Linear Algebra

Holberton

Machine Learning

Why

Social media features

Sentiment analysis

Face recognition

Product recommendations

Medical analysis

Stock market prediction

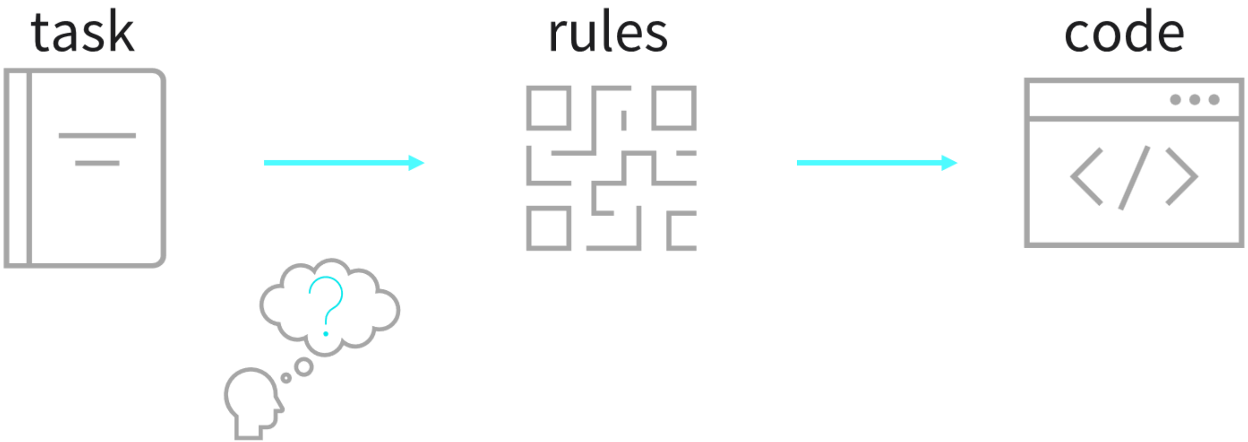
What

“A computer program is said to **learn** from **experience** E with respect to some class of tasks T and **performance** measure P, if its performance at tasks in T, as measured by P, improves with experience E„

