], the subproblem obtained when the set of depots to open is fixed is a CARP with multiple depots (MDCARP), which is also a NP-hard problem recently studied by Hu et al. [121] and Krushinsky and Van Woensel [131]. The uncapacitated case, the multiple depot RPP has been considered in Fernández and Rodríguez-Pereira [91], Chen et al. [49], and Fernández et al. [88,89].

In the next years we may expect some progress in the exact solution of the CARP on undirected and mixed graphs mainly. This will lead to the development of exact methods capable of solving small/medium instances of MDCARPs and capacitated LARPs or, at least, the ability to provide good lower bounds by possibly using aggregate or set covering/packing formulations such as the ones summarized in Belenguer et al. [26]. It is through the use of metaheuristics/matheuristics by which we think that a good number of variants or generalizations are going to be addressed successfully. These models will represent better the problems of this type that we find in practice. In particular, we expect some progress in more complex problems that, besides the conditions on the number, type and costs of the vehicles, the number of trips per vehicle, and so on, take into account the stochasticity in the demands or travel times, consider a planning horizon divided into several periods, deal with several objectives simultaneously or consider also inventory decisions, to mention just a few.

3.5 | ARPs with drones

As pointed out by many authors, the use of aerial, ground, and maritime drones provides new opportunities for improving the service in routing applications because of their higher speeds, reduced costs, and safety improvements. The idea in drone ARPs is to use drones to optimize the coverage or service (imaging, inspection, surveillance, etc.) of certain edges in a pipeline, street, railroad, or power lines network.

Usually, drone delivery problems are modeled as extensions of well-known node routing problems and have been extensively studied in the last few years (see, e.g., the excellent survey by Otto et al. [179]). However, the extensions of classical ARPs when drones are used instead of ground vehicles have received little attention, possibly because of their greater difficulty. Some papers dealing with aerial or ground drones and ARPs are those by Easton and Burdisck [79], Sipahioglu et al. [212], Oh et al. [175,176], Dille and Singh [71], Yazici et al. [225], Chow [53], and Li et al. [142]. A brief description of them can be found in Campbell et al. [40].

In routing problems, the streets, roads, or railway lines are represented by means of arcs or edges of a graph and their real shape is usually ignored. The reason is that vehicles have to traverse the edges (or arcs) from one of their vertices to the other one. While this does not seem to be a big problem in node routing because the service has to be performed in the vertices, it can represent a serious limitation in arc routing when drones are used to perform the service of the edges of the graph. This is because the drones may travel directly between any two points in the plane and, therefore, do not need to follow the edges of the graph and may start their service at any point of them.

As the example in Figure 3 (taken from [40]) shows, the shape of the lines to service must be taken into account in ARPs with drones. This, and the fact that drones may start and end the service of an edge at any point of it, make a big difference with respect to the way of modeling and solving classical ARPs. In fact, Drone ARPs are continuous optimization problems with an infinite and noncountable number of feasible solutions. In [40] this drawback is circumvented by approximating the curved lines by means of polygonal chains and allowing the drones to enter and leave each curve only at the points of the polygonal chain. The authors substitute each curve by their endpoints and a set of segments defined by some intermediate points. Associated with each segment there is a service cost such that the sum of the costs associated with these segments coincides with the service cost of the curve. Hence, the segments define the set of required edges of the transformed graph and the set of nonrequired edges defines a complete graph with costs given by the Euclidean distances.

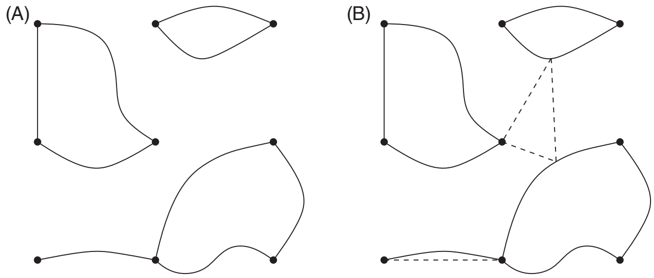

FIGURE 3 A drone ARP instance (A) and its optimal solution (B)

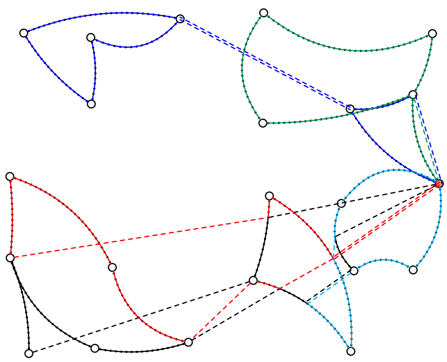

FIGURE 4 Drone ARP solution with five vehicles [Color figure can be viewed at wileyonlinelibrary.com]

The drone RPP is introduced in Campbell et al. [40]. Given a set of lines, each one with an associated service cost, and a point called the depot, and assuming that the cost of deadheading between any two points is the Euclidean distance, the drone RPP consists of finding a minimum cost tour starting and ending at the depot and traversing all the given lines. For this problem, authors propose an algorithm that exactly solves several RPP instances with an increasing number of intermediate points. The algorithm is capable of solving instances with up to 92 original lines and 16 connected components.

The extension of the drone RPP to the case where K drones are available and the length of their routes is limited by a maximum distance is also introduced and briefly discussed in [40]. Authors point out that in the length constrained K-drones rural postman problem (LC K-DRPP) two or more drones may be necessary to serve a given customer (edge) and that, unlike what happens with the split delivery vehicle routing problem, two drones can share the service of more than one edge, as can be seen in Figure 4, where red and black drones, for example, share the service of two edges. Note also that an edge can be serviced by more than two drones, as happens with the red, black and cyan drones. A matheuristic for the solution of the LC K-DRPP is proposed in [41]. The procedure includes an optimization procedure that exactly solves RPP instances associated with each single route. The algorithm has been tested on instances with up to 137 original lines and with 2-6 drones.

An ILP formulation of the LC K-DRPP and a polyhedral study of its solutions are presented in [42]. Based on the introduced families of valid inequalities, and in the use of the upper bounds provided by the above commented matheuristic, the authors propose a branch-and-cut algorithm that produces good computational results.

From the previous comments, the reader will realize that only a few ARPs with drones have been studied so far. There are many aspects that are worth being studied both for their practical applications and for their theoretical interest. For example, servicing arcs (for imaging or inspection) and nodes (for delivery) simultaneously, considering recharge or battery replacement points for the drones ([4]), several depots, capacities for the drones, combining trucks and drones [144], and so on. In general, all the classical ARPs have their associated (and different) versions for drones that will need to be studied and solved in the near future. Furthermore, the possibility of drones of servicing areas close enough to their trajectories makes modeling the service of a given surface (e.g., by inspecting it) by means of straight or curved lines another field of research of great interest. Note that this also makes the study of combined problems of the type "close enough" (see Section 3) and ARPs with drones an important and attractive research subject. The recent papers by Oruc and Kara [178], Singgih et al. [211], and di Placido et al. [202] already consider this possibility. And we should not forget that the advances in the design and manufacture of new types of drones will cause new problems to arise that will have to be addressed.

ARPs with a single drone are also related to the problems of designing paths for cutting machines or drawing plotters as will be discussed in Section 5.1.7.

4 | FUTURE DEVELOPMENTS IN HANDLING DIFFICULT/NEW/RELEVANT ASSUMPTIONS

While Section 3 is devoted to the discussion of ARPs that we consider relevant in the near future, in this section we present several features that can be common to many of these ARPs and are important from both the theoretical and practical point of view. Some applications require the services to be performed within a certain time window. In other applications, the data can change in a dynamic fashion or there can be some uncertainty in the data. Considering several commodities or time-dependent costs are very common features in most real-life problems.

4.1 | Time windows

In ARPs with time windows, the service performed on an edge must begin within a certain time interval. If the service cannot be performed outside of this interval, it is called a hard time window. Most of the ARPs treated in literature consider this type of time window. When the service can be carried out outside of this interval but with a greater cost, it is called a soft time window.

Most of the works that deal with time windows on ARPs focus on the CARP or variants of it [1,82,103,110,128,133,168,223]. Only a few more recent papers deal with single vehicle problems like the CPP [9,221], or the RPP [164,165]) A variant of the Windy RPP including zigzag service has also been addressed in Lum et al. [150] and Nossack et al. [174]. Several articles consider particular real-life applications of ARPs with time windows, such as mobile mapping [220], city-courier and scheduling problems [48], railroad maintenance [136,180], and snow removal [193].

As for the solution methods proposed, the ones most commonly used are column generation [1,103,128,136,193] and metaheuristics such as iterated local search [220], scatter search [48], ALNS [165], simulated annealing [82], and GRASP with path relinking [133]. Heuristics have also been proposed in [128,150,168,223], while only few papers propose other methods like cutting-plane [164], branch-and-cut [174], or matheuristics [136].

Many of the cited papers transform the problem into a node routing problem before attempting to solve it. In particular, very few attempts have been made at solving ARPs with time windows exactly using an arc routing formulation (only [9]). In the future, we may see more efforts trying to find efficient arc routing formulations, at least for single vehicle ARPs with time windows, since this would also be of help for solving multi-vehicle problems like the CARP with time windows. There are other variants of ARPs with time windows that have not been studied yet, like the OARP or the TOARP, so this could also be a line for future research.

4.2 | Dynamic changes

In deterministic ARPs all the data associated with the instance of the problem being solved are known in advance. However, unexpected events that modify the instance during the implementation of the solution may occur. Such disruptions include vehicle breakdowns, blocked roads, changes in the costs or the demands and more. These events may cause the current solution to become suboptimal or make it impossible to carry out. In such cases, the routes must be redesigned in order to adapt to the new situation.

Several different types of disruptions have been considered recently in the literature. The appearance of new tasks has been addressed by Archetti et al. [18], Liu et al. [143], Padungwech [181], and Padungwech et al. [182]. The possible cancelation of tasks has also been studied in [18,143]. Liu et al. [143], Monroy-Licht et al. [166], and Yacizi et al. [225] take into consideration the breakdown of vehicles. Some edges may be blocked unexpectedly, for example, because of road works or accidents, and then the corresponding edge should disappear from the graph. This is considered in Liu et al. [143], Mei et al. [160], and Yacizi et al. [225]. If the vehicles have a limited capacity, sudden variations in the demand of the customers with respect to the initial data can render the solution infeasible. This type of disruption is considered in [143,160]. Finally, the cost/length/traversal time of the edges may change over time, as taken into account by Liu et al. [143] and Tagmouti et al. [217].

The usual approach for dealing with disruptions consists of applying heuristics [18,160,166,181,225] or metaheuristics (variable neighborhood descent in [225] and memetic algorithms in [143]) to reoptimize the route. These heuristics may be applied every time a disruption takes place or once every time a certain number of occurrences have taken place. Obtaining a new solution that can overcome the disruptions in a short computing time is critical in these situations, since vehicles are already on the road and a delay in the response may have negative consequences for the service costs and customer satisfaction. For this reason, exact methods are not usually applied for this type of problem. However, the increase in performance of current computers may make the use of other approaches, such as matheuristic algorithms, possible. Furthermore, new technologies can nowadays provide nearly instant updates on road conditions and travel times, so we can expect new works that consider this kind of dynamic information. Moreover, most of the works on dynamic ARPs focus on one or two types of disruptions only, the only exception being Liu et al. [143], in which several different types of events are considered. Future works could develop solution methods that can deal with multiple types of events and dynamic changes simultaneously.

4.3 | Stochastic data

In real-world applications of ARPs, it is not uncommon that some of the data are not exactly known in advance. When trying to incorporate this feature in the solution of the problems, this can be approached in two different ways. One is treating the data as deterministic and designing procedures that can adapt the solutions whenever unexpected changes in the data appear (see Section 4.2). The other approach consists of considering that the data follow a certain probability distribution. The solution procedures take this uncertainty into account and try to find solutions that are as robust as possible.

There are many works on the CARP that consider uncertainty in the demands of the arcs. Fleury et al. [92] tackle this problem using an evolutionary algorithm. A branch-and-price method is presented by Christiansen et al. [54], while Laporte et al. [137] propose an ALNS procedure. Beraldi et al. [31] present an integer formulation and solve the problem using a branch-and-cut algorithm and a heuristic. Monte-Carlo simulation and a randomized version of the saving heuristic are proposed by González-Martín et al. [106]. In Chen et al. [52] it is not the size of the demand that is considered stochastic, but its existence.

Some other papers deal with uncertainty in both demands and travel times or costs. This is the case of Mei et al. [157], who test existing metaheuristics for the CARP with this version of the problem and compare them using a repair-based robustness measure. An evolutionary method is developed by Mei et al. [160], while Liu et al. [143] propose a genetic algorithm.

Recently, uncertainty has also been considered within other contexts in the arc routing area. Renaud et al. [194] study the close enough ARP but consider that the reading of the meters can fail with a certain probability. They propose a cutting-plane procedure to solve the problem, as well as some heuristics. Majumder et al. [152] deal with a biobjective CPP in which profit is maximized while travel times are minimized. Both profits and travel times are considered to be stochastic. The CARP with uncertain demands is also addressed by MacLachlan et al. [151], but in this case vehicles can collaborate to handle the negative effects of uncertainty. It seems that, even if uncertainty has been extensively studied for the CARP, only some recent papers have taken into account this feature in other ARPs, so this is something that could be addressed in future works.

4.4 | Time and load-dependent costs

Many works on ARPs assume the traversal cost of the arcs to be constant, but this is usually not true in many real-life applications. Traversal times can change during the day due to traffic or weather conditions, for example.

