Measuring Resistance for a Voltage Divider Circuit

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Abstract

In this experiment we calculate and measure the unknown resistance of the second resistor in a passive voltage divider circuit, consisting of a $5V$ DC source and 2 resistors, all connected in series. Known resistor's are acquired to find the the values of unknown resistors, using formulas derived from Ohm's law and Kirchhoff's voltage law. The voltage values across both the resistors are known and specified in the initial condition by the instructor. The calculated values of the unknown resistor is then analyzed apropos to the measured value, which is calculated by setting up the circuit and measuring appropriate parameters using an electrical multimeter. The percentage error in the calculated and measured R_1 and R_2 values were calculated to be 1.016 % and 6.942 % respectively. A graph is generated to analyze how the voltage is related to the equivalent resistance of the circuit, which it turns out to be a straight line with a very negligible slope.

1 Objective

The objective of this laboratory experiment is to learn designing a simple passive voltage divider circuit, and measuring the appropriate unknown electrical values across the resistor using the use of an electrical multimeter. It is verified if the two resistors connected in series for a voltage divider circuit are related using the Kirchhoff's Voltage Law and Ohm's Law.

2 Hypothesis

If a voltage divider circuit is constructed to scale down any amount of source voltage, then the two resistors connected in series when simplified into the circuit's equivalent resistance, has to demonstrate a linear relationship with the source voltage and completely obey Ohm's Law.

3 Data Analysis

1. According to Kirchhoff's voltage law, the algebraic sum of all the voltages around any closed loop is equal to zero. In a voltage divider circuit, the single loop structure with two resistors connected in series allows to use the law to prepare a basic equation as shown below.

$$-V_s + V_1 + V_2 = 0, \tag{1}$$

Where V_s is the 5V DC source, V_1 is the voltage across the first resistor, and V_2 is the voltage across the second resistor.

2. The circuit setup is drawn and shown below:

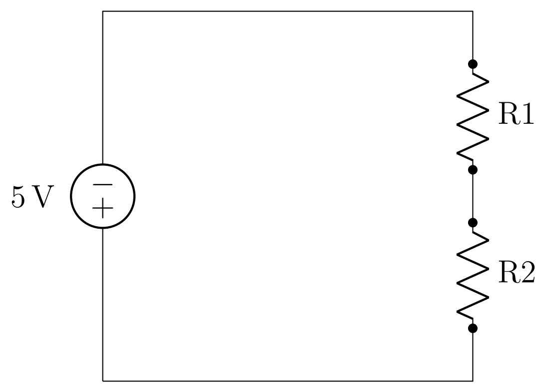


Figure: 1 Schematic diagram of the voltage divider circuit.

3. The Ohms Law can be used to substitute the values of V_1 and V_2 as shown below.

$$V_1 = IR_1, \tag{2}$$

$$V_2 = IR_2, \tag{3}$$

4. Substituting Equation (2) and (3) in (1) yeilds the following.

$$V_s = \text{I } R_1 + \text{I } R_2$$

$$V_s = \text{I } (R_1 + R_2)$$

$$I = \frac{V_s}{R_1 + R_2}$$

5. According to the Ohms law, I can be substituted as either V_1/R_1 or V_2/R_2 depending on the variable that needs to be calculated. The current flowing in a series - single loop circuit is the same throughout.

$$\frac{V_1}{R_1} = \frac{V_s}{R_1 + R_2}$$

$$V_1 = \frac{V_s R_1}{R_1 + R_2} \tag{4}$$

6. Equation 4 derived above serves as the endpoint to calculating the unknown values for resistances. The initial conditions for the voltages across the two resistors were specified by instructor. Resistor R_1 was acquired and considered as known. The goal is to find the R_2 knowing the values of V_s, R_1, and V_1.

7. For first case, the known resistor (R_1) was picked to be 3.30 kΩ. The multimeter was used to verify the validity of the manufacturers assertion. The multimeter read its values as $3.25k\Omega$. However, the multimeter measured R_1 value is not used here in this step, as it is preserved to be discussed further down the analysis. The specified value for V_1 was $3.00V$, which was used to calculate the value for unknown resistance using Equation (4). Rearranging the equation to bring R_2 on other side gives:

$$R_2 = \frac{R_1(V_s - V_1)}{V_1}$$

$$R_2 = \frac{(3.30k\Omega)(5.00V - 2.00V)}{(3.00V)}$$

$$R_2 = 2.20 \text{ k}\Omega$$

Similarly, values for unknown resistances for several provided cases are calculated and represented in Table 1 below. These values of R_2 server as theoretical values.

Table 1: Representation of measured and calculated variables for the circuit.

