Fast 
$$\overline{PQ} = \frac{\partial x}{\partial u} du \mathbf{i} + \frac{\partial y}{\partial u} du \mathbf{j}$$

$$\overline{PR} = \frac{\partial x}{\partial v} dv \mathbf{i} + \frac{\partial y}{\partial v} dv \mathbf{j}$$

$$dA = |\overline{PQ} \times \overline{PR}|$$

$$= \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du dv$$