The Chaos Game

Generating Fractals from Randomness

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Introduction

What is the Chaos Game?

The Chaos Game is a method of generating fractals from seemingly random data points by implementing a simple re-iterated rule.

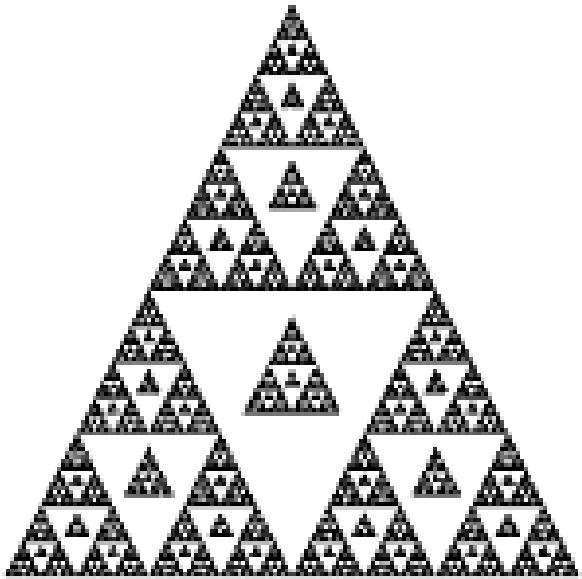

Figure: Example of the Sierpinski's Triangle, the simplest method of playing the chaos game.

Theory

The Chaos Game has been used to replicate things in nature. The best example of this is Michael Barnsley's fern.

Figure: Micheal Barnsley enjoying some tea.

Figure: The Barnsley Fern increasing in complexity with additional iterations

Procedures for Generating a Sierpinski Triangle

1. Generate an equation for calculating a midpoint of two points

$$(0.5 * (P[0] + Q[0]), 0.5 * (P[1] + Q[1])) \quad (1)$$

2. Define the vertices of your shape (for the Sierpinski Triangle, define an equaliateral triangle

$$vertices = [(0, 0), (2, 2 * np.sqrt(3)), (4, 0)] \quad (2)$$

3. Generate a random vertex and calculate the midpoint between your current point and that randomly chosen vertes

for i in range (1, n):

$$x[i], y[i] = midpoint(vertices[randint(0, 2)], (x[i-1], y[i-1])) \quad (3)$$

Analysis

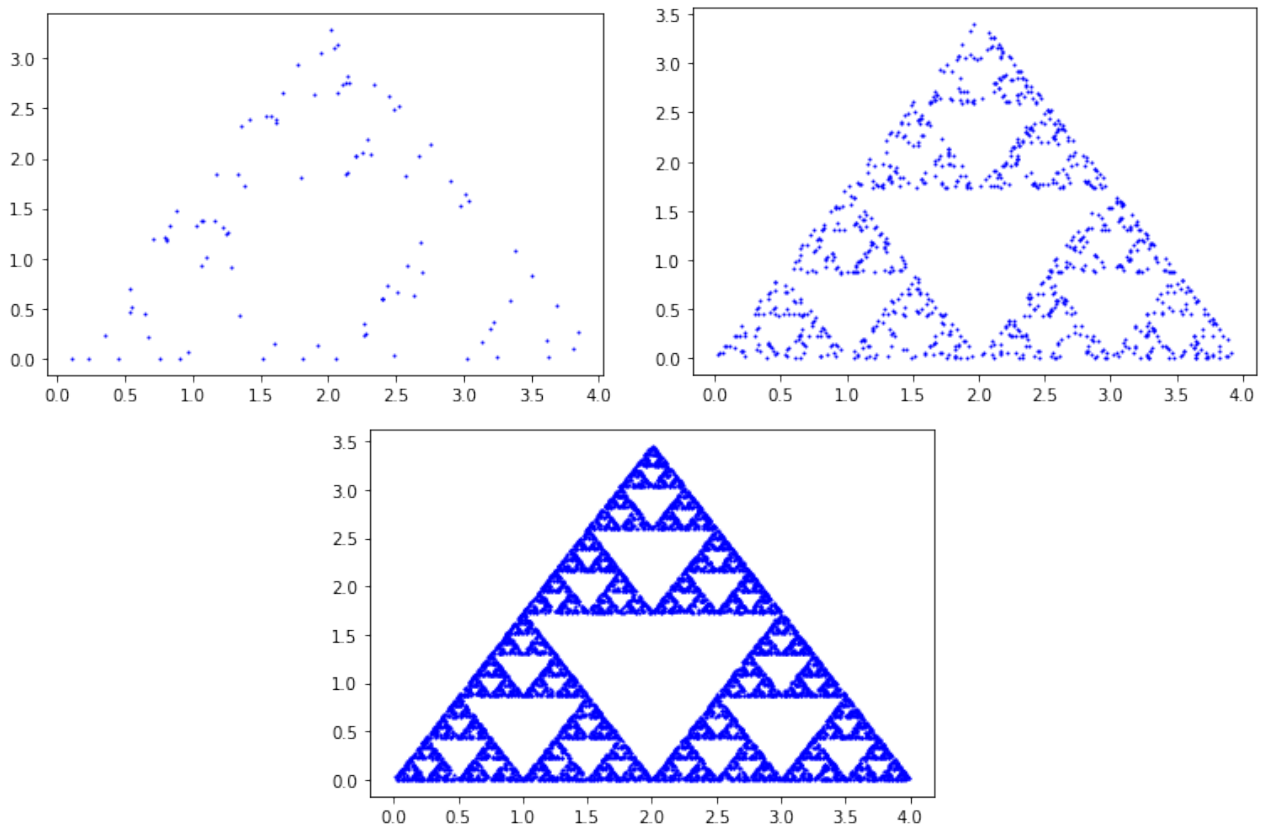

Figure: Sierpinski's Triangle, iterated 100, 1000, and 10000 times respectively.

Here you can appreciate the increased complexity developed with subsequent iterations of the function.

Conclusions

In complex systems such as neuroscience where the biological data that can be collected is infinite when you consider:

- *connectomics*
- *cell and receptor mechanics*
- *firing rates and synchrony*
- *comorbidities and diversity of disease*

The Chaos Game and coding as a mechanism to execute this serve as an important tool for organizing copious amounts data that wouldn't be able to be analyzed by conventional means.