The Mandelbrot Set

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Abstract

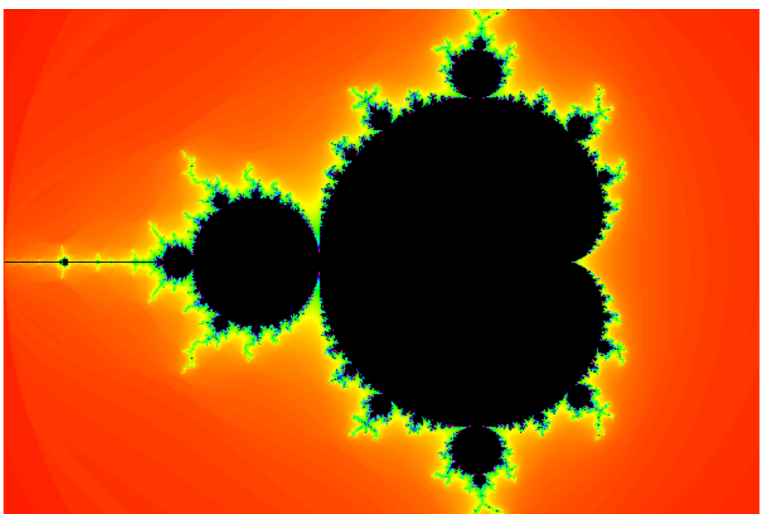
This lab formally introduces the Mandelbrot set. This was briefly mentioned in a previously lab on bifurcation, however in this lab we expand upon the concept and introduce the set. The Mandelbrot set is a plot of the following recursive function:

$$z_{n+1} = z_n^2 + c \tag{1}$$

For each point initial point, c, if the function settles on an equilibrium point than that number, c, is a part of the set. If however, the function ascends to infinity, it is not a part of the set.

1 Introduction

Discovered by Dr Adrien Douady, and named for Dr. Benoit Mandelbrot, the Mandelbrot set is a fractal, otherwise known as a defined shape with an infinite perimeter. It's shape is iterated on smaller and smaller scales as you specify initial values of c with greater and greater accuracy. The function can be iterated infinitely and the detail of the set will only increase.



2 Theory

The Mandelbrot set works as follows (such that $z_0 = 0$) :

1. Take a number and plug it in for c, solve.
2. Take the result and plug it in for c, solve.
3. Repeat.

When following these rules, should the result diverge, either into infinity or descend to zero, they will not be a part of the set. If the numbers are bound then they are a part of the set. For example, let's look at a number in the set, -1.

$$z_{n+1} = 0^2 + (-1) \tag{2}$$

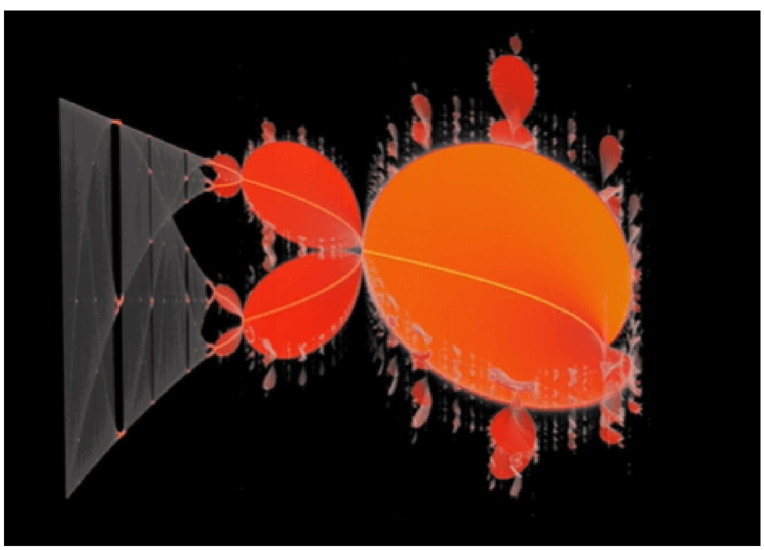
This is -1. Now let's iterate the function again with our new z_n.

$$z_{n+1} = -1^2 + (-1) \tag{3}$$

This is 0. Now let's iterate the function again with our new z_n.

$$z_{n+1} = 0^2 + (-1) \tag{4}$$

The answer is again -1. This value is bound to those limits and is therefore part of the set. This can be done for any value and, when done infinitely the Mandelbrot set appears.
The Mandelbrot set is also connected to the bifurcation diagram, as we saw in a previous lab. There are points in the Mandelbrot set where the function results in a finite number, and these values correspond to the periodicity within the bifurcation diagram.



3 Procedures

```
[1] import torch
    import matplotlib.pyplot as plt
```

```
[2] xmin = -2.4
    xmax = 1.2
    ymin = -1.2
    ymax = 1.2
    maxiter = 256

    w = 3200
    h = 2400
    dpi = 1000
```

```
[3] x = torch.linspace(xmin, xmax, w).cuda()
    y = torch.linspace(ymin, ymax, h).cuda()

    cx, cy = torch.meshgrid([x,y])

    zx = 0*cx
    zy = 0*cy

    M = torch.zeros((w,h)).cuda() #Represent the color of the pixel, ininitialy 0

    #z = z**2 + c == (zx+zyi)**2 + (cx+cyi) == (zx**2-zy**2+cx)+(2*zx*zy+cy)i == (zxn)+(zyn)i, complex numbers are mapped real->x and complex->y
    for i in range(maxiter):
        zx2 = zx**2
        zy2 = zy**2
        #inf is a tensor containing all the points for which zx2+zy2>4
        inf = (zx2+zy2)>4 #if zx2+zy2>4 then sqrt(zx2+zy2)>2, i.e. point's distance from (0,0) is >2, i.e. it will escape to inifinity;
        M[inf] = i #for all the points escaping to infinity, store the iteration which is plotted as a color
        zxn = zx2 - zy2 + cx
        zyn = 2*zx*zy + cy
        zx = zxn
        zy = zyn
```

4 Conclusions

The Mandelbrot set is a fractal resultant from a simple equation and hearkens back to the previous concepts we have discussed within nonlinear dynamical systems. The bifurcation diagram, which we learned about previously is directly tied to the Mandelbrot set and their relevance is seen within nonlinear systems.