fdasrsf Documentation

Release 2.4.1

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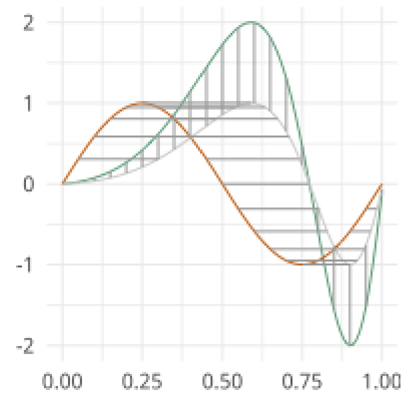
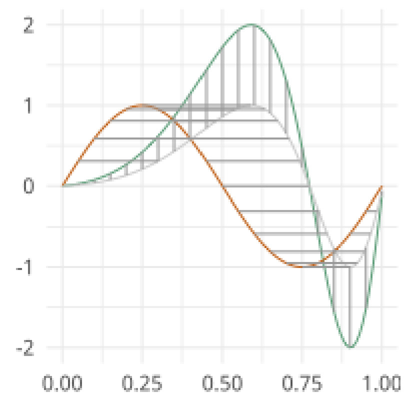


A python package for functional data analysis using the square root slope framework and curves using the square root velocity framework which performs pair-wise and group-wise alignment as well as modeling using functional component analysis and regression.

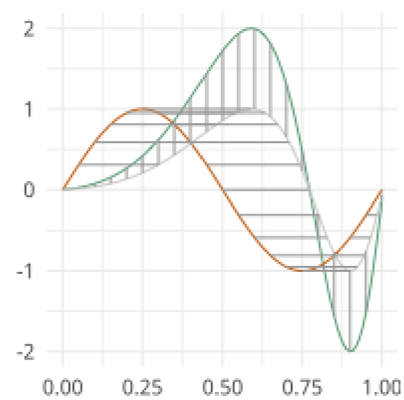


CHAPTER

ONE

USER GUIDE

Contents:



1.1 Elastic Functional Alignment

Otherwise known as time warping in the literature is at the center of elastic functional data analysis. Here our goal is to separate out the horizontal and vertical variability of the functional data

[1]:
```
import fdasrsf as fs
import numpy as np
```

Load in our example data

[2]:
```
data = np.load('../../bin/simu_data.npz')
time = data['arr_1']
f = data['arr_0']
```

We will then construct the `fdawarp` object

[3]:
```
obj = fs.fdawarp(f,time)
```

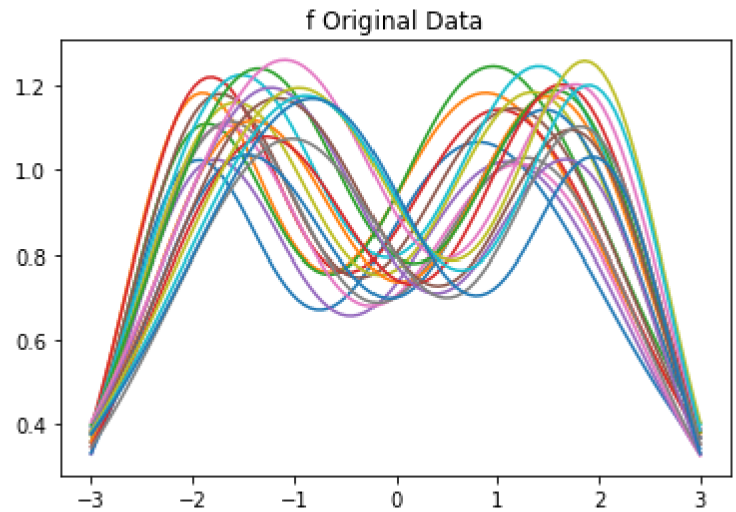
Next we will align the functions using the elastic framework

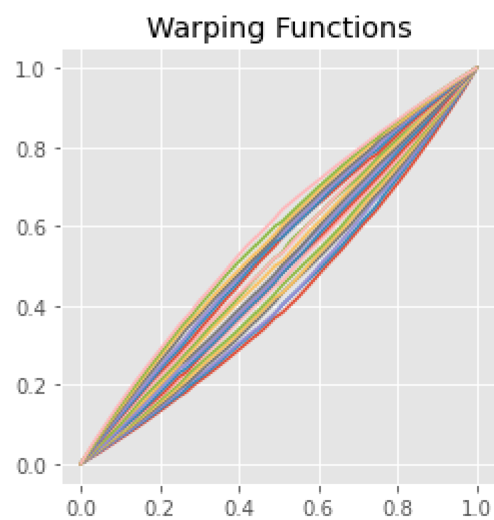
[4]:
```
obj.srsf_align(parallel=True)
```