Time-dependent cost functions have been used in some works since the early 00s. It is usually considered that a time-dependent cost function should satisfy the first in first out rule, that is, if two vehicles reach the beginning of an arc at different times, the first one that arrives will always be the first one to finish traversing this arc. Ichoua et al. [125] introduced a model for time-dependent travel times in which they divide the planning horizon in several intervals and consider that the speed of the vehicle changes when the boundary between two consecutive intervals is crossed. Although this model was initially proposed in the context of VRPs, it has also been widely used for ARPs. Another way of representing time-dependent costs consists of constructing a road timetable as proposed by Eglese et al. [84].

Several different ARPs using time-dependent functions have been studied, such as the CPP [214], the directed CPP [221], the RPP [39,218,228], the CARP [155,215-217], the prize-collecting ARP [34,226], and the hierarchical CPP [46].

In recent years, advances in technology have made it easier to obtain more accurate estimations of travel times. In order to incorporate this huge amount of information to the problems, new ways of modeling time-dependent functions and algorithms capable of dealing with huge amounts of information will be needed. Data updates about traffic and travel times in real time can also suggest to use this information in a dynamic fashion (see Section 4.2).

In addition to considering time-dependent travel times or costs, there are other data that can also vary with time. Lu et al. [147] study an orienteering problem in which not only the travel times are time-dependent, but also the profits associated with the arcs. In Ahabchanu et al. [2], the mixed capacitated GRP with time-dependent demands is considered. Nossack et al. [174] present the windy RPP with time-dependent zigzag option. In this problem, both sides of the streets of a network must be cleaned. To do so, the vehicle can traverse the street twice, once in each direction, or only once performing a zigzag. However, the zigzag service can only be performed during certain hours.

Another factor that can affect the cost of the routes is the load of the vehicle. The higher the load, the greater the fuel consumption, and as a consequence, the polluting emissions. Therefore, the cost of traversing an arc depends on which arcs have been serviced previously by the route. This is addressed in the CPP with load-dependent costs (CPP-LC), introduced by Corberán et al. [56], and in the cumulative CARP, studied by Rivera and Lenis [199]. In [56] it is proved that the CPP-LC is strongly NP-hard even when it is defined on a weighted tree. For this problem, the authors provide two formulations, one based on a pure arc-routing representation, and a second one based on a transformation of the problem into a node-routing problem, but they could only solve very small instances optimally in a reasonable time. Hence, ARPs with load-dependent costs are very difficult problems that may receive greater attention in the near future.

4.5 | Split service

Applications where the service on an arc can be shared by two different vehicles are not so frequent as those of split delivery in the node routing context. The first of such applications is introduced by Mullaseril et al. [169], where the authors define the split delivery capacitated rural postman problem with time windows applied to the planning of the feed distribution to the cattle in a livestock ranch, although it should really be considered a split delivery CARP (SD-CARP) with Time Windows. This same problem was later studied by Gueguen [110], who transforms it into a node routing problem and solves it using column generation.

The SD-CARP is studied by Labadi et al. [132], who solve it using a memetic algorithm, and Belenguer et al. [27], who discuss its application to waste collection, propose a cutting-plane algorithm to obtain lower bounds, and propose an evolutionary local search method reinforced by a multistart procedure and a variable neighborhood descent.

There still seems to be room for improvement in the solution of the SD-CARP with or without time windows. No recent papers have tried sophisticated metaheuristics other than [27,132]. Only two formulations have been proposed for these problems, one based on a node routing transformation [110] and an incomplete one that is used only to obtain lower bounds [27]. A new complete pure arc routing formulation could help to solve instances of moderate size using exact methods.

More recently, split service has been considered in problems with several commodities by Muyldermans and Pang [171] and Zbib and Laporte [230]. Here a fleet of vehicles with several compartments must collect several different types of waste at each arc. Two or more vehicles can collect waste from the same arc, although all the waste of a single type in the arc must be collected by the same vehicle.

Another problem that can be seen as a split service problem is the K-drones arc routing problem (K-DARP). In the K-DARP, a set of required arcs must be traversed by a fleet of drones. Since drones can leave or enter any arc at any point, part of an arc can be traversed, and thus serviced, by a drone, while the other part is traversed and serviced by another one. This problem has been recently addressed by Campbell et al. [41].

The study of both multi-commodity problems and drone ARPs has only started in recent years, so more works on these subjects are expected in the near future (see Sections 35. and 4.7).

4.6 | Handling different objectives

Typical objectives in ARPs include, for example, minimizing the total cost/length/duration of the routes, the makespan, the number of vehicles used or maximizing the profit collected. However, many real applications require solutions that achieve a balance between two or more of these objectives.

Since it is usually not possible to optimize both objectives simultaneously, the most common approach consists of obtaining a set of nondominated solutions defining the Pareto frontier. To do this, many authors use epsilon-constraint methods. In these procedures, only one objective is optimized, while a minimum or maximum level of epsilon is set for the other objectives by means of a constraint. By changing the value of epsilon, different nondominated solutions can be obtained. Another popular procedure consists of adding all the objectives in the objective function multiplying each one by a certain weight. Different solutions can be obtained by changing the weights assigned to each objective.

In addition to minimizing the total cost of the route, most papers in the literature also try to balance the length of the routes. This is mostly done by minimizing the makespan, as in Fleury et al. [93], Lacomme et al. [134], Mei et al. [158], Grandinetti et al. [107], Shang et al. [208,209], Mandal et al. [154], and Amini et al. [8]. Other functions to achieve the balancing of the routes are also explored in Halvorsen-Weare and Savelsbergh [115], where the difference between the longest and shortest routes, the sum of the differences between the distance of each route and the average distance or the sum of the differences between the distance of each route and a desired target distance are also considered.

Recently, other objectives other than the balancing of routes have also been considered. Zachariadis and Kiranoudis [227] maximize the gross profit collected while minimizing the total travel time. Huber [124] considers minimizing the sum of delivery times in order to try to perform the services as soon as possible. Corberán et al. [64] and Lum et al. [148] incorporate different aesthetic measures to make the routes visually appealing in addition to minimizing the makespan. A nonlinear dispersion metric that tries to keep vehicles as far from each other as possible is also observed by Dhein et al. [70], while Fröhlich and Dörner [96] study the route inconsistency in order to construct routes that are as different from each other as possible. All these alternative objectives have been studied only recently, and will probably receive more attention in the near future.

As for the solution methods most commonly used for multiobjective ARPs, different variants of evolutionary algorithms seem to be the most frequent ones [8,70,93,134,154,158,208,209]. Other heuristic and metaheuristic methods are proposed in [96,107,148,227]. Very few works consider the use of exact procedures [64], so this could be another future line of research.

4.7 | Handling several commodities

Multi-commodity ARPs have been mainly addressed in the context of waste collection. There are different types of waste, which can be collected by vehicles equipped with several different compartments. This family of problems was first introduced by Muyldermans and Pang [171], who propose the multi-compartment capacitated arc routing problem (MCCARP). They solve it by means of a local search heuristic and compare the results with those obtained using vehicles with just one compartment. In this version of the problem, each arc has certain demand for each type of commodity. While the demand of a single commodity in an arc must be serviced by one single vehicle, several vehicles can service the same arc as long as they collect different types of waste. This problem has later been called the commodity split MCCARP (CS-MCCARP) in literature, while the problem in which the same vehicle must collect all the types of waste of the arc is called the no split MCCARP (NS-MCCARP).

Zbib and Laporte [230] study the CS-MCCARP and solve it using a three phase heuristic. The NS-MCCARP has been studied by Eydi and Javazi [87], Zbib [229], and Zbib and Wøhlk [232]. In [87] a variant of the NS-MCCARP is described in which two objectives (total cost and number of vehicles) are minimized and the demand on the arcs is uncertain. The authors present a multiobjective genetic algorithm that uses the Pareto ranking technique hybridized with stochastic simulation. Several different variants of a path-scanning construction heuristic are proposed and compared in [229], while a multi-move descent algorithm is presented in [232].

Kiilerich and Wøhlk [130] consider several variants of the MCCARP, including the above-mentioned CS- and NS-MCCARP, as well as the multi-day CS-MCCARP and the coordinated CARP. Formulations for these problems are proposed and a set of large-size instances based on real data is presented. Wøhlk and Laporte [224] also study the coordinated CARP. In this problem, each vehicle can collect only one type of waste, and the routes of the vehicle must be coordinated so that each arc is serviced with a certain frequency. A multi-phase districting-based heuristic is described.

The literature on ARPs with several commodities is scarce and most of the references are quite recent, so we can expect further works on these problems in the near future. In particular, although some formulations have been proposed, no attempt to solve these problems exactly or to obtain good lower bounds has been made to date, so this could be a promising line of research.

5 | APPLICATIONS

Applications of the large family of vehicle routing problems (VRPs) abound in society. The development of models and solution methods for variants of the VRP has been tremendous since the start in 1950.

In the literature, there is a fairly clear dichotomy between node routing and ARPs. It might seem that the categorization is straightforward. VRPs are defined on graphs. If the application is such that all service is located on nodes, it is a case of node routing. Conversely, if the service is located on the links, it is an ARP. In practice, it is not so simple.

One may argue that there are applications that are clear cases of arc routing. Street cleaning, winter gritting, and snow clearing are examples of applications where segments of a street network must be serviced. It would seem natural to model such applications as an ARP where the abstract graph representation is derived more or less directly from the concrete street network with only minor modifications.

However, as described in Section 2, an ARP may be transformed to a node routing VRP, thus making it solvable by node routing algorithms. Conversely, a node routing problem may be transformed into an ARP. A main motivation for the latter type of transformation is abstraction. For household applications like a postal service, newspaper delivery, and municipal collection of waste, the basic demand is located in points, so a node routing model would seem appropriate. However, the sheer size of such node routing instances, and in fact all types of node routing instances, may call for abstraction through demand aggregation, thus transforming them to ARP instances. Another motivation is that, if the service is done with trucks, for example, once it enters a street segment, the truck cannot turn around and must traverse it completely to the next street intersection. Hence, in this case, the demand can be associated with the street segment.

Particularly in dense urban areas, one may assume that aggregating demand per street segment for household type of delivery or pickup will not introduce large errors. This is the background for the fact that historically, applications with demand that is fundamentally point based such as a postal service, newspaper delivery, and waste collection are presented as typical cases of the ARP in the literature. For less dense rural and combined areas, simple aggregation of demand per street segment may be crude. More sophisticated aggregation heuristics may leave many demand points unaggregated for a given instance, thus forming an instance of some variant of the GRP that combines node and arc routing.

5.1 | Applications where service is applied to arcs

The applications in this section are all ones where a service is being applied along the edge or arc in the graph used to model the application. In most applications, the edge or arc represents a section of a road network. In these cases, an arc routing model is the most natural and appropriate way to model the activity.

5.1.1 | Street cleaning

Street cleaning operations include road sweeping where roadside kerbs need to be swept and cleaned from detritus. This is usually performed by a special vehicle equipped with a rotating brush on one side that can move along the roadside and sweep material into a container on the vehicle that can later be emptied at a waste site. A key modeling consideration is that if the vehicle only has a brush on one side, then each road section must be treated by the vehicle passing once in each direction of travel. A paper by Eglese and Murdock [85] describes a heuristic for road sweeping in a rural area using a model that depends on this feature of the formulation.

Various constraints may be relevant in different applications that require a more complicated model than a RPP. For example, several vehicles may be needed to cover the roads to be swept and these vehicles may have limited capacities so that the length of road each vehicle treats is limited before it must travel to a site to empty the waste. In some urban areas, time window constraints may be relevant. Bodin and Kursh [36] describe a street sweeping application where some streets in New York city can only be swept when there are parking restrictions preventing parked cars from obstructing the sweeper. More recently, Blazquez et al. [35] give an application to street sweeping where the sweeper routes must also take account of additional constraints such as prohibitions for some types of turns.

The objective is normally related to minimizing the cost of the operation which is closely related to amount of deadheading needed. The length of roadside kerb swept before the sweeper is full may vary according to the type of road, the time of year and prevailing weather conditions, so this is a particularly challenging feature to model effectively.

5.1.2 | Salt spreading

This operation is part of winter road network maintenance activities in many countries. It is sometimes referred to as winter gritting. It may be combined with snow removal in some circumstances, which is covered in the next subsection. Surveys and more detailed descriptions of the operation and models used can be found in Perrier et al. [185,188] and Eglese et al. [83]. The key operation is to spread a de-icing agent, such as salt, on to the surface of roads to prevent ice forming and so making the roads dangerously slippery for traffic. Most vehicles used for this operation can spray the salt on to both sides of the carriageway if there is no central barrier, so it is possible to model some roads as edges that can be treated in any direction. Other roads with one-way restrictions or with central reservations need to be modeled as arcs.

The objective is normally to plan the routes of the vehicles carrying out the operation as efficiently as possible so that the total distance traveled is minimized. This means minimizing the deadheading, the distance traveled when the vehicle needs to travel along an edge or arc without treating it. If roads are treated too early before the temperature falls, then the salt may be blown or washed away. If roads are treated too late, then accidents may occur when vehicles slide on ice that has formed on untreated sections of road. This means that time constraints are normally important additional constraints to add to the basic arc routing model. Roads are often divided into different categories depending on their importance, so roads in higher categories should usually be treated before those in lower categories, when a decision is made to treat roads in an area. Instead of a strict priority system of classification, roads may be divided into deadline classes where roads in higher categories must be treated within shorter times from the start of the operation. This gives the option for roads in lower categories to be treated within the deadline of a higher category if it is efficient to do so [139].

