Comparing SSA model to experimental results

Code for forecasting extinction risk under different levels of temporal autocorrelation for a specific time series and time scale. This notebook contains all the code used to generate the simulations and create Fig. 5; for easy replication of that figure, download the required MX files from the repository beforehand (files). Subfigures from this notebook were compiled using Adobe Illustrator 2024.

Background Functions

Run this section once to define the TPC and the necessary functions for the SSA model. Before running, define the directory you want to import to or export from.

```
directory = "/directory/";

(*parameters for K and lactin2 model fit*)
K = 5000;
a = 0.044;
b = -1.774;
tmax = 35.254;
deltat = 5.435;

(*lactin2 equation*)
netgrowth[temp_] := Module[
    {a = 0.044,
     b = -1.774,
     tmax = 35.254,
     deltat = 5.435,
     est},
    est = Exp[a * temp] - Exp[a * tmax - ((tmax - temp) / deltat)] + b;
    est
   ];

(*define birth function*)
bTemp[temp_] := Module[
    {a = 0.044,
     birth},
    birth = Exp[a * temp];
```

```
    birth
   ];
rmax = FindMaximum[netgrowth[x], x];
topt = x /. rmax[[2]];
rmax = rmax[[1]];
d1 = netgrowth[topt] / K;

(*define death function, adjusted so that K declines over time*)
dTemp[temp_, n_, t_] := Module[
    {a = 0.044,
     b = -1.774,
     tmax = 35.254,
     deltat = 5.435,
     topt = 25.0341,
     d1 = 0.000103037},
    Exp[a * tmax - ((tmax - temp) / deltat)] - b + (d1 + d1 * t / 56) * n
   ];

(*calculate persistence boundaries*)
lactin2[T_, {a_, b_, tmax_, δT_}] := Exp[a T] - Exp[a tmax - ((tmax - T) / δT)] + b;
paramsfit = {0.044, -1.774, 35.254, 5.435};
w[T_] := lactin2[T, paramsfit];
divisions = 40;
σTrange = Range[0.01, 8.01, 8 / divisions];
μTrange = Range[10, 35, 25 / divisions];

Clear[T];
moments = Flatten[ParallelTable[Module[{mean, var, skew, kurt},
      mean = NExpectation[w[T], T ≈ NormalDistribution[μT, σT]];
      var = NExpectation[(w[T] - mean) ^ 2, T ≈ NormalDistribution[μT, σT]];
      skew =
       NExpectation[((w[T] - mean)) ^ 3, T ≈ NormalDistribution[μT, σT]] / var ^ (3 / 2);
      kurt =
       NExpectation[(w[T] - mean) ^ 4, T ≈ NormalDistribution[μT, σT]] / (var ^ 2) - 3;
      {μT, σT, mean, var, skew, kurt, If[mean > 0, Log10[var / mean], 10]}],
     {σT, σTrange}, {μT, μTrange}], 1];
```

SSA Model

Run simulations and store extinction outcomes. Choose which temperature time series to use by changing the value of 'series' (temps = white noise, temps2 = pink noise, temps3 = brown noise). The SSA occurs in the ParallelDo loop over the parameters specified in 'μTrange' and 'σTrange.' This code is quite slow; if you have already downloaded the MX files, skip to the next block of code.

```
(*experimental temperature time series*)
tempsequence = {28.105, 20.9738, 21.7771, 31.756, 28.6008, 28.3445, 28.4703,
   23.9669, 30.6391, 24.6075, 27.7707, 30.1283, 24.8433, 24.1295, 23.2893,
   27.3607, 32.3506, 20.818, 25.63, 25.3925, 29.9094, 23.1105, 25.0783,
   26.2814, 25.2352, 24.9217, 25.5506, 26.0331, 26.4504, 16.71, 23.5496,
   22.7367, 21.6555, 24.0484, 26.198, 29.3465, 33.29, 30.9471, 25., 24.21,
   24.5286, 26.7995, 23.6345, 20.6535, 28.2229, 28.878, 18.6904, 19.6306,
   25.1567, 27.46, 21.2635, 25.3137, 22.8324, 25.9516, 26.623, 19.0529,
   22.3352, 26.3655, 25.4714, 26.1152, 23.802, 27.2633, 23.2005, 27.5613,
   25.7098, 34.1517, 17.6494, 22.0094, 20.2919, 22.4387, 26.7107, 31.3096,
   21.3992, 23.4638, 22.2293, 21.5297, 24.7648, 26.5362, 24.2902, 29.5213,
   28.7365, 18.244, 24.6863, 19.3609, 29.0262, 23.0192, 21.895, 27.6648,
   25.79, 30.3694, 21.122, 27.1676, 22.9265, 24.4494, 20.4787, 26.8895,
   22.1208, 25.8705, 26.9808, 27.8792, 27.0735, 23.7186, 27.9906, 29.182,
   22.6393, 23.377, 24.37, 23.8848, 19.8717, 29.7081, 20.0906, 22.54};
tempsequence2 = {19.6306, 20.818, 21.122, 23.7186, 28.3445, 29.3465, 27.6648,
   27.0735, 24.7648, 30.3694, 27.5613, 27.46, 26.1152, 27.1676, 24.0484,
   25.4714, 25.8705, 23.0192, 26.4504, 29.7081, 25.63, 32.3506, 26.9808,
   25.3137, 28.878, 22.7367, 29.182, 29.9094, 26.7995, 25.3925, 22.4387,
   23.4638, 24.2902, 25.1567, 24.9217, 22.9265, 26.623, 27.8792, 23.2893,
   22.6393, 20.4787, 23.9669, 25.5506, 24.5286, 19.8717, 26.8895, 26.2814,
   29.5213, 31.756, 31.3096, 26.5362, 34.1517, 30.1283, 28.6008, 33.29,
   29.0262, 28.7365, 28.2229, 28.4703, 24.21, 24.4494, 30.9471, 25., 27.2633,
   25.9516, 23.377, 21.895, 24.6863, 24.8433, 24.6075, 20.0906, 21.5297,
   25.79, 23.1105, 21.3992, 17.6494, 22.54, 22.3352, 25.2352, 21.2635,
   26.3655, 18.6904, 20.2919, 24.37, 23.6345, 25.0783, 23.5496, 22.2293,
   21.7771, 22.0094, 22.1208, 19.0529, 16.71, 18.244, 21.6555, 23.2005,
   24.1295, 27.3607, 23.802, 19.3609, 20.6535, 25.7098, 26.7107, 26.198,
   28.105, 30.6391, 23.8848, 26.0331, 27.9906, 27.7707, 20.9738, 22.8324};
tempsequence3 = {22.9265, 22.8324, 23.377, 23.6345, 23.802, 23.8848, 25.1567,
   24.6863, 23.2893, 22.4387, 21.895, 20.4787, 20.9738, 21.6555, 20.0906, 21.3992,
   20.6535, 22.2293, 23.2005, 23.5496, 25.2352, 26.3655, 25.63, 25.7098, 24.6075,
   24.2902, 22.54, 24.0484, 24.4494, 23.4638, 24.9217, 25., 23.7186, 23.1105,
   22.7367, 22.1208, 22.3352, 21.5297, 21.2635, 19.3609, 19.6306, 18.6904,
   18.244, 16.71, 17.6494, 19.0529, 20.2919, 20.818, 21.7771, 21.122, 19.8717,
   22.0094, 23.0192, 24.1295, 25.3137, 26.2814, 25.9516, 26.0331, 27.5613,
   26.8895, 26.4504, 25.79, 26.198, 25.3925, 26.7107, 30.1283, 29.9094, 28.878,
   27.7707, 28.3445, 28.105, 27.9906, 26.5362, 29.0262, 29.182, 30.6391, 30.9471,
   31.3096, 30.3694, 32.3506, 33.29, 34.1517, 31.756, 29.7081, 27.6648, 28.7365,
   27.1676, 28.2229, 29.5213, 26.9808, 26.1152, 27.2633, 26.623, 27.46, 27.3607,
   27.8792, 28.4703, 29.3465, 27.0735, 25.8705, 25.5506, 28.6008, 26.7995,
   25.4714, 24.8433, 24.7648, 25.0783, 23.9669, 24.37, 24.21, 24.5286, 22.6393};
```

```
(*add zero to end of each series to
 keep algorithm from crashing at final step*)
temps = Join[(tempsequence - 25) / 3.5, {0}]; (*white*)
temps2 = Join[(tempsequence2 - 25) / 3.5, {0}]; (*pink*)
temps3 = Join[(tempsequence3 - 25) / 3.5, {0}]; (*brown*)

series = temps; (*pick series here*)

(*delineate parameter range*)
divisions = 40;
σTrange = Range[0.01, 8.01, 8 / divisions];
μTrange = Range[10, 35, 25 / divisions];
output = {};
reps = 40;(*40*)

SetSharedVariable[output];
ParallelDo[
  extinct = 0;
  Do[
   n = 500;
   t = 0;
   tcount = 1;
   T = series[[tcount]] * σ + μ;
   Tinterval = tnext = 0.5;

   While[t < 56,
    evector = {bTemp[T] * n, dTemp[T, n, t] * n};
    t += RandomReal[ExponentialDistribution[Total[evector]]];
    event = RandomChoice[evector → {1, 2}];
    If[event == 1, n++, n--];
    If[n == 0, extinct++; Break[]];
    If[t ≥ tnext, tcount++;
     T = series[[tcount]] * σ + μ;
     tnext += Tinterval]], {reps}];
  If[n < 10 && n ≠ 0, extinct++];
  AppendTo[output, {μ, σ, N[1 - extinct / reps]}], {μ, μTrange}, {σ, σTrange}];

Which[series == temps, color = "white",
  series == temps2, color = "pink", series == temps3, color = "brown"];
Export[directory <> "SSA2_" <> color <> ".m", output, "MX"]
```

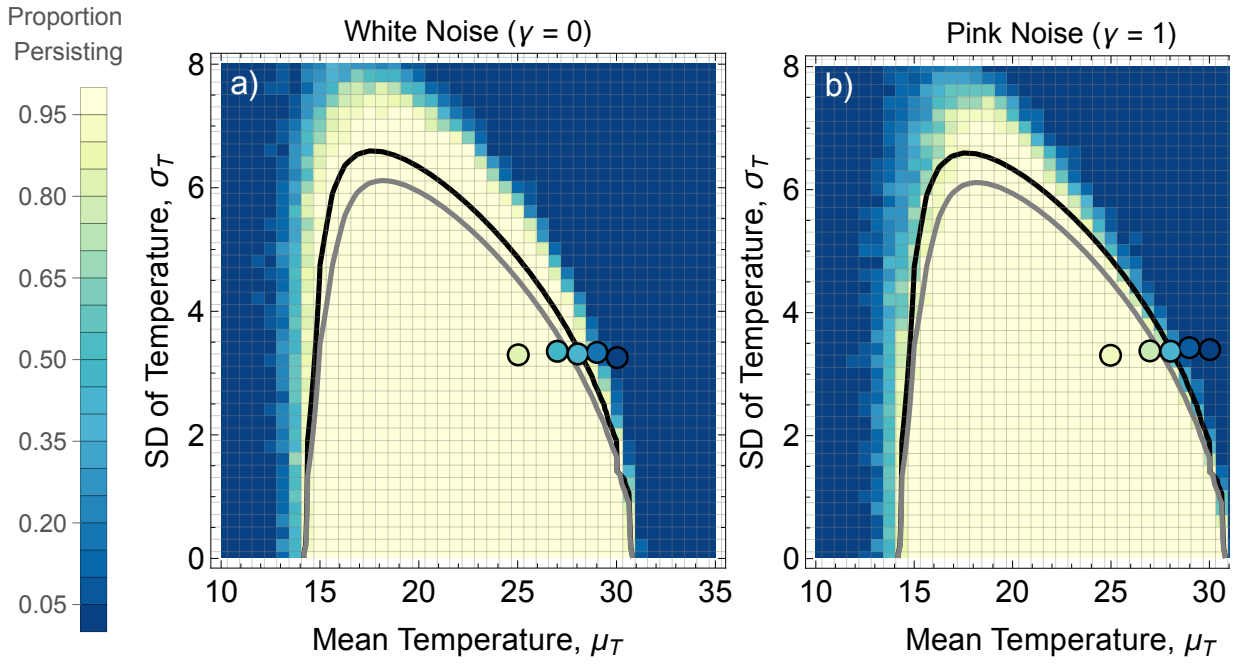
Download the needed MX files and generate plots shown in Fig. 5a-c.