-- Tom Mitchel

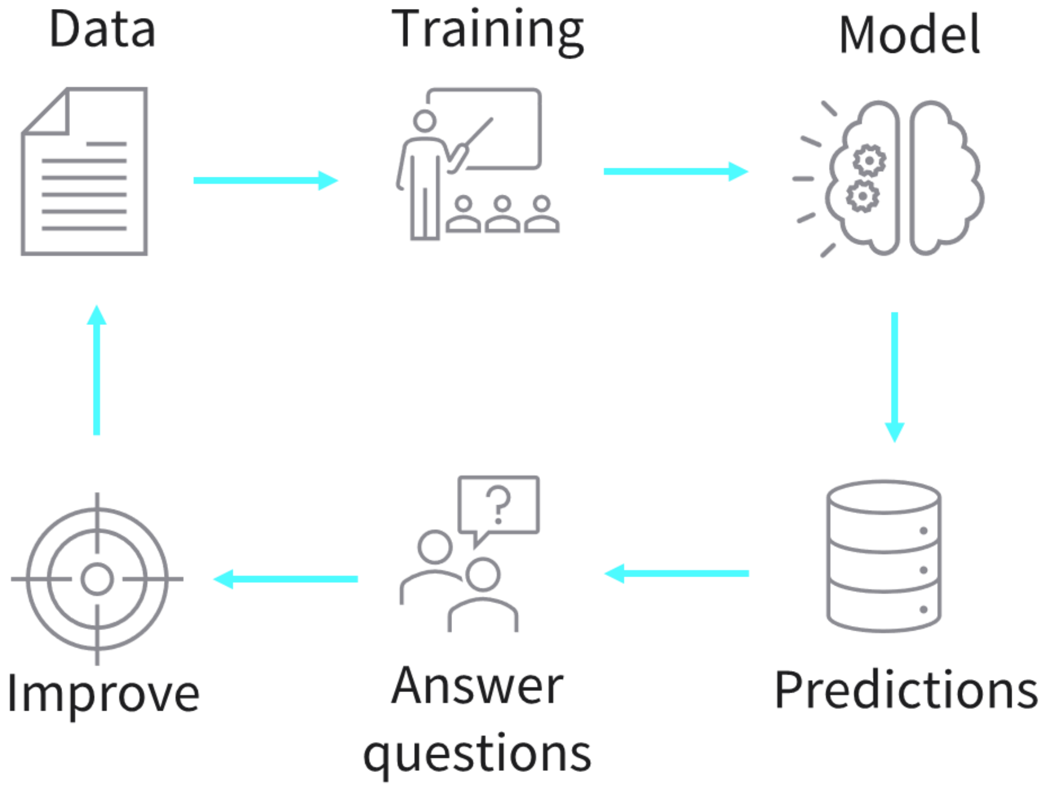
How

- For some problems we cannot simply write down the rules

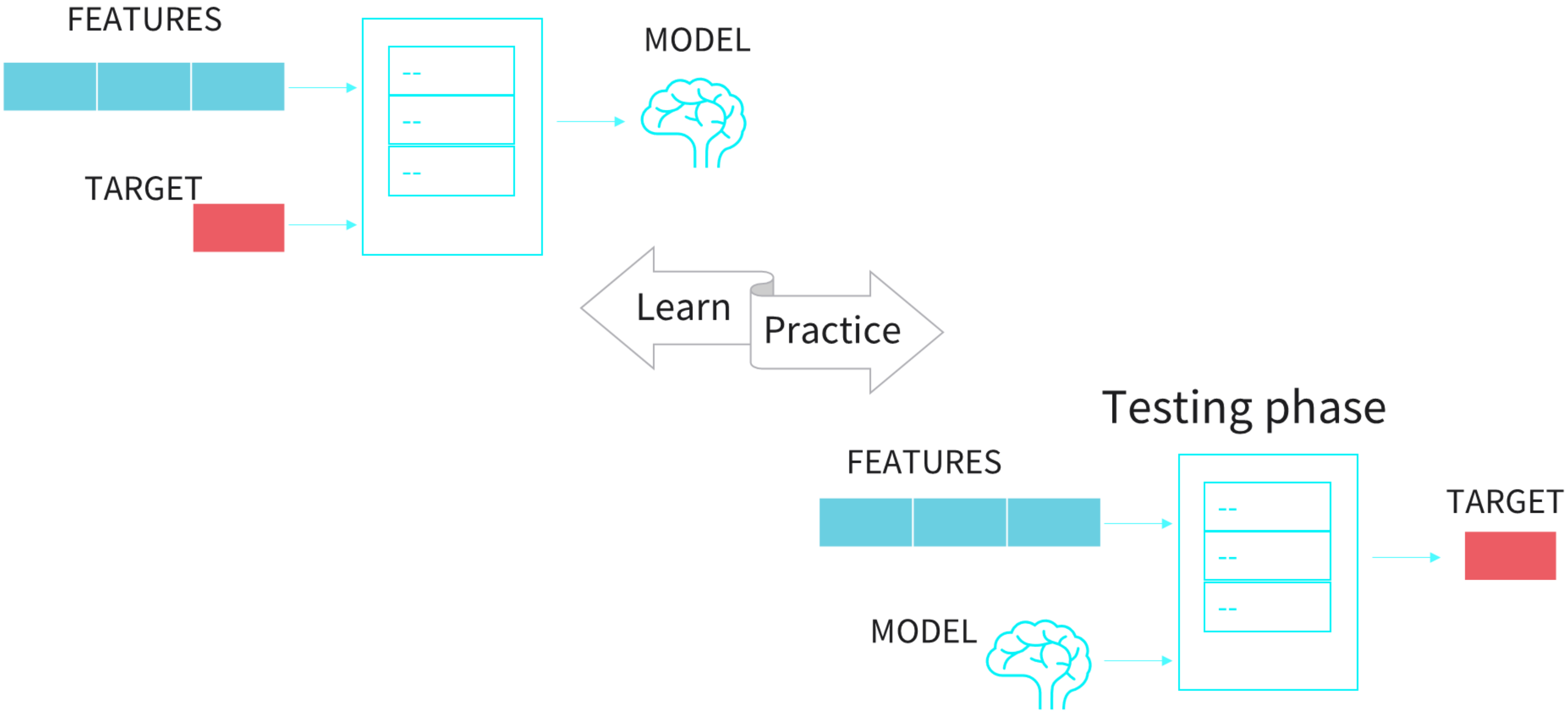


- What if computer systems could learn through trial and error?

Machine Learning Framework



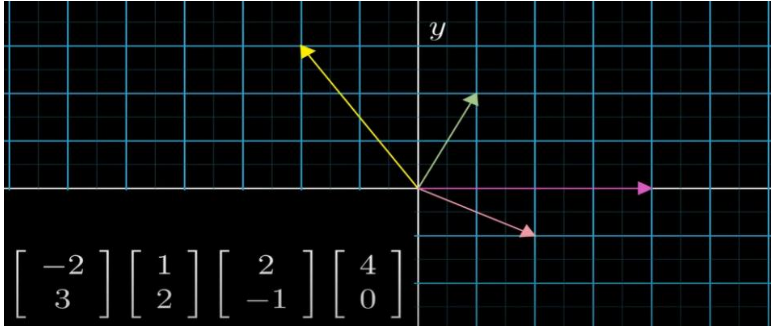
Machine Learning Framework

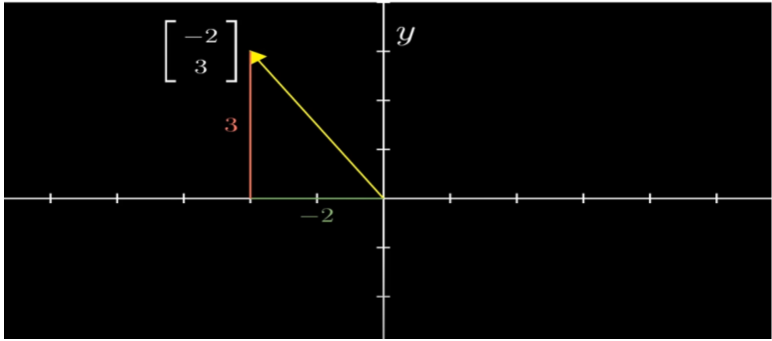


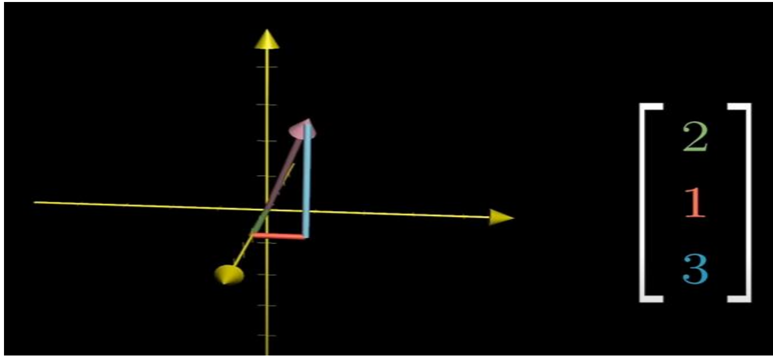
Introduction to Vectors and Matrices

Vectors

- Cornerstone of linear algebra
- Physics: arrows pointing in space
- CS: list of numbers
- Math: generalize both views







Key elements

- Length and direction of vectors

The Matrix

- A m by n matrix is a function of two variables with domains $\{1, 2, \ldots m\}$ and $\{1, 2, \ldots n\}$
- Matrices can be used to store data