Traditionally, salt spreading routes have been planned in advance for a set of roads for which the operator is responsible. However, thermal mapping combined with on-line sensors opens up the opportunity of a more flexible dynamic approach where particular roads are treated depending on the prevailing weather conditions, for example, Handa et al. [116]. Another variation is where service costs are time-dependent as weather reports indicate bad weather passing across a region and where each road section has a preferred treatment time, with associated penalties if the service is carried out earlier or later, as described for example in Tagmouti et al. [217].

Exact solution methods have been proposed for some applications for relatively small problems [215], but most practical applications involve the use of a heuristic approach. Some of the practical applications described consider other features than simply minimizing the distance traveled by the vehicles. For example, Lotan et al. [146] involve both depot location and routing for an application in Antwerp, Belgium. Muyldermans et al. [170] are concerned with designing appropriate sectors for planning salt spreading operations.

5.1.3 | Snow clearing

This is another operation associated with winter maintenance and in some circumstances may be combined with salt spreading. It is usually carried out by vehicles equipped with metal blades and is often referred to as snow plowing. Reviews of snow plowing models can be found in Perrier et al. [186,187,189] and Campbell et al. [43].

From a modeling perspective, a key difference between snow clearing and salt spreading is that snow plowing can normally only be applied to one lane of a highway at a time, whereas salt may be spread across the whole width of a carriageway in one pass. This means that in general the graphs representing the underlying road network for snowplowing consist of directed arcs and sometimes a road section may be represented by several arcs, depending on the number of lanes involved. Additionally, the operation of plowing or pushing snow to one side of the vehicle's path means that in some models there are penalties to discourage plowing straight across a crossroad (and thus leaving a bank of snow across the adjoining road), as well as constraints or penalties associated with prohibited U-turns or other route options. There are also applications where teams of vehicles are needed to plow multiple lanes simultaneously in echelons for example, Salazar-Aguilar et al. [204]. As for salt spreading operations, snow clearing is often applied to road networks where the roads have been divided into categories depending on their importance or urgency to be treated.

Several different objectives may be considered. Typically, the total distance or deadheading distance is to be minimized, but time to complete the operation may be used or some weighted combination of these together with penalties describing relevant soft constraints. Solution methods include both exact and heuristic methods. Campbell et al. [43] note that "recent works show a shift from heuristic to mathematical programming-based approaches, as well as efforts to include more of the real-world complexity in snow plow routing." They describe some of the more recent models used for snow plowing applications. In addition, they provide a description of a case study for snow plowing in Centennial, Colorado. The case study provides results showing the improvements from using new "optimized" routes, particularly in reducing the range of times for the new routes and thus giving a significant reduction of 24% for the completion time for all routes. The case description also emphasizes the importance of linking the new routes to the route guidance available to drivers through GPS or Sat-Nav units so that drivers can follow unfamiliar routes.

Some recent examples of snow plowing application papers include Dussault et al. [77,78]. These use a model based on a windy rural postman problem formulation. Their formulation allows two special features to be taken into account. The first is that it is difficult or impossible to plow some steep streets uphill. The second is that it is quicker to deadhead a road segment that has already been plowed rather than deadheading it before plowing. Heuristics are developed to address these variants. More recently, Castro-Campos et al. [45] study the same problems and show that some cases can be solved in polynomial time. Furthermore, they propose faster heuristics for the plowing with precedence problem.

Another recent application in this area is Quirion-Blais et al. [192]. The problem is formulated as a min-max problem with multiple vehicles on a mixed graph with hierarchy. The objective is to minimize the latest finishing time for each priority class. Additional constraints such as turn restrictions are also included. The solution method involves a transformation to an equivalent node routing problem. A metaheuristic approach is developed and tested on three large real-world networks.

5.1.4 | Road marking

Road marking is another operation which is natural to formulate as an ARP. However there do not seem to be many applications reported in this area. Amaya et al. [7] is one example where this problem is addressed. It is modeled as a CARP with refill points and multiple loads. A further complication is that two types of vehicle are involved: a servicing vehicle of limited capacity that marks the road and a refilling vehicle which needs to meet the servicing vehicles in order to replenish them. The models and solution methods are tested on randomly generated test instances as well as a real road network from a region in Canada.

Salazar-Aguilar et al. [205] model the same application, where both certain nodes and arcs require service and consideration is given to synchronizing the routes for the two types of vehicle. Their model can be described as a synchronized GRP, which they use to test different replenishment policies.

5.1.5 | Street mapping

Vansteenwegen et al. [220] describe an application where a mapping van is required to travel along roads in an area to photograph streets and road signs. The objective is to minimize the number of days needed for all the required roads to be photographed. A particular feature of this application is to avoid taking pictures in the direction of the sun as far as possible as the results may not be usable. The problem is modeled as a CARP with soft time windows. The solution approach is to convert the problem to a VRP with soft time windows which is then solved by a metaheuristic algorithm. The method is tested on benchmark instances and real-life problem instances.

5.1.6 | Road maintenance

One type of operation associated with road maintenance is an inspection service. In this application, a set of roads must be visited by a fleet of vehicles in order to visually check the operational status of the road, to report any defects and carry out other checks. There is a limit on the time available for each vehicle each day and so a model can be built based on a CARP where the time available limits the length of each route. Chen et al. [51] describe such an application based on the road network in Shanghai. The time to service each road or to deadhead along a road can be highly variable due to traffic conditions, accidents, technicians' skills, and other factors. The authors decided to use a stochastic arc routing model where the service and deadheading times are not fixed, but are uncertain, following a given probability distribution. A chance-constrained programming model is developed and solved using a branch-and-cut algorithm. This approach can only solve small problems exactly, so an adaptive large neighborhood search algorithm is used to solve larger problems and tested on a set of generated instances. In a subsequent paper, Chen et al. [50] describe how this application can be modeled using robust optimization. This enables them to minimize the worst-case service cost over the uncertainty without being overly conservative. The approach is compared to their previous model and sensitivity tests are carried out on a road network in Shanghai with a total of 93 arcs divided into two sectors, each with their own depot. The paper illustrates managerial insights that can be gained by varying the level of robustness and the fleet size.

A similar type of inspection problem occurs for rail track maintenance as described in Lannez et al. [136]. However, the inspection in this case consists of using special trains to travel along sections of track to perform ultrasonic inspections to detect defects in rails. A deterministic model is used to minimize the total deadheading required. The presence of several complicating constraints, such as a maximum shift duration and a maximum daily inspection distance, led to the formulation being described as a rich ARP. A matheuristic approach is proposed which can solve a problem based on a dataset provided by the French national railway company describing a network with over 2000 arcs.

Huang and Lin [122] describe an application to road resurfacing work. This is another operation where it is natural to base models on a CARP formulation. In the application described here, an additional feature is that roads to be resurfaced require a series of different treatments using different machinery in a particular order. For example, the old surface must normally be removed before a new surface is laid down. In this study, time windows are used to impose these precedence constraints. The objective is to minimize the total expected costs. The solution approach is to transform the problem into an equivalent node routing problem and then to use an Ant Colony Optimization algorithm to find a solution. The method is tested on adapted benchmark problems and applied to a case based on part of the road network in Kaohsiung City in Taiwan.

Yet another form of road maintenance is provided by road watering in open pit mines as described in Riquelme-Rodríguez et al. [196,197]. Watering is required to reduce harmful airborne dust particles. A key feature of this application is that evaporation of the water means that the operation must be repeated periodically (see Section 3.1). Other similar applications include dust suppression in forest roads and watering plants alongside streets. The rate of evaporation of the water may be different on different roads and leads to an ARP with inventory decisions and constraints. In a subsequent paper, Riquelme-Rodríguez et al. [198] extend their model to a LARP where the best location for the depot is also to be determined.

5.1.7 | Plotters, printers, and cutting machines

The paper by Dewil et al. [69] is a good survey of problems arising in laser cutting, mainly on the cutting path problem, which consists of finding a path minimizing the time required to cut all parts from a sheet. Extending a previous classification of the different types of problems related to cutting path proposed by Hoeft and Palekar [120], Dewil et al. describe six classes of cutting path problems: the continuous cutting problem (CCP), where the tool can enter at any point on the perimeter of the object but must cut the entire object before moving to the next one and, hence, the tool must enter and exit each object at the same point; the endpoint cutting problem, in which the tool can cut the contour by sections using only a few specified points to enter and exit; the intermittent cutting problem (ICP), where the object can be cut into sections and there is no restriction on the points that can be used to enter or exit; the touring polygons problem, which is a variant of the CCP where the sequence of objects is given beforehand; and two final classes that can be modeled as a traveling salesman problem and a generalized traveling salesman problem.

Although many of the above problems could be modeled and solved as variations of the RPP, most of the papers reviewed in [69] propose heuristic or exact solution algorithms based on node routing methods. Hence, some classes of cutting path problems offer the possibility of being addressed as ARPs. Among them, the intermittent cutting problem deserves a special attention. Note that the ICP is, possibly, the most general cutting path problem and is a very hard problem to solve. In fact, there are only two articles mentioned in [69] related to the ICP. The first one is a paper by Garfinkel and Webb [98], which introduces and provides some properties of an extension of the RPP, the crossing RPP, in which the postman can enter and leave the edges of an undirected graph by any point (as in the drone RPP), but no solution method is proposed. The other article is

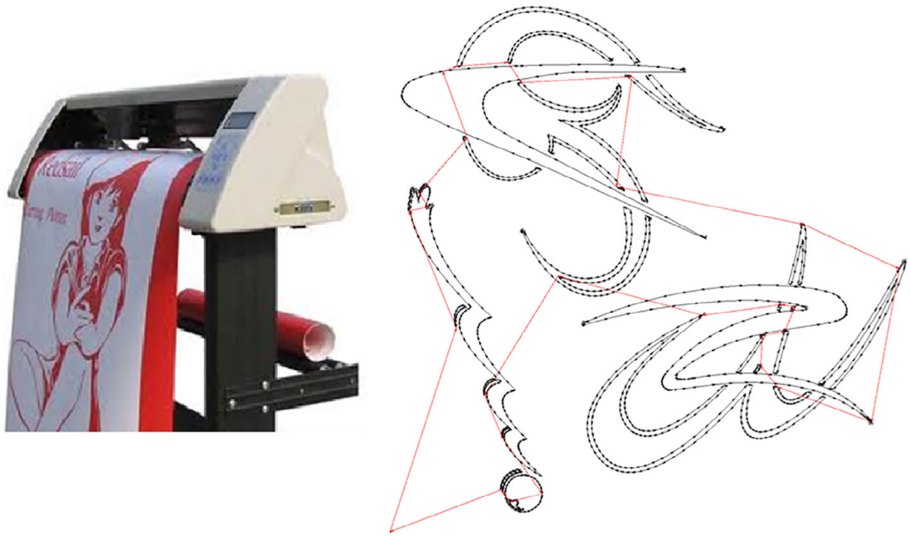

FIGURE 5 Cutting plotter [Color figure can be viewed at wileyonlinelibrary.com]

by Rodrigues and Soeiro [200], which present a memetic algorithm for the RPP and apply it to the solution of a cutting path problem appearing in an industrial application.

Related problems in the context of cutting plotters (see Figure 5) were already mentioned in [62]. Cutting plotters are equipped with a razor that can be moved in two directions and is used to print and cut signs, stickers, and so on. The goal is to plan the movements of the paper (cloth or other material) and the razor to cut all the drawings in the shortest time. Since the cost of moving the paper in one direction and the opposite may be different, nonsymmetric costs for the edges have to be considered, which leads to a problem that can be modeled as a windy RPP.

In our opinion, the new possibilities offered by the use of 3D printers are very interesting. Iori and Novellani [126] define three-dimensional printing as a production technology that allows three-dimensional objects with complex shapes to be constructed by adding one or more materials layer by layer. Basically, the 3D object is divided into many layers that, in turn, are decomposed into segments that must be traversed by the tool (a nozzle in this case) to be printed. Hence, as authors point out, the time needed to produce a 3D object with this technology is very large and, therefore, is essential to minimize the printing time. Similarly to the cutting/drawing problems described above, the problem here consists of optimizing the path that the tool has to follow to build the three-dimensional object. But, as noted by Iori and Novellani [126], the graph to be printed here is divided into clusters (representing the layers to be printed) that have to be visited sequentially and involves a huge number of edges. For this problem, authors propose several heuristics whose behavior compares favorably with CURA, a popular software for 3DP path definition. Fok et al. [94] also deal with this problem for which they propose a procedure based on the Frederickson algorithm for the RPP [95].

Campbell et al. [40] point out that ARPs with a single drone share some characteristics with the problems of generating paths for laser cutting machines or drawing plotters. We think that the intermittent cutting problem, and other problems related to the search for paths for laser cutting machines, drawing plotters, and 3D printers, could be addressed through the methods proposed in [40,41]. This is another area with a great future whose importance will be multiplied with the new applications and possibilities offered by 3D printers.

5.2 | Arc routing applications where demand is point based

Last mile logistics for pickup, delivery or service at households have similar characteristics. Municipal waste collection, postal service, newspaper delivery, and meter reading are examples that were discussed in the arc routing literature early on. Demand is point based, so node routing models would seem natural. However, the total number of nodes and the high density of demand along links in the street network in question may render arc routing models the best choice, particularly for urban areas. As an example, Figure 6 (taken from Hasle [119]) shows a road topology-based aggregation of service points in a dense urban part of Oslo, where green crosses mark the original points and red lines mark aggregates. In this section, such applications are discussed.