In[ ]:=
```
SSAwhite = Import[directory <> "SSA2_white.m", "MX"];
SSApink = Import[directory <> "SSA2_pink.m", "MX"];
```

```
SSAbrown = Import[directory <> "SSA2_brown.m", "MX"];

wresults =
  {{25.0284, 3.29531, 0.83333}, {27.0072, 3.3534, 0.5}, {28.0454, 3.31001, 0.41667},
   {29.0142, 3.33436, 0.16667}, {30.0345, 3.25052, 0}};
presults = {{24.9767, 3.30643, 0.91667},
   {26.9774, 3.37776, 0.75}, {28.0292, 3.37251, 0.41667},
   {28.9799, 3.43065, 0.08333}, {30.0191, 3.39953, 0}};
bresults = {{25.0567, 3.4065, 1}, {27.0608, 3.3472, 0.16667},
   {28.0491, 3.39566, 0}, {29.0499, 3.39271, 0}, {29.9115, 3.44396, 0}};

newmap[x_] := Blend[{RGBColor["#ffffd9"], RGBColor["#edf8b1"], RGBColor["#c7e9b4"],
    RGBColor["#7fcdbb"], RGBColor["#41b6c4"], RGBColor["#4eb3d3"],
    RGBColor["#2b8cbe"], RGBColor["#0868ac"], RGBColor["#084081"]}, 1 - x];

SSAwhiteplot = Show[ListContourPlot[SSAwhite, InterpolationOrder → 0, Contours → 19,
    ColorFunction → newmap, PlotLegends → Placed[BarLegend[Automatic, LegendLabel →
        "Proportion\n Persisting", LabelStyle → {Darker[Gray], 13}], Left],
    PlotLabel → Style["White Noise (γ = 0)", 15], ImageSize → 310,
    Frame → True, FrameStyle → 16,
    FrameLabel → {"Mean Temperature, μT", "SD of Temperature, σT"},
    GridLines → {μTrange + .5 × 25 / 40, σTrange + .5 × 8 / 40},
    GridLinesStyle → Directive[Opacity[0.4], Thickness[0.0001]],
    Epilog →
     {{Text[Style["a)", White, 17], {11.2, 7.7}]},
      Table[{Directive[Black], PointSize[.045],
        Point[{point[[1]], point[[2]]}]}, {point, wresults}],
      Table[{Directive[newmap[point[[3]]]], PointSize[.035],
        Point[{point[[1]], point[[2]]}]}, {point, wresults}]}],
   ListContourPlot[moments[[1 ;;, {1, 2, 7}]], InterpolationOrder → 3,
    Contours → {0, Log10[2]}, ContourStyle → {{Thickness[0.01], Opacity[1], Gray},
      {Thickness[0.01], Opacity[1], Black}}, ContourShading → None]];

SSApinkplot = Show[ListContourPlot[SSApink, InterpolationOrder → 0, Contours → 19,
    ColorFunction → newmap, (*PlotLegends→Placed[Automatic,Left],*)PlotLabel →
     Style["Pink Noise (γ = 1)", 15], ImageSize → 310, Frame → True, FrameStyle → 16,
    FrameLabel → {"Mean Temperature, μT", "SD of Temperature, σT"},
    GridLines → {μTrange + .5 × 25 / 40, σTrange + .5 × 8 / 40},
    GridLinesStyle → Directive[Opacity[0.4], Thickness[0.0001]],
    Epilog →
     {{Text[Style["b)", White, 17], {11.2, 7.7}]},
      Table[{Directive[Black], PointSize[.045],
        Point[{point[[1]], point[[2]]}]}, {point, presults}],
      Table[{Directive[newmap[point[[3]]]], PointSize[.035],
```

```
        Point[{point[[1]], point[[2]]}]}, {point, presults}]}],
   ListContourPlot[moments[[1 ;;, {1, 2, 7}]], InterpolationOrder → 3,
    Contours → {0, Log10[2]}, ContourStyle → {{Thickness[0.01], Opacity[1], Gray},
      {Thickness[0.01], Opacity[1], Black}}, ContourShading → None]];

SSAbrownplot = Show[ListContourPlot[SSAbrown, InterpolationOrder → 0, Contours → 19,
    ColorFunction → newmap, (*PlotLegends→Placed[Automatic,Left],*)
    PlotLabel → Style["Brown Noise (γ = 2)", 15],
    ImageSize → 310, Frame → True, FrameStyle → 16,
    FrameLabel → {"Mean Temperature, μT", "SD of Temperature, σT"},
    GridLines → {μTrange + .5 × 25 / 40, σTrange + .5 × 8 / 40},
    GridLinesStyle → Directive[Opacity[0.4], Thickness[0.0001]],
    Epilog →
     {{Text[Style["c)", White, 17], {11.2, 7.7}]},
      Table[{Directive[Black], PointSize[.045],
        Point[{point[[1]], point[[2]]}]}, {point, bresults}],
      Table[{Directive[newmap[point[[3]]]], PointSize[.035],
        Point[{point[[1]], point[[2]]}]}, {point, bresults}]}],
   ListContourPlot[moments[[1 ;;, {1, 2, 7}]], InterpolationOrder → 3,
    Contours → {0, Log10[2]}, ContourStyle → {{Thickness[0.01], Opacity[1], Gray},
      {Thickness[0.01], Opacity[1], Black}}, ContourShading → None]];

SSAs = GraphicsRow[
  {SSAwhiteplot, SSApinkplot, SSAbrownplot}, Spacings → 0, ImageSize → 1000]
(*Export[directory<>"SSAs.pdf",SSAs,ImageResolution→1000]*)
```

Out[ ]=

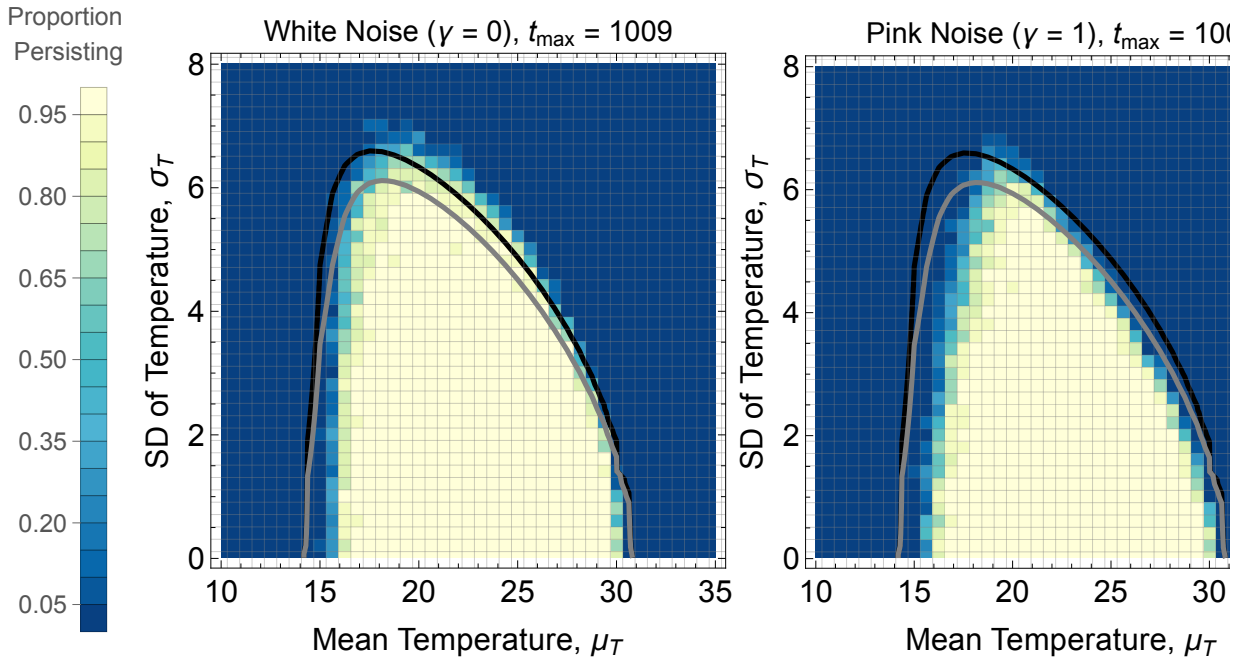


Generate additional plots over a longer timescale to see how the envelope of persistence contracts when tmax is greater; use to generate Fig S5.

```
SSAwhite2 = Import[directory <> "SSA2_t1009_20reps_white.m", "MX"];
SSApink2 = Import[directory <> "SSA2_t1009_20reps_pink.m", "MX"];
SSAbrown2 = Import[directory <> "SSA2_t1009_20reps_brown.m", "MX"];

SSAwhiteplot2 =
  Show[ListContourPlot[SSAwhite2, InterpolationOrder → 0, Contours → 19,
    ColorFunction → newmap, PlotLegends → Placed[BarLegend[Automatic, LegendLabel →
        "Proportion\n Persisting", LabelStyle → {Darker[Gray], 13}], Left],
    PlotLabel → Style["White Noise (γ = 0), tmax = 1009", 15],
    ImageSize → 310, Frame → True, FrameStyle → 16,
    FrameLabel → {"Mean Temperature, μT", "SD of Temperature, σT"},
    GridLines → {μTrange + .5 × 25 / 40, σTrange + .5 × 8 / 40},
    GridLinesStyle → Directive[Opacity[0.4], Thickness[0.0001]](*,
    Epilog→
     {{Text[Style["a)",White,17],{11.2,7.7}]},
      Table[{Directive[Black],PointSize[.045],
        Point[{point[[1]],point[[2]]}]},{point,wresults}],
      Table[{Directive[newmap[point[[3]]]],PointSize[.035],
        Point[{point[[1]],point[[2]]}]},{point,wresults}]}*)],
   ListContourPlot[moments[[1 ;;, {1, 2, 7}]], InterpolationOrder → 3,
    Contours → {0, Log10[2]}, ContourStyle → {{Thickness[0.01], Opacity[1], Gray},
```

```
      {Thickness[0.01], Opacity[1], Black}}, ContourShading → None]];

SSApinkplot2 = Show[ListContourPlot[SSApink2, InterpolationOrder → 0, Contours → 19,
    ColorFunction → newmap, (*PlotLegends→Placed[Automatic,Left],*)
    PlotLabel → Style["Pink Noise (γ = 1), t_max = 1009", 15],
    ImageSize → 310, Frame → True, FrameStyle → 16,
    FrameLabel → {"Mean Temperature, μ_T", "SD of Temperature, σ_T"},
    GridLines → {μTrange + .5 × 25 / 40, σTrange + .5 × 8 / 40},
    GridLinesStyle → Directive[Opacity[0.4], Thickness[0.0001]](*,
    Epilog→
     {{Text[Style["b)",White,17],{11.2,7.7}]},
      Table[{Directive[Black],PointSize[.045],
        Point[{point[[1]],point[[2]]}]},{point,presults}],
      Table[{Directive[newmap[point[[3]]]],PointSize[.035],
        Point[{point[[1]],point[[2]]}]},{point,presults}]}*)],
   ListContourPlot[moments[[1 ;;, {1, 2, 7}]], InterpolationOrder → 3,
    Contours → {0, Log10[2]}, ContourStyle → {{Thickness[0.01], Opacity[1], Gray},
      {Thickness[0.01], Opacity[1], Black}}, ContourShading → None]];

SSAbrownplot2 =
  Show[ListContourPlot[SSAbrown2, InterpolationOrder → 0, Contours → 19,
    ColorFunction → newmap, (*PlotLegends→Placed[Automatic,Left],*)
    PlotLabel → Style["Brown Noise (γ = 2), t_max = 1009", 15],
    ImageSize → 310, Frame → True, FrameStyle → 16,
    FrameLabel → {"Mean Temperature, μ_T", "SD of Temperature, σ_T"},
    GridLines → {μTrange + .5 × 25 / 40, σTrange + .5 × 8 / 40},
    GridLinesStyle → Directive[Opacity[0.4], Thickness[0.0001]](*,
    Epilog→
     {{Text[Style["c)",White,17],{11.2,7.7}]},
      Table[{Directive[Black],PointSize[.045],
        Point[{point[[1]],point[[2]]}]},{point,bresults}],
      Table[{Directive[newmap[point[[3]]]],PointSize[.035],
        Point[{point[[1]],point[[2]]}]},{point,bresults}]}*)],
   ListContourPlot[moments[[1 ;;, {1, 2, 7}]], InterpolationOrder → 3,
    Contours → {0, Log10[2]}, ContourStyle → {{Thickness[0.01], Opacity[1], Gray},
      {Thickness[0.01], Opacity[1], Black}}, ContourShading → None]];

SSAs = GraphicsRow[
  {SSAwhiteplot2, SSApinkplot2, SSAbrownplot2}, Spacings → 0, ImageSize → 1000]
(*SSAs=GraphicsGrid[{{SSAwhiteplot,SSApinkplot,SSAbrownplot},
   {SSAwhiteplot2,SSApinkplot2,SSAbrownplot2}},Spacings→0,ImageSize→1000]*)
Export[directory <> "SSAs_t1009.pdf", SSAs, ImageResolution → 1000];
```

Out[ ]=



Compare results to the running mean

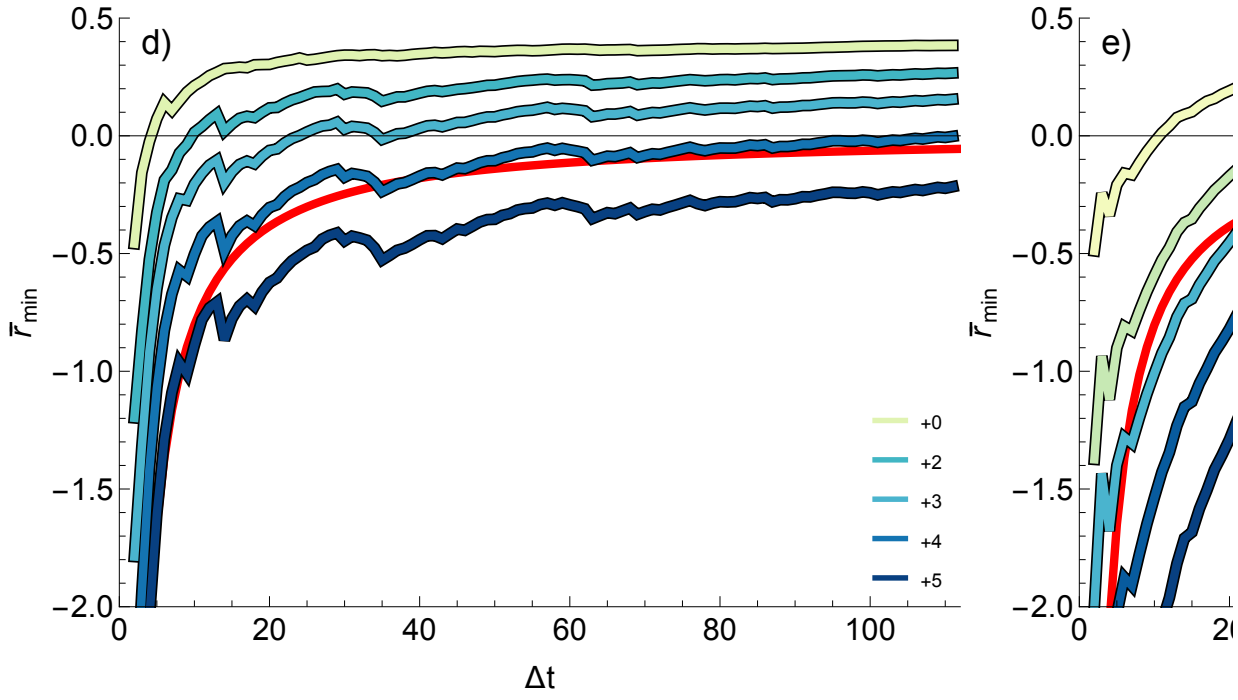
Use this code to calculate the running mean and generate Fig. 5d-f.

