HAB model

2024-04-02

Model

- n polygons $i = 1, \ldots, n$
- t time in weeks: Week 0 is the week of 1 January 2018, $t = 0, \ldots, T$
- q_t is the seasonal indicator: 1 if the week is in the months of December-July, 0 otherwise
- V_i volume to a depth of 15m
- P_{ijt} = proportion of mass in polygon j moving to polygon i in week t.

  Mass conservation implies

  $$W_{jt} = \sum_{i=1}^{n} P_{ijt} \leq 1 \qquad \text{for all } j, t$$

  where the sum is less than 1 when mass is lost at the edges of the set of polygons.
- Retention: $W_{jt} \equiv \sum_{i=1}^{n} P_{ijt}$ proportion of mass in polygon j retained by the system (i.e. not lost at the edges) over the one week period at time t
- $R_{it} \equiv P_{iit}$ = proportion of mass remaining resident in polygon i in week t
- $\mathbf{x}_{it}$ covariates in polygon i at time t (temperature, salinity, etc.)
- At time step t the epoch indicator $E_t | E_{t-1}$ is drawn

  $$\begin{aligned} E_0 &\sim \text{Bernoulli}(\tfrac{1}{2}) \\ E_t | E_{t-1} = 0 &\sim \text{Bernoulli}(\tau_0 q_t) \qquad \text{for } t > 0 \\ E_t | E_{t-1} = 1 &\sim \text{Bernoulli}(\tau_1 q_t) \end{aligned}$$

  where τ_0 is the probability of initiation of an epoch favourable to algal blooms (τ_0 is low), and τ_1 is the probability of the persistence of that epoch.

  (Q: do we need spatial autocorrelation as well? how can we include covariates?)
- π_{it} bloom initiation probability in polygon i at time t

  $$\text{logit}(\pi_{it}) = \mu_0 + \mathbf{x}_{it}^T \beta_0 + \lambda_0 \,\text{logit}(R_{it})$$

- At time step t in each polygon i the innovation indicator is drawn

  $$I_{it} \sim \text{Bernoulli}(\pi_{it} E_t)$$

  An innovation in biomass only occurs in a polygon if t is in a favourable epoch ($E_t \neq 0$).
- New biomass innovation in polygon i at time t

  $$\begin{aligned} A_{it} | I_{it} = 0 &\sim \delta_0 \\ A_{it} | I_{it} = 1 &\sim \text{Gamma}(a, b) \end{aligned}$$

  δ_0 is a point mass at zero: we get no new biomass if $I_{it} = 0$.
- Baseline algal mass distribution $M_{i0} = 0$ for all i: i.e. no mass at time $t = 0$.
- Time evolution of algal mass M_{it} for $t > 0$

  $$M_{it} = e^{\eta_{it}} \left[A_{it} + e^{-\gamma(1-E_t) + E_t(\alpha + \mathbf{x}_{it}^T \beta_2 + \lambda_2 \,\text{logit}\, R_{it})} \sum_{j=1}^{n} P_{ijt} e^{E_t(\mathbf{x}_{jt}^T \beta_1 + \lambda_1 \,\text{logit}\, R_{jt})} M_{j,t-1} \right]$$

where

$$\eta_{it} \sim \text{Normal}(0, \sigma_\eta^2)$$

We may not need both of (λ_1, β_1) (growth before transport) and (λ_2, β_2) (growth after transport).

$\alpha > 0$ is the basic (covariate independent) growth rate of blooms in a favourable epoch $E_t = 1$, $\gamma > 0$ is the decay rate of blooms in an unfavourable epoch $E_t = 0$.

- Observation of algal density Y_{it}

$$\begin{aligned} Y_{it} | M_{it} = 0 &\sim \delta_0 \\ \log Y_{it} | M_{it} > 0, \sigma_\varepsilon^2 &\sim \text{Normal}(\log(M_{it}/V_i), \sigma_\varepsilon^2) \end{aligned}$$

Summary of quantities

Indexing

- $i = 1, \ldots, n$ - labels of polygons
- $t = 0, \ldots, T$ - labels of weeks

Fixed

- q_t seasonal indicator (1 if the week is in the months of December-July, 0 otherwise)
- V_i volume of polygon i to a depth of 15m

Observed/Modelled separately

- $\mathbf{x}_{it}$ - covariates (temperature, salinity, etc)
- P_{ijt} - proportion of mass in polygon j moving to polygon i in week t
- Y_{it} - observed density (in a subset of polygons at a subset of times)

Latent (to be estimated)

