Ordinary Differential Equations

ODE

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Introduction to Ordinary Differential Equations

The simplest form of an ODE is the following:

$$(dx/dt) = ax \tag{1}$$

Here in this equation "t" is a real variable and "d" is the derivative of the function x=x(t). This can be expanded to the following:

$$x'(t) = ax(t) \tag{2}$$

Calculus allows us to derive the following solution to the equation:

$$f'(t) = aK^{at} = af(t) \tag{3}$$

Therefore the solution can be identified as f(t)=K^{at}.

Ordinary Differential Equations (ODE)

In this equation k is any constant derived via a certain initial condition.

If a doesn't equal 0, than the equation is stable (to a point) and the initial condition has some degree of freedom. Zero is therefore, sometimes referred to as the bifurcation point, otherwise known as the point where chaos can emerge from the system given the slightest change in the initial condition.

ODE- Expanding beyond the basics

For example, let's examine a system of two differential equations in two unknown functions: $x_1'(t) = a_1 x_1, x_2'(t) = a_2 x_2$
Immediately these functions can be solved because there is no specific relationship identified between them.

$$x_1(t) = K_1^{a_1 t} \tag{4}$$

$$x_2(t) = K_2^{a_2 t} \tag{5}$$

Although this is a simple system, more complicated systems can be reduced to this form.

Newton's Second Law

The concept between the physical force field and the mathematical concept of differential equations is Newton's second law: "F = m a". We say that x(t) denotes the position vector of the particle at time t, where x: R -¿ R' is a sufficiently differentiable curve. Since the acceleration vector is the second derivative of x(t) with respect to time, one can say that:

$$a(t) = \ddot{x}(t) \tag{6}$$

Newton's second law states:

$$F(x(t)) = m\ddot{x}(t) \tag{7}$$

Thus we obtain a second order differential equation:

$$\ddot{x} = \frac{1}{m}F(x) \tag{8}$$

Conclusions

In this lab we analyzed the importance of initial conditions in the context of dynamical systems. We looked at an example with a particle moving through a two dimensional plane. The real applications of this are seen in the documentary featuring the women behind the ENIAC computer.

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