Analyzing a DC Circuit using Kirchhoff's Laws

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Abstract

In this experiment we calculate and measure the unknown voltage, and current values in a passive DC circuit, consisting of a $5V$, and $3V$ DC source and 5 resistors, all connected in parallel using the Node and Mesh analysis techniques. The circuit is simulated using Multisim to get the Actual measured values for the experiment which acts as an ideal case throughout the experiment. The experimental measured values of the voltages and currents, which is calculated by setting up the circuit on breadboard and measuring appropriate parameters using an electrical multimeter, is then analyzed apropos to the Multisim recorded value. The average percentage error in the experimental and measured Multisim values is known to be 40.0 %.

1 Objective

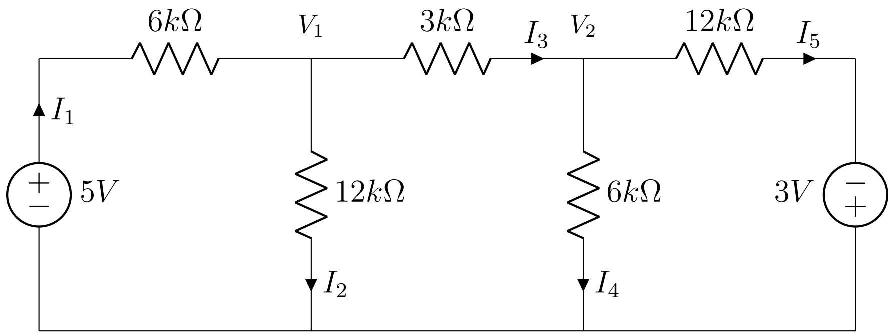
The objective of this laboratory experiment is to learn and analyze a DC circuit involving three meshes using both Node and Mesh analysis, while also comparing the values with the one received from analysis of the Schematic on Multisim. The circuit is also then built on a breadboard to compare theoretical values with the experimental values, accounting for tolerance analysis and errors.

2 Hypothesis

If a DC circuit with three mesh is constructed, then the algebraic sum of the current leaving the node has to equal the sum of current entering the node, and the algebraic sum of voltages around any closed look has to equal zero. The charge, and voltage is known to be conserved when the appropriate sum values of current, and voltage is calculated at the point of connection of two or more circuit elements.

3 Data Analysis

1. A parallel DC circuit as shown below in the Schematic diagram is to be analyzed by finding voltage, and current values. It is possible to analyze it by using Kirchhoff's Laws, which is by Mesh and Node Analysis. Before setting up the circuit in bread board, the unknown parameters will be calculated first using formulas.



Circuit 1

2. Using the Node Analysis, the appropriate values for V_1, V_2, I_1, I_2, I_3, I_4, I_5, and I_6 are calculated below.

According to Kirchhoff's Current Law, the current entering the Node must equal the current leaving the node. Thus for Node 1,

$$-I_1 + I_2 + I_3 = 0$$

$$\frac{-(5V - V_1)}{6k\Omega} + \frac{V_1}{12k\Omega} + \frac{(V_1 - V_2)}{3k\Omega}$$

Solving it gives Equation 1,

$$7V_1 - 4V_2 = 10 \tag{1}$$

Now, for Node 2,

$$-I_3 + I_4 + I_5 = 0$$

$$\frac{-((V_1 - V_2)}{3k\Omega} + \frac{V_2}{6k\Omega} + \frac{((V_2 - (-3))}{12k\Omega}$$

Solving it gives Equation 2,

$$-4V_1 + 7V_2 = -3 \tag{2}$$

The system of two equation – Equation 1 and 2 is solved in a calculator and the values found are listed below.

$V_1 = 1.7576V$

$V_2 = 0.5758V$

$I_1 = \dfrac{5V - V_1}{6k\Omega} = \dfrac{5V - 1.7576V}{6k\Omega} = 0.540mA$

$$I_2 = \frac{V_1}{12k\Omega} = \frac{1.7576V}{12k\Omega} = 0.1460mA$$

$$I_3 = \frac{V_1 = V_2}{3k\Omega} = \frac{1.7576V - 0.5758V}{3k\Omega} = 0.3940mA$$

$$I_4 = \frac{V_2}{6k\Omega} = \frac{0.5758}{6k\Omega} = 0.00507mA$$

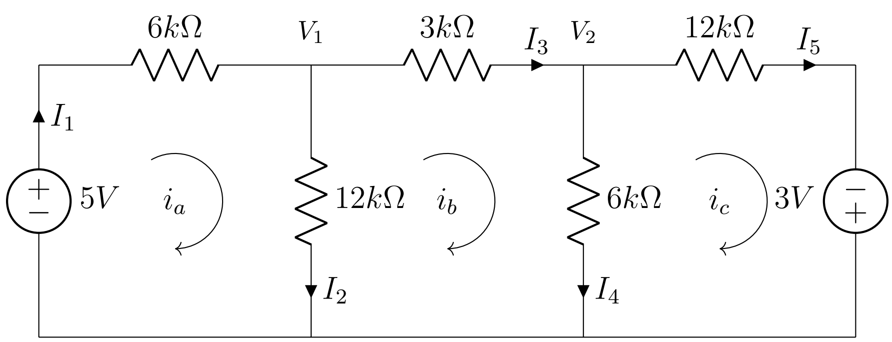
$$I_5 = \frac{V_2 + 3}{12k\Omega} = 0.298mA$$

The values found using Node analysis are recorded in Table 1 below.

