0.1 Naive Bayes

0.1.1 Probability Laws

Bayes' Theorem

$$P(Y|X) = \frac{P(Y \cap X)}{P(X)} = \frac{P(X|Y) \times P(Y)}{P(X)}$$

Law of total probability

$$P(A) = P(A \cap B) + P(A \cap \neg B)$$

0.1.2 Naive Bayes

Suppose the categorical outcome variable Y takes on values in the set $\{y_1, y_2, \ldots, y_k\}$ and there are m feature variables $X_1, X_2, \ldots, X_m$. By Bayes Theorem, for $j = 1, 2, \ldots, k$,

$$\begin{aligned}
&P(Y = y_j|X_1 = x_1, X_2 = x_2, \ldots, X_m = x_m) \\
&= \frac{P(X_1 = x_1, X_2 = x_2, \ldots, X_m = x_m|Y = y_j) \times P(Y = y_j)}{P(X_1 = x_1, X_2 = x_2, \ldots, X_m = x_m)}
\end{aligned}$$

Assume Conditional Independence

$$\begin{aligned}
&P(X_1 = x_1, X_2 = x_2, \ldots, X_m = x_m|Y = y_j) \\
&= P(X_1 = x_1|Y = y_j)P(X_2 = x_2|Y = y_j) \ldots P(X_m = x_m|Y = y_j) \\
&= \prod_{i=1}^{m} P(X_i = x_i|Y = y_j)
\end{aligned}$$

Ignore Denominator

$$P(X_1 = x_1, X_2 = x_2, \ldots, X_m = x_m)$$

Finally

For $j = 1, 2, \ldots, k$,

$$\begin{aligned}
&P(Y = y_j|X_1 = x_1, X_2 = x_2, \ldots, X_m = x_m) \\
&\propto P(Y = y_j) \times \prod_{i=1}^{m} P(X_i = x_i|Y = y_j)
\end{aligned}$$

0.1.3 Numerical Underflow

To prevent probability scores from becoming too small to be accurately stored in a computer, we can take logarithm on both sides,

$$\log P(Y = y_j|X_1 = x_1, X_2 = x_2, \ldots, X_m = x_m)$$

$$\propto \log P(Y = y_j) + \sum_{i=1}^{m} \log P(X_i = x_i | Y = y_j)$$

0.1.4 R Implementation

```r
library(e1071)
data <- data.frame(
  watch_lectures = c(TRUE, TRUE, FALSE, TRUE, FALSE, TRUE),
  does_tutorials = c(TRUE, FALSE, TRUE, FALSE, FALSE, TRUE),
  prior_exp = c(FALSE, FALSE, FALSE, TRUE, TRUE, FALSE),
  grades_for_dsa1101 = c('A-', 'B+', 'C', 'B+', 'B-', 'A'),
  stringsAsFactors = TRUE
)
train <- data[1:4,]
test <- data[5:6,]
model <- naiveBayes(grades_for_dsa1101 ~ watch_lectures + does_tutorials
                    + prior_exp, train, laplace = 0)
predict(model, test)

## [1] B+ A-
## Levels: A A- B- B+ C
```

0.1.5 Calculation Intensive Exam Questions & Solutions

```r
table_to_naiveBayes <- function(features, feature_categories, )

## Error:  <text>:1:63:  unexpected ')'
## 1:  table_to_naiveBayes <- function(features, feature_categories, )
##                                                                   ^
```

Prediction from Table of Data