In[ ]:=
```
ExtTime[t_, α_, N0_] :=
  r /. NSolve[1 == N0 r / α Exp[r t] / (((r / α) - N0) + N0 Exp[r t]), r, Reals][[1]];

white = Table[{i, Min[MovingAverage[w[tempsequence], i]]},
   {i, 2, Length[tempsequence] - 1}];
pink = Table[{i, Min[MovingAverage[w[tempsequence2], i]]},
   {i, 2, Length[tempsequence2] - 1}];
brown = Table[{i, Min[MovingAverage[w[tempsequence3], i]]},
   {i, 2, Length[tempsequence3] - 1}];

white2 = Table[{i, Min[MovingAverage[w[tempsequence + 2], i]]},
   {i, 2, Length[tempsequence] - 1}];
pink2 = Table[{i, Min[MovingAverage[w[tempsequence2 + 2], i]]},
   {i, 2, Length[tempsequence2] - 1}];
brown2 = Table[{i, Min[MovingAverage[w[tempsequence3 + 2], i]]},
   {i, 2, Length[tempsequence3] - 1}];

white3 = Table[{i, Min[MovingAverage[w[tempsequence + 3], i]]},
   {i, 2, Length[tempsequence] - 1}];
```

```
pink3 = Table[{i, Min[MovingAverage[w[tempsequence2 + 3], i]]},
   {i, 2, Length[tempsequence2] - 1}];
brown3 = Table[{i, Min[MovingAverage[w[tempsequence3 + 3], i]]},
   {i, 2, Length[tempsequence3] - 1}];

white4 = Table[{i, Min[MovingAverage[w[tempsequence + 4], i]]},
   {i, 2, Length[tempsequence] - 1}];
pink4 = Table[{i, Min[MovingAverage[w[tempsequence2 + 4], i]]},
   {i, 2, Length[tempsequence2] - 1}];
brown4 = Table[{i, Min[MovingAverage[w[tempsequence3 + 4], i]]},
   {i, 2, Length[tempsequence3] - 1}];

white5 = Table[{i, Min[MovingAverage[w[tempsequence + 5], i]]},
   {i, 2, Length[tempsequence] - 1}];
pink5 = Table[{i, Min[MovingAverage[w[tempsequence2 + 5], i]]},
   {i, 2, Length[tempsequence2] - 1}];
brown5 = Table[{i, Min[MovingAverage[w[tempsequence3 + 5], i]]},
   {i, 2, Length[tempsequence3] - 1}];

imsize = 310;
arat = .7;

wplot = Show[ListLinePlot[ParallelTable[{t, ExtTime[t, α, N0]},
     {t, 2, Length[tempsequence]}], PlotStyle → {Thickness[0.01], Red},
    PlotRange → {{0, Length[tempsequence]}, {-2, .5}},
    Frame → {True, True, False, False}, FrameStyle → 16,
    FrameLabel → {"Δt", "r̄min"}, AspectRatio → arat, ImageSize → imsize,
    Epilog → {Text[Style["d)", Black, 18], {5, .4}]}],
   ListLinePlot[{white, white2, white3, white4, white5},
    PlotStyle → {{Thickness[.014], Black}},
    PlotRange → {{0, Length[tempsequence]}, {-2, .5}}],
   ListLinePlot[{white, white2, white3, white4, white5},
    PlotLegends → Placed[{"+0", "+2", "+3", "+4", "+5"}, {Right, Bottom}],
    PlotRange → {{0, Length[tempsequence]}, {-2, .5}},
    PlotStyle →
     Table[{Directive[newmap[point[[3]]], Thickness[.01]}, {point, wresults}]]];

pplot = Show[
   ListLinePlot[ParallelTable[{t, ExtTime[t, α, N0]}, {t, 2, Length[tempsequence]}],
    PlotStyle → {Thickness[0.01], Red},
    PlotRange → {{0, Length[tempsequence]}, {-2, .5}},
    Frame → {True, True, False, False}, FrameStyle → 16,
    FrameLabel → {"Δt", "r̄min"}, AspectRatio → arat, ImageSize → imsize,
    Epilog → {Text[Style["e)", Black, 18], {5, .4}]}],
```

```
    ListLinePlot[{pink, pink2, pink3, pink4, pink5},
     PlotStyle → {{Thickness[.014], Black}},
     PlotRange → {{0, Length[tempsequence]}, {-2, .5}}],
    ListLinePlot[{pink, pink2, pink3, pink4, pink5},
     PlotLegends → Placed[{"+0", "+2", "+3", "+4", "+5"}, {Right, Bottom}],
     PlotRange → {{0, Length[tempsequence]}, {-2, .5}},
     PlotStyle →
      Table[{Directive[newmap[point[[3]]], Thickness[.01]}, {point, presults}]]];

bplot = Show[
    ListLinePlot[ParallelTable[{t, ExtTime[t, α, N0]}, {t, 2, Length[tempsequence]}],
     PlotStyle → {Thickness[0.01], Red},
     PlotRange → {{0, Length[tempsequence]}, {-2, .5}},
     Frame → {True, True, False, False}, FrameStyle → 16,
     FrameLabel → {"Δt", "r̄min"}, AspectRatio → arat, ImageSize → imsize,
     Epilog → {Text[Style["f)", Black, 18], {5, .4}]}],
    ListLinePlot[{brown, brown2, brown3, brown4, brown5},
     PlotStyle → {{Thickness[.014], Black}},
     PlotRange → {{0, Length[tempsequence]}, {-2, .5}}],
    ListLinePlot[{brown, brown2, brown3, brown4, brown5},
     PlotLegends → Placed[{"+0", "+2", "+3", "+4", "+5"}, {Right, Bottom}],
     PlotRange → {{0, Length[tempsequence]}, {-2, .5}},
     PlotStyle →
      Table[{Directive[newmap[point[[3]]], Thickness[.01]}, {point, bresults}]]];

exttimeplots = GraphicsRow[{wplot, pplot, bplot}, Spacings → 0, ImageSize → 1500]

Export[directory <> "exttimeplots.pdf", exttimeplots, ImageResolution → 1000];
```

Out[ ]=



Plot Experimental Results

Use this code to generate plots in the supplemental material S4. You will first need to download 'experimentalresults.csv.'

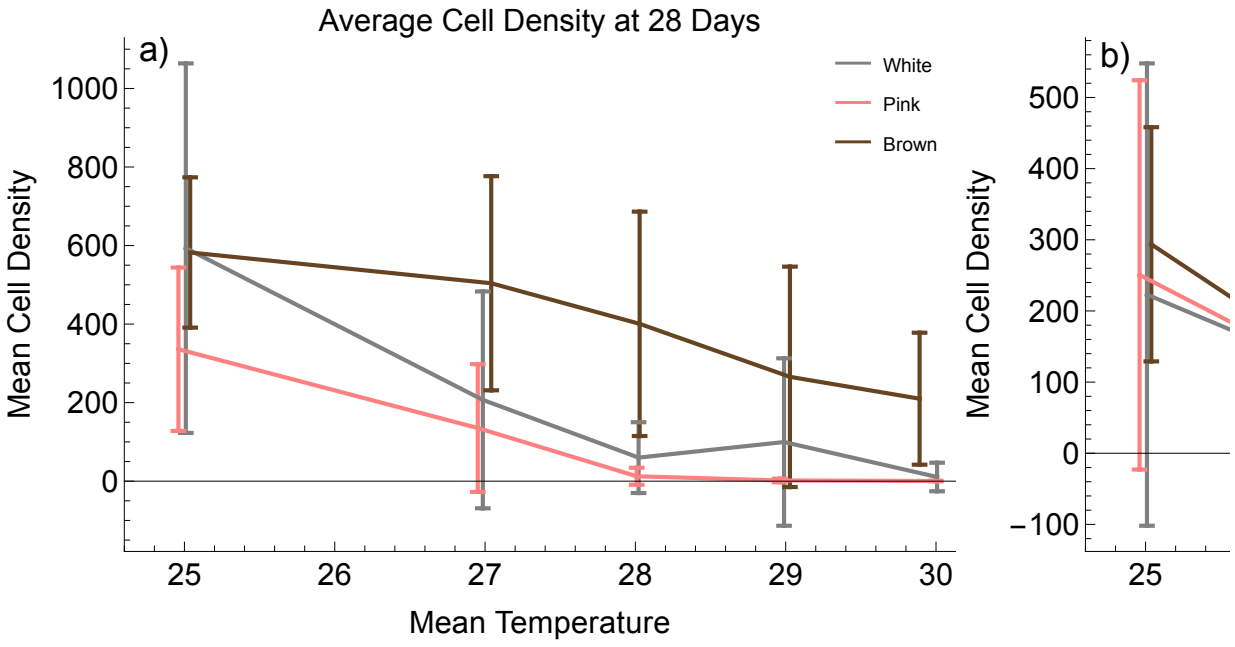
In[ ]:=
```
results = Import[directory <> "experimentalresults.csv", "CSV"];

(*label all data*)
wmeantemps = results[[All, 1]];
wsdtemps = results[[All, 2]];
w28persist = results[[All, 3]];
w56persist = results[[All, 4]];
w28ext = results[[All, 5]];
w56ext = results[[All, 6]];
w28mpop = results[[All, 7]];
w56mpop = results[[All, 8]];
w28sdpop = results[[All, 9]];
w56sdpop = results[[All, 10]];

pmeantemps = results[[All, 11]];
psdtemps = results[[All, 12]];
p28persist = results[[All, 13]];
```

```
p56persist = results[[All, 14]];
p28ext = results[[All, 15]];
p56ext = results[[All, 16]];
p28mpop = results[[All, 17]];
p56mpop = results[[All, 18]];
p28sdpop = results[[All, 19]];
p56sdpop = results[[All, 20]];

bmeantemps = results[[All, 21]];
bsdtemps = results[[All, 22]];
b28persist = results[[All, 23]];
b56persist = results[[All, 24]];
b28ext = results[[All, 25]];
b56ext = results[[All, 26]];
b28mpop = results[[All, 27]];
b56mpop = results[[All, 28]];
b28sdpop = results[[All, 29]];
b56sdpop = results[[All, 30]];

w28extsd = results[[All, 31]];
p28extsd = results[[All, 32]];
b28extsd = results[[All, 33]];
w56extsd = results[[All, 34]];
p56extsd = results[[All, 35]];
b56extsd = results[[All, 36]];

(*pop density plots*)
w28density = Transpose[{wmeantemps, Around @@@ Transpose[{w28mpop, w28sdpop}]}];
p28density = Transpose[{pmeantemps, Around @@@ Transpose[{p28mpop, p28sdpop}]}];
b28density = Transpose[{bmeantemps, Around @@@ Transpose[{b28mpop, b28sdpop}]}];
w56density = Transpose[{wmeantemps, Around @@@ Transpose[{w56mpop, w56sdpop}]}];
p56density = Transpose[{pmeantemps, Around @@@ Transpose[{p56mpop, p56sdpop}]}];
b56density = Transpose[{bmeantemps, Around @@@ Transpose[{b56mpop, b56sdpop}]}];

thickness = .005;

density28 = ListLinePlot[{w28density, p28density, b28density},
   PlotRange → {{24.6, 30.13}, Automatic},
   PlotLabel → Style["Average Cell Density at 28 Days", 16],
   PlotStyle → {{Thickness[thickness], Gray},
     {Thickness[thickness], Pink}, {Thickness[thickness], Darker[Brown]}},
   PlotLegends → Placed[{"White", "Pink", "Brown"}, {Right, Top}],
   Frame → {True, True, False, False}, FrameStyle → 16,
   FrameLabel → {"Mean Temperature", "Mean Cell Density"},
```