Case	V_1 (V)	V_2 (V)	R_1 (kΩ)	R_2(*calculated*) (kΩ)	I (μ A)
1	3.000	2.000	3.300	2.200	909.1
2	2.000	3.000	15.00	22.50	133.3
3	4.000	1.000	20.00	5.000	200.0
4	1.000	4.000	25.00	100.0	40.00

8. Now that all the unknown values for resistors are known, it is fairly easy to find the current flowing through the circuit in all cases using the Ohm's Law. For the first case, $R_1 = 3.3k\Omega$ and $V_1 = 3.00V$. Thus, the current flowing is calculated as below.

$$I = \frac{V_1}{R_1}$$

$$I = \frac{3.00V}{3.3k\Omega}$$

$$I = 909.1\mu A$$

Similarly, the current for all different cases is calculated and is already represented in Table 1.

9. Next goal is to simplify the circuit to study its behaviors. It is important to find the equivalent resistances for all the cases. For the first case, values from Table 1 for resistance are picked and the R_{eq} is calculated as below.

For case 1,

$$R_{eq} = R_1 + R_2$$

$$R_{eq} = 3.30k\Omega + 2.20k\Omega$$

$$R_{eq} = 5.50k\Omega$$

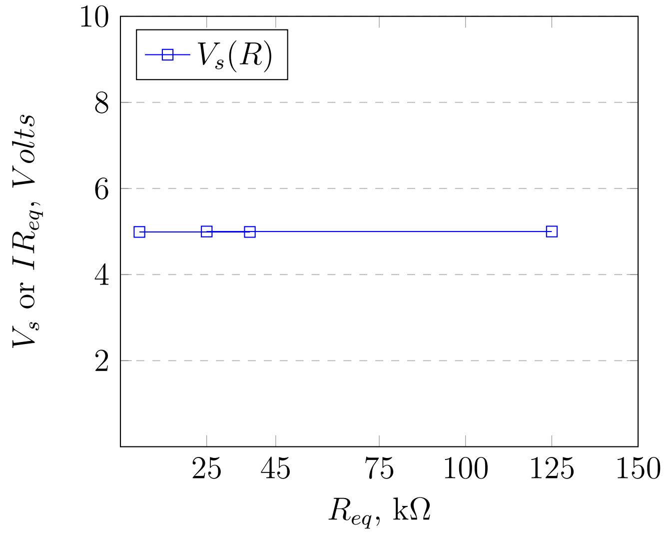
Similarly, R_{eq} are calculated for all the cases and presented in Table 2 below.

Table 2: Representation of the equivalent resistances.

$Case$	R_{eq} ($k\Omega$)	IR_{eq} (V)
1	5.50	4.999
2	37.50	4.999
3	25.00	5.000
4	125.00	5.000

10. Knowing the values for resistance and the DC power source, it is crucial to verify the calculation are accurate as per the Ohm's Law. Since there is a linear relationship between the Voltage and Resistance in the Ohm's Law, the graph is plotted for V_s as a function of R_{eq} - i.e. $V_s(R_{eq})$, and show below.

Graph of V_s as a function of R_{eq}



11. The graph above turns out to be a straight line because the Ohm's Law is simply a linear equation of the form $y = mx + b$. In the acquired case, the y value is voltage source, the m value is slope(constant) corresponding to I(that remains constant throughout an entire case). Thus, plotting it gives a straight line. Since voltage is depending on the current, and where current is constant, the only variable that incorporates the change in voltage is the resistance value. Additional, it also validates Kirchhoffs's law as the voltage values never crosses the max 5V mark for all the cases, as the algebraic sum of all the voltage around any closed loop is equal to zero. That clearly validates the hypothesis for this experiment.

12. It is important to address for calculations the potential error prone source or techniques used. As advised by the course instructor, the value of manufacturer specified resistance was not directly taken for use, rather, it was measured using the multimeter while performing the experiment to do basic error analysis. The given value for the first resistor, and the multimeter calculated values are presented in the table 3 below.

Table 3: Representation of given R_1 value and measured R_1 value using multimeter.

$R_{1,given}(k\Omega)$	$R_{1,measured}$ $(k\Omega)$
3.300	3.250
15.00	14.85
20.00	19.80
25.00	24.89

13. The percent error from the two different resistance values from Table 3 is calculated and shown below.

$$\%Error = \frac{TheoreticalValue - ExperimentalValue}{TheoreticalValue} * 100$$

$$\%Error = \frac{R_{1,given} - R_{1,measured}}{R_{1,given}} * 100$$

For Case 1, the percent error is calculated below:

$$\%Error = \frac{(3.300V - 3.250V)}{(3.300V)} * 100$$

$$\%Error = 1.515\ \%$$

14. The percent error in known and measured R_1 values is calculated and represented in the table below:

Table 4: Error in known R_1 value, and measured R_1 value using multimeter.