- E_t epoch indicator
- I_{it} innovation indicator
- A_{it} innovation biomass
- M_{it} mass in polygon i at time t

Derived

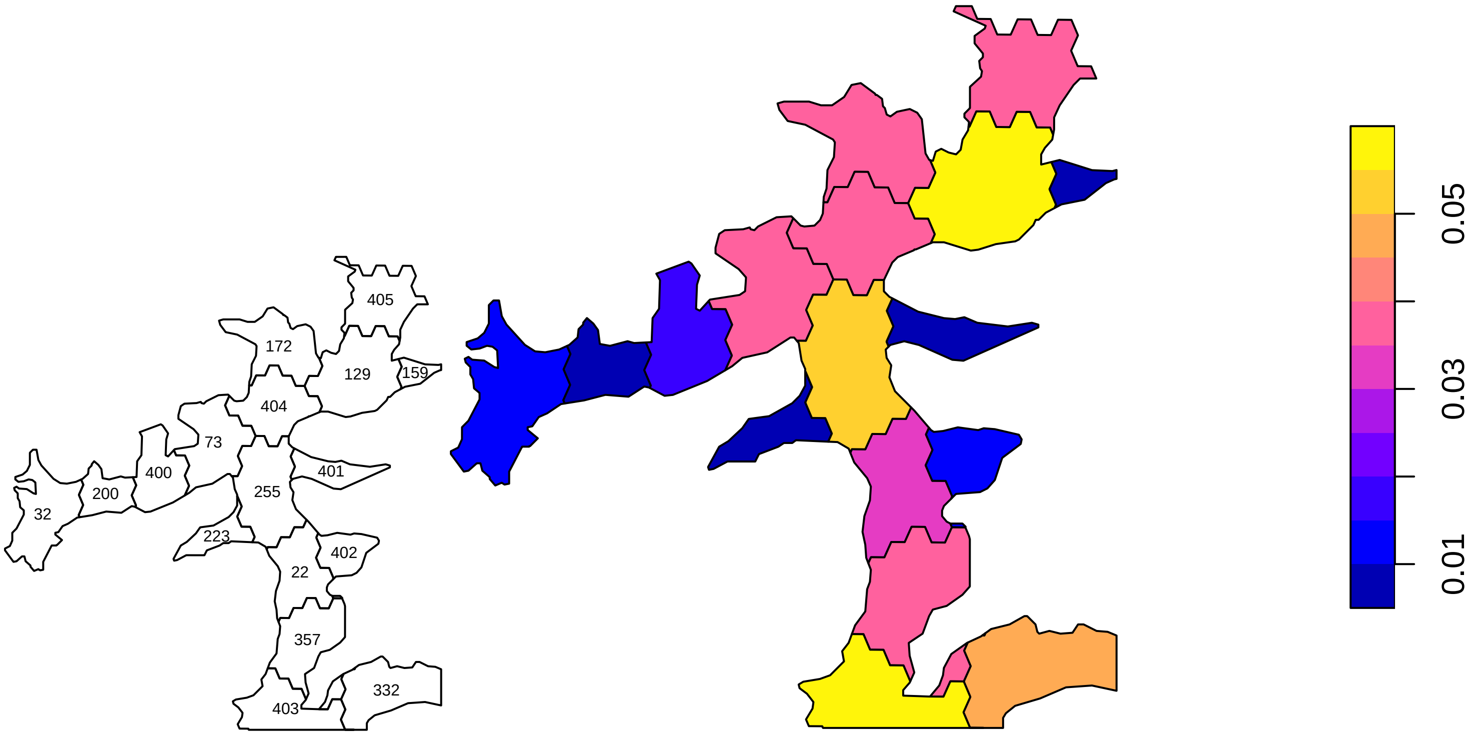
- R_{it} - residency: derived from $\mathbf{P}_t$: $R_{it} = P_{iit}$
- π_{it} - bloom initiation probability: $\text{logit}(\pi_{it}) = \mu_0 + \mathbf{x}_{it}^T \beta_0 + \lambda_0 \,\text{logit}(R_{it})$

Parameters (to be estimated)