5.2.1 | Newspaper delivery and postal service

Newspaper circulation numbers are dropping, as people more and more use alternative channels for news. Still, there are many major newspapers with high circulation numbers. Large volumes of newsprint are processed in printing presses, and then

FIGURE 6 A dense urban part of Oslo [Color figure can be viewed at wileyonlinelibrary.com]

packaged, loaded on to vehicles, and distributed from the print works to readers, typically in a multi-echelon distribution chain. Either, a newspaper copy is delivered to the home of the reader via subscription, or the reader buys it from a retailer. The final delivery to subscribers is made by carriers, either on foot, by bicycle, by car, or a combination.

Arc routing models are relevant for optimizing the final delivery to the doormat by carriers. Newspapers are perishable goods. Subscribers expect to find their fresh morning newspaper on the doormat before breakfast. There is limited time between printing and delivery. Normally, carrier routes should take less than 2 hours, and routes should have balanced duration. In addition to low operational cost, "visual beauty" on the map is a typical desirable characteristic for a carrier routing plan: routes should be compact and nonoverlapping. For successful automatic route design, a detailed electronic road network (GIS) and good models for travel and service time are crucial.

Golden et al. [105] give an account of the newspaper delivery supply chain and what the authors define as the newspaper distribution problem (NDP). They present an industrial background that also focuses on the serious challenges that newspaper companies started to face in the mid-1990s, and continue with a literature survey of VRP algorithms for the NDP. All nine papers discussed focus on the first echelon. The authors present three case studies, two large dailies in the United States and a European newspaper, that all utilize routing software systems for planning and operation.

In a chapter of the 2014 book on arc routing edited by Corberán and Laporte, Hasle [119] discusses logistics for newspaper distribution and surveys the relevant literature. No papers that focus on last mile newspaper delivery were found at the time. A new literature search has not disclosed more recent papers either. However, newspaper delivery by carriers is mentioned as a relevant application in many papers on ARPs, GRPs, and in papers on route balance [156] and visual beauty aspects [201]. Hasle also discusses node routing vs. arc routing and advocates the use of the Mixed Capacitated General Routing Problem (MCGRP, see for instance [38]) as the basis for adequate models for many real-life last mile logistics applications. A case study describing the development of an electronic carrier guide and a web-based service for carrier route design for a Norwegian solution vendor is presented. As subscribers became fewer and their requirements more versatile, tactical carrier routing plans that visited all households became less useful. It became necessary to replace the paper-based delivery guide with handheld computers or smartphones.

With the falling trend of subscriptions in many parts of the world, newspapers and distribution companies have tried to compensate by utilizing their last mile distribution system also for other products, for example, from Internet shopping, to create more revenue. This change also adds dynamics to the routing problems and further strengthens the need for an electronic delivery guide that specifies which subscribers to visit and what products to deliver today.

Postal service and newspaper distribution bear a resemblance. The last mile delivery is performed by postal carriers that start at a depot, normally the post office, and visit the mailboxes of virtually every house in the surrounding postal area. The goal of route design for mail carriers is to create a minimum cost, balanced set of routes that visit all mailboxes in the area. In rural areas, carriers normally drive cars. In cities, they typically walk or use small vehicles. Constraints on route duration and weight are common. Addressed mail is sorted at the post office before delivery.

In a book chapter [141], Levy and Bodin describe the design of city carrier routes for the US Postal Service. Route duration should be approximately 5 hours for each mailman, after 3 hours of work at the office. The chapter focuses on revision (route adjustment) for four types of routes. An algorithm based on graph partitioning is described. The algorithm was implemented in a system and successfully used by the U.S. Postal Service. Twenty years later, Irnich [127] presents modeling and solution methods for real-world postman problems that extend the windy rural postman problem with aspects such as turning and crossing restrictions, alternative modes, and zigzagging. The solution algorithm is based on a transformation to the asymmetric TSP.

In a more recent publication, Gussmagg-Pfliegl et al. [112] study the tactical mail carrier route design problem. Car, moped, bicycle, and walking are the alternative modes available. The authors use a node routing model and propose a two-phase solution approach that combines exact and heuristic methods. Computational experiments on real-life instances show improvements up to 9.5% over current practice at the time.

Like newspaper subscriptions, traditional mail volumes are decreasing. In some countries, carriers no longer deliver mail every weekday. Internet shopping with home delivery is booming, and last mile delivery drives cost and is a bottleneck. There are pilot studies on autonomous vehicles for last mile distribution of newspapers, mail, small packages, and products from Internet shopping such as foodboxes. If they succeed, the cost of last mile deliveries will drop significantly. Demand will be highly dynamic, and increase. However, it is difficult for some to envisage how autonomous vehicles will replace human carriers in reaching the doorstep of every type of household. Joint distribution by human carriers and autonomous vehicles will create novel routing problems. Arc routing models may or may not become more adequate for last mile applications in the future, depending on the level of consolidation of many types of demand that will be possible.

5.2.2 | Meter reading

In parts of the world, utility companies use special employees to read consumption meters of every household. Given the volumes, this is of course extremely costly, and the use of optimization technology has a huge savings potential. Earlier, visual inspection was needed. Every meter had to be visited for close up inspection at regular intervals. Again, the demand for service is point based, but the number and density of visits for urban and suburban areas made arc routing a natural modeling choice for route design. Also, in meter reading, alternative transport modes and workload balancing are important factors in the routing problem.

In their book chapter, Eglese et al. [83] summarize the most important papers on meter reading from the 1970s until the late 2000s and describe current practice in the utility industry. Utility meters have undergone a substantial technological development that has drastically changed the utility meter routing problem in the past decade or so. Meters were equipped with RFID tags such that meter reading could be performed from a distance, for instance by vehicles driving through the area, or a number of stationary transceivers. The new meter technology created novel and interesting optimization problems such as the Close Enough ARP (see Section 3.3), and solutions for optimization technology vendors.

In contrast, meter reading has never been an interesting routing application in countries like Norway. By and large, households only have electricity meters. The utility companies have trusted the consumers to correctly report their consumption at given intervals. Recently, old electricity meters have been replaced by automatic meter readers (AMRs) that continuously record detailed electricity consumption data and communicate those data to the energy supplier. As of 2019, all meters in Norway have been replaced by AMRs.

One would expect that AMR technology will remove the need for routing technology for meter reading worldwide in the future. Network inspection will still be important to utility companies, with new interesting applications for arc routing of drones (see Section 3.5).

5.2.3 | Waste collection

There is a substantial number of waste management inspired publications in the routing literature, also due to the very large costs involved and the importance to human well-being. For municipal curbside waste collection, arc routing models have been studied for a long time. One or more waste bins are located close to each household. Regularly, with an interval of a few days, vehicles follow predefined routes that cover a given area to pick up and compress the waste of every household on the route. When full, or at the end of the working day, the vehicles go to the treatment plant or the disposal site for discharge. There are two basic variants of municipal waste collection systems: single stream and source separation. In single stream systems, there is only one fraction of waste that is collected by single compartment vehicles. In source separation systems, households separate the waste in a small number of fractions. Each fraction may be collected by single compartment vehicles. Alternatively, all fractions are picked up simultaneously by multi-compartment vehicles. There are subvariants and hybrids of these main types.

The CARP is the general basis for municipal waste collection route design. The periodic nature, the need to visit intermediate facilities, waste fractions, and inventory aspects that may follow from waste bin sensors, call for richer models. Also relevant are working time regulations, driving restrictions, and opening times. Narrow streets may call for a two-tiered system where small vehicles service the first mile and transfer the waste to intermediate storage locations or larger vehicles.

Ghiani et al. give a survey of routing in waste collection applications in [102]. They describe solid waste management systems, refer to survey papers, and discuss node routing vs. arc routing also by discussing relevant papers. Finally, they present a case study based on waste collection in a part of Lisboa. A matheuristic is proposed for solving an integer programming model based on a mixed graph. Bing et al. [33] contrast practice with waste modeling issues discussed in the literature and recommend a holistic view for future OR in this area. Based on information gathered in a waste collection research project in Denmark, Kiilerich and Wøhlk [130] introduce five new CARP variants and present large-scale instances inspired from real operations in five urban and rural areas. Based on the same Danish cases information, Zbib and Wøhlk [231] compare transport requirements for single stream and source separation systems. They find that the "sort, then collect by multi-compartment vehicles" system outperforms the others in terms of driven distance, and that the "sort, then collect by single compartment vehicles" system outperforms the others in terms of number of routes.

Recently, waste bins are equipped with sensors that measure and communicate filling degree. There are still technical challenges, but when reliable, this "smart bin" technology may change waste collection systems drastically. Instead of a static, frequency-based collection pattern, bins can be emptied when necessary. Instead of a tactical route plan, routes may be designed dynamically, maybe on a daily basis, according to current demand. Inventory aspects may be important for the routing problem, given reliable information on filling rates. The density of collection points on demand-based waste collection routes will be lower, but arc routing models will presumably be relevant, at least in urban areas.

Some cities with old centers that were built in a time when waste collection systems were primitive or nonexistent, have serious challenges with conventional waste collection systems. In the medieval city center of Bergen, they started operating a pipe based, pneumatic waste collection system in 2016. The underground suction system, also called the world's largest central vacuum cleaner, transports both residential and commercial waste to terminals twice per day. Arc routing is irrelevant for such waste collection systems, but OR models for network design and extension may be useful. The costs and difficulties involved in the installation of the pipe infrastructure limit widespread use of this technology.

5.3 | Applications where a mixed arc and node model is relevant

Prins and Bouchenoua [190] study the MCGRP that combines node and arc routing on a mixed graph. They re-baptize the problem the node, edge, and ARP and motivate their study by applications in waste collection. In certain cases, most requests may be aggregated and adequately modeled as located on streets, but some large point-based demands exist (for instance at schools or hospitals) that should be modeled on vertices, hence a need for a general routing model on a mixed graph.

Hasle [119] argues that a pure arc routing model for applications where demand is fundamentally point based may be a crude abstraction. In practice, it is better in such cases to apply a qualified aggregation heuristic for demand that takes proximity, capacity, temporal constraints, and other relevant aspects into account to reduce the number of requests. Often, the result will be a combination of aggregated requests located on arcs or edges, and point based requests that have not been aggregated, and probably should not be aggregated. His conclusion is that the MCGRP is a suitable basis for an adequate model for such applications.

As mentioned in Section 3.5, there are applications where drones need to service both nodes and arcs, for instance as reported by Singgih et al. [211]. A few more examples are briefly described in Section 5.4.

5.4 | Other types of application

As mentioned in Sections 3.1, 3.3, and 3.5, network surveillance and inspection is an important potential application area for arc routing. Infrastructure in the form of roads, railroads, power lines, water supply, gas, and sewage pipe networks is vital to a well-functioning society. Surveillance or inspection at regular intervals is needed for detection of potential problems. In case of major failures and disasters, time to find the sources of failure is critical to minimize harm. Conventional inspection methods involve human efforts to a large degree. Hence they are very expensive and time consuming. Recent technological advances, primarily in robotics and autonomy, have changed the way network surveillance and inspection operations are performed. Further technology development (more autonomy, higher speed, better sensors), and increased usage and competition, is expected to lower cost and efficiency of such operations even more in the near future. As a result, we expect an increasing interest in arc routing models and solution methods that will provide the basis for optimization based planning tools.

Power line inspections are typically performed annually, traditionally by foot, by car, or by helicopter. Recently, man-operated drones equipped with camera and sensors have been used to inspect power lines with much lower cost and higher efficiency. Similarly, drones are now used to check the condition of roads, bridges, and railway tracks. Drones with gas sensors are used for leak detection of gas pipes. There are crawling robots for checking leaks, corrosion, and sludge in pipes.

Planning of network surveillance and inspection operations naturally lead to some ARPs where the abstract graph is derived directly from the physical network. Utility networks normally have a tree structure, whereas road networks and bridge structures have a more complex topology. The use of drones or robots may fundamentally change the associated ARPs, as discussed in Section 3.5. Often, inspection can be done within a certain distance from the physical link, service may be started from any point, and the vehicles may need frequent refueling/recharging.

Despite the high potential impact, only a few application oriented papers on arc routing in network inspection and surveillance have been published, as far as we can see. Liu et al. [144] investigate a system with a ground vehicle and a drone for power line inspection of large areas. The ground vehicle can only stop at predefined potential rendezvous places, where the drone can be launched or retrieved. They define the combined routing problem for the ground vehicle and the drone, with the goal of minimizing the completion time, and call it the two-layer point-arc routing problem. It may be seen as an arc routing and single vehicle variant of the VRP with drones. They propose a heuristic and perform computational experiments based on a real-life case in China where they also investigate sensitivity with respect to drone inspection speed and battery capacity. In a short conference paper, Harris et al. [117] discuss inspection of truss bridges with multiple robots where the goal is to design efficient and balanced routes. They model the problem as a min-max k-windy CPP and propose a genetic algorithm for solving it.