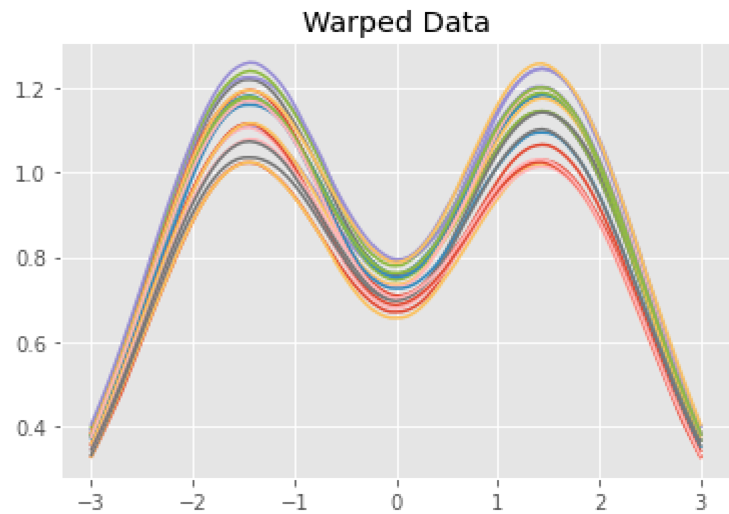
```
Initializing...
Compute Karcher Mean of 21 function in SRSF space...
updating step: r=1
updating step: r=2
```

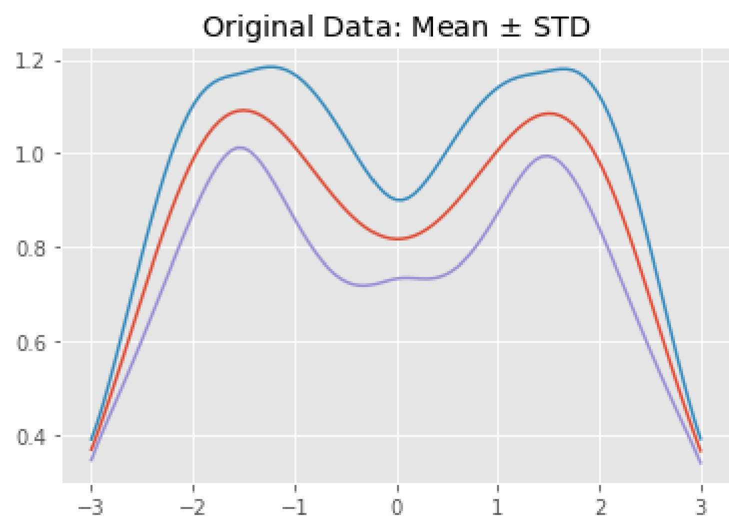
Display plots demonstrating the alignment