Case	*Error* (%)
1	1.515
2	1.000
3	1.000
4	0.440

15. The average percentage error in the R_1 values can be calculated as belows:

$$Average\%Error = \frac{(1.515 + 1.000 + 1.000 + 0.440)}{(4)}$$

$$Average\%Error = 1.016\ \%$$

16. While the experiment was performed in the laboratory, the calculated values are not quite what one would expect to get. However, the fact that the know values for R_1 were little off from the manufacturer's asserted value, also influenced the final Unknown resistance value to be a little off from the calculated R_2 value in Table 1.

17. The calculated R_2 values for Table 1, and the measured R_2 values using the multimeter in the lab are represented in the table 5 below.

Table 5: Error in known R_2 value, and measured R_2 value using multimeter.

Case	$R_{2,calculated}$ (kΩ)	$R_{2,measured}$ (kΩ)
1	2.200	2.100
2	22.50	23.90
3	5.000	4.300
4	100.0	97.00

18. The percent error from the two different resistance (R_2) values from Table 5 is calculated and shown below.

$$\%Error = \frac{TheoreticalValue - ExperimentalValue}{TheoreticalValue} * 100$$

$$\%Error = \frac{R_{2,calculated} - R_{2,measured}}{R_{2,calculated}} * 100$$

For Case 1, the percent error is calculated below:

$$\%Error = \frac{(2.200V - 2.100V)}{(2.200V)} * 100$$

$$\%Error = 4.545\ \%$$

19. The percent error in calculated and measured R_2 values is calculated and represented in the table below:

Table 6: Error in known R_2 value, and measured R_2 value using multimeter.

$Case$	$Error$ (%)
1	4.545
2	6.222
3	14.00
4	3.000

15. The average percentage error in the R_2 values can be calculated as belows:

$$Average\%Error = \frac{(4.545 + 6.222 + 14.00 + 3.000)}{(4)}$$

$$Average\%Error = 6.942\ \%$$

4 Conclusions

In this experiment, the DC source voltage was related to both voltages across the resistors, using the Kirchhoff's Voltage Law and the Ohm's Law as shown in Equation 4. The passive voltage divider circuit takes a $5V$ DC source and scales it down to a specific ratio of input voltage with respect to the output voltage. The ratio is depended on the choice of the selected R_1 and R_2. The purpose of the experiment was to learn designing a circuit, and also to verify that the divider circuits are used to produce different voltage levels from a common voltage source, while the current flowing still remains the same for all the resistor. To verify the hypothesis, it was started with calculating the unknown resistance (R_2) values, which were calculated using the derived Equation 4. When a graph was plotted for the DC voltage source as a function of the equivalent resistance, a linearity was observed in the graph, meaning the graph turned out to be a straight line. The values for V_s coincided with the product of I and R_{eq}, which suggests that if the resistors are connected in series in passive voltage divider circuit, after solving the circuit, it still has to obey the Ohm's Law. However, when performing the experiment in laboratory, there were errors observed in the experiment and measured R_1 and R_2 values. The average percent error in R_1 was known to be 1.016 %, and that of R_2 was 6.942 % . The percent error in R_1 suggests that instead of taking into account the tolerance using the color band, the direct difference was measured using a multimeter which still gave average of 6.492 % difference. That occurs because when the resistor tries to limit the flow of current, the energy is wasted and dissipated in terms of heat. The same holds true for average error in R_2 values.

Much of the variants of multimeters used from the laboratory didn't seem to work until after they were changed frequently. The multimeter in itself had the values fluctuating and a good approximation was taken into account for the measured values. That influenced the resistance values on top of its already specified tolerance values by manufacturer. For the purposes of this experiment, all errors cited above are not significant. The accuracy of the final results were acceptable. The unknown resistance was determine using the Kirchhoff's Law and Ohm's Law. Experimental measurements were within 7 % error. Graphical technique was used to validate the hypothesis that the circuit has to follow Ohm's Law, because it demonstrated a linear relationship. The product of I and R_{eq} never crossed the 5 Volts mark. The voltage within the same loop turned out to be equaling zero. This experiment was successful because objective was achieved as the circuit did scale down the voltage according to the specified conditions of V_1 and V_2. A passive voltage divider circuit was successfully designed and the unknown parameters were calculated verifying that they followed the Kirchhoff's voltage law and the Ohm's Law.

References

[1] J. David Irwin and R. Mark Nelms, *Basic Engineering Circuit Analysis*, (John Wiley and Sons, Inc., 11th edition, 2015).

[2] *Percent Error*, available at `https://tinyurl.com/arth-percent-difference_value`.