Agriculture is a not much investigated, but promising, ARP application area. Arc routing models for planning of operations in animal farms were investigated in several papers in the 1990s. Dror et al. [76] discuss the operations of a large cattle yard in Arizona in a chapter of the arc routing book edited by Dror [75]. The authors focus on arc routing models for pen inspection and feed delivery, where capacity, time windows, and split delivery are important aspects. Today, the use of drones and robots in agriculture is rapidly increasing. They are used to automate a variety of tasks such as plowing, seeding, inspection, irrigation, weed control, spraying, harvesting, picking, and also feeding. Several robots may collaborate on performing a given task. The optimal planning of such operations may be formulated as ARPs. Relevant aspects are capacity, refueling/recharging, split delivery, and close enough service. For feeding robots, storages for various types and qualities of fodder may call for adding inventory aspects. Also for applications in agriculture we expect an increasing interest in planning tools, and hence an increase in targeted arc routing methods.

In the context of high level political visions such as smart cities and mobility-as-a-service, there is currently a drive towards the use of more on-demand public transport, particularly for the first/last mile. A possible approach is to use nontimetabled, flexible mini-buses and more or less dynamic routing based on current demand. In dense urban areas, such mini-buses cannot stop at any doorstep. Suitable points or road segments where passengers may embark or disembark must be selected such that the walking distance to/from any household in the area is within a certain limit from a selected potential stop. We believe that a variant of the close enough ARP could be useful in determining the selection of stops and road segments.

The recovery phase of disaster operations management [6,97] involves routing problems. A paper by de la Torre et al. [67] provides a survey and an analysis of OR models for routing of vehicles within disaster-affected regions, but does not mention arc routing. Many tasks in the recovery phase are naturally modeled as ARPs; however, some of these have been discussed above. Examples are disaster assessment, debris removal, rebuilding of roads, bridges, and key infrastructure. Oruc and Kara [178] study an extension of the GRP that they call the postdisaster assessment routing problem. It concerns early assessment of damage to populated areas, infrastructure, and facilities by teams equipped with drones and motorcycles, for further planning of post disaster actions. They develop two biobjective mathematical models with accompanying approximation methods, as response time is critical. An experimental study is conducted on data from Istanbul. Kasaei and Salman [129] study two ARPs for clearing blocked roads, one for minimizing the time to reconnect the road network, the other for maximizing the total benefit gained by reconnecting network components within a time limit. Sahin et al. [203] focus on debris removal from blocked roads, define the so-called debris removal problem in the response phase problem as an extension of the GRP. Akbari and Salman [3] propose a method for generating a synchronized work schedule for road clearing teams to restore connectivity that minimizes makespan. They provide a MIP model for the so-called multi-vehicle synchronized arc routing problem to restore network connectivity, and also develop and investigate a heuristic approach.

6 | ARC ROUTING LIBRARIES AND INSTANCE REPOSITORIES

There are some instance repositories of many ARPs that provide interesting and useful information about the behavior and limits of some of the best-known existing solution algorithms and that are also available to compare the performance of new exact and approximate methods. Among them, we can cite the repository described in Willemse and Joubert [222] consisting of undirected and mixed CARPs under time restrictions with intermediate facilities. Authors point out that the benchmark sets include realistic instances that are based on actual road networks and that are consistent in size with waste collection instances found in practice.

Updated lower and upper bound values and integer solutions for well-known CARP benchmark instances, including the KSHS, GDB, BBCM, EGL, EGL-large, and BMCV instances, are reported in the web site https://logistik.bwl.uni-mainz.de/forschung/benchmarks/, maintained by Stefan Irnich. The data for the CARP instances can be found there and in the web site https://www.uv.es/belengue/carp.html, maintained by J.M. Belenguer.

The web site http://optimization.dk/index.html, maintained by Sanne Wøhlk, contains information and data of instances for the CARP and some variations that have their origin in a waste collection problem (see Section 5.2.3). These large-scale instances are described in Kiilerich and Wøhlk [130], and are based on real-life networks and waste data from five areas in Denmark and cover rural as well as urban areas.

In https://www.uv.es/corberan/instancias.htm, data for different types of ARPs defined on undirected, directed, mixed, and windy graphs can be found, as well as information on optimal or lower and upper bound values (when the optimal value is unknown) for the instances. In particular, this web site provides the data, characteristics, and information of instances of the mixed CPP (undirected, mixed, and windy) RPP (undirected, directed, mixed, and windy) GRP, maximum benefit CPP, stacker crane problem, close enough and distance-constrained close enough ARP, orienteering and TOARP, hierarchical windy postman problem, min-max K-vehicles windy RPP, profitable windy RPP, and drone RPP.

Finally, we mention the library of arc routing instances and algorithms created by Oliver Lum and hosted at https://github.com/Olibear/ArcRoutingLibrary. The library is described in Lum et al. [149] and presented as "an open source, arc routing Java library that has a flexible graph architecture with solvers for several uncapacitated arc routing problems and the ability to dynamically generate and visualize real-world street networks." Implementations of well-known algorithms for the exact or approximate solution of the (undirected, directed, and mixed) CPP and the (directed and windy) RPP, as well as some other utilities are available at that web site.

7 | CONCLUSIONS AND PERSPECTIVES

The paper has presented a broad survey of developments in arc routing. Following the brief historical introduction, the emphasis has been on discussing the variants of ARPs that are of particular interest at this stage of research into the area.

Section 3 dealt with particular types of problem that are of current and growing research interest. For many of the types of problem included, a conclusion is that progress in solving simpler classic ARPs, such as the RPP and the CARP, will be able to be incorporated into approaches to solve the more complex and demanding problems are tackled here. This will lead to advances in the size of problems that can be solved using exact methods and to enhanced metaheuristics and matheuristic algorithms that can be used to support decision making for real-world problems.

Section 4 considered several features that are relevant to modeling ARPs of many different types. In these areas, the conclusions are varied and point towards further extensions of models that have not yet been researched or the opportunity to make use of recent advances in the availability of data and communications to make new methods and approaches possible. For example, the availability of current tracking data for traffic and vehicles can be used to provide information for dynamic ARPs and then modern forms of communication can enable the results to be implemented in time to be effective.

Section 5 surveyed the use of ARPs in many different practical applications. Some of these, such as waste collection, have been a focus for application of ARPs for many years, while others are relatively new. In many applications, there is a decision to be made about the level of aggregation of demand that will affect whether the model to be used is based on a node routing or arc routing formulation, or whether a mixed approach is required leading to some form of GRP.

Some applications that have been the focus of studies will become less important. For example, applications in meter reading will reduce as more homes are equipped with smart meters that do not need a visit or local monitoring. On the other hand, new applications in areas such as the use of drones and in 3D printing will require further research. It is interesting to see the emergence of a challenging ARP, the intermittent cutting problem, which may be relevant to both of these new applications.

Section 6 gives information on where to find standard data sets together with best-known solutions and bounds from existing methods.

The review has demonstrated that the study of ARPs is an important area of research and that it will continue to have a vibrant future in motivating the technical challenges of discrete optimization and modeling more realistic representations of operations in the real world.

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ORCID

Ángel Corberán https://orcid.org/0000-0002-0664-4232
Isaac Plana https://orcid.org/0000-0002-0516-4608

REFERENCES

[1] H.M. Afsar, *A branch-and-price algorithm for capacitated arc routing problem with flexible time windows*, Electron. Notes Discrete Math. **36** (2010), 319–326.

[2] C. Ahabchane, A. Langevin, and M. Trépanier, *The mixed capacitated general routing problem with time-dependent demands*, WARP3 Proceedings, Pizzo, Italy, 2019.

[3] V. Akbari and F.S. Salman, *Multi-vehicle synchronized arc routing problem to restore post-disaster network connectivity*, Eur. J. Oper. Res. **257** (2017), 625–640.

[4] M. Alapati, Drone routing for damage assessment on power distribution systems under hilf events, Ph.D. thesis, University of North Carolina at Charlotte, 2018.

[5] M. Albareda-Sambola, "*Location-routing and location-arc routing*," *Location Science*, G. Laporte *et al.* (eds), Springer, Berlin, 2015, pp. 399–418.

[6] N. Altay and W.G. Green, *OR/MS research in disaster operations management*, Eur. J. Oper. Res. **175** (2006), 475–493.

[7] C.A. Amaya, A. Langevin, and M. Trépanier, *A heuristic method for the capacitated arc routing problem with refill points and multiple loads*, J. Oper. Res. Soc. **61** (2010), 1095–1103.

[8] A. Amini, R. Tavakkoli-Moghaddam, and S. Ebrahimnejad, *A bi-objective transportation-location arc routing problem*, Transp. Lett. (2019), 1–15, https://doi.org/10.1080/19427867.2019.1679405.

[9] U.F. Aminu and R. Eglese, *A constraint programming approach to the Chinese postman problem with time windows*, Comput. Oper. Res. **33** (2006), 3423–3431.

[10] J. Aráoz, E. Fernández, and C. Franquesa, *GRASP and path relinking for the clustered prize-collecting arc routing problem*, J. Heuristics **19** (2013), 343–371.

[11] J. Aráoz, E. Fernández, and C. Franquesa, *The generalized arc routing problem*, TOP **25** (2017), 497–525.

[12] C. Arbib, M. Servilio, C. Archetti, and M.G. Speranza, *The directed profitable location rural postman problem*, Eur. J. Oper. Res. **236** (2014), 811–819.

[13] C. Archetti, L. Bertazzi, D. Laganá, and F. Vocaturo, *The undirected capacitated general routing problem with profits*, Eur. J. Oper. Res. **257** (2017), 822–833.

[14] C. Archetti, Á. Corberán, I. Plana, J.M. Sanchis, and M.G. Speranza, *The team orienteering arc routing problem*, Transp. Sci. **48** (2014), 442–457.

[15] C. Archetti, Á. Corberán, I. Plana, J.M. Sanchis, and M.G. Speranza, *A matheuristic for the team orienteering arc routing problem*, Eur. J. Oper. Res. **245** (2015), 392–401.

[16] C. Archetti, Á. Corberán, I. Plana, J.M. Sanchis, and M.G. Speranza, *A branch-and-cut algorithm for the orienteering arc routing problem*, Comput. Oper. Res. **66** (2016), 95–104.

[17] C. Archetti, D. Feillet, A. Hertz, and M.G. Speranza, *The undirected capacitated arc routing problem with profits*, Comput. Oper. Res. **37** (2010), 1860–1869.

[18] C. Archetti, G. Guastaroba, and M.G. Speranza, *Reoptimizing the rural postman problem*, Comput. Oper. Res. **40** (2013), 1306–1313.

[19] C. Archetti, G. Guastaroba, and M.G. Speranza, *An ILP refined tabu search for the directed profitable rural postman problem*, Discrete Appl. Math. **163** (2014), 3–16.

[20] C. Archetti and M.G. Speranza, "*Arc routing problems with profits*," *Arc Routing: Problems, Methods, and Applications*. MOS-SIAM Series on Optimization, Á. Corberán and G. Laporte (eds), SIAM, Philadelphia, 2014, pp. 281–299.

[21] A. Assad, *Leonhard Euler: A brief appreciation*, Networks **49** (2007), 190–198.

[22] T. Ávila, Á. Corberán, I. Plana, and J.M. Sanchis, *A branch-and-cut algorithm for the profitable windy rural postman problem*, Eur. J. Oper. Res. **249** (2016), 1092–1101.

[23] T. Ávila, Á. Corberán, I. Plana, and J.M. Sanchis, *A new branch-and-cut algorithm for the generalized directed rural postman problem*, Transplant Sci **50** (2016), 750–761.

[24] T. Ávila, Á. Corberán, I. Plana, and J.M. Sanchis, *Formulations and exact algorithms for the distance-constrained generalized directed rural postman problem*, EURO J. Comput. Optim. **5** (2017), 339–365.

[25] B. Behdani and J.C. Smith, *An integer-programming-based approach to the close-enough traveling salesman problem*, INFORMS J. Comput. **26** (2014), 415–432.

[26] J.M. Belenguer, E. Benavent, and S. Irnich, "*The capacitated arc-routing problem: Exact methods*," *Arc Routing: Problems, Methods, and Applications*. MOS-SIAM Series on Optimization, Á. Corberán and G. Laporte (eds), SIAM, Philadelphia, 2014, pp. 183–221.

[27] J.M. Belenguer, E. Benavent, N. Labadi, C. Prins, and M. Reghioui, *Split-delivery capacitated arc-routing problem: Lower bound and metaheuristic*, Transp. Sci. **44** (2010), 206–220.

[28] E.J. Beltrami and L.D. Bodin, *Networks and vehicle routing for municipal waste collection*, Networks **4** (1974), 65–94.

[29] E. Benavent, Á. Corberán, L. Gouveia, M.C. Mourão, and L.S. Pinto, *Profitable mixed capacitated arc routing and related problems*, TOP **23** (2015), 244–274.

[30] E. Benavent, Á. Corberán, D. Laganá, and F. Vocaturro, *The periodic rural postman problem with irregular services on mixed graphs*, Eur. J. Oper. Res. **276** (2019), 826–839.

[31] P. Beraldi, M.E. Bruni, D. Laganà, and R. Musmanno, *The mixed capacitated general routing problem under uncertainty*, Eur. J. Oper. Res. **240** (2015), 382–392.

[32] N. Bianchessi, Á. Corberán, I. Plana, M. Reula, and J.M. Sanchis, On the min-max close-enough arc routing problem, 2020, Submitted.

[33] X. Bing, J.M. Bloemhof, T. Rodrigues-Pereira, C.Y.W.A.P. Barbosa-Povoa, and J.G.A.J. van der Vorst, *Research challenges in municipal solid waste logistics management*, Waste Manage. **48** (2016), 584–592.

[34] D. Black, R. Eglese, and S. Wøhlk, *The time-dependent prize-collecting arc routing problem*, Comput. Oper. Res. **40** (2013), 526–535.