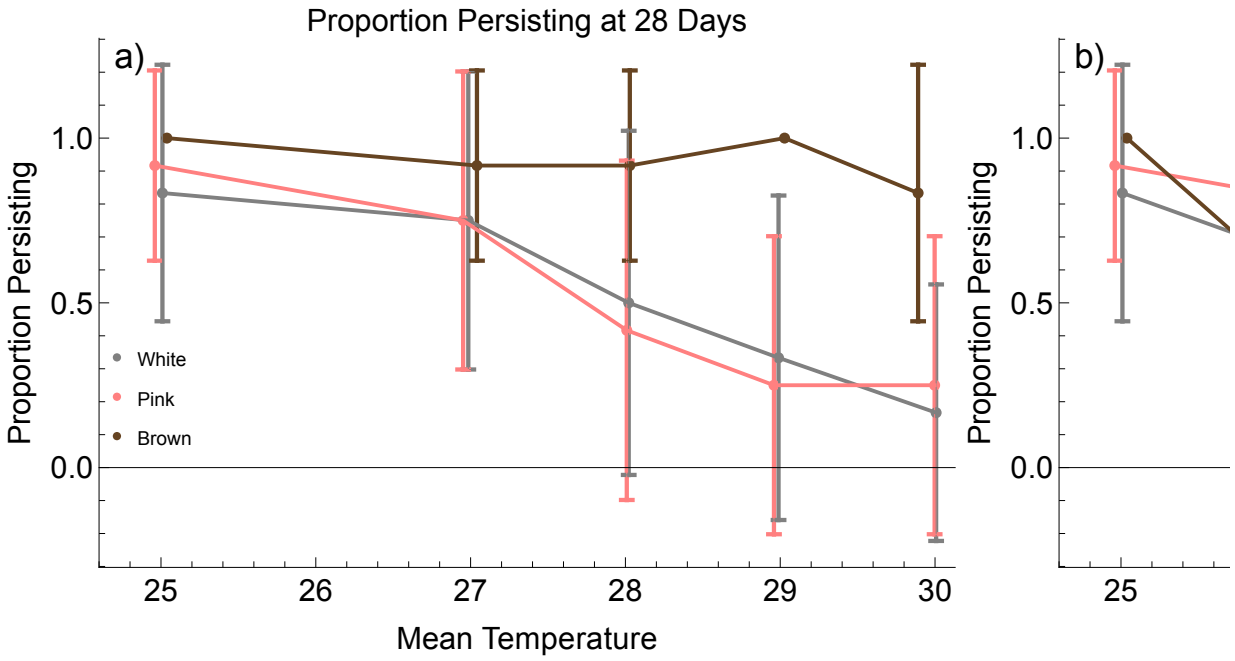
```
    LabelStyle → Directive[Black], ImageSize → 500,
    Epilog → {Text[Style["a)", Black, 18], {24.8, 1100}]}];
density56 = ListLinePlot[
    {w56density, p56density, b56density}, PlotRange → {{24.6, 30.13}, Automatic},
    PlotLabel → Style["Average Cell Density at 56 Days", 16],
    PlotStyle → {{Thickness[thickness], Gray},
      {Thickness[thickness], Pink}, {Thickness[thickness], Darker[Brown]}},
    PlotLegends → Placed[{"White", "Pink", "Brown"}, {Right, Top}],
    Frame → {True, True, False, False}, FrameStyle → 16,
    FrameLabel → {"Mean Temperature", "Mean Cell Density"},
    LabelStyle → Directive[Black], ImageSize → 500,
    Epilog → {Text[Style["b)", Black, 18], {24.8, 560}]}];
densities = GraphicsRow[{density28, density56}, Spacings → 0, ImageSize → 1000]

(*persistence plots*)
w28p = Transpose[{wmeantemps, Around @@@ Transpose[{w28persist, w28extsd}]}];
p28p = Transpose[{pmeantemps, Around @@@ Transpose[{p28persist, p28extsd}]}];
b28p = Transpose[{bmeantemps, Around @@@ Transpose[{b28persist, b28extsd}]}];
w56p = Transpose[{wmeantemps, Around @@@ Transpose[{w56persist, w56extsd}]}];
p56p = Transpose[{pmeantemps, Around @@@ Transpose[{p56persist, p56extsd}]}];
b56p = Transpose[{bmeantemps, Around @@@ Transpose[{b56persist, b56extsd}]}];

persist28 = Show[ListPlot[{w28p, p28p, b28p}, PlotRange → {{24.6, 30.13}, Automatic},
     PlotLabel → Style["Proportion Persisting at 28 Days", 16],
     PlotStyle → {{Thickness[thickness], Gray},
       {Thickness[thickness], Pink}, {Thickness[thickness], Darker[Brown]}},
     PlotLegends → Placed[{"White", "Pink", "Brown"}, {Scaled[{0, 0.32}], {0, 0.5}}],
     Frame → {True, True, False, False}, FrameStyle → 16,
     FrameLabel → {"Mean Temperature", "Proportion Persisting"},
     LabelStyle → Directive[Black], ImageSize → 500,
     Epilog → {Text[Style["a)", Black, 18], {24.8, 1.25}]}],
   ListLinePlot[{w28p, p28p, b28p}, PlotStyle → {Gray, Pink, Darker[Brown]}]];
persist56 = Show[ListPlot[{w56p, p56p, b56p}, PlotRange → {{24.6, 30.13}, Automatic},
     PlotLabel → Style["Proportion Persisting at 56 Days", 16],
     PlotStyle → {{Thickness[thickness], Gray},
       {Thickness[thickness], Pink}, {Thickness[thickness], Darker[Brown]}},
     PlotLegends → Placed[{"White", "Pink", "Brown"}, {Right, Top}],
     Frame → {True, True, False, False}, FrameStyle → 16,
     FrameLabel → {"Mean Temperature", "Proportion Persisting"},
     LabelStyle → Directive[Black], ImageSize → 500,
     Epilog → {Text[Style["b)", Black, 18], {24.8, 1.25}]}],
   ListLinePlot[{w56p, p56p, b56p}, PlotStyle → {Gray, Pink, Darker[Brown]}]];
persistence = GraphicsRow[{persist28, persist56}, Spacings → 0, ImageSize → 1000]
```

```
(*extinction plots*)
w28e = Transpose[{wmeantemps, Around @@@ Transpose[{w28ext, w28extsd}]}];
p28e = Transpose[{pmeantemps, Around @@@ Transpose[{p28ext, p28extsd}]}];
b28e = Transpose[{bmeantemps, Around @@@ Transpose[{b28ext, b28extsd}]}];
w56e = Transpose[{wmeantemps, Around @@@ Transpose[{w56ext, w56extsd}]}];
p56e = Transpose[{pmeantemps, Around @@@ Transpose[{p56ext, p56extsd}]}];
b56e = Transpose[{bmeantemps, Around @@@ Transpose[{b56ext, b56extsd}]}];

extinct28 = Show[ListPlot[{w28e, p28e, b28e}, PlotRange → {{24.6, 30.13}, Automatic},
     PlotLabel → Style["Proportion Extinct at 28 Days", 16],
     PlotStyle → {{Thickness[thickness], Gray},
       {Thickness[thickness], Pink}, {Thickness[thickness], Darker[Brown]}},
     PlotLegends → Placed[{"White", "Pink", "Brown"}, {Scaled[{0, 0.77}], {0, 0.5}}],
     Frame → {True, True, False, False}, FrameStyle → 16,
     FrameLabel → {"Mean Temperature", "Proportion Persisting"},
     LabelStyle → Directive[Black], ImageSize → 500,
     Epilog → {Text[Style["a)", Black, 18], {24.8, 1.25}]}],
    ListLinePlot[{w28e, p28e, b28e}, PlotStyle → {Gray, Pink, Darker[Brown]}]];
extinct56 = Show[ListPlot[{w56e, p56e, b56e}, PlotRange → {{24.6, 30.13}, Automatic},
     PlotLabel → Style["Proportion Extinct at 56 Days", 16],
     PlotStyle → {{Thickness[thickness], Gray},
       {Thickness[thickness], Pink}, {Thickness[thickness], Darker[Brown]}},
     PlotLegends → Placed[{"White", "Pink", "Brown"}, {Scaled[{0, 0.77}], {0, 0.5}}],
     Frame → {True, True, False, False}, FrameStyle → 16,
     FrameLabel → {"Mean Temperature", "Proportion Persisting"},
     LabelStyle → Directive[Black], ImageSize → 500,
     Epilog → {Text[Style["b)", Black, 18], {24.8, 1.25}]}],
    ListLinePlot[{w56e, p56e, b56e}, PlotStyle → {Gray, Pink, Darker[Brown]}]];
extinct = GraphicsRow[{extinct28, extinct56}, Spacings → 0, ImageSize → 1000]

Export[directory <> "densities.pdf", densities, "PDF", ImageResolution → 1000];
Export[directory <> "persist.pdf", persistence, "PDF", ImageResolution → 1000];
Export[directory <> "extinct.pdf", extinct, "PDF", ImageResolution → 1000];
```

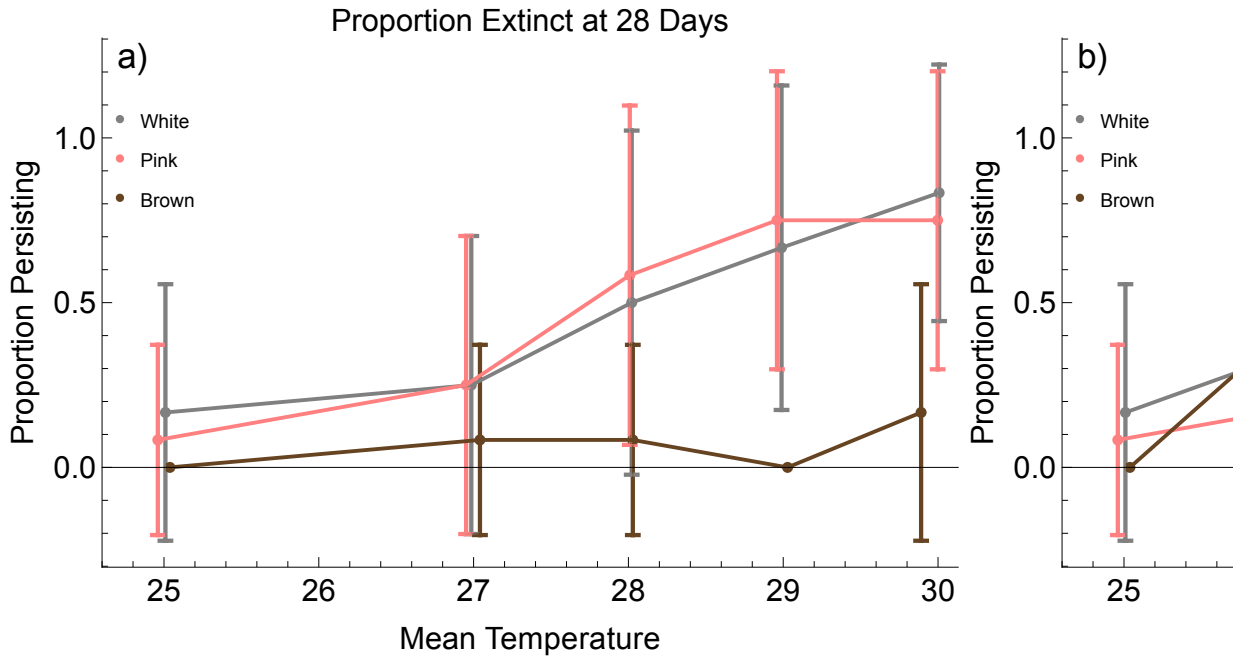
Out[ ]=


Average Cell Density at 28 Days
a)
b)
White
Pink
Brown
Mean Cell Density
Mean Temperature
1000
800
600
400
200
0
500
300
100
−100
25
26
27
28
29
30

Out[ ]=


Proportion Persisting at 28 Days
a)
b)
White
Pink
Brown
Proportion Persisting
Mean Temperature
1.0
0.5
0.0
25
26
27
28
29
30

Out[ ]=



SSA Comparison Across Different Time Series

Use this code to generate plots S6-8 (uses downloaded MX files of SSA outputs for these time series; you can calculate the spectral exponents using code in MethodsFigures.nb, and you can run the SSA model above with these time series to get these same results, but again, the code is quite slow).

