Logistic Maps

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Abstract

This lab introduces the concept of logistic maps and bifurcation. Logistic maps are the graphing of a reiterated function that results in repeated bifurcations and ultimately chaotic behavior. These functions are very simple non-linear equations that can be defined as any parabolic equation. The repeated iterations result in repeated bifurcations and ultimately chaos, however the chaos is not as unpredictable as one would assume.

1 Introduction

The equation we looked at in this lab is the following:

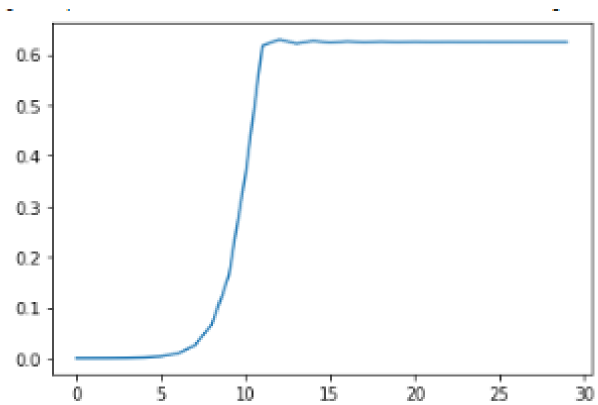
$$x_{n+1} = rx_n(1 - x_n) \tag{1}$$

In this equation "r" is the growth rate of variable x.
"X" will be whatever it is you're measuring, you're independent variable. It could be the population, temperature, etc.
We add the term "(1-x_n)" to represent environmental constraints that restrict the variable x from exponentially increasing.
The variable "n" is obviously the amount of iterations.
Now let's look at the result of manipulating the variables.

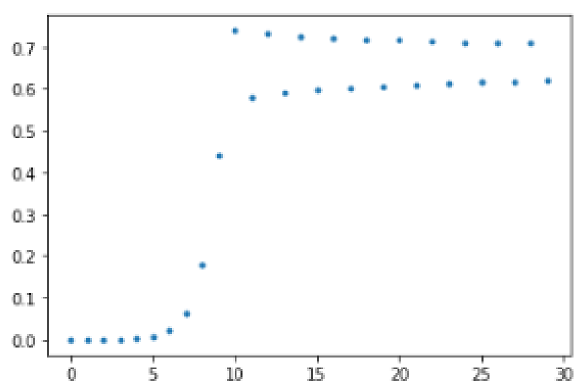
2 Theory

For this example, we will call our x variable a population. The easiest thing to manipulate is the x value, or initial value. This will affect initial population of our sample.
Now the interesting thing about manipulating the x value is that the population doesn't exponentially grow or decrease, due to our second term. What actually happens is that the populations level out over a relatively short period of time, regardless of the initial value.

So our initial value is negligible. Now let's manipulate r, the growth rate. Obviously if the growth rate is less than one, the population will exponentially decline until ultimate extinction of the species. If the growth rate is greater than one the population will increase, correct? Not exactly.
Increasing the growth rate typically results in an equilibrium constant, that is mentioned above. The population typically levels out after a couple of generations.



However the growth rate isn't that simple. Once you increase the growth rate to a certain number, typically around 3, the population doesn't reach an equilibrium. It bifurcates and populations fluctuate from year to year. This bifurcation never reaches an equilibrium. The population will continue to fluctuate.



Increasing r continues to increase the amount of bifurcations and once r= 3.57 the population is completely unpredictable and chaotic.
This unpredictability becomes stable at certain values for r. For example at 3.83 there is an equilibrium with a period of 3 years. Then it follows the same bifurcation as it increases, going to a period of 6 and 12, and then ultimately chaos again. Theoretically you can find an r value to give you the period of any real number.

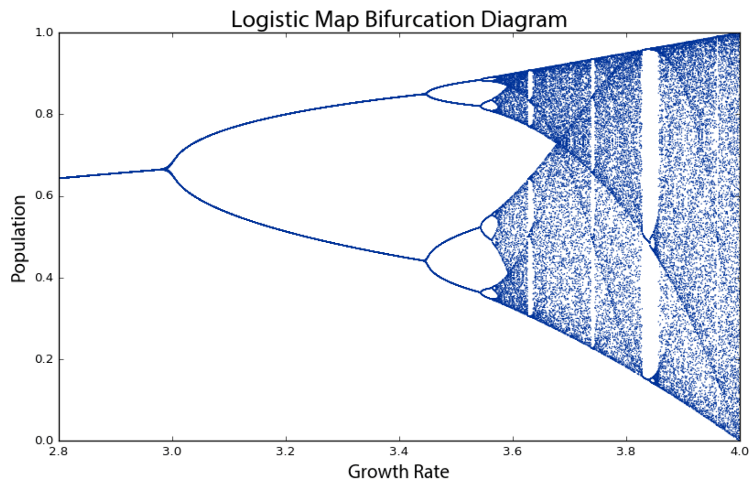


Figure 1: Bifurcation diagram. Here you can clearly see the chaos returning to periods of 3, when r goes to 3.83