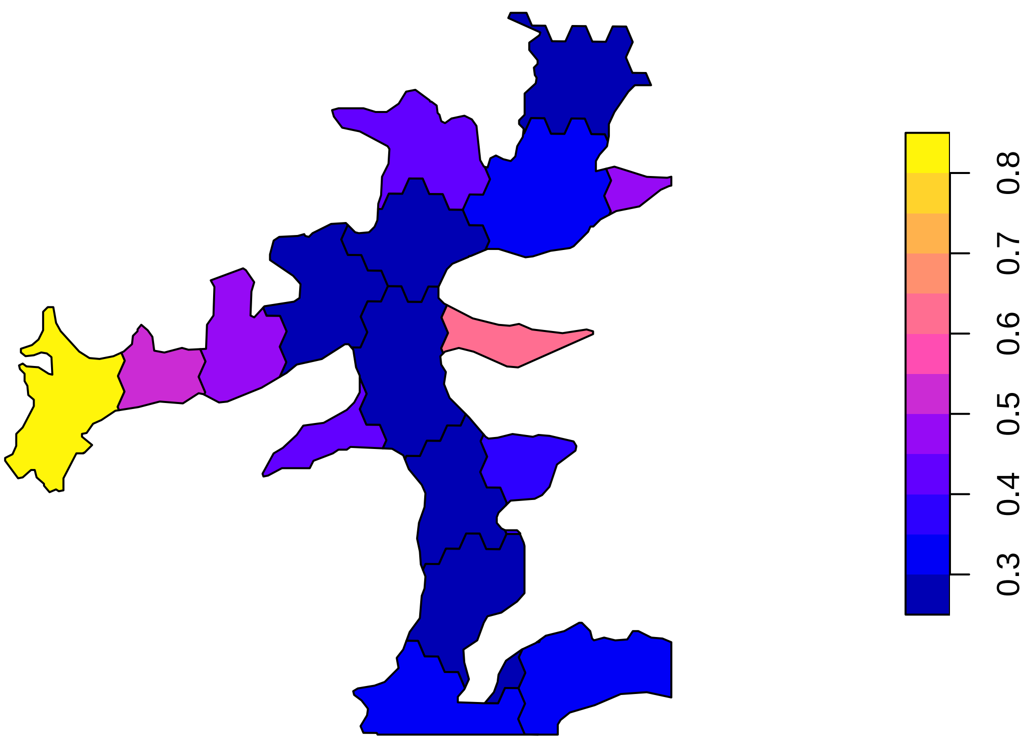
- τ_0 growth epoch initiation probability
- τ_1 growth epoch persistence probability
- (α_0, α_1) baseline log growth parameters in epochs 0 (decay) and 1 (growth)
- μ_0 baseline bloom initiation probability parameter
- (λ_0, β_0) parameters of bloom initiation probability
- (λ_1, β_1) parameters of pre-transport growth
- (λ_2, β_2) parameters of post-transport growth
- σ_η - mass evolution error
- σ_ε - density observation error