In[3192]:=

```
(*Fig S6; all time series listed out below, tempsequence,
tempsequence2, and tempsequence3 are used in the main paper*)
tempsequence = {28.105012956566068`, 20.973777168683974`, 21.777119582710675`,
   31.756000496201537`, 28.600816667020446`, 28.344521382791513`,
   28.47034266137066`, 23.96687349828974`, 30.63909206823437`,
   24.607518315140997`, 27.770735127101812`, 30.12831827439933`,
   24.843283381565694`, 24.129453086270793`, 23.289282561098656`,
   27.360714125686286`, 32.350579224955645`, 20.818000065734736`,
   25.63004329427447`, 25.392481684859003`, 29.90935651497313`,
   23.110450057296283`, 25.078338674047963`, 26.281372248801993`,
   25.235173241485796`, 24.921661325952037`, 25.550587396135597`,
   26.03312650171026`, 26.450446653600267`, 16.710015292562446`,
   23.549553346399733`, 22.736717716225108`, 21.655478617208487`,
   24.048419981102587`, 26.19796757263583`, 29.34653377145162`,
   33.289984707437554`, 30.947121810389547`, 25.`, 24.21002216148944`,
   24.52858572096878`, 26.79954635260587`, 23.634488914372888`,
   20.65346622854838`, 28.222880417289325`, 28.8780378716445`,
```

```
    18.690399182412833`, 19.630578094766086`, 25.156716618434306`,
    27.460010808216847`, 21.263503166426503`, 25.313748228765167`,
    22.832426306719285`, 25.951580018897413`, 26.622977130100125`,
    19.052878189610453`, 22.33516086609751`, 26.365511085627112`,
    25.47141427903122`, 26.115237773875315`, 23.80203242736417`,
    27.263282283774892`, 23.20045364739413`, 27.561328293508662`,
    25.709825270812722`, 34.151719694604665`, 17.649420775044355`,
    22.009434104564036`, 20.29191678038176`, 22.438671706491338`,
    26.710717438901344`, 31.309600817587167`, 21.399183332979554`,
    23.463754792880312`, 22.229264872898188`, 21.52965733862934`,
    24.764826758514204`, 26.536245207119688`, 24.290174729187278`,
    29.521278215385287`, 28.736496833573497`, 18.243999503798463`,
    24.686251771234833`, 19.36090793176563`, 29.026222831316026`,
    23.01917912323498`, 21.894987043433932`, 27.66483913390249`,
    25.78997783851056`, 30.369421905233914`, 21.1219621283555`,
    27.167573693280715`, 22.926540726162184`, 24.449412603864403`,
    20.478721784614713`, 26.889549942703717`, 22.120769481240764`,
    25.870546913729207`, 26.98082087676502`, 27.879230518759236`,
    27.073459273837816`, 23.718627751198007`, 27.990565895435964`,
    29.181999934265264`, 22.639285874313714`, 23.377022869899875`,
    24.36995670572553`, 23.884762226124685`, 19.87168172560067`,
    29.70808321961824`, 20.09064348502687`, 22.539989191783153`};
tempsequence2 = {19.630578094766086`, 20.818000065734736`, 21.1219621283555`,
    23.718627751198007`, 28.344521382791513`, 29.34653377145162`,
    27.66483913390249`, 27.073459273837816`, 24.764826758514204`,
    30.369421905233914`, 27.561328293508662`, 27.460010808216847`,
    26.115237773875315`, 27.167573693280715`, 24.048419981102587`,
    25.47141427903122`, 25.870546913729207`, 23.01917912323498`,
    26.450446653600267`, 29.70808321961824`, 25.63004329427447`,
    32.350579224955645`, 26.98082087676502`, 25.313748228765167`,
    28.8780378716445`, 22.736717716225108`, 29.181999934265264`,
    29.90935651497313`, 26.79954635260587`, 25.392481684859003`,
    22.438671706491338`, 23.463754792880312`, 24.290174729187278`,
    25.156716618434306`, 24.921661325952037`, 22.926540726162184`,
    26.622977130100125`, 27.879230518759236`, 23.289282561098656`,
    22.639285874313714`, 20.478721784614713`, 23.96687349828974`,
    25.550587396135597`, 24.52858572096878`, 19.87168172560067`,
    26.889549942703717`, 26.281372248801993`, 29.521278215385287`,
    31.756000496201537`, 31.309600817587167`, 26.536245207119688`,
    34.151719694604665`, 30.12831827439933`, 28.600816667020446`,
    33.289984707437554`, 29.026222831316026`, 28.736496833573497`,
    28.222880417289325`, 28.47034266137066`, 24.21002216148944`,
    24.449412603864403`, 30.947121810389547`, 25.`, 27.263282283774892`,
    25.951580018897413`, 23.377022869899875`, 21.894987043433932`,
```

```
    24.686251771234833`, 24.843283381565694`, 24.607518315140997`,
    20.09064348502687`, 21.52965733862934`, 25.78997783851056`,
    23.110450057296283`, 21.399183332979554`, 17.649420775044355`,
    22.539989191783153`, 22.33516086609751`, 25.235173241485796`,
    21.263503166426503`, 26.365511085627112`, 18.690399182412833`,
    20.29191678038176`, 24.36995670572553`, 23.634488914372888`,
    25.078338674047963`, 23.549553346399733`, 22.229264872898188`,
    21.777119582710675`, 22.009434104564036`, 22.120769481240764`,
    19.052878189610453`, 16.710015292562446`, 18.243999503798463`,
    21.655478617208487`, 23.20045364739413`, 24.129453086270793`,
    27.360714125686286`, 23.80203242736417`, 19.36090793176563`,
    20.65346622854838`, 25.709825270812722`, 26.710717438901344`,
    26.19796757263583`, 28.105012956566068`, 30.63909206823437`,
    23.884762226124685`, 26.03312650171026`, 27.990565895435964`,
    27.770735127101812`, 20.973777168683974`, 22.832426306719285`};
tempsequence3 = {22.926540726162184`, 22.832426306719285`, 23.377022869899875`,
    23.634488914372888`, 23.80203242736417`, 23.884762226124685`,
    25.156716618434306`, 24.686251771234833`, 23.289282561098656`,
    22.438671706491338`, 21.894987043433932`, 20.478721784614713`,
    20.973777168683974`, 21.655478617208487`, 20.09064348502687`,
    21.399183332979554`, 20.65346622854838`, 22.229264872898188`,
    23.20045364739413`, 23.549553346399733`, 25.235173241485796`,
    26.365511085627112`, 25.63004329427447`, 25.709825270812722`,
    24.607518315140997`, 24.290174729187278`, 22.539989191783153`,
    24.048419981102587`, 24.449412603864403`, 23.463754792880312`,
    24.921661325952037`, 25.`, 23.718627751198007`, 23.110450057296283`,
    22.736717716225108`, 22.120769481240764`, 22.33516086609751`,
    21.52965733862934`, 21.263503166426503`, 19.36090793176563`,
    19.630578094766086`, 18.690399182412833`, 18.243999503798463`,
    16.710015292562446`, 17.649420775044355`, 19.052878189610453`,
    20.29191678038176`, 20.818000065734736`, 21.777119582710675`,
    21.1219621283555`, 19.87168172560067`, 22.009434104564036`,
    23.01917912323498`, 24.129453086270793`, 25.313748228765167`,
    26.281372248801993`, 25.951580018897413`, 26.03312650171026`,
    27.561328293508662`, 26.889549942703717`, 26.450446653600267`,
    25.78997783851056`, 26.19796757263583`, 25.392481684859003`,
    26.710717438901344`, 30.12831827439933`, 29.90935651497313`,
    28.8780378716445`, 27.770735127101812`, 28.344521382791513`,
    28.105012956566068`, 27.990565895435964`, 26.536245207119688`,
    29.026222831316026`, 29.181999934265264`, 30.63909206823437`,
    30.947121810389547`, 31.309600817587167`, 30.369421905233914`,
    32.350579224955645`, 33.289984707437554`, 34.151719694604665`,
    31.756000496201537`, 29.70808321961824`, 27.66483913390249`,
    28.736496833573497`, 27.167573693280715`, 28.222880417289325`,
```

```
    29.521278215385287`, 26.98082087676502`, 26.115237773875315`,
    27.263282283774892`, 26.622977130100125`, 27.460010808216847`,
    27.360714125686286`, 27.879230518759236`, 28.47034266137066`,
    29.34653377145162`, 27.073459273837816`, 25.870546913729207`,
    25.550587396135597`, 28.600816667020446`, 26.79954635260587`,
    25.47141427903122`, 24.843283381565694`, 24.764826758514204`,
    25.078338674047963`, 23.96687349828974`, 24.36995670572553`,
    24.21002216148944`, 24.52858572096878`, 22.639285874313714`};

Z0 = {25.156716618434306`, 20.478721784614713`, 23.549553346399733`,
    27.167573693280715`, 30.63909206823437`, 23.634488914372888`,
    28.736496833573497`, 22.438671706491338`, 19.87168172560067`,
    28.600816667020446`, 24.21002216148944`, 19.36090793176563`,
    25.951580018897413`, 25.709825270812722`, 25.550587396135597`,
    27.990565895435964`, 22.539989191783153`, 24.129453086270793`,
    26.281372248801993`, 23.01917912323498`, 21.1219621283555`,
    33.289984707437554`, 29.026222831316026`, 23.463754792880312`,
    20.29191678038176`, 24.52858572096878`, 30.12831827439933`,
    23.20045364739413`, 21.655478617208487`, 24.921661325952037`,
    18.243999503798463`, 24.764826758514204`, 23.718627751198007`,
    23.80203242736417`, 27.879230518759236`, 24.843283381565694`,
    29.34653377145162`, 34.151719694604665`, 31.756000496201537`,
    26.889549942703717`, 27.561328293508662`, 26.03312650171026`,
    29.521278215385287`, 26.79954635260587`, 26.365511085627112`,
    22.832426306719285`, 23.110450057296283`, 21.263503166426503`,
    20.65346622854838`, 29.90935651497313`, 26.450446653600267`,
    22.229264872898188`, 25.47141427903122`, 21.399183332979554`,
    30.947121810389547`, 24.36995670572553`, 22.639285874313714`,
    27.360714125686286`, 18.690399182412833`, 21.777119582710675`,
    25.392481684859003`, 24.607518315140997`, 22.009434104564036`,
    23.289282561098656`, 32.350579224955645`, 30.369421905233914`,
    24.290174729187278`, 28.8780378716445`, 25.078338674047963`,
    28.105012956566068`, 29.181999934265264`, 21.52965733862934`,
    26.115237773875315`, 22.926540726162184`, 28.344521382791513`,
    26.19796757263583`, 19.052878189610453`, 28.222880417289325`,
    24.686251771234833`, 27.073459273837816`, 27.263282283774892`,
    25.63004329427447`, 20.973777168683974`, 24.048419981102587`,
    22.33516086609751`, 16.710015292562446`, 31.309600817587167`,
    24.449412603864403`, 23.96687349828974`, 25.235173241485796`, 25.`,
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Z1 = {24.449412603864403`, 26.622977130100125`, 24.921661325952037`,
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    25.078338674047963`, 27.360714125686286`, 24.290174729187278`,
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    17.649420775044355`, 21.399183332979554`, 20.478721784614713`,
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    30.12831827439933`, 29.521278215385287`, 26.450446653600267`};
Z2 = {22.438671706491338`, 20.65346622854838`, 19.87168172560067`,
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Z0two = {23.549553346399733`, 23.80203242736417`, 28.8780378716445`,
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Z2two = {27.167573693280715`, 24.686251771234833`, 25.`, 24.290174729187278`,
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thickness = .005;
Tmax = 30.92;
Tmin = 14.26;

compareserieswhite = ListLinePlot[{tempsequence, Z0, Z0two},
    InterpolationOrder → 0, ImageSize → 450, AspectRatio → .3, PlotStyle →
     {{Thickness[thickness], Gray}, {Thickness[thickness], Darker[Gray]}, Black},
    GridLines → {None, {Tmin, Tmax}}, GridLinesStyle → {Dashed, Thin},
    PlotLegends → Placed[{"series1", "series2", "series3"}, {Left, Bottom}],
    PlotRange → {{-.2, 113}, {13, 35}}, Frame → {True, True, False, False},
    FrameLabel → {"Time Step (12 hr)", "Temperature (°C)"}, FrameStyle → 16];
compareseriespink =
  ListLinePlot[{tempsequence2, Z1, Z1two}, InterpolationOrder → 0, ImageSize → 450,
    PlotStyle → {{Thickness[thickness], Pink}, {Thickness[thickness], Red}, Black},
    GridLines → {None, {Tmin, Tmax}}, GridLinesStyle → {Dashed, Thin},
    PlotLegends → Placed[{"series1", "series2", "series3"}, {Left, Bottom}],
    PlotRange → {{-.2, 113}, {13, 35}}, Frame → {True, True, False, False},
    FrameLabel → {"Time Step (12 hr)", "Temperature (°C)"}, FrameStyle → 16];
compareseriesbrown =
  ListLinePlot[{tempsequence3, Z2, Z2two}, InterpolationOrder → 0,
    ImageSize → 450, PlotStyle → {{Thickness[thickness], Brown},
      {Thickness[thickness], Darker[Darker[Brown]]}, Black},
    GridLines → {None, {Tmin, Tmax}}, GridLinesStyle → {Dashed, Thin},
    PlotLegends → Placed[{"series1", "series2", "series3"}, {Left, Bottom}],
    PlotRange → {{-.2, 113}, {13, 35}}, Frame → {True, True, False, False},
    FrameLabel → {"Time Step (12 hr)", "Temperature (°C)"}, FrameStyle → 16];

white1 = ListLinePlot[{tempsequence},
    InterpolationOrder → 0, ImageSize → 495, AspectRatio → .3, PlotStyle →
     {{Thickness[thickness], Gray}, {Thickness[thickness], Darker[Gray]}, Black},
    GridLines → {None, {Tmin, Tmax}}, GridLinesStyle → {Dashed, Thin},
```
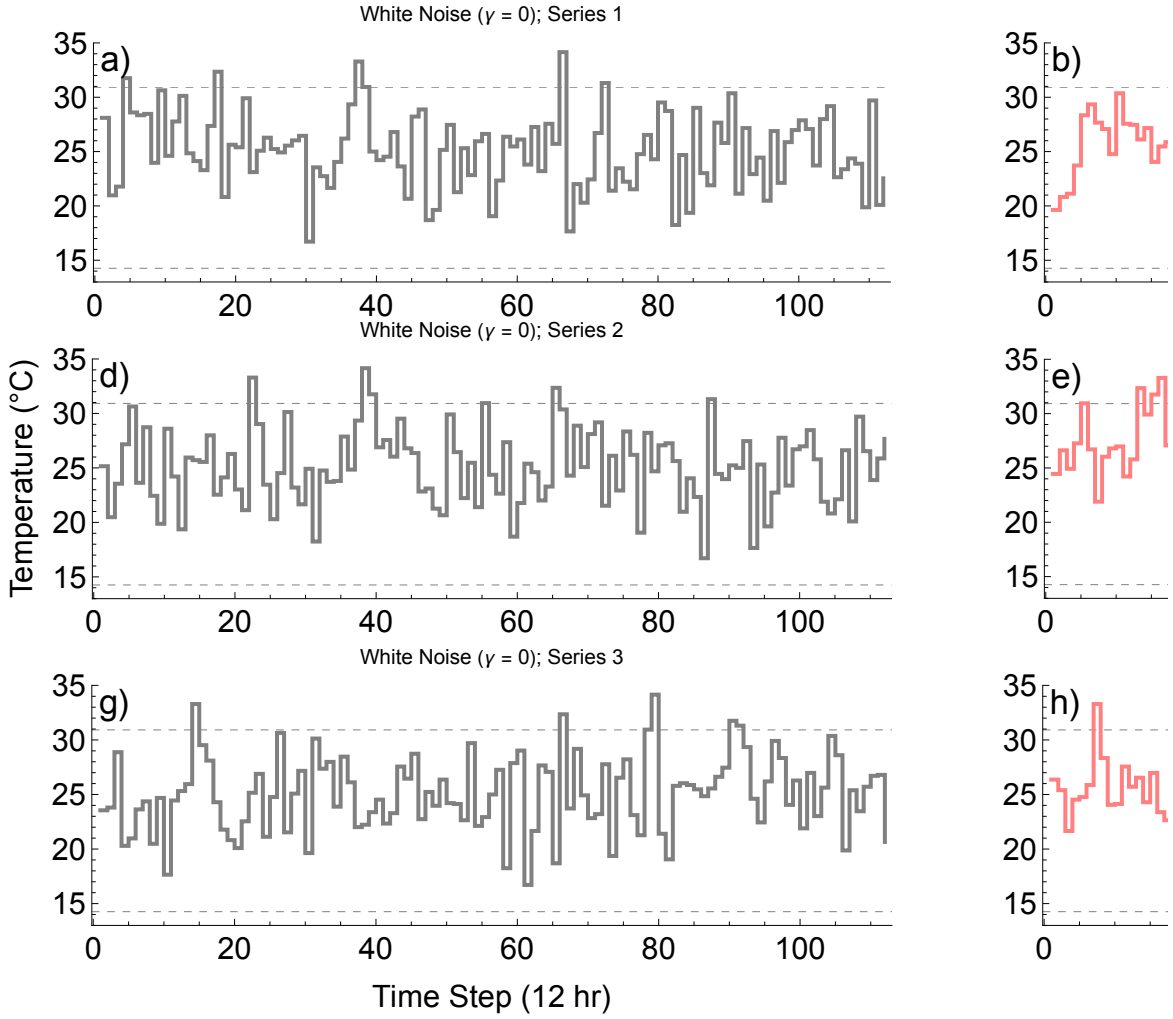

```
    PlotLabel → "White Noise (γ = 0); Series 1", PlotRange → {{-.2, 113}, {13, 35}},
    Frame → {True, True, False, False}, FrameLabel → {None, None}, FrameStyle → 16,
    Epilog → {Text[Style["a)", Black, 18], {3, 33.5}]}];
white2 = ListLinePlot[{Z0}, InterpolationOrder → 0,
    ImageSize → 520, AspectRatio → .3, PlotStyle →
     {{Thickness[thickness], Gray}, {Thickness[thickness], Darker[Gray]}, Black},
    GridLines → {None, {Tmin, Tmax}}, GridLinesStyle → {Dashed, Thin},
    PlotLabel → "White Noise (γ = 0); Series 2",
    PlotRange → {{-.2, 113}, {13, 35}}, Frame → {True, True, False, False},
    FrameLabel → {None, "Temperature (°C)"}, FrameStyle → 16,
    Epilog → {Text[Style["d)", Black, 18], {3, 33.5}]}];
white3 = ListLinePlot[{Z0two},
    InterpolationOrder → 0, ImageSize → 450, AspectRatio → .3, PlotStyle →
     {{Thickness[thickness], Gray}, {Thickness[thickness], Darker[Gray]}, Black},
    GridLines → {None, {Tmin, Tmax}}, GridLinesStyle → {Dashed, Thin},
    PlotLabel → "White Noise (γ = 0); Series 3",
    PlotRange → {{-.2, 113}, {13, 35}}, Frame → {True, True, False, False},
    FrameLabel → {"Time Step (12 hr)", None}, FrameStyle → 16,
    Epilog → {Text[Style["g)", Black, 18], {3, 33.5}]}];

pink1 = ListLinePlot[{tempsequence2}, InterpolationOrder → 0, ImageSize → 495,
    AspectRatio → .3, PlotStyle → {{Thickness[thickness], Pink}},
    GridLines → {None, {Tmin, Tmax}}, GridLinesStyle → {Dashed, Thin},
    PlotLabel → "Pink Noise (γ = 1); Series 1", PlotRange → {{-.2, 113}, {13, 35}},
    Frame → {True, True, False, False}, FrameLabel → {None, None}, FrameStyle → 16,
    Epilog → {Text[Style["b)", Black, 18], {3, 33.5}]}];
pink2 = ListLinePlot[{Z1}, InterpolationOrder → 0, ImageSize → 495,
    AspectRatio → .3, PlotStyle → {{Thickness[thickness], Pink}},
    GridLines → {None, {Tmin, Tmax}}, GridLinesStyle → {Dashed, Thin},
    PlotLabel → "Pink Noise (γ = 1); Series 2", PlotRange → {{-.2, 113}, {13, 35}},
    Frame → {True, True, False, False}, FrameLabel → {None, None}, FrameStyle → 16,
    Epilog → {Text[Style["e)", Black, 18], {3, 33.5}]}];
pink3 =
  ListLinePlot[{Z1two}, InterpolationOrder → 0, ImageSize → 450, AspectRatio → .3,
    PlotStyle → {{Thickness[thickness], Pink}}, GridLines → {None, {Tmin, Tmax}},
    GridLinesStyle → {Dashed, Thin}, PlotLabel → "Pink Noise (γ = 1); Series 3",
    PlotRange → {{-.2, 113}, {13, 35}}, Frame → {True, True, False, False},
    FrameLabel → {"Time Step (12 hr)", None}, FrameStyle → 16,
    Epilog → {Text[Style["h)", Black, 18], {3, 33.5}]}];

brown1 = ListLinePlot[{tempsequence3}, InterpolationOrder → 0, ImageSize → 495,
    AspectRatio → .3, PlotStyle → {{Thickness[thickness], Darker[Brown]}},
    GridLines → {None, {Tmin, Tmax}}, GridLinesStyle → {Dashed, Thin},
    PlotLabel → "Brown Noise (γ = 2); Series 1", PlotRange → {{-.2, 113}, {13, 35}},
```

```
    Frame → {True, True, False, False}, FrameLabel → {None, None}, FrameStyle → 16,
    Epilog → {Text[Style["c)", Black, 18], {3, 33.5}]}];
brown2 = ListLinePlot[{Z2}, InterpolationOrder → 0, ImageSize → 495,
    AspectRatio → .3, PlotStyle → {{Thickness[thickness], Darker[Brown]}},
    GridLines → {None, {Tmin, Tmax}}, GridLinesStyle → {Dashed, Thin},
    PlotLabel → "Brown Noise (γ = 2); Series 2", PlotRange → {{-.2, 113}, {13, 35}},
    Frame → {True, True, False, False}, FrameLabel → {None, None}, FrameStyle → 16,
    Epilog → {Text[Style["f)", Black, 18], {3, 33.5}]}];
brown3 = ListLinePlot[{Z2two}, InterpolationOrder → 0, ImageSize → 450,
    AspectRatio → .3, PlotStyle → {{Thickness[thickness], Darker[Brown]}},
    GridLines → {None, {Tmin, Tmax}}, GridLinesStyle → {Dashed, Thin},
    PlotLabel → "Brown Noise (γ = 2); Series 3",
    PlotRange → {{-.2, 113}, {13, 35}}, Frame → {True, True, False, False},
    FrameLabel → {"Time Step (12 hr)", None}, FrameStyle → 16,
    Epilog → {Text[Style["i)", Black, 18], {3, 33.5}]}];

compareSSAseries = GraphicsGrid[{{white1, pink1, brown1}, {white2, pink2, brown2},
    {white3, pink3, brown3}}, ImageSize → 1500, Spacings → -25]
Export[directory <> "compareseries.pdf",
  compareSSAseries, "PDF", ImageResolution → 800];
```

