Logistic Maps

Creating Chaos

Amanda Rojas

[1]PhD Graduate Student Center for Complex Systems and Brain Sciences Florida Atlantic University

Nonlinear Dynamics, Fall 2020

Introduction

What is a logistic map?

Logistic maps are the graphing of a reiterated function that results in repeated bifurcations and ultimately chaotic behavior.

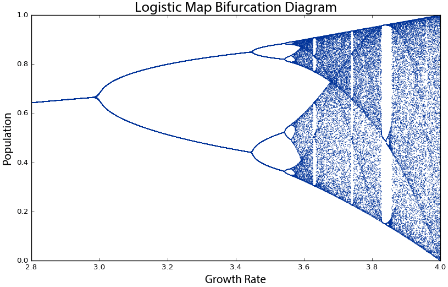

Figure: Bifurcation diagram

Theory

These functions are very simple non-linear equations that can be defined as any parabolic equation.
The repeated iterations of the function result in repeated bifurcations and ultimately chaos. The example function we will look at in this lab is the following:

$$x_{n+1} = rx_n(1 - x_n) \tag{1}$$

Plotting the increasing of r (the growth rate) vs the equilibrium factor of each r, results in the Mandelbrot set, a well studied fractal.

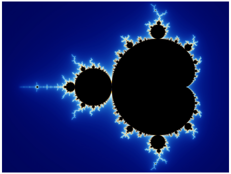

Figure: Mandelbrot set

Theory Continued

The chaos does have some organization though. The period doubling does occur in the beginning and form chaos, however at around 3.83, the periodicity returns to 3, however as r grows again it doubles to 6, 12, 24, etc.

In fact any real number above 2 can be found as the equilibrium constant if you have the correct r value.

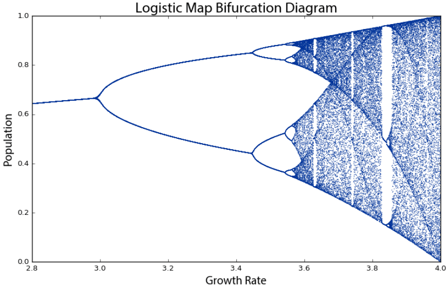

Figure: Here you can easily see the period of 3 when r= 3.83

Conclusions

As we have seen previously, complex systems can be created from a simple equation.
Logistic maps have been used to replicate nature, fractals, and nonlinear systems like the brain.
It seems that this period bifurcation and the constants that regulate it are a law of nature and manipulating the terms and variables involved can give us some insight into nonlinear systems.

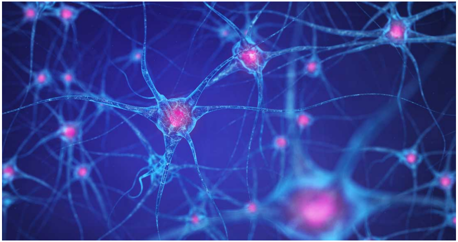

Figure: The complexity of systems neuroscience can benefit from a nonlinear approach.