[35] C.A. Blazquez, A. Beghelli, and V.P. Meneses, *A novel methodology for determining low-cost fine particulate matter street sweeping routes*, J. Air Waste Manage. Assoc. **62** (2012), 242–251.

[36] L.D. Bodin and S.J. Kursh, *A computer-assisted system for the routing and scheduling of street sweepers*, Oper. Res. **26** (1987), 525–537.

[37] R. Borges-Lopes, F. Plastria, C. Ferreira, and B. Sousa-Santos, *Location-arc routing problem: Heuristic approaches and test instances*, Comput. Oper. Res. **43** (2014), 309–317.

[38] A. Bosco, D. Laganà, R. Musmanno, and F. Vocaturo, *Modeling and solving the mixed capacitated general routing problem*, Optim. Lett. **7** (2013), 1451–1469.

[39] T. Calogiuri, G. Ghiani, E. Guerriero, and R. Mansini, *A branch-and-bound algorithm for the time-dependent rural postman problem*, Comput. Oper. Res. **102** (2019), 150–157.

[40] J.F. Campbell, Á. Corberán, I. Plana, and J.M. Sanchis, *Drone arc routing problems*, Networks **72** (2018), 543–559.

[41] J.F. Campbell, Á. Corberán, I. Plana, J.M. Sanchis, and P. Segura, A matheuristic for the length constrained k-drones rural postman problem, 2019, Submitted.

[42] J.F. Campbell, Á. Corberán, I. Plana, J.M. Sanchis, and P. Segura, Drones and the rural postman problem, 2020, Submitted.
[43] J.F. Campbell, A. Langevin, and N. Perrier, "*Advances in vehicle routing for snow plowing*," *Arc Routing: Problems, Methods, and Applications*. MOS-SIAM Series on Optimization, Á. Corberán and G. Laporte (eds), SIAM, Philadelphia, 2014, pp. 321–350.
[44] F. Carrabs, C. Cerrone, R. Cerulli, and M. Gaudioso, *A novel discretization scheme for the close enough traveling salesman problem*, Comput. Oper. Res. **78** (2017), 163–171.
[45] R. Castro-Campos, C.A. Rodríguez, and F. Zaragoza-Martínez, Plowing with precedence in polynomial time, Networks, 2021, in press.
[46] M.K. Çodur and M. Yılmaz, *A time-dependent hierarchical Chinese postman problem*, Central Eur. J. Oper. Res. **28** (2018), 337–366.
[47] C. Cerrone, R. Cerulli, B.L. Golden, and R. Pentangelo, "*A flow formulation for the close-enough arc routing problem*," *Optimization and Decision Science: Methodologies and Applications. ODS 2017*, A. Sforza and C. Sterle (eds), Springer, New York, 2017, pp. 539–546.
[48] T.S. Chang and H.M. Yen, *City-courier routing and scheduling problems*, Eur. J. Oper. Res. **223** (2012), 489–498.
[49] H. Chen, T. Cheng, and J. Shawe-Taylor, *A balanced route design for min-max multiple-depot rural postman problem (MMMDRPP): A police patrolling case*, Int. J. Geogr. Inform. Sci. **32** (2018), 169–190.
[50] L. Chen, M. Gendreau, M.H. Hà, and A. Langevin, *A robust optimization approach for the road network daily maintenance routing problem with uncertain service time*, Transp. Res. E: Logist. Transp. Rev. **85** (2016), 40–51.
[51] L. Chen, M.H. Hà, A. Langevin, and M. Gendreau, *Optimizing road network daily maintenance operations with stochastic service and travel times*, Transp. Res. E: Logist. Transp. Rev. **64** (2014), 88–102.
[52] S. Chen, B.L. Golden, R. Wong, and H. Zhong, *Arc-routing models for small-package local routing*, Transp. Sci. **43** (2009), 43–55.
[53] J.Y.J. Chow, *Dynamic UAV-based traffic monitoring as a stochastic arc-inventory routing policy*, Int. J. Transp. Sci. Technol. **5** (2016), 167–185.
[54] C.H. Christiansen, J. Lysgaard, and S. Wøhlk, *A branch-and-price algorithm for the capacitated arc routing problem with stochastic demands*, Oper. Res. Lett. **37** (2009), 392–398.
[55] M. Colombi and R. Mansini, *New results for the directed profitable rural postman problem*, Eur. J. Oper. Res. **238** (2014), 760–773.
[56] Á. Corberán, G. Erdoğan, G. Laporte, I. Plana, and J.M. Sanchis, *The Chinese postman problem with load-dependent costs*, Transp. Sci. **52** (2018), 370–385.
[57] Á. Corberán, E. Fernández, C. Franquesa, and J.M. Sanchis, *The windy clustered prize-collecting arc-routing problem*, Transp. Sci. **45** (2011), 317–334.
[58] Á. Corberán and G. Laporte, "*Arc routing: Problems, methods and applications*," *MOS-SIAM Series on Optimization*SIAM, Philadelphia, 2014.
[59] Á. Corberán, I. Plana, M. Reula, and J.M. Sanchis, *A matheuristic for the distance-constrained close-enough arc routing problem*, TOP **27** (2019), 312–326.
[60] Á. Corberán, I. Plana, M. Reula, and J.M. Sanchis, On the distance-constrained close enough arc routing problem, 2020, Submitted.
[61] Á. Corberán, I. Plana, A. Rodríguez-Chía, and J.M. Sanchis, *A branch-and-cut algorithm for the maximum benefit Chinese postman problem*, Math. Program. **141** (2013), 21–48.
[62] Á. Corberán, I. Plana, and J.M. Sanchis, "*The rural postman problem on directed, mixed, and windy graphs*," *Arc Routing: Problems, Methods, and Applications*. MOS-SIAM Series on Optimization, Á. Corberán and G. Laporte (eds), SIAM, Philadelphia, 2014, pp. 101–128.
[63] Á. Corberán and C. Prins, *Recent results on arc routing problems: An annotated bibliography*, Networks **56** (2010), 50–69.
[64] Á. Corberán, B.L. Golden, O. Lum, I. Plana, and J.M. Sanchis, *Aesthetic considerations for the min-max k-windy rural postman problem*, Networks **70** (2017), 216–232.
[65] W.P. Coutinho, A. Subramanian, R.Q. do Nascimento, and A.A. Pessoa, *A branch-and-bound algorithm for the close enough traveling salesman problem*, INFORMS J. Comput. **28** (2016, 765), 752.
[66] T. Cura, *An artificial bee colony approach for the undirected capacitated arc routing problems with profits*, Int. J. Oper. Res. **17** (2013), 483–508.
[67] L.E. de la Torre, I.S. Dolinskaya, and K.R. Smilowitz, *Disaster relief routing: Integrating research and practice*, Socioecon. Plann. Sci. **46** (2012), 88–97.
[68] A. Del-Pia and C. Filippi, *A variable neighborhood descent algorithm for a real waste collection problem with mobile depots*, Int. Trans. Oper. Res. **13** (2006), 125–141.
[69] R. Dewil, P. Vansteenwegen, and D. Cattrysse, *A review of cutting path algorithms for laser cutters*, Int. J. Adv. Manuf. Technol. **87** (2016), 1865–1884.
[70] G. Dhein, O.C.B. de Araújo, and G. Cardoso, *Genetic local search algorithm for a new bi-objective arc routing problem with profit collection and dispersion of vehicles*, Expert Syst. Appl. **92** (2018), 276–288.
[71] M. Dille and S. Singh, *Efficient aerial coverage search in road networks*, AIAA Conference on Guidance, Navigation, and Control (GNC) Conference, Boston, 2013.
[72] J. Dong, N. Yang, and M. Chen, "*Heuristic approaches for a TSP variant: The automatic meter reading shortest tour problem*," *Extending the Horizons: Advances in Computing, Optimization, and Decision Technologies*. Springer, US, 2007, pp. 145–163.
[73] M. Drexl, On some generalized routing problems, Ph.D. thesis, Rheinisch-Westfälische Technische Hochschule, Aachen University, 2007.
[74] M. Drexl, *On the generalized directed rural postman problem*, J. Oper. Res. Soc. **65** (2014), 1143–1154.
[75] M. Dror, *Arc Routing: Theory, Solutions and Applications*, Kluwer Academic Publishers, Boston/Dordrecht/London, 2000.
[76] M. Dror, J.M.Y. Leung, and P.A. Mullaseril, "*Livestock feed distribution and arc traversal problems*," *Arc Routing: Theory, Solutions and Applications*, M. Dror (ed.), Kluwer Academic Publishers, Boston/Dordrecht/London, 2000, pp. 443–474.
[77] B. Dussault, B.L. Golden, C. Groër, and E. Wasil, *Plowing with precedence: A variant of the windy postman problem*, Comput. Oper. Res. **40** (2013), 1047–1059.
[78] B. Dussault, B.L. Golden, and E. Wasil, *The downhill plow problem with multiple plows*, J. Oper. Res. Soc. **65** (2014), 1465–1474.
[79] K. Easton and J.A. Burdick, *Coverage algorithm for multi-robot boundary inspection*, Proceedings of the 2005 IEEE International Conference on Robotics and Automation, Barcelona, Spain, 2005.
[80] J. Edmonds, *Paths, trees and flowers*, Can. J. Math. **17** (1965), 449–467.
[81] J. Edmonds and E. Johnson, *Matching, Euler tours and the Chinese postman problem*, Math. Program. **5** (1973), 88–124.
[82] R. Eglese, *Routeing winter gritting vehicles*, Discrete Appl. Math. **48** (1994), 231–244.
[83] R. Eglese, B.L. Golden, and E. Wasil, "*Route optimization for meter reading and salt spreading*," *Arc Routing: Problems, Methods, and Applications*. MOS-SIAM Series on Optimization, Á. Corberán and G. Laporte (eds), SIAM, Philadelphia, 2014, pp. 303–320.
[84] R. Eglese, W. Maden, and A. Slater, *A road timetable to aid vehicle routing and scheduling*, Comput. Oper. Res. **33** (2006), 3508–3519.
[85] R. Eglese and H. Murdock, *Routeing road sweepers in a rural area*, J. Oper. Res. Soc. **42** (1991), 281–288.
[86] J. Euchi and H. Chabchoub, *Hybrid metaheuristics for the profitable arc tour problem*, J. Oper. Res. Soc. **62** (2011), 2013–2022.