Out[3216]=



In[3219]:=

```
(*generate compison plots of SSA outcomes, Figs S7-8*)
(*import SSA outputs for the three different sets of time series above*)
SSAwhite = Import[directory <> "SSA2_white.m", "MX"];
SSApink = Import[directory <> "SSA2_pink.m", "MX"];
SSAbrown = Import[directory <> "SSA2_brown.m", "MX"];

(*draw persistence boundaries*)
lactin2[T_, {a_, b_, tmax_, δT_}] := Exp[a T] - Exp[a tmax - ((tmax - T) / δT)] + b;
paramsfit = {0.044, -1.774, 35.254, 5.435};
w[T_] := lactin2[T, paramsfit];
divisions = 40;
σTrange = Range[0.01, 8.01, 8 / divisions];
μTrange = Range[10, 35, 25 / divisions];

Clear[T];
moments = Flatten[ParallelTable[Module[{mean, var, skew, kurt},
```

```
      mean = NExpectation[w[T], T ≈ NormalDistribution[μT, σT]];
      var = NExpectation[(w[T] - mean)^2, T ≈ NormalDistribution[μT, σT]];
      skew =
       NExpectation[((w[T] - mean))^3, T ≈ NormalDistribution[μT, σT]] / var^(3 / 2);
      kurt =
       NExpectation[(w[T] - mean)^4, T ≈ NormalDistribution[μT, σT]] / (var^2) - 3;
      {μT, σT, mean, var, skew, kurt, If[mean > 0, Log10[var / mean], 10]}],
     {σT, σTrange}, {μT, μTrange}], 1];

newmap[x_] := Blend[{RGBColor["#ffffd9"], RGBColor["#edf8b1"], RGBColor["#c7e9b4"],
     RGBColor["#7fcdbb"], RGBColor["#41b6c4"], RGBColor["#4eb3d3"],
     RGBColor["#2b8cbe"], RGBColor["#0868ac"], RGBColor["#084081"]}, 1 - x];

(*plot outputs*)
SSAwhiteplot =
  Show[ListContourPlot[SSAwhite, InterpolationOrder → 0, Contours → 19,
    ColorFunction → newmap, PlotLegends → Placed[BarLegend[Automatic, LegendLabel →
        "Proportion\n Persisting", LabelStyle → {Darker[Gray], 13}], Left],
    PlotLabel → Style["White Noise (γ = 0); Series 1", 15],
    ImageSize → 310, Frame → True, FrameStyle → 16,
    FrameLabel → {"Mean Temperature, μT", "SD of Temperature, σT"},
    GridLines → {μTrange + .5 × 25 / 40, σTrange + .5 × 8 / 40},
    GridLinesStyle → Directive[Opacity[0.4], Thickness[0.0001]],
    Epilog →
     {{Text[Style["a)", White, 17], {11.2, 7.7}]}}],
   ListContourPlot[moments⟦1 ;;, {1, 2, 7}⟧, InterpolationOrder → 3,
    Contours → {0, Log10[2]}, ContourStyle → {{Thickness[0.01], Opacity[1], Gray},
      {Thickness[0.01], Opacity[1], Black}}, ContourShading → None]];

SSApinkplot = Show[ListContourPlot[SSApink, InterpolationOrder → 0, Contours → 19,
    ColorFunction → newmap, (*PlotLegends→Placed[Automatic,Left],*)
    PlotLabel → Style["Pink Noise (γ = 1); Series 1", 15],
    ImageSize → 310, Frame → True, FrameStyle → 16,
    FrameLabel → {"Mean Temperature, μT", "SD of Temperature, σT"},
    GridLines → {μTrange + .5 × 25 / 40, σTrange + .5 × 8 / 40},
    GridLinesStyle → Directive[Opacity[0.4], Thickness[0.0001]],
    Epilog →
     {{Text[Style["b)", White, 17], {11.2, 7.7}]}}],
   ListContourPlot[moments⟦1 ;;, {1, 2, 7}⟧, InterpolationOrder → 3,
    Contours → {0, Log10[2]}, ContourStyle → {{Thickness[0.01], Opacity[1], Gray},
      {Thickness[0.01], Opacity[1], Black}}, ContourShading → None]];

SSAbrownplot = Show[ListContourPlot[SSAbrown, InterpolationOrder → 0, Contours → 19,
    ColorFunction → newmap, (*PlotLegends→Placed[Automatic,Left],*)
```

```
    PlotLabel → Style["Brown Noise (γ = 2); Series 1", 15],
    ImageSize → 310, Frame → True, FrameStyle → 16,
    FrameLabel → {"Mean Temperature, μT", "SD of Temperature, σT"},
    GridLines → {μTrange + .5 × 25 / 40, σTrange + .5 × 8 / 40},
    GridLinesStyle → Directive[Opacity[0.4], Thickness[0.0001]],
    Epilog →
     {{Text[Style["c)", White, 17], {11.2, 7.7}]}}],
   ListContourPlot[moments[[1 ;;, {1, 2, 7}]], InterpolationOrder → 3,
    Contours → {0, Log10[2]}, ContourStyle → {{Thickness[0.01], Opacity[1], Gray},
      {Thickness[0.01], Opacity[1], Black}}, ContourShading → None]];

SSAwhite2 = Import[directory <> "SSA_timeseries2_white.m", "MX"];
SSApink2 = Import[directory <> "SSA_timeseries2_pink.m", "MX"];
SSAbrown2 = Import[directory <> "SSA_timeseries2_brown.m", "MX"];

SSAwhiteplot2 =
  Show[ListContourPlot[SSAwhite2, InterpolationOrder → 0, Contours → 19,
    ColorFunction → newmap, PlotLegends → Placed[BarLegend[Automatic, LegendLabel →
        "Proportion\n Persisting", LabelStyle → {Darker[Gray], 13}], Left],
    PlotLabel → Style["White Noise (γ = 0); Series 2", 15],
    ImageSize → 310, Frame → True, FrameStyle → 16,
    FrameLabel → {"Mean Temperature, μT", "SD of Temperature, σT"},
    GridLines → {μTrange + .5 × 25 / 40, σTrange + .5 × 8 / 40},
    GridLinesStyle → Directive[Opacity[0.4], Thickness[0.0001]],
    Epilog →
     {{Text[Style["d)", White, 17], {11.2, 7.7}]}}],
   ListContourPlot[moments[[1 ;;, {1, 2, 7}]], InterpolationOrder → 3,
    Contours → {0, Log10[2]}, ContourStyle → {{Thickness[0.01], Opacity[1], Gray},
      {Thickness[0.01], Opacity[1], Black}}, ContourShading → None]];

SSApinkplot2 = Show[ListContourPlot[SSApink2, InterpolationOrder → 0, Contours → 19,
    ColorFunction → newmap, (*PlotLegends→Placed[Automatic,Left],*)
    PlotLabel → Style["Pink Noise (γ = 1); Series 2", 15],
    ImageSize → 310, Frame → True, FrameStyle → 16,
    FrameLabel → {"Mean Temperature, μT", "SD of Temperature, σT"},
    GridLines → {μTrange + .5 × 25 / 40, σTrange + .5 × 8 / 40},
    GridLinesStyle → Directive[Opacity[0.4], Thickness[0.0001]],
    Epilog →
     {{Text[Style["e)", White, 17], {11.2, 7.7}]}}],
   ListContourPlot[moments[[1 ;;, {1, 2, 7}]], InterpolationOrder → 3,
    Contours → {0, Log10[2]}, ContourStyle → {{Thickness[0.01], Opacity[1], Gray},
      {Thickness[0.01], Opacity[1], Black}}, ContourShading → None]];

SSAbrownplot2 =
```

```
  Show[ListContourPlot[SSAbrown2, InterpolationOrder → 0, Contours → 19,
    ColorFunction → newmap, (*PlotLegends→Placed[Automatic,Left],*)
    PlotLabel → Style["Brown Noise (γ = 2); Series 2", 15],
    ImageSize → 310, Frame → True, FrameStyle → 16,
    FrameLabel → {"Mean Temperature, μT", "SD of Temperature, σT"},
    GridLines → {μTrange + .5 × 25 / 40, σTrange + .5 × 8 / 40},
    GridLinesStyle → Directive[Opacity[0.4], Thickness[0.0001]],
    Epilog →
     {{Text[Style["f)", White, 17], {11.2, 7.7}]}}],
   ListContourPlot[moments[[1 ;;, {1, 2, 7}]], InterpolationOrder → 3,
    Contours → {0, Log10[2]}, ContourStyle → {{Thickness[0.01], Opacity[1], Gray},
      {Thickness[0.01], Opacity[1], Black}}, ContourShading → None]];

SSAwhite3 = Import[directory <> "SSA_timeseries3_white.m", "MX"];
SSApink3 = Import[directory <> "SSA_timeseries3_pink.m", "MX"];
SSAbrown3 = Import[directory <> "SSA_timeseries3_brown.m", "MX"];

SSAwhiteplot3 =
  Show[ListContourPlot[SSAwhite3, InterpolationOrder → 0, Contours → 19,
    ColorFunction → newmap, PlotLegends → Placed[BarLegend[Automatic, LegendLabel →
        "Proportion\n Persisting", LabelStyle → {Darker[Gray], 13}], Left],
    PlotLabel → Style["White Noise (γ = 0); Series 3", 15],
    ImageSize → 310, Frame → True, FrameStyle → 16,
    FrameLabel → {"Mean Temperature, μT", "SD of Temperature, σT"},
    GridLines → {μTrange + .5 × 25 / 40, σTrange + .5 × 8 / 40},
    GridLinesStyle → Directive[Opacity[0.4], Thickness[0.0001]],
    Epilog →
     {{Text[Style["g)", White, 17], {11.2, 7.7}]}}],
   ListContourPlot[moments[[1 ;;, {1, 2, 7}]], InterpolationOrder → 3,
    Contours → {0, Log10[2]}, ContourStyle → {{Thickness[0.01], Opacity[1], Gray},
      {Thickness[0.01], Opacity[1], Black}}, ContourShading → None]];

SSApinkplot3 = Show[ListContourPlot[SSApink3, InterpolationOrder → 0, Contours → 19,
    ColorFunction → newmap, (*PlotLegends→Placed[Automatic,Left],*)
    PlotLabel → Style["Pink Noise (γ = 1); Series 3", 15],
    ImageSize → 310, Frame → True, FrameStyle → 16,
    FrameLabel → {"Mean Temperature, μT", "SD of Temperature, σT"},
    GridLines → {μTrange + .5 × 25 / 40, σTrange + .5 × 8 / 40},
    GridLinesStyle → Directive[Opacity[0.4], Thickness[0.0001]],
    Epilog →
     {{Text[Style["h)", White, 17], {11.2, 7.7}]}}],
   ListContourPlot[moments[[1 ;;, {1, 2, 7}]], InterpolationOrder → 3,
    Contours → {0, Log10[2]}, ContourStyle → {{Thickness[0.01], Opacity[1], Gray},
      {Thickness[0.01], Opacity[1], Black}}, ContourShading → None]];
```