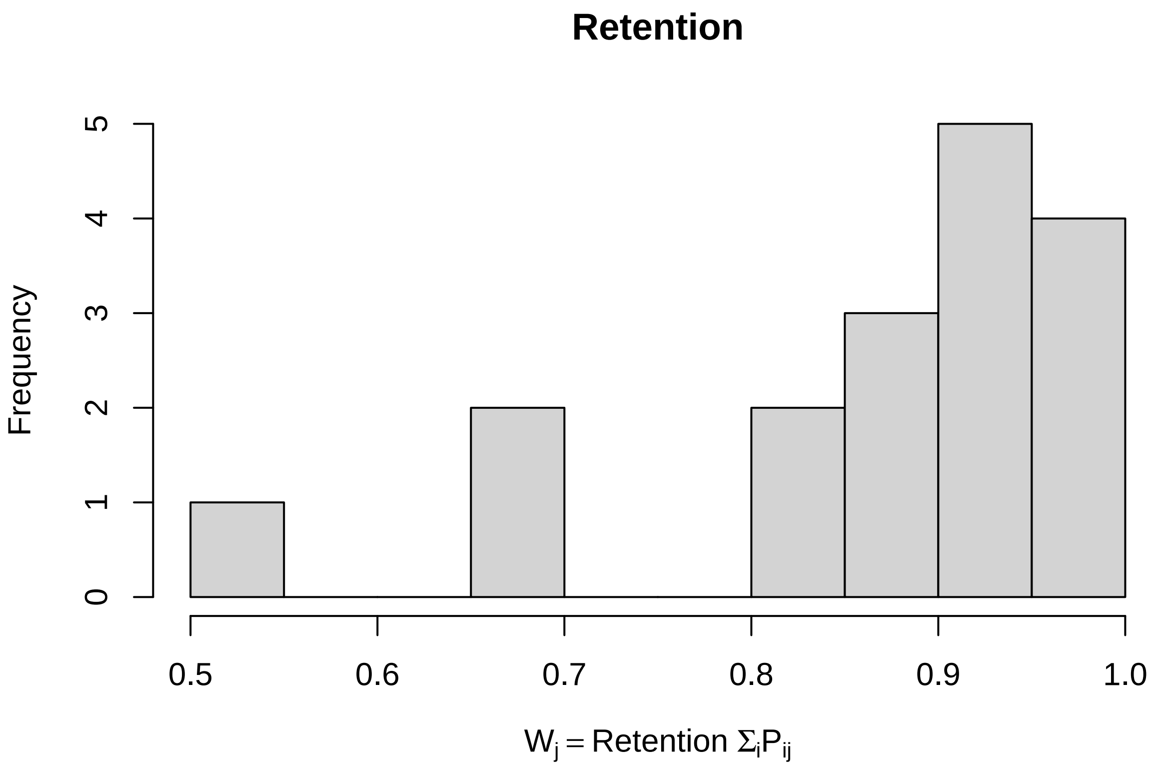
Data in the Nydia area

Volume to 15m (cubic km)