[87] A. Eydi and L. Javazi, *Model and solution approach for multi objective-multi commodity capacitated arc routing problem with fuzzy demand*, J. Ind. Syst. Eng. **5** (2012), 208–229.
[88] E. Fernández, D. Fontana, and M.G. Speranza, *On the collaboration uncapacitated arc routing problem*, Comput. Oper. Res. **67** (2016), 120–131.
[89] E. Fernández, G. Laporte, and J. Rodríguez-Pereira, *A branch-and-cut algorithm for the multidepot rural postman problem*, Transp. Sci. **52** (2018), 353–369.
[90] E. Fernández, G. Laporte, and J. Rodríguez-Pereira, *Exact solution of several families of location-arc routing problems*, Transp. Sci. **53** (2019), 1313–1333. https://doi.org/10.1287/trsc.2018.0881.
[91] E. Fernández and J. Rodríguez-Pereira, *Multi-depot rural postman problem*, TOP **25** (2017), 340–372.
[92] G. Fleury, P. Lacomme, and C. Prins, "*Evolutionary algorithms for stochastic arc routing problems*," *Applications of Evolutionary Computing*Springer Berlin Heidelberg, Berlin, Heidelberg, 2004, pp. 501–512.
[93] G. Fleury, P. Lacomme, C. Prins, and M. Sevaux, *A memetic algorithm for a bi-objective and stochastic CARP*, Proceedings of The Sixth Metaheuristics International Conference, Vienna, Austria, 2005.
[94] K.Y. Fok, C.T. Cheng, and C.K. Tse, *A refinement process for nozzle path planning in 3D printing*, 2017 IEEE International Symposium on Circuits and Systems (ISCAS), Baltimore, MD, 2017, pp. 1–4.
[95] G. Frederickson, *Approximation algorithms for some postman problems*, J. ACM **26** (1979), 538–554.
[96] G.E.A. Fröhlich and K.F. Dörner, *Metaheuristics for the multi-objective and periodic node, edge, arc routing problem considering costs and route inconsistency*, WARP3 Proceedings, Pizzo, Italy, 2019.
[97] G. Galindo and R. Batta, *Review of recent developments in OR/MS research in disaster operations management*, Eur. J. Oper. Res. **230** (2013), 201–211.
[98] R. Garfinkel and I. Webb, *On crossings, the crossing postman problem, and the rural postman problem*, Networks **34** (1999), 173–180.
[99] D. Gavalas, C. Konstantopoulos, K. Mastakas, G. Pantziou, and N. Vathis, *Efficient metaheuristics for the mixed team orienteering problem with time windows*, Algorithms **9** (2016), 2–26.
[100] M. Gendreau, G. Laporte, and F. Semet, *The covering tour problem*, Oper. Res. **45** (1997), 568–576.
[101] G. Ghiani, G. Improta, and G. Laporte, *The capacitated arc routing problem with intermediate facilities*, Networks **37** (2001), 134–143.
[102] G. Ghiani, M.C. Mourão, L.S. Pinto, and D. Vigo, "*Routing in waste collection applications*," *Arc Routing: Problems, Methods, and Applications*. MOS-SIAM Series on Optimization, Á. Corberán and G. Laporte (eds), SIAM, Philadelphia, 2014, pp. 351–370.
[103] N. Golbaharan, An application of optimization to the snow removal problem: A column generation approach, Ph.D. thesis, Linköping University (Sweden), 2001.
[104] B. Golden and R. Wong, *Capacitated arc routing problems*, Networks **11** (1981), 305–315.
[105] B.L. Golden, A.A. Assad, and E.A. Wasil, "*Routing vehicles in the real world: Applications in the solid waste, beverage, food, diary, and newspaper industries*," *The Vehicle Routing Problem*, P. Toth and D. Vigo (eds), SIAM, Philadelphia, 2002, pp. 245–286.
[106] S. Gonzalez-Martin, A.A. Juan, D. Riera, M.G. Elizondo, and J.J. Ramos, *A simheuristic algorithm for solving the arc routing problem with stochastic demands*, J. Simul. **12** (2018), 53–66.
[107] L. Grandinetti, F. Guerriero, D. Laganá, and O. Pisacane, *An optimization-based heuristic for the multi-objective undirected capacitated arc routing problem*, Comput. Oper. Res. **39** (2012), 2300–2309.
[108] I. Gribkovskaia, Ø. Halskau, and G. Laporte, *The bridges of Königsberg, a historical perspective*, Networks **49** (2007), 199–203.
[109] M. Guan, *Graphic programming using odd or even points*, Chin. Math. **1** (1962), 273–277.
[110] C. Gueguen, Méthodes de résolution exacte pour les problèmes de tournées de véhicules, Ph.D. thesis, Ecole centrale de Paris, 1999.
[111] D.J. Gulczynski, J.W. Heath, and C.C. Price, "*The close enough traveling salesman problem: A discussion of several heuristics*," *Perspectives in Operations Research*Springer, New York, 2006, pp. 217–283.
[112] E. Gussmagg-Pfliegl, F. Tricoire, K.F. Doerner, R.F. Hartl, and S. Irnich, "*Heuristics for a real-world mail delivery problem*," *Applications of Evolutionary Computation. EvoApplications 2011. Lecture Notes in Computer Science*, vol. **6625**, C. Di Chio *et al.* (eds), Springer, Amsterdam, 2011, pp. 481–490.
[113] M.H. Hà, N. Bostel, A. Langevin, and L.M. Rousseau, *An exact algorithm for close enough traveling salesman problem*, Proceedings of the 1st International Conference on Operations Research and Enterprise Systems, 2012, pp. 233–238.
[114] M.H. Hà, N. Bostel, A. Langevin, and L.M. Rousseau, *Solving the close enough arc routing problem*, Networks **63** (2014), 107–118.
[115] E.E. Halvorsen-Weare and M.W. Savelsbergh, *The bi-objective mixed capacitated general routing problem with different route balance criteria*, Eur. J. Oper. Res. **251** (2016), 451–465.
[116] H. Handa, L. Chapman, and X. Yao, *Dynamic salting route optimisation using evolutionary computation*, 2005 IEEE Congress on Evolutionary Computation, 2005, pp. 158–165.
[117] N. Harris, S. Liu, S.J. Louis, and J.H. La, *A genetic algorithm for multi-robot routing in automated bridge inspection*, Proceedings of the Genetic and Evolutionary Computation Conference Companion, GECCO '19, ACM, New York, NY, 2019, pp. 369–370.
[118] H. Hashemi-Doulabi and A. Seifi, *Lower and upper bounds for location-arc routing problems with vehicle capacity constraints*, Eur. J. Oper. Res. **224** (2013), 189–208.
[119] G. Hasle, "*Arc routing applications in the newspaper industry*," *Arc Routing: Problems, Methods, and Applications*. MOS-SIAM Series on Optimization, Á. Corberán and G. Laporte (eds), SIAM, Philadelphia, 2014, pp. 371–396.
[120] J. Hoeft and U. Palekar, *Heuristics for the plate-cutting traveling salesman problem*, IIE Trans. **29** (1997), 719–731.
[121] H. Hu, T. Liu, N. Zhao, Y. Zhou, and D. Min, *A hybrid genetic algorithm with perturbation for the multi-depot capacitated arc routing problem*, J. Appl. Sci. **13** (2013), 32–39.
[122] S.H. Huang and P.C. Lin, *Multi-treatment capacitated arc routing of construction machinery in Taiwan's smooth road project*, Autom. Construct. **21** (2012), 210–218.
[123] S.H. Huang and T. Lin, *Using ant colony optimization to solve periodic arc routing problem with refill points*, J. Ind. Prod. Eng **31** (2014), 441–451.
[124] S. Huber, *Strategic decision support for the bi-objective location-arc routing problem*, 49th Hawaii International Conference on System Sciences, IEEE Computer Society Press, Koloa, 2016, pp. 1407–1416.
[125] S. Ichoua, M. Gendreau, and J.Y. Potvin, *Vehicle dispatching with time-dependent travel times*, Eur. J. Oper. Res. **144** (2003), 379–396.
[126] M. Iori and S. Novellani, *Optimizing the nozzle path in the 3D printing process*, Design Tools and Methods in Industrial Engineering. ADM 2019. Lecture Notes in Mechanical Engineering., Springer, Cham, 2020, pp. 912–924.
[127] S. Irnich, *Solution of real-world postman problems*, Eur. J. Oper. Res. **190** (2008), 52–67.

[128] E.L. Johnson and S. Wøhlk, *Solving the capacitated arc routing problem with time windows using column generation*, Tech. report, Aarhus School of Business, 2009.

[129] M. Kasaei and F.S. Salman, *Arc routing problems to restore connectivity of a road network*, Transp. Res. E: Logist. Transp. Rev. **95** (2016), 177–206.

[130] L. Kiilerich and S. Wøhlk, *New large-scale data instances for CARP and new variations of CARP*, INFOR: Inform. Syst. Oper. Res. **56** (2018), 1–32.

[131] D. Krushinsky and T. van Woensel, *Lower and upper bounds for location-arc routing problems with vehicle capacity constraints*, Eur. J. Oper. Res. **244** (2015), 100–109.

[132] N. Labadi, C. Prins, and M. Reghioui, "*An evolutionary algorithm with distance measure for the split delivery capacitated arc routing problem*," *Recent Advances in Evolutionary Computation for Combinatorial Optimization. Studies in Computational Intelligence*, vol. **153**, C. Cotta and J. Hemert (eds), Springer Berlin Heidelberg, Berlin, Heidelberg, 2008, pp. 275–294.

[133] N. Labadi, C. Prins, and M. Reghioui, "*GRASP with path relinking for the capacitated arc routing problem with time windows*," *Advances in Computational Intelligence in Transport, Logistics, and Supply Chain Management*, A. Fink and F. Rothlauf (eds), Springer, Berlin Heidelberg, 2008, pp. 111–135.

[134] P. Lacomme, C. Prins, and M. Sevaux, *A genetic algorithm for a bi-objective capacitated arc routing problem*, Comput. Oper. Res. **33** (2006), 3473–3493.

[135] D. Laganà, R. Paradiso, and F. Vocaturo, *Exact solution of a periodic multi-vehicle arc routing problem*, WARP3 Proceedings, Pizzo, Italy, 2019.

[136] S. Lannez, C. Artigues, J. Damay, and M. Gendreau, *A railroad maintenance problem solved with a cut and column generation matheuristic*, Networks **66** (2015), 40–56.

[137] G. Laporte, R. Musmanno, and F. Vocaturo, *An adaptive large neighbourhood search heuristic for the capacitated arc-routing problem with stochastic demands*, Transp. Sci. **44** (2010), 125–135.

[138] J. Lenstra and A. Rinnooy Kan, *On general routing problems*, Networks **6** (1976), 273–280.

[139] A.N. Letchford and R. Eglese, *The rural postman problem with deadline classes*, Eur. J. Oper. Res. **105** (1998), 390–400.

[140] A.N. Letchford and A. Oukil, *Exploiting sparsity in pricing routines for the capacitated arc routing problem*, Comput. Oper. Res. **36** (2009), 2320–2327.

[141] L. Levy and L. Bodin, "*Scheduling the postal carriers for the United States Postal Service: An application of arc partitioning and routing*," *Vehicle Routing: Methods and Studies*, B.L. Golden and A. Assad (eds), North-Holland, Amsterdam, 1988, pp. 359–394.

[142] M. Li, L. Zhen, S. Wang, W. Lv, and X. Qu, *Unmanned aerial vehicle scheduling problem for traffic monitoring*, Comput. Ind. Eng. **122** (2018), 15–23.

[143] M. Liu, H.K. Singh, and T. Ray, *A memetic algorithm with a new split scheme for solving dynamic capacitated arc routing problems*, 2014 IEEE Congress Evolutionary Computation (CEC), 2014, pp. 595–602.

[144] Y. Liu, J. Shi, Z. Liu, J. Huang, and T. Zhou, *Two-layer routing for high-voltage powerline inspection by cooperated ground vehicle and drone*, Energies **12** (2019), 1–20.

[145] H. Longo, M. Poggi de Aragão, and E. Uchoa, *Solving capacitated arc routing problems using a transformation to the CVRP*, Comput. Oper. Res. **33** (2006), 1823–1837.

[146] T. Lotan, D. Cattrysse, and D. van Oudheusden, *Wintergritting in the province of Antwerp – a combined locationand routing problem*, JORBEL Belgian J. Oper. Res. Stat. Comput. Sci. **36** (1996), 141–157.

[147] Y. Lu, G. Josse, T. Emrich, U. Demiryurek, M. Renz, C. Shahabi, and M. Schubert, *Scenic routes now: Efficiently solving the time-dependent arc orienteering problem*, Proceedings of the 2017 ACM on Conference on Information and Knowledge Management, CIKM '17, ACM, New York, NY, 2017, pp. 487–496.

[148] O. Lum, C. Cerrone, B.L. Golden, and E. Wasil, *Partitioning a street network into compact, balanced, and visually appealing routes*, Networks **69** (2017), 290–303.

[149] O. Lum, B.L. Golden, and E. Wasil, *OAR Lib: An open source arc routing library*, Math. Prog. Comput. **11** (2019), 587–629.

[150] O. Lum, R. Zhang, B.L. Golden, and E. Wasil, *A hybrid heuristic procedure for the windy rural postman problem with zigzag time windows*, Comput. Oper. Res. **88** (2017), 247–257.

[151] J. MacLachlan, Y. Mei, J. Branke, and M. Zhang, *Genetic programming hyper-heuristics with vehicle collaboration for uncertain capacitated arc routing problems*, Evol Comput (2019), 1–29, https://doi.org/10.1162/evco_a_00267.

[152] S. Majumder, S. Kar, and T. Pal, *Uncertain multi-objective Chinese postman problem*, Soft Comput. **23** (2019), 11557–11572.

[153] C. Malandraki and M. Daskin, *The maximum benefit Chinese postman problem and the maximum benefit traveling salesman problem*, Eur. J. Oper. Res. **65** (1993), 218–234.

[154] S.K. Mandal, D. Pacciarelli, A. Løkketangen, and G. Hasle, *A memetic NSGA-II for the bi-objective mixed capacitated general routing problem*, J. Heuristics **21** (2015), 359–390.

[155] R. Martinelli, T.A. Pham, T. Vidal, A. Baffa, M.H. Hà, and X.H. Nguyen, *Time-dependent shortest path optimization and capacitated arc routing problems*, WARP3 Proceedings, Pizzo, Italy, 2019.

[156] P. Matl, R.F. Hartl, and T. Vidal, *Workload equity in vehicle routing: The impact of alternative workload resources*, Comput. Oper. Res. **110** (2019), 116–129.

[157] Y. Mei, K. Tang, and X. Yao, *Capacitated arc routing problem in uncertain environments*, IEEE Congress on Evolutionary Computation, 2010, pp. 1–8.

[158] Y. Mei, K. Tang, and X. Yao, *Decomposition-based memetic algorithm for multi-objective capacitated arc routing problem*, IEEE Trans. Evol. Comput. **15** (2011), 151–165.

[159] Y. Mei, K. Tang, and X. Yao, *A memetic algorithm for periodic capacitated arc routing problem*, IEEE Trans. Syst. Man Cybern. B **41** (2011), 1654–1667.

[160] Y. Mei, K. Tang, and X. Yao, "*Evolutionary computation for dynamic capacitated arc routing problem*," *Evolutionary Computation for Dynamic Optimization Problems*. Studies in Computational Intelligence, S. Yang and X. Yao (eds), Springer, Berlin, 2013, pp. 377–401.

[161] W.K. Mennell, Heuristics for solving three routing problems: Close-enough traveling salesman problem, close-enough vehicle routing problem, sequence-dependent team orienteering problem, Ph.D. thesis, University of Maryland, College Park, 2009.

[162] E. Minieka, *The Chinese postman problem for mixed networks*, Manage. Sci. **25** (1979), 643–648.

[163] I. Monroy, C. Amaya, and A. Langevin, *The periodic capacitated arc routing problem with irregular services*, Discrete Appl. Math. **161** (2013), 691–701.

[164] M. Monroy-Licht, C.A. Amaya, and A. Langevin, *The rural postman problem with time windows*, Networks **64** (2014), 169–180.