Table 1: Recorded values by calculation using the Node Analysis.

$Variable$	$Value$
V_1	1.7576 V
V_2	0.5758 V
I_1	0.5400 mA
I_2	0.1460 mA
I_3	0.3940 mA
I_4	0.09597 mA
I_5	0.2980 mA

3. The circuit schematic is redrawn to represent the current in the mesh considered while performing the calculations.



Circuit 2

4.Using the Mesh Analysis, the appropriate values for V_1, V_2, I_1, I_2, I_3, I_4, I_5, and I_6 are calculated below.

According to Kirchhoff's Voltage Law, in a closed loop series path the algebraic sum of all the voltages around any closed loop in a circuit is equal to zero. Thus, for Mesh 1 the current flowing within the loop is called i_a,

$$\sum V = 0$$

$$-5V + (6k\Omega)i_a + (12k\Omega)(i_a - i_b) = 0$$

$$18i_a - 12i_b = 5 \tag{3}$$

For Mesh 2, the current flowing within the loop is called i_b,

$$(3k\Omega)i_b + (6k\Omega)(i_b - i_c) + (12\Omega)(i_b - i_a) = 0$$

$$-12i_a + 21i_b - 6i_c = 5 \tag{4}$$

For Mesh 3, the current flowing within the loop is called i_c,

$$(12k\Omega)i_c - 3V + (6k\Omega)(i_c - i_b) = 0$$

$$-6i_b + 18i_c = 3 \tag{5}$$

The system of three equation – Equation 3, 4 and 5 is solved in a calculator and the values found are listed below.

$i_a = 0.5404mA$

$i_b = 0.394mA$

$i_c = 0.298mA$

$I_1 = i_a = 0.5404mA$

$I_2 = i_a - i_b = 0.1464mA$

$I_3 = i_b = 0.3940mA$

$I_4 = i_b - i_c = 0.0096960mA$

$I_5 = i_c = 0.2980mA$

The values found using Mesh analysis are recorded in Table 2 below.

Table 2: Recorded values by calculation using the Mesh Analysis.

$Variable$	$Value$
V_1	1.7576 V
V_2	0.5758 V
I_1	0.5400 mA
I_2	0.1460 mA
I_3	0.3940 mA
I_4	0.09597 mA
I_5	0.2980 mA

5. The same circuit is simulated in Multitism and the observed values are recorded in Table 3 below. The resulting circuit in Multisim is also shown below.

Table 3: Recorded values by simulation in Multisim.

$Variable$	$Value$
V_1	1.7576 V
V_2	0.5757 mV
I_1	0.5404 mA
I_2	0.1465 mA
I_3	0.3939 mA
I_4	0.09596 mA
I_5	0.2980 mA

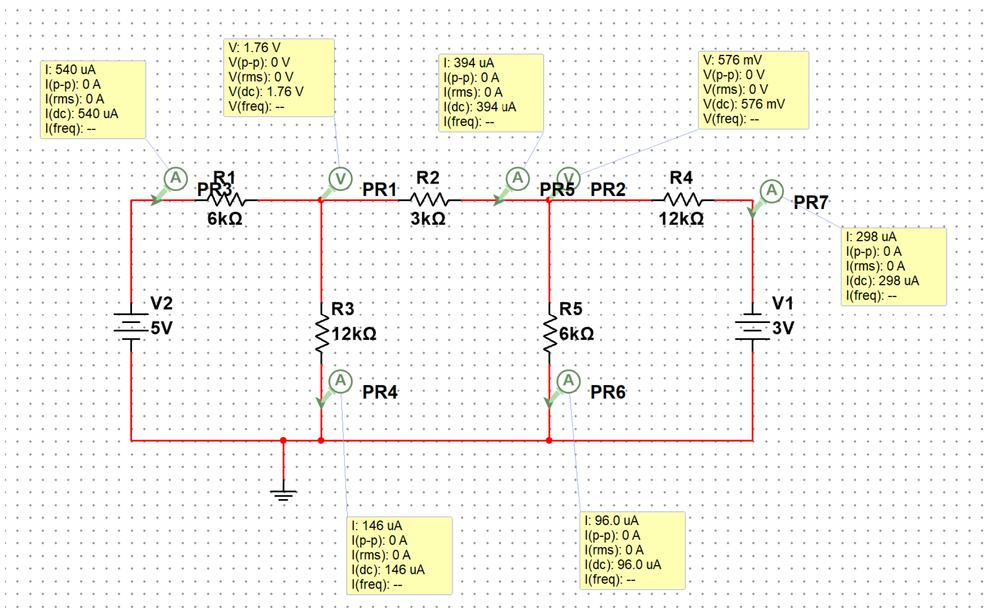


Figure 1: Circuit 1 simulated in Multisim.