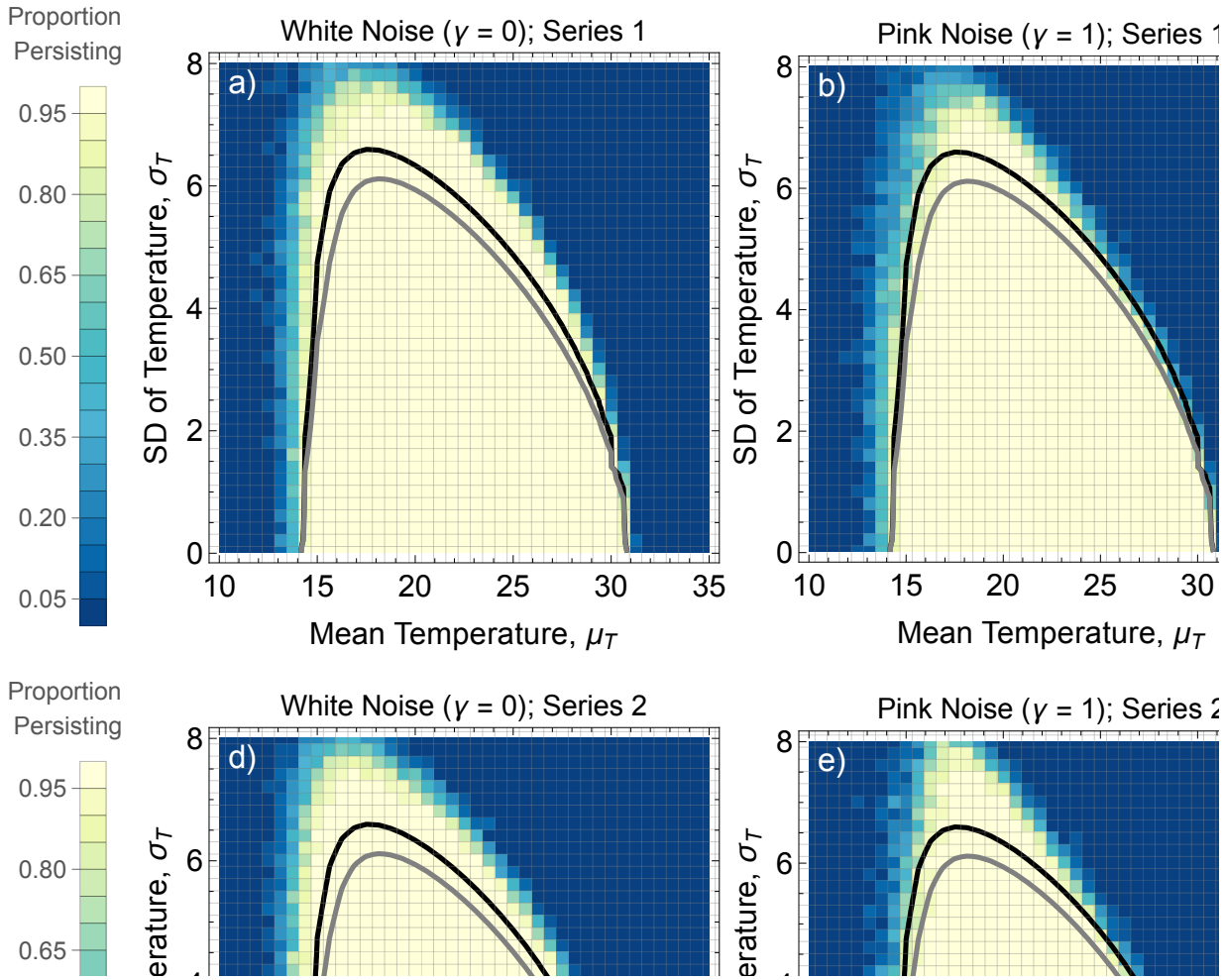
```
SSAbrownplot3 =
  Show[ListContourPlot[SSAbrown3, InterpolationOrder → 0, Contours → 19,
    ColorFunction → newmap, (*PlotLegends→Placed[Automatic,Left],*)
    PlotLabel → Style["Brown Noise (γ = 2); Series 3", 15],
    ImageSize → 310, Frame → True, FrameStyle → 16,
    FrameLabel → {"Mean Temperature, μT", "SD of Temperature, σT"},
    GridLines → {μTrange + .5 × 25 / 40, σTrange + .5 × 8 / 40},
    GridLinesStyle → Directive[Opacity[0.4], Thickness[0.0001]],
    Epilog →
     {{Text[Style["i)", White, 17], {11.2, 7.7}]}}],
   ListContourPlot[moments[[1 ;;, {1, 2, 7}]], InterpolationOrder → 3,
    Contours → {0, Log10[2]}, ContourStyle → {{Thickness[0.01], Opacity[1], Gray},
      {Thickness[0.01], Opacity[1], Black}}, ContourShading → None]];

SSAs = GraphicsGrid[{{SSAwhiteplot, SSApinkplot, SSAbrownplot},
   {SSAwhiteplot2, SSApinkplot2, SSAbrownplot2},
   {SSAwhiteplot3, SSApinkplot3, SSAbrownplot3}}, Spacings → 0, ImageSize → 1000]
(*Export[directory<>"S7.pdf",SSAs,"PDF",ImageResolution→800]*)

(*DIFFERENCE PLOTS*)
(*matrices are not indexed in the same order for some reason;
sort outputs by the same index here*)
sortedw1 = SortBy[SSAwhite, {#[[1]], #[[2]]} &];
sortedw2 = SortBy[SSAwhite2, {#[[1]], #[[2]]} &];
sortedw3 = SortBy[SSAwhite3, {#[[1]], #[[2]]} &];
sortedp1 = SortBy[SSApink, {#[[1]], #[[2]]} &];
sortedp2 = SortBy[SSApink2, {#[[1]], #[[2]]} &];
sortedp3 = SortBy[SSApink3, {#[[1]], #[[2]]} &];
sortedb1 = SortBy[SSAbrown, {#[[1]], #[[2]]} &];
sortedb2 = SortBy[SSAbrown2, {#[[1]], #[[2]]} &];
sortedb3 = SortBy[SSAbrown3, {#[[1]], #[[2]]} &];

(*calculate differences between outputs*)
diffw12 = sortedw1[[All, 3]] - sortedw2[[All, 3]];
SSAw12 = Transpose[{sortedw1[[All, 1]], sortedw1[[All, 2]], diffw12}];
diffp12 = sortedp1[[All, 3]] - sortedp2[[All, 3]];
SSAp12 = Transpose[{sortedw1[[All, 1]], sortedw1[[All, 2]], diffp12}];
diffb12 = sortedb1[[All, 3]] - sortedb2[[All, 3]];
SSAb12 = Transpose[{sortedw1[[All, 1]], sortedw1[[All, 2]], diffb12}];

diffw13 = sortedw1[[All, 3]] - sortedw3[[All, 3]];
SSAw13 = Transpose[{sortedw1[[All, 1]], sortedw1[[All, 2]], diffw13}];
diffp13 = sortedp1[[All, 3]] - sortedp3[[All, 3]];
```