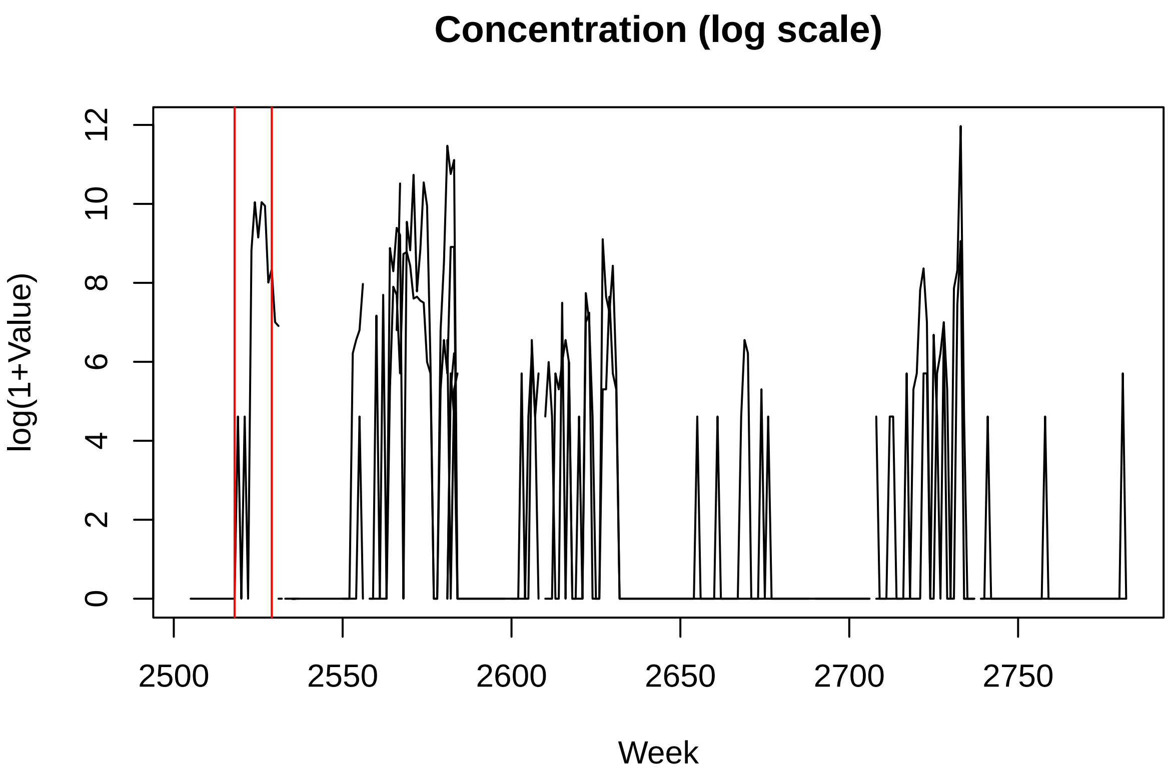


1 week water residency



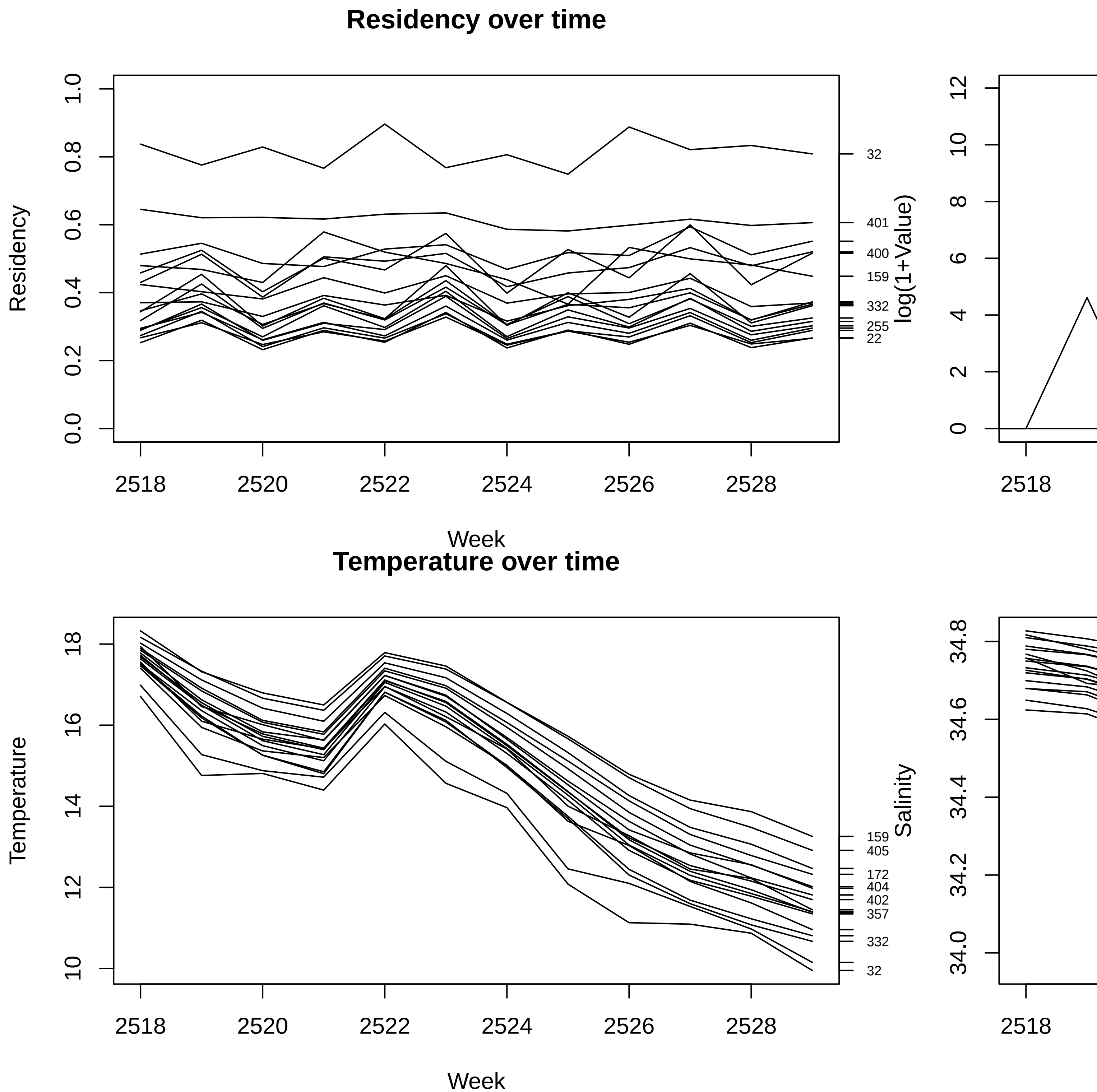


Data on events

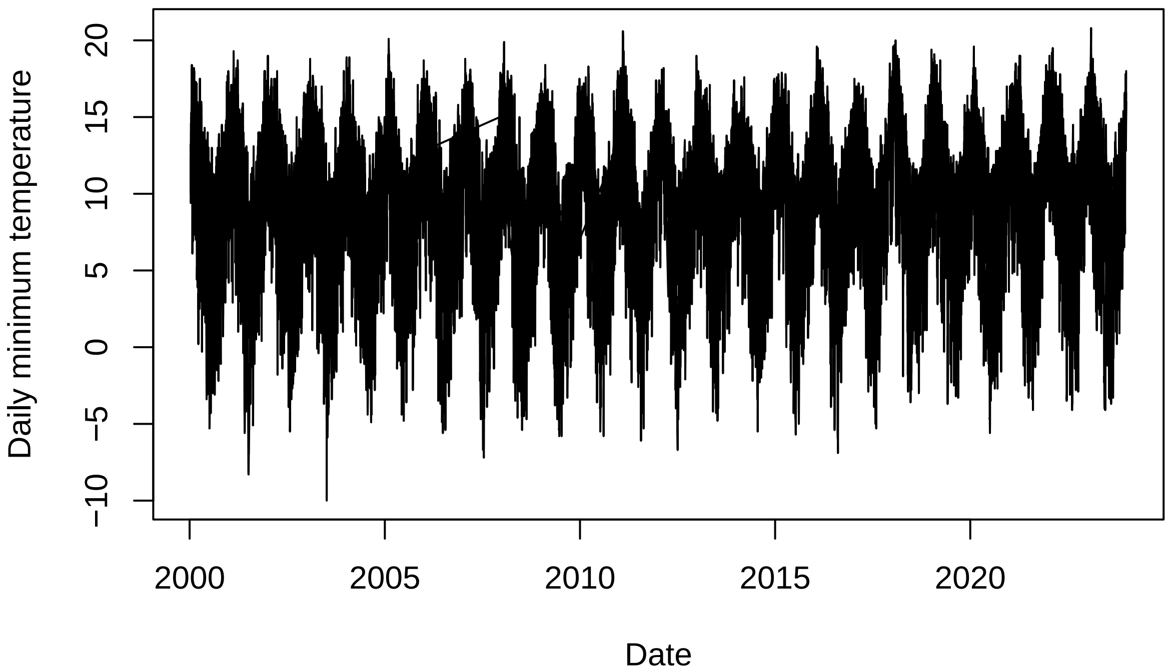