[165] M. Monroy-Licht, C.A. Amaya, and A. Langevin, *Adaptive large neighborhood search algorithm for the rural postman problem with time windows*, Networks **70** (2017), 44–59.
[166] M. Monroy-Licht, C.A. Amaya, A. Langevin, and L.M. Rousseau, *The rescheduling arc routing problem*, Int. Trans. Oper. Res. **24** (2017), 1325–1346.
[167] M.C. Mourão and L.S. Pinto, *An updated annotated bibliography on arc routing problems*, Networks **70** (2017), 144–194.
[168] P.A. Mullaseril, Capacitated rural postman problem with time windows and split delivery, Ph.D. thesis, University of Arizona, 1996.
[169] P.A. Mullaseril, M. Dror, and J. Leung, *Split-delivery routeing heuristics in livestock feed distribution*, J. Oper. Res. Soc. **48** (1997), 107–116.
[170] L. Muyldermans, D. Cattrysse, D. van Oudheusden, and T. Lotan, *Districting for salt spreading operations*, Eur. J. Oper. Res. **139** (2002), 521–532.
[171] L. Muyldermans and G. Pang, *A guided local search procedure for the multi-compartment capacitated arc routing problem*, Comput. Oper. Res. **37** (2010), 1662–1673.
[172] G. Nagy and S. Salhi, *Location-routing: Issues, models and methods*, Eur. J. Oper. Res. **177** (2007), 649–672.
[173] Y. Nobert and J. Picard, *An optimal algorithm for the mixed Chinese postman problem*, Networks **27** (1996), 95–108.
[174] J. Nossack, B.L. Golden, E. Pesch, and R. Zhang, *The windy rural postman problem with a time-dependent zigzag option*, Eur. J. Oper. Res. **258** (2017), 1131–1142.
[175] H. Oh, S. Kim, A. Tsourdos, and B.A. White, *Coordinated road-network search route planning by a team of UAVs*, Int. J. Syst. Sci. **45** (2014), 825–840.
[176] H. Oh, H.S. Shin, A. Tsourdos, B.A. White, and P. Silson, *Coordinated road-network search for multiple UAVs using Dubins path*, Advances in Aerospace Guidance, Navigation and Control, Springer Berlin Heidelberg, 2011, pp. 55–66.
[177] C. Orloff, *A fundamental problem in vehicle routing*, Networks **4** (1974), 35–64.
[178] B.E. Oruc and B.Y. Kara, *Post-disaster assessment routing problem*, Transp. Res. B: Methodol. **116** (2018), 76–102.
[179] A. Otto, N. Agatz, J.F. Campbell, B.L. Golden, and E. Pesch, *Optimization approaches for civil applications of unmanned aerial vehicles (UAVs) or aerial drones: A survey*, Networks **72** (2018), 411–458.
[180] M.L. P. Pelikánová, J. Fink, *Winter road maintenance in Czech Republic*, WARP3 Proceedings, 2019.
[181] W. Padungwech, Heuristic algorithms for dynamic capacitated arc routing, Ph.D. thesis, School of Mathematics, Cardiff University, 2018.
[182] W. Padungwech, J. Thompson, and R. Lewis, *Effects of update frequencies in a dynamic capacitated arc routing problem*, Networks (2021) in press.
[183] C. Papadimitriou, *On the complexity of edge traversing*, J. ACM **23** (1976), 544–554.
[184] D. Pecin and E. Uchoa, *Comparative analysis of capacitated arc routing formulations for designing a new branch-cut-and-price algorithm*, Transp. Sci. **53** (2019), 1673–1694.
[185] N. Perrier, J. Campbell, A. Langevin, and M. Gendreau, "*Vehicle routing models and algorithms for winter road spreading operations*," *Hybrid Algorithms for Service, Computing and Manufacturing Systems: Routing and Scheduling Solutions*, J.R. Montoya-Torres, A.A. Juan, J.H. Huatuco, J. Faulin, and G.L. Rodriguez-Verjan (eds), IGI Global, Hershey, PA, 2012, pp. 15–45.
[186] N. Perrier, A. Langevin, and J.F. Campbell, *A survey of models and algorithms for winter road maintenance. Part I: System design for spreading and plowing*, Comput. Oper. Res. **33** (2006), 209–238.
[187] N. Perrier, A. Langevin, and J.F. Campbell, *A survey of models and algorithms for winter road maintenance. Part II: System design for snow disposal*, Comput. Oper. Res. **33** (2006), 239–262.
[188] N. Perrier, A. Langevin, and J.F. Campbell, *A survey of models and algorithms for winter road maintenance. Part III: Vehicle routing and depot location for spreading*, Comput. Oper. Res **34** (2007), 211–257.
[189] N. Perrier, A. Langevin, and J.F. Campbell, *A survey of models and algorithms for winter road maintenance. Part IV: Vehicle routing and fleet sizing for plowing and snow disposal*, Comput. Oper. Res **34** (2007), 258–294.
[190] C. Prins and S. Bouchenoua, "*A memetic algorithm solving the VRP, the CARP and general routing problems with nodes, edges and arcs*," *Recent Advances in Memetic Algorithms. Studies in Fuzziness and Soft Computing 166*, W.E. Hart, J.E. Smith, and N. Krasnogor (eds), Springer, Berlin, 2005, pp. 351–370.
[191] C. Prodhon and C. Prins, *A survey of recent research on location-routing problems*, Eur. J. Oper. Res. **238** (2014), 1–17.
[192] O. Quirion-Blais, A. Langevin, and M. Trépanier, *A case study of combined winter road snow plowing and de-icer spreading*, Can. J. Civil Eng. **44** (2017), 1005–1013.
[193] G. Razmara, Snow removal routing problems: Theory and applications, Ph.D. thesis, Linköping University, Sweden, 2004.
[194] A. Renaud, N. Absi, and D. Feillet, *The stochastic close-enough arc routing problem*, Networks **69** (2017), 205–221.
[195] J. Riera-Ledesma and J.J. Salazar-González, *Solving the team orienteering arc routing problem with a column generation approach*, Eur. J. Oper. Res. **262** (2017), 14–27.
[196] J.P. Riquelme-Rodriguez, M. Gamache, and A. Langevin, *Adaptive large neighborhood search for the periodic capacitated arc routing problem with inventory constraints*, Networks **64** (2014), 125–139.
[197] J.P. Riquelme-Rodriguez, M. Gamache, and A. Langevin, *Periodic capacitated arc-routing problem with inventory constraints*, J. Oper. Res. Soc. **65** (2014), 1840–1852.
[198] J.P. Riquelme-Rodríguez, M. Gamache, and A. Langevin, *Location arc routing problem with inventory constraints*, Comput. Oper. Res. **76** (2016), 84–94.
[199] J.C. Rivera and S.A. Lenis, *The cumulative capacitated arc routing problem*, VeRoLog 2019 Proceedings, Seville, Spain, 2019.
[200] A.M. Rodrigues and J. Soeiro, *Cutting path as a rural postman problem: Solutions by memetic algorithms*, Int. J. Combin. Optim. Probab. Inform. **3** (2012), 22–37.
[201] D.G. Rossit, D. Vigo, F. Tohmé, and M. Frutos, *Visual attractiveness in routing problems: A review*, Comput. Oper. Res. **103** (2019), 13–34.
[202] D.D. Russo, C. Cerrone, and A. di Placido, *The mixed-constrained routing problem – A combination of CEARP and CETSP*, WARP3 Proceedings, Pizzo, Italy, 2019.
[203] H. Sahin, B.Y. Kara, and O.E. Karasan, *Debris removal during disaster response: A case for Turkey*, Socioecon. Plan. Sci. **53** (2016), 49–59.
[204] M.A. Salazar-Aguilar, A. Langevin, and G. Laporte, *Synchronized arc routing for snow plowing operations*, Comput. Oper. Res. **39** (2012), 1432–1440.
[205] M.A. Salazar-Aguilar, A. Langevin, and G. Laporte, *The synchronized arc and node routing problem: Application to road marking*, Comput. Oper. Res. **40** (2013), 1708–1715.
[206] S.E. Schaeffer, R.Z. Ríos-Mercado, and E. Fernández, The windy prize-collecting rural postman problem: An ant-colony based heuristic, Tech. report, UANL Mexico, 2014.
[207] A. Shafahi and A. Haghani, *Generalized maximum benefit multiple Chinese postman problem*, Transp. Res. C **55** (2015), 261–272.

[208] R. Shang, J. Wang, L. Jiao, and Y. Wang, *An improved decomposition-based memetic algorithm for multi-objective capacitated arc routing problem*, Appl. Soft Comput. **19** (2014), 343–361.

[209] R. Shang, Y. Wang, J. Wang, L. Jiao, S. Wang, and L. Qi, *A multi-population cooperative coevolutionary algorithm for multi-objective capacitated arc routing problem*, Inform. Sci. **277** (2014), 609–642.

[210] R. Shuttleworth, B.L. Golden, S. Smith, and E.A. Wasil, "*Advances in meter reading: Heuristic solution of the close enough traveling salesman problem over a street network*," *The Vehicle Routing Problem: Lastest Advances and New Challenges*, B.L. Golden, S. Raghavan, and E.A. Wasil (eds), Springer, New York, 2008, pp. 487–501.

[211] I.K. Singgih, J. Lee, and B.I. Kim, "*Node and edge surveillance using drones considering required camera altitude and battery charging*," *Advances in Production Management Systems. Smart Manufacturing for Industry 4.0. APMS 2018*Springer Nature, Switzerland, 2018, pp. 55–66.

[212] A. Sipahioglu, G. Kirlik, O. Parlaktuna, and A. Yazici, *Energy constrained multi-robot sensor-based coverage path planning using capacitated arc routing approach*, Robot. Autom. Syst. **58** (2010), 529–538.

[213] W. Souffriau, P. Vansteenwegen, G. van den Berghe, and D. van Oudheusden, *The planning of cycle trips in the province of East Flanders*, Omega **39** (2011), 209–213.

[214] J. Sun, G. Tan, and G. Hou, *A new integer programming formulation for the Chinese postman problem with time dependent travel times*, Int. J. Comput. Inform. Eng. **5** (2011), 410–414.

[215] M. Tagmouti, M. Gendreau, and J.Y. Potvin, *Arc routing problems with time-dependent service costs*, Eur. J. Oper. Res. **181** (2007), 30–39.

[216] M. Tagmouti, M. Gendreau, and J.Y. Potvin, *A variable neighborhood descent heuristic for arc routing problems with time-dependent service costs*, Comput. Ind. Eng. **59** (2010), 954–963.

[217] M. Tagmouti, M. Gendreau, and J.Y. Potvin, *A dynamic capacitated arc routing problem with time-dependent service costs*, Transp. Res. C **19** (2011), 20–28.

[218] G. Tan and J. Sun, "*An integer programming approach for the rural postman problem with time dependent travel times*," *Computing and Combinatorics*Springer Berlin Heidelberg, Berlin, Heidelberg, 2011, pp. 414–431.

[219] D.V. Thomaz, G.V. Loch, C.T. Scarpin, and C. Schenekemberg. A Mathematical Model for the Periodic Capacitated Arc Routing Problem with Time Windows, in IEEE Latin America Transactions, vol. 16, 2018 pp. 2567–2573.

[220] P. Vansteenwegen, W. Souffriau, and K. Sörensen, *Solving the mobile mapping van problem: A hybrid metaheuristic for capacitated arc routing with soft time windows*, Comput. Oper. Res. **37** (2010), 1870–1876.

[221] H.F. Wang and Y.P. Wen, *Time-constrained Chinese postman problems*, Comput. Math. Appl. **44** (2002), 375–387.

[222] E.J. Willemse and J.W. Joubert, *Benchmark dataset for undirected and mixed capacitated arc routing problems under time restrictions with intermediate facilities*, Data Brief **8** (2016), 972–977.

[223] S. Wøhlk, Contributions to arc routing, Ph.D. thesis, University of Southern Denmark, 2005.

[224] S. Wøhlk and G. Laporte, *A districting-based heuristic for the coordinated capacitated arc routing problem*, Comput. Oper. Res. **111** (2019), 271–284.

[225] A. Yazici, G. Kirlik, O. Parlaktuna, and A. Sipahioglu, *A dynamic path planning approach for multirobot sensor-based coverage considering energy constraints*, IEEE Trans. Cybern. **44** (2014), 305–314.

[226] V.F. Yu and S.W. Lin, *Iterated greedy heuristic for the time-dependent prize-collecting arc routing problem*, Comput. Ind. Eng. **90** (2015), 54–66.

[227] E. Zachariadis and C. Kiranoudis, *Local search for the undirected capacitated arc routing problem with profits*, Eur. J. Oper. Res. **210** (2011), 358–367.

[228] R. Zanotti, R. Mansini, G. Ghiani, and E. Guerriero, *A kernel search approach for the time-dependent rural postman problem*, WARP3 Proceedings, Pizzo, Italy, 2019.

[229] H. Zbib, Variants of the path scanning construction heuristic for the no-split multi-compartment capacitated arc routing problem, Tech. report, Aarhus University, 2017.

[230] H. Zbib and G. Laporte, *The commodity-split multi-compartment arc routing problem*, WARP3 Proceedings, Pizzo, Italy, 2019.

[231] H. Zbib and S. Wøhlk, *A comparison of the transport requirements of different curbside waste collection systems in Denmark*, Waste Manage. **87** (2019), 21–32.

[232] H. Zbib and S. Wøhlk, *A multi-move decent algorithm for the no-split multi-compartment capacitated arc routing problem*, VeRoLog 2017 Proceedings, Amsterdam, Netherlands, 2017.

[233] Y. Zhang, Y. Mei, K. Tang, and K. Jiang, *Memetic algorithm with route decomposing for periodic capacitated arc routing problem*, Appl. Soft Comput. **52** (2017), 1130–1142.

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