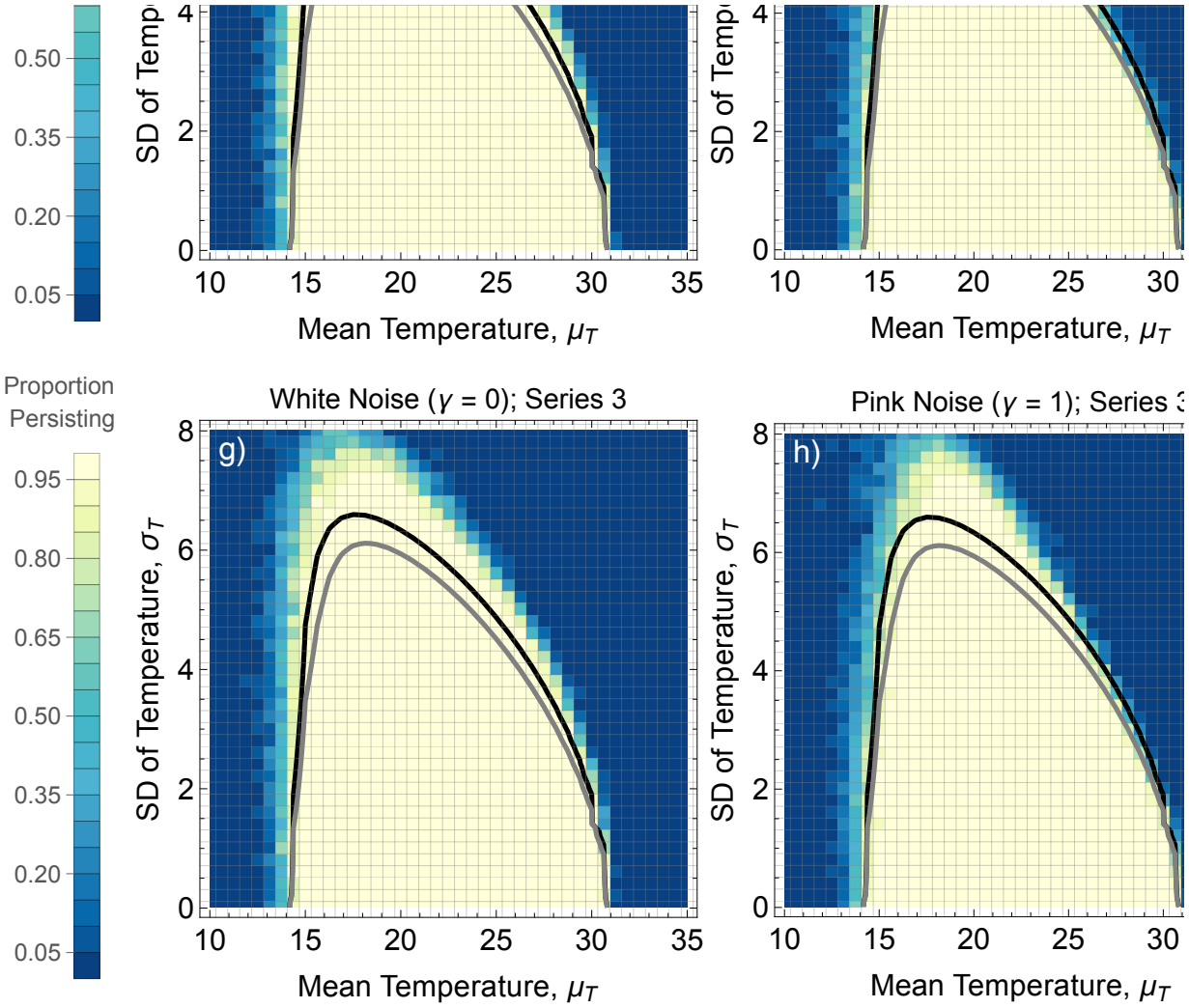
```
SSAp13 = Transpose[{sortedw1[[All, 1]], sortedw1[[All, 2]], diffp13}];
diffb13 = sortedb1[[All, 3]] - sortedb3[[All, 3]];
SSAb13 = Transpose[{sortedw1[[All, 1]], sortedw1[[All, 2]], diffb13}];

(*align color scales across plots;
attempt to make 0 approximately white (not quite successful, but close)*)
vals = {SSAw12[[All, 3]], SSAp12[[All, 3]],
   SSAb12[[All, 3]], SSAw13[[All, 3]], SSAp13[[All, 3]], SSAb13[[All, 3]]};
maxAbs = Max[Abs[vals]];
minVal = -maxAbs;
maxVal = maxAbs;
contours = Subdivide[minVal, maxVal, 19];
newmap2[z_] := Module[{v = z}, v = Clip[v, {minVal, maxVal}];
   Which[v < 0, Blend[{Blue, White}, (v - minVal) / (0 - minVal)],
    v > 0, Blend[{White, Red}, (v - 0) / (maxVal - 0)], True, White]];

(*plot outputs*)
compwhite2 =
  Show[ListContourPlot[SSAw12, InterpolationOrder → 0, Contours → 19, PlotRange →
     {minVal, maxVal}, ColorFunction → newmap2, ColorFunctionScaling → False,
    PlotLegends → Placed[BarLegend[Automatic, All], Left], (*Placed[
     Automatic,Left]*)PlotLabel → Style["White Noise (γ = 0); Series 1-2", 15],
    ImageSize → 310, Frame → True, FrameStyle → 16,
    FrameLabel → {"Mean Temperature, μT", "SD of Temperature, σT"},
    GridLines → {μTrange + .5 × 25 / 40, σTrange + .5 × 8 / 40},
    GridLinesStyle → Directive[Opacity[0.4], Thickness[0.0001]],
    Epilog →
     {{Text[Style["a)", Black, 17], {11.2, 7.7}]}}],
   ListContourPlot[moments[[1 ;;, {1, 2, 7}]], InterpolationOrder → 3,
    Contours → {0, Log10[2]}, ContourStyle → {{Thickness[0.01], Opacity[1], Gray},
      {Thickness[0.01], Opacity[1], Black}}, ContourShading → None]];
comppink2 =
  Show[ListContourPlot[SSAp12, InterpolationOrder → 0, Contours → 19, PlotRange →
     {minVal, maxVal}, ColorFunction → newmap2, ColorFunctionScaling → False,
    PlotLegends → Placed[BarLegend[Automatic, All], Left],
    PlotLabel → Style["Pink Noise (γ = 1); Series 1-2", 15],
    ImageSize → 310, Frame → True, FrameStyle → 16,
    FrameLabel → {"Mean Temperature, μT", "SD of Temperature, σT"},
    GridLines → {μTrange + .5 × 25 / 40, σTrange + .5 × 8 / 40},
    GridLinesStyle → Directive[Opacity[0.4], Thickness[0.0001]],
    Epilog →
     {{Text[Style["b)", Black, 17], {11.2, 7.7}]}}],
   ListContourPlot[moments[[1 ;;, {1, 2, 7}]], InterpolationOrder → 3,
    Contours → {0, Log10[2]}, ContourStyle → {{Thickness[0.01], Opacity[1], Gray},
```

```
      {Thickness[0.01], Opacity[1], Black}}, ContourShading → None]];
compbrown2 =
  Show[ListContourPlot[SSAb12, InterpolationOrder → 0, Contours → 19, PlotRange →
     {minVal, maxVal}, ColorFunction → newmap2, ColorFunctionScaling → False,
    PlotLegends → Placed[BarLegend[Automatic, All], Left],
    PlotLabel → Style["Brown Noise (γ = 1); Series 1-2", 15],
    ImageSize → 310, Frame → True, FrameStyle → 16,
    FrameLabel → {"Mean Temperature, μT", "SD of Temperature, σT"},
    GridLines → {μTrange + .5 × 25 / 40, σTrange + .5 × 8 / 40},
    GridLinesStyle → Directive[Opacity[0.4], Thickness[0.0001]],
    Epilog →
     {{Text[Style["c)", Black, 17], {11.2, 7.7}]}}],
   ListContourPlot[moments[[1 ;;, {1, 2, 7}]], InterpolationOrder → 3,
    Contours → {0, Log10[2]}, ContourStyle → {{Thickness[0.01], Opacity[1], Gray},
      {Thickness[0.01], Opacity[1], Black}}, ContourShading → None]];

compwhite3 =
  Show[ListContourPlot[SSAw13, InterpolationOrder → 0, Contours → 19, PlotRange →
     {minVal, maxVal}, ColorFunction → newmap2, ColorFunctionScaling → False,
    PlotLegends → Placed[BarLegend[Automatic, All], Left], (*Placed[
     Automatic,Left]*)PlotLabel → Style["White Noise (γ = 0); Series 1-3", 15],
    ImageSize → 310, Frame → True, FrameStyle → 16,
    FrameLabel → {"Mean Temperature, μT", "SD of Temperature, σT"},
    GridLines → {μTrange + .5 × 25 / 40, σTrange + .5 × 8 / 40},
    GridLinesStyle → Directive[Opacity[0.4], Thickness[0.0001]],
    Epilog →
     {{Text[Style["d)", Black, 17], {11.2, 7.7}]}}],
   ListContourPlot[moments[[1 ;;, {1, 2, 7}]], InterpolationOrder → 3,
    Contours → {0, Log10[2]}, ContourStyle → {{Thickness[0.01], Opacity[1], Gray},
      {Thickness[0.01], Opacity[1], Black}}, ContourShading → None]];
comppink3 =
  Show[ListContourPlot[SSAp13, InterpolationOrder → 0, Contours → 19, PlotRange →
     {minVal, maxVal}, ColorFunction → newmap2, ColorFunctionScaling → False,
    PlotLegends → Placed[BarLegend[Automatic, All], Left],
    PlotLabel → Style["Pink Noise (γ = 1); Series 1-3", 15],
    ImageSize → 310, Frame → True, FrameStyle → 16,
    FrameLabel → {"Mean Temperature, μT", "SD of Temperature, σT"},
    GridLines → {μTrange + .5 × 25 / 40, σTrange + .5 × 8 / 40},
    GridLinesStyle → Directive[Opacity[0.4], Thickness[0.0001]],
    Epilog →
     {{Text[Style["e)", Black, 17], {11.2, 7.7}]}}],
   ListContourPlot[moments[[1 ;;, {1, 2, 7}]], InterpolationOrder → 3,
    Contours → {0, Log10[2]}, ContourStyle → {{Thickness[0.01], Opacity[1], Gray},
      {Thickness[0.01], Opacity[1], Black}}, ContourShading → None]];
```

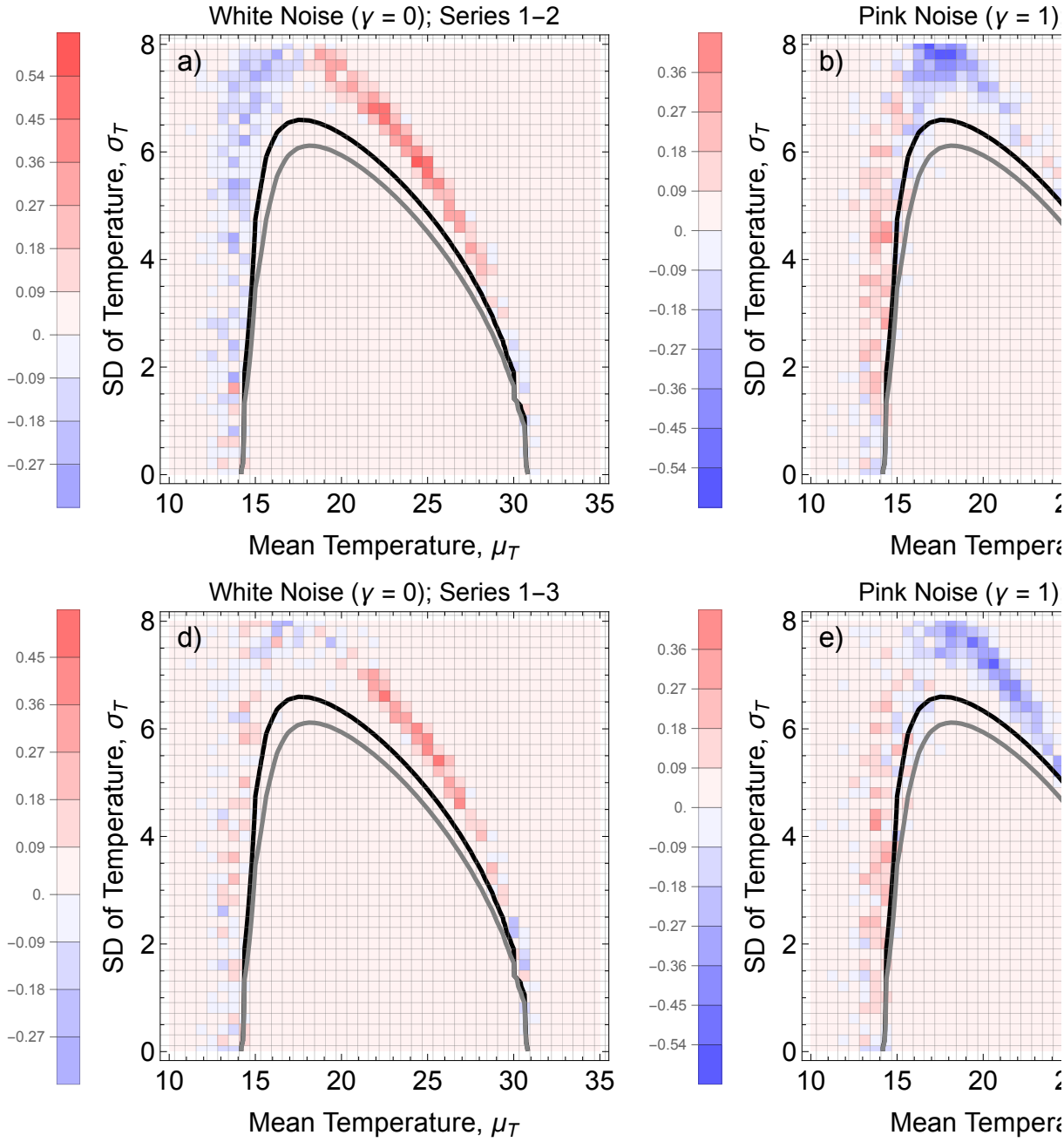
```
compbrown3 =
  Show[ListContourPlot[SSAb13, InterpolationOrder → 0, Contours → 19, PlotRange →
     {minVal, maxVal}, ColorFunction → newmap2, ColorFunctionScaling → False,
    PlotLegends → Placed[BarLegend[Automatic, All], Left],
    PlotLabel → Style["Brown Noise (γ = 2); Series 1-3", 15],
    ImageSize → 310, Frame → True, FrameStyle → 16,
    FrameLabel → {"Mean Temperature, μT", "SD of Temperature, σT"},
    GridLines → {μTrange + .5 × 25 / 40, σTrange + .5 × 8 / 40},
    GridLinesStyle → Directive[Opacity[0.4], Thickness[0.0001]],
    Epilog →
     {{Text[Style["f)", Black, 17], {11.2, 7.7}]}}],
   ListContourPlot[moments[[1 ;;, {1, 2, 7}]], InterpolationOrder → 3,
    Contours → {0, Log10[2]}, ContourStyle → {{Thickness[0.01], Opacity[1], Gray},
      {Thickness[0.01], Opacity[1], Black}}, ContourShading → None]];

SSAdiffs = GraphicsGrid[{{compwhite2, comppink2, compbrown2},
   {compwhite3, comppink3, compbrown3}}, Spacings → 2, ImageSize → 1150]
(*Export[directory<>"S8.pdf",SSAdiffs,"PDF",ImageResolution→800]*)
```

Out[3246]=




0.50
0.35
0.20
0.05
SD of Temp
4
2
0
10
15
20
25
30
35
Mean Temperature, μ_T
Proportion Persisting
0.95
0.80
0.65
0.50
0.35
0.20
0.05
White Noise (γ = 0); Series 3
Pink Noise (γ = 1); Series 3
g)
h)
SD of Temperature, σ_T
8
6
4
2
0

Out[3280]=


White Noise (γ = 0); Series 1–2
Pink Noise (γ = 1)
a)
b)
SD of Temperature, σT
Mean Temperature, μT
White Noise (γ = 0); Series 1–3
Pink Noise (γ = 1)
d)
e)
SD of Temperature, σT
Mean Temperature, μT