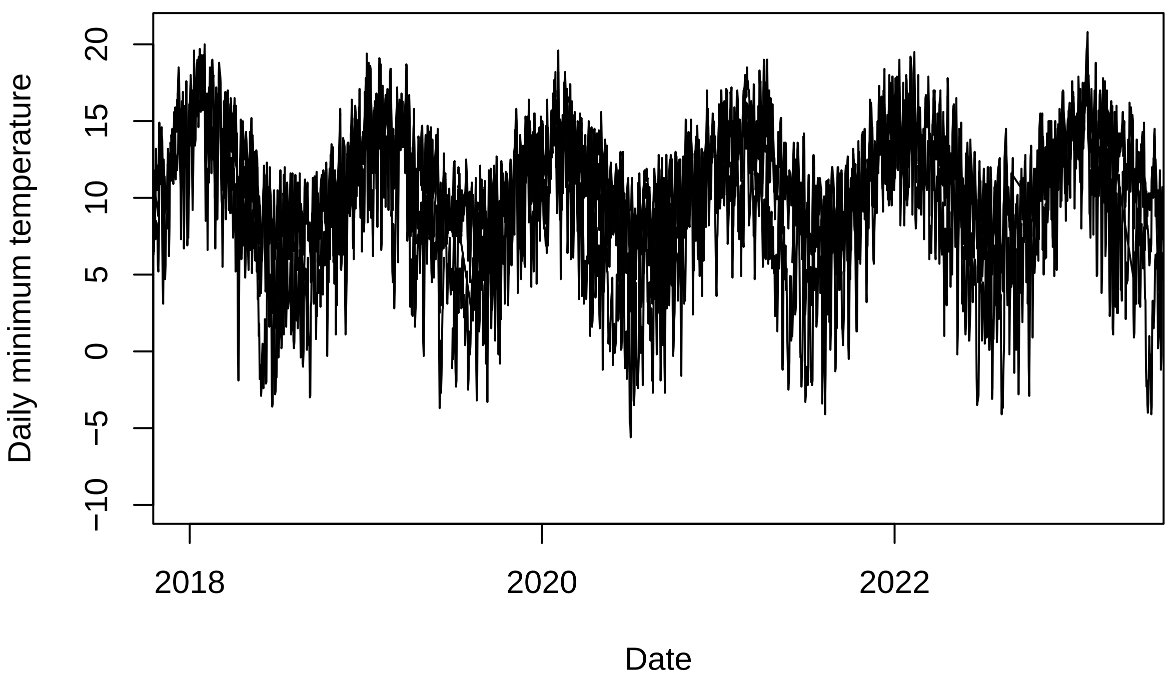
Covariates

Residency: we have matrices P_{ijt} for the weeks 2518-2529 (dates 2018-04-02-2018-06-18)