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \underbrace{x\begin{bmatrix} a \\ c \end{bmatrix} + y\begin{bmatrix} b \\ d \end{bmatrix}} = \begin{bmatrix} ax+by \\ cx+dy \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}_{\text{Shear}}\left(\underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{Rotation}}\begin{bmatrix} x \\ y \end{bmatrix}\right) = \underbrace{\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{Composition}}\begin{bmatrix} x \\ y \end{bmatrix}$$

Linear transformations

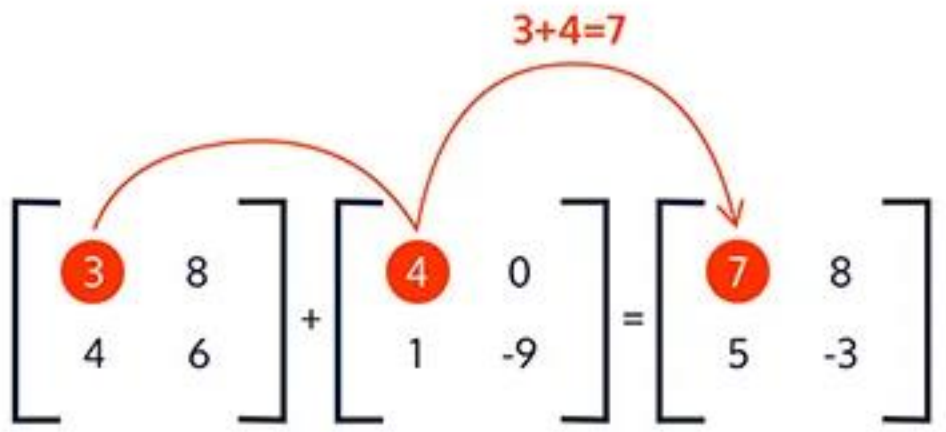
- A linear transform can be represented by an array of coefficients
- The composition of linear transforms yields the product of their matrices

$$\overbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}^{M_2}\overbrace{\begin{bmatrix} e & f \\ g & h \end{bmatrix}}^{M_1} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

Key Operations

Matrix addition

- In matrix addition, corresponding matrix entries are added together



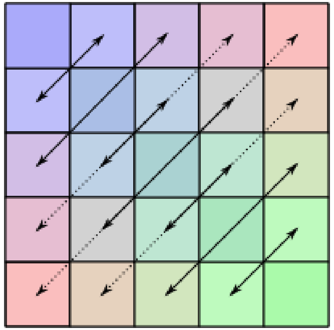
Matrix transpose

- Flip a matrix over its diagonal
- Switch the row and column indices

A

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

- A square matrix whose transpose is equal to itself is called a symmetric matrix $A = A^T$



Matrix dot product: directional multiplication

- **Add vectors**: accumulate the growth contained in several vectors
- **Multiply by a constant**: make an existing vector stronger
- **Dot product**: Apply the directional growth of one vector to another

