Lab Report 2- The Chaos Game

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Abstract

The chaos game is a method of generating fractals, or a defined shape with an infinite perimeter, from perceived chaos and randomness. Fundamentally it's about taking a seemingly random strategy with a single consistently implemented rule in order to generate order. Essentially infinite fractals can be created with the addition of constrictions such as: don't choose the vertex that was just chosen, don't choose the vertex clockwise to the vertex just chosen, etc. The Chaos Game demonstrates in a very simple and beautiful way the way that researchers and nature can decrease entropy and create order within chaos in the most simple way with the least energy expenditure. All that is required is consistency and time.

1 Introduction

The most common example of the Chaos Game is Sierpinski's Triangle.
It works by taking a shape and choosing a random point within that shape. Then chose a random vertex in the shape and find the midpoint between your randomly chosen point and vertex. Then choose another random vertex and find the midpoint between those two, and so on and so forth. This can be done for any shape except those with four vertices (square, rectangle, parallelogram, etc.). For those shapes another step must be added, which is that you may not choose a previously chosen vertex for example.

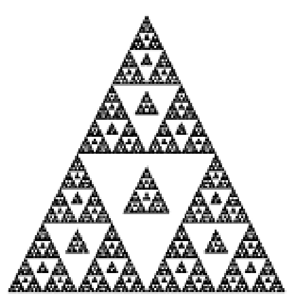

Figure 1: Example of the Sierpinski's Triangle, the simplest method of playing the chaos game.

2 Theory

The Chaos Game is fundamentally a method of slowly organizing a massive amount of data in the simplest fashion available. This makes sense for biological systems trying to conserve energy and maximize efficiency of their systems.
Scientists have attempted to use Chaos Theory to study biological systems and have found examples of the chaos game in nature. The most famous of these is Barnsley's Fern.

Figure 2: Barnsley's fern, which is an example of mathematics replicating nature.

Scientists have also analyzed DNA and different protein sequences with the Chaos Game and have found a certain amount of order within the copious amounts of biological data that is DNA. (Almeida et al., 2001)

3 Procedures

The Chaos Game (for generating the Sierpinski Triangle) works as follows:

1. Generate an equation for calculating the midpoint (defined here as P,Q) of two points (here 0,0 and 1,1).

$$(0.5 * (P[0] + Q[0]), 0.5 * (P[1] + Q[1])) \tag{1}$$

2. Then define the vertices of that shape (3 for the Sierpinski Triangle). In addition, you'll want your triangle to be an equilateral triangle in order to generate the Sierpinski Triangle.

$$vertices = [(0, 0), (2, 2 * np.sqrt(3)), (4, 0)] \tag{2}$$

3. Then you will choose a random vertex and find the midpoint between your position and that vertex and plot a point. Continue this n amount of times.

$$n = 1000 * 100 \tag{3}$$

$$x = [0] * n \tag{4}$$

$$y = [0] * n \tag{5}$$

$$x[0] = random() \tag{6}$$

$$y[0] = random() \tag{7}$$

for i in range (1, n):

$$x[i], y[i] = midpoint(vertices[randint(0, 2)], (x[i-1], y[i-1])) \tag{8}$$

4 Analysis

Then you just continue the last step in the equation, and theoretically, you can do this endlessly as fractals continue to create the shape on smaller and smaller scales.

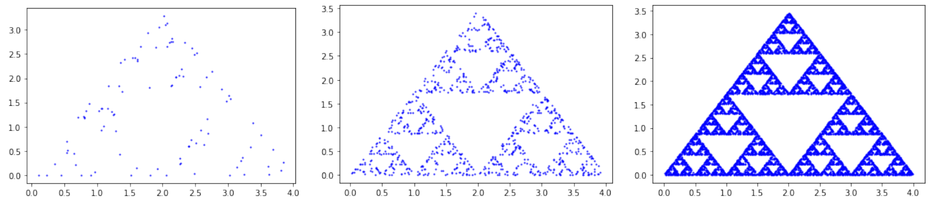

Figure 3: Sierpinski's Triangle, iterated 100, 1000, and 10000 times respectively.

The Chaos Game can also be expanded upon, if you add another condition, or change the shape to a more complex one (for example, see figure 2 above). These conditions change the fractal that you end up with and fundamentally, the organization of the data.

5 Conclusion

In complex systems such as neuroscience where the biological data that can be collected is infinite when you consider connectomics, cell and receptor mechanics, firing rates and synchrony, as well as diversity within disease states, the Chaos Game and coding as a mechanism to execute this serve as an important tool for organizing copious amounts data that wouldn't be able to be analyzed by conventional means.

The Chaos Game allows us to appreciate this in a visual sense and the different implementations of the Chaos Game show us it's application in nature.

6 References

1. Barnsley, M.F.(1993).Fractals everywhere. Boston: Acad.Press.
2. Almeida, J. S., Carrico, J. A., Maretzek, A., Noble, P. A., Fletcher, M. (2001). Analysis of genomic sequences by Chaos Game Representation. Bioinformatics, 17(5), 429-437.