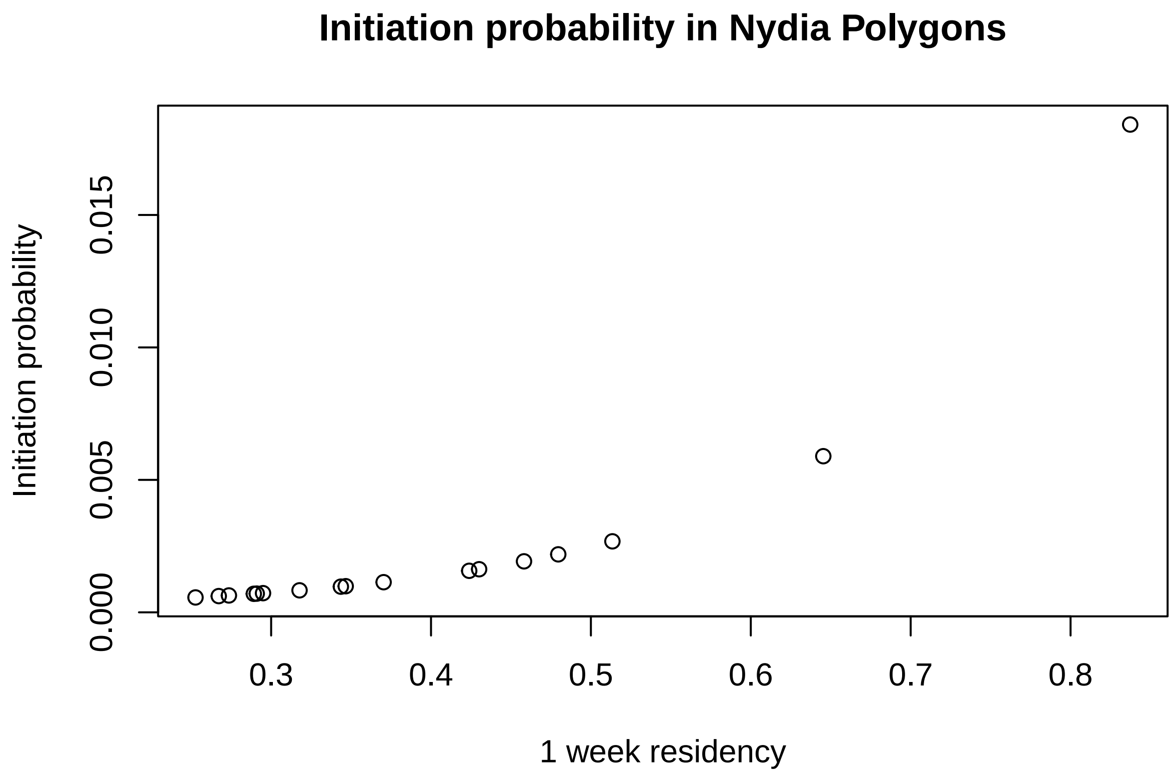


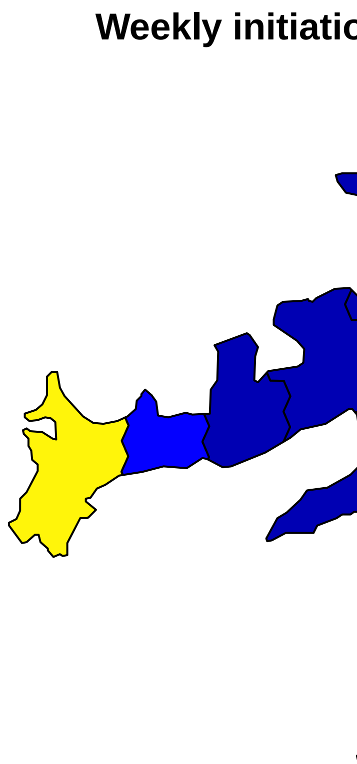
Temperature over time


Daily minimum temperature
20
15
10
5
0
−5
−10
2000
2005
2010
2015
2020
Date


Daily minimum temperature
20
15
10
5
0
−5
−10
2018
2020
2022
Date

