Euler's Method

Approximating ODEs

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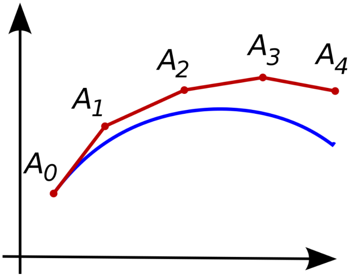
Introduction

In this lab we study Euler's Method, which is another technique used to study differential equations. Euler's Method uses the idea of linear approximation in order to estimate the solution to an ODE given initial conditions.

Euler's method is based on approximation and therefore has a certain margin of error. The local error, or the error per step, is directly proportional to the global error, or the error at a given time.



Theory

Euler's method is defined by the following equation:

$$x_{n+1} = x_n + hf(t_n, x_n) \tag{1}$$

You begin by computing the function $\mathrm{f}(\mathrm{t}_0, x_0)$, or$f(t, x)$.

This results in the slope of the tangential line at point (t, x).

Next you multiply this slope by h, which is the step size, or change in t. This results in a change in the x value and a new value to repeat the process.

A common thought is to decrease h in order to reduce the step size. This is beneficial except for the fact that it increases the necessary computations required to estimate the result.

There are modifications to Euler's method that have made it more accurate and reduce the power needed to compute with said accuracy. Below are example of modifications to the Euler method:

The Trapezoid Rule:

$$x_{k+1} = x_k + \frac{1}{2}h[f(t_k, x_k) + f(t_k + h, x_k f(t_k, x_k))] \tag{2}$$

The Modified Euler Method:

$$x_0 = \alpha \tag{3}$$

$$x_{i+1} = x_i + \frac{1}{2}h[f(t_i, x_i) + f(t_{i+1}, x_I + hf(t_i, x_i))] \tag{4}$$

The Heun Method:

$$x_0 = \alpha \tag{5}$$

$$x_{i+1} = x_i + \frac{h}{4}[f(t_i, x_i) + 3f(t_i + \frac{2}{3}h, x_i + \frac{2}{3}hf(t_i, x_i))] \tag{6}$$

Conclusions

Euler's method is a valuable computational tool for calculating derivatives giving initial conditions. In order to increase accuracy while maintaining the computational power needed, one must implement different modifications to Euler's method, like the ones we discussed here. Ultimately Euler's method is an important tool for accurately solving differential equations.