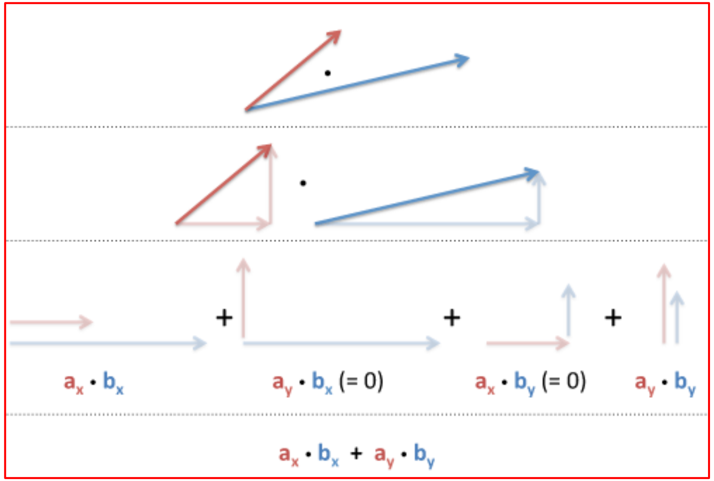
Dot product

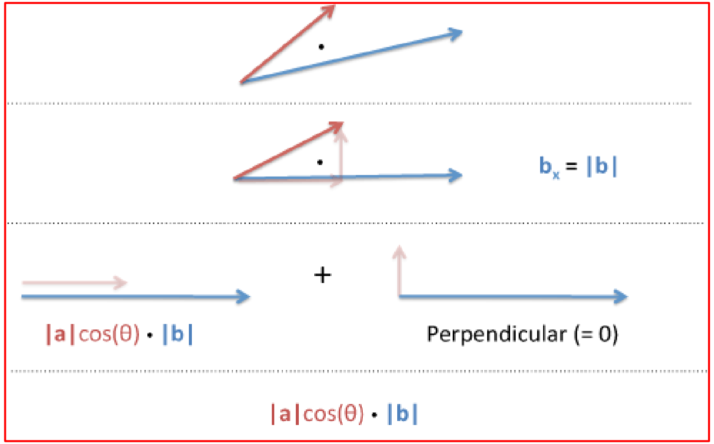
- Rectangular Coordinates: Component-by-component overlap

- $A \cdot B = \sum(A[i,j] \cdot B[i,j])$

- Polar coordinates: Projection

- How much energy is actually going in our original direction?



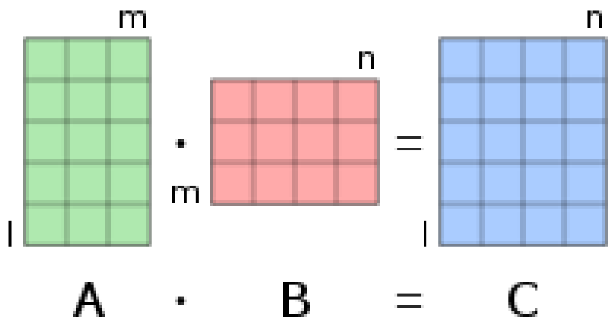


Matrix multiplication

- A binary operation that produces a matrix from two matrices

- The number of columns in the first matrix must be equal to the number of rows in the second matrix

- The resulting matrix has the number of rows of the first and the number of columns of the second matrix



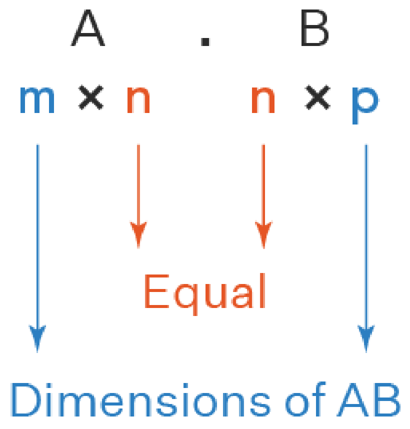
$$\boxed{C_{ij} = \Sigma_1^n (A_{ik} \cdot B_{kj})}$$

Matrix multiplication

$$\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & 1 & 5 \\ -2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} & & \\ & & \end{bmatrix}$$

$$(2 \times 2) \qquad (2 \times 3) \qquad (2 \times 3)$$

Matrix multiplication vs dot product

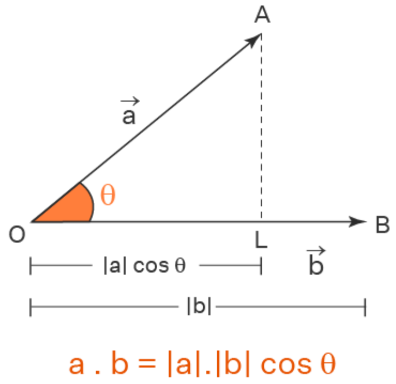


Matrix multiplication

- Combining two matrices to produce a new matrix.
- The number of columns in the first matrix must match the number of rows in the second matrix.
- The product of matrix multiplication is a new matrix

Dot product

- An operation between two vectors to produce a scalar
- The vectors must have the same number of elements



Any questions?