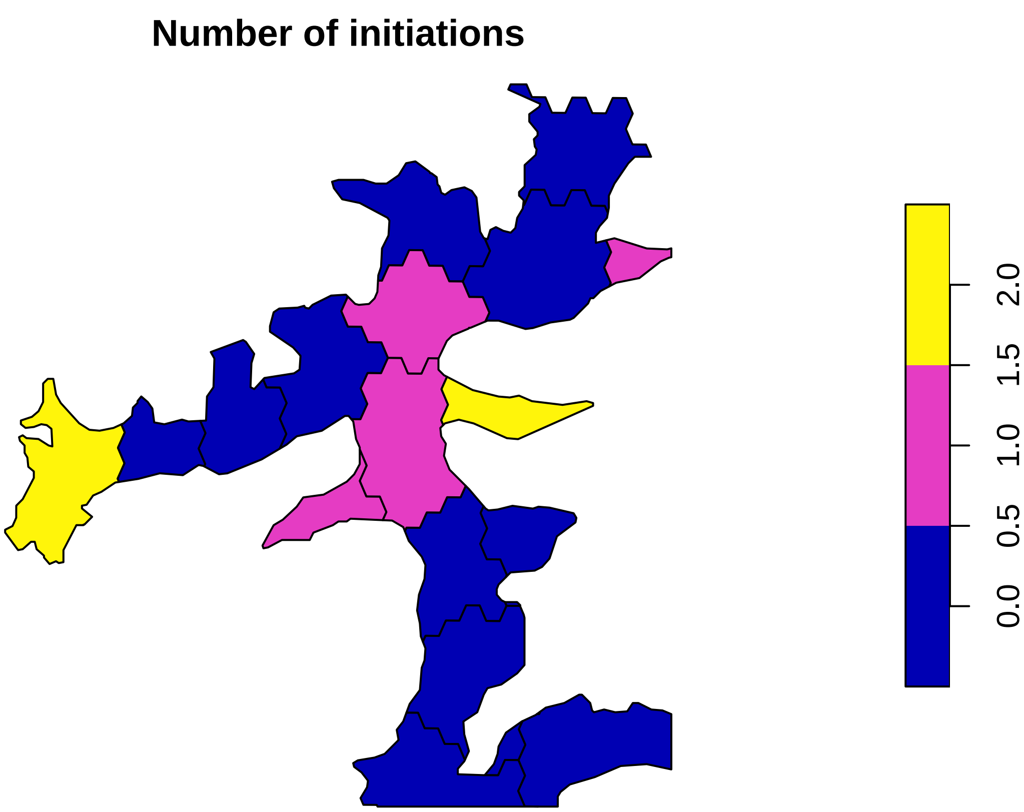
Model implementation

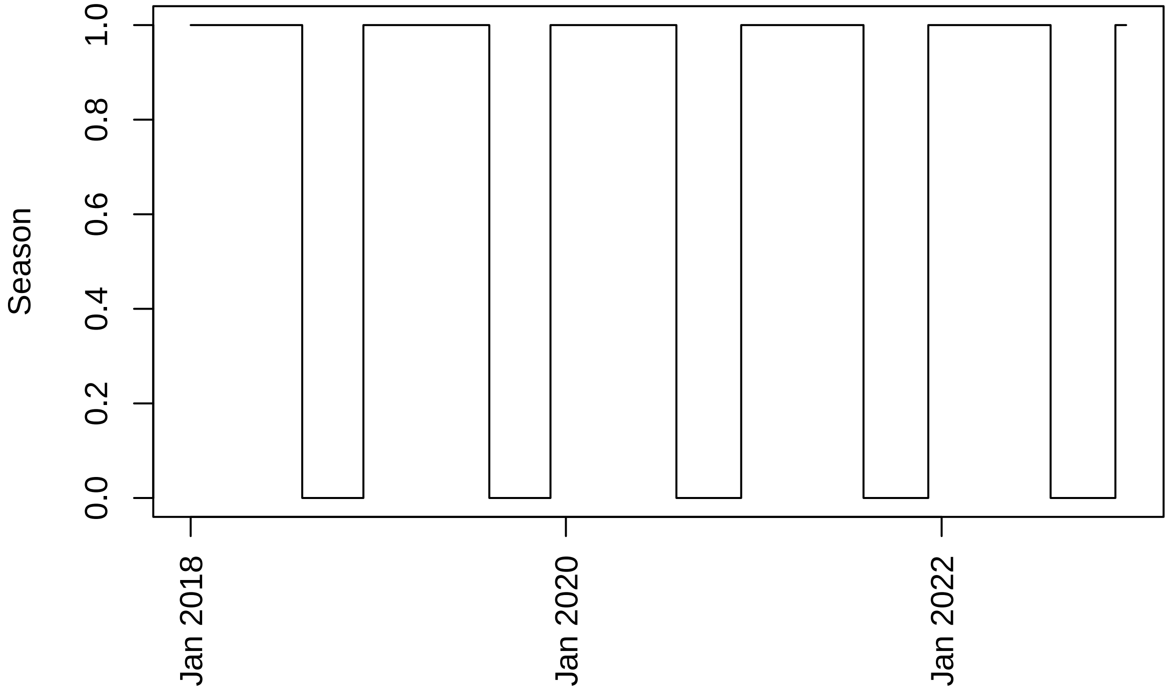





```
##  [1] 0 2 0 0 1 0 0 1 1 0 0 0 2 0 0 1 0

## Warning: st_centroid assumes attributes are constant over geometries
```





Season
1.0
0.8
0.6
0.4
0.2
0.0
Jan 2018
Jan 2020
Jan 2022