3 Procedures

Logistic Map (Lab 4)

```
#imports number and plot libraries
import numpy as np
import matplotlib.pyplot as plt
```

```
#defines logistic map
def logistic(r, x):
  return r * x * (1 - x)
```

```
x= 0.5
```

```
r= 3.5
```

```
x= logistic(r, x)
```

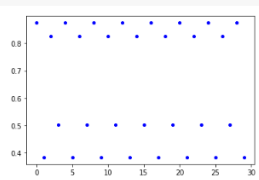
```
x
```

```
0.875
```

```
for i in range(30):

#defines logistic relationship
  plt.plot(i,x,'b.',markersize=8)

  x= logistic(r,x)
```



Vectorized Logistic Map

```
#defines different r values
n = 10
#generates values of n between the defined ranges
r = np.linspace(2.5, 4.0, n)
```

```
#populations are initialized to the same level
x = 1e-5 * np.ones(n)
```

```
#interates thee function 30 times
iterations = 30
```

```
x= logistic(r, x)
```

```
x.shape
```

```
(10,)
```

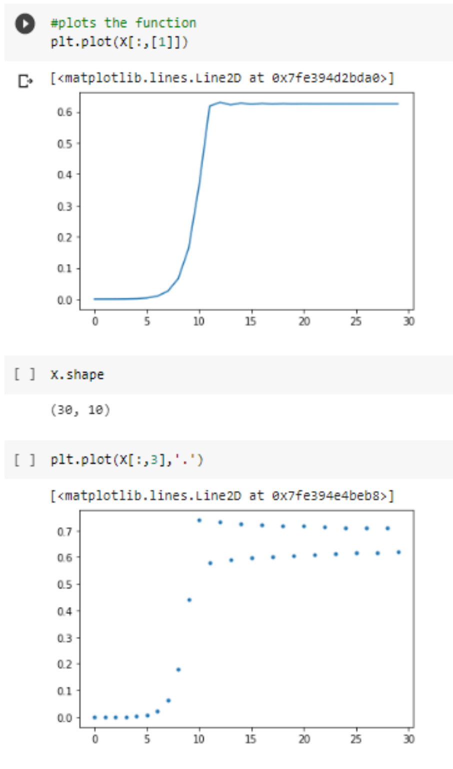
```
x
```

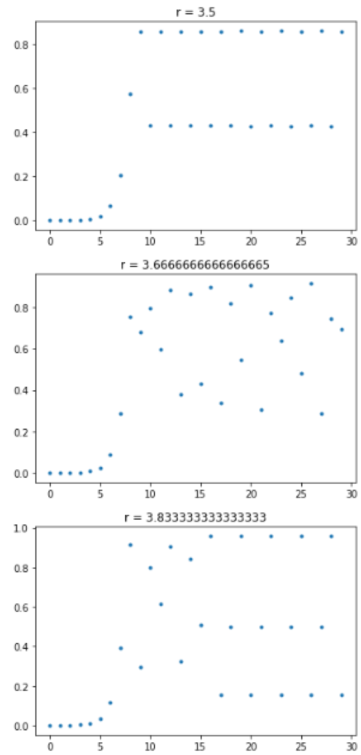
```
array([2.499975e-05, 2.666640e-05, 2.833305e-05, 2.999970e-05,
       3.166635e-05, 3.333300e-05, 3.499965e-05, 3.666630e-05,
       3.833295e-05, 3.999960e-05])
```

```
#creates a number library of the iterations done
X = np.zeros((iterations,n))
```

```
for i in range(1,iterations):

    x = logistic(r, x)
#iterates and completes the function
    X[i,:] = x
```





4 Conclusions

As we have seen previously, complex systems can be created from a simple equation. Logistic maps have been used to replicate nature, fractals, and nonlinear systems like the brain. It seems that this period bifurcation and the constants that regulate it are a law of nature and manipulating the terms and variables involved can give us some insight into nonlinear systems.