6. Now, due to the lack of sufficient variety of Resistors in the laboratory, the resistors of exact values were not taken while building the circuit on breadboard. Resistors of $3.3k\Omega$ were taken for $3k\Omega$, $6.8k\Omega$ for $6k\Omega$, and $10k\Omega$ for $12k\Omega$.

The values for the parameters recorded while conducting the experiment are laid down below in Table 4.

Table 4: Recorded values by calculation using the circuit built on breadboard.

$Variable$	$Value$
V_1	1.720 V
V_2	0.645 V
I_1	0.444 mA
I_2	0.257 mA
I_3	0.184 mA
I_4	0.160 mA
I_5	0.472 mA

7. Since the values acquired by solving the circuit using Node analysis, Mesh Analysis, and Multisim were extremely close, It is appealing to compare Multisim recorded values with Circuit built on breadboard to find the percentage error.

The percentage difference is calculated for Multisim values, and measured values from Table 3 and 4 to interpret the error.

$$\%Error = \frac{TheoreticalValue - ExperimentalValue}{TheoreticalValue} * 100$$

$$\%Error = \frac{V_{1,multisim} - V_{1,measured}}{V_{1,multisim}} * 100$$

For V_1, the percent error is calculated below:

$$\%Error = \frac{(1.7576V - 1.720V)}{(1.7576V)} * 100$$

$$\%Error = 2.139\ \%$$

Similarly the percentage difference is calculated for all Voltage, and Current values is recorded and represented in the table below.

Table 5: Percentage difference between Multisim and Actual values.

$Variable$	$\%Difference$
V_1	2.14
V_2	12.0
I_1	17.8
I_2	70.0
I_3	53.0
I_4	66.7
I_5	58.3

8. The average percentage difference between the Multisim recorded values, and the Actual measured values is 40.0 %.

4 Conclusions

In this experiment, the circuit constructed in Multisim acted as an approximate reference to the values one was trying to achieve by building the circuit in laboratory. The acquired circuit was analyzed using Node, and Mesh analysis, which infact included the use of Kirchhoff's Current and Voltage law. They simply state that the algebraic some of current entering and leaving the node is zero, and also the sum of electromotive forces in a loop equals the sum of potential drops in the loop. These two techniques developed a system of equation which is easily solvable using a calculator, or Matlab. The values were received from Node analysis, Mesh analysis, and Multisim Analysis, were very close to each other verifying of the assumptions taken in the first place using Kirchhoff's laws. One of the noticable features of using this technique is to be able to be consistent with the answer as long as the standard conventions are followed throughout the calculation. Once the problem has been solved, some currents have a positive value, and the direction arbitrarily chosen is the one of the actual current. In the solution some currents may have a negative value, in which case the actual current flows in a direction opposite that of the arbitrary initial choice. At the end, the negative value in a answer indicated the polarity assumed at the beginning to one in the real circuit. When the circuit was built on breadboard, the resistors taken were not already near to the ones specified in the circuit diagram, and thus it created large amount of average error. In Node Analysis, the variables in the circuit were selected to be the node voltages. The voltages are defined with respect to a common point in the circuit where two or more circuit elements connect. In this circuit, it was observed while analysis that the electric cell gives the charge a electromotive force, and then the resistance dissipated that force. However, in resistance if the direction is opposite of the current's direction, this electric resistance adds to the electromotive force. Thus, this finally concludes the validity of the Kirchhoff's Laws, indicating that the Voltage and current are conserved. They are not created, not destroyed.

Now for the case where circuit was constructed on breadboard, much of the variants of multimeters used from the laboratory didn't seem to work until after they were changed frequently. The multimeter in itself had the values fluctuating and a good approximation was taken into account for the measured values. That influenced the resistance values on top of its already specified tolerance values by manufacturer. For the purposes of this experiment, all errors cited above are not significant, considering the fact that the exact values of resistors were unavailable in the laboratory. The resistor values used were approximately off by 30 % from the original needed resistor value.The accuracy of the final results were acceptable. The unknown currents were determine using the Multisim and recording made in the laboratory. The error in experimental measurements were 40 %, i.e. within 41 % error. Kirchoff's Law technique was used to validate the hypothesis that sum the current entering and leaving the node is equal to zero, and sum of voltage around any closed path is zero. A passive DC circuit was successfully designed and the unknown parameters were calculated verifying that they followed the Kirchhoff's law.

References

[1] J. David Irwin and R. Mark Nelms, *Basic Engineering Circuit Analysis*, (John Wiley and Sons, Inc., 11th edition, 2015).

[2] *Percent Error*, available at `https://tinyurl.com/arth-percent-difference`.