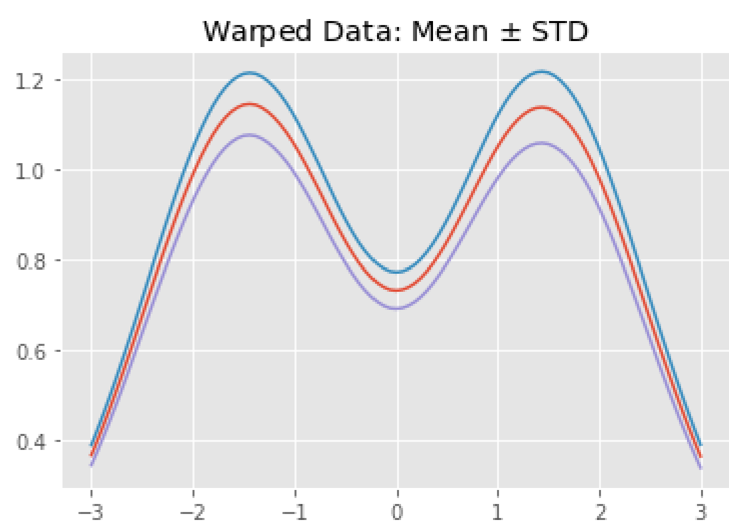
[5]:
```
obj.plot()
```

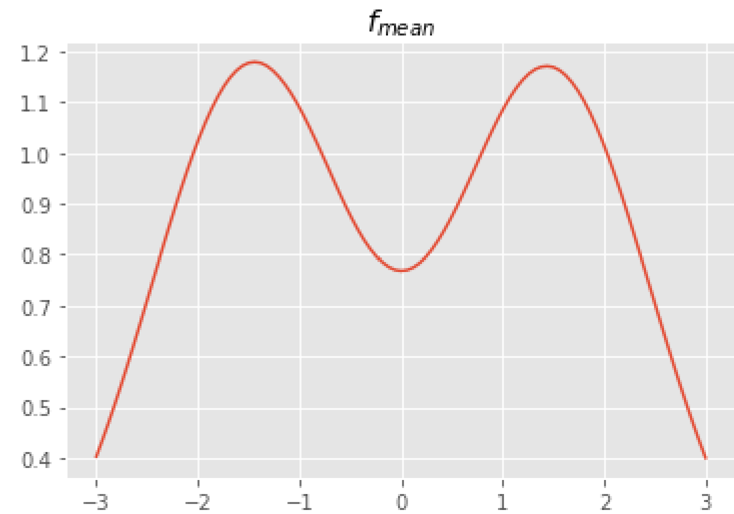


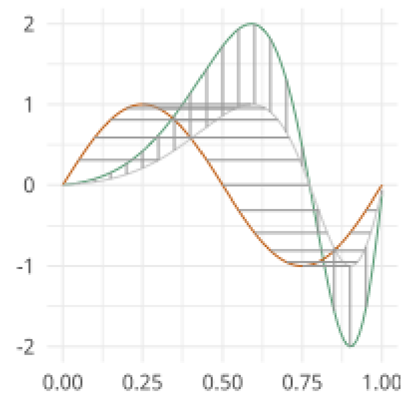



Warped Data
1.2
1.0
0.8
0.6
0.4
−3
−2
−1
0
1
2
3


Original Data: Mean ± STD
1.2
1.0
0.8
0.6
0.4
−3
−2
−1
0
1
2
3


Warped Data: Mean ± STD
1.2
1.0
0.8
0.6
0.4
−3
−2
−1
0
1
2
3





1.2 Elastic Functional Principal Component Analysis

After we have aligned our data we can compute functional principal component analysis (fPCA) on the aligned data, warping functions, and jointly

[1]:
```
import fdasrsf as fs
import numpy as np
```

We will load in our example data again and compute the alignment

[2]:
```
data = np.load('../../bin/simu_data.npz')
time = data['arr_1']
f = data['arr_0']
obj = fs.fdawarp(f,time)
obj.srsf_align(parallel=True)
```

```
Initializing...
Compute Karcher Mean of 21 function in SRSF space...
updating step: r=1
updating step: r=2
```

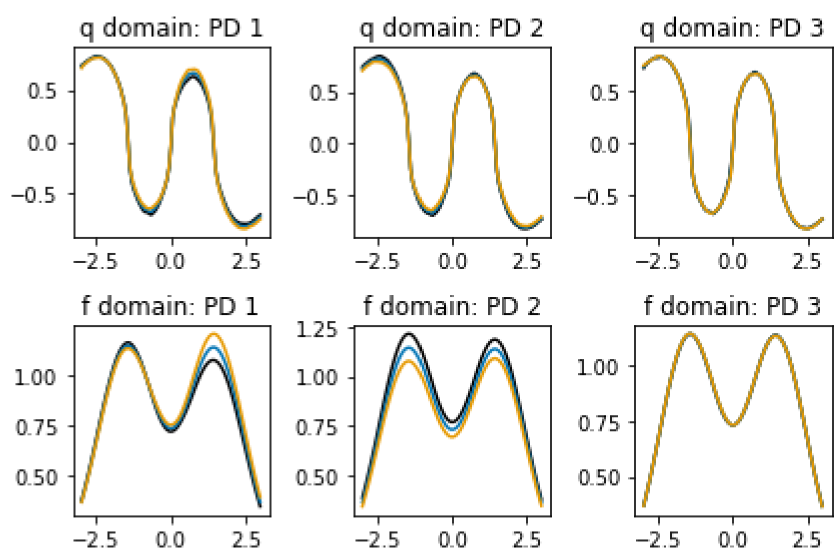
1.2.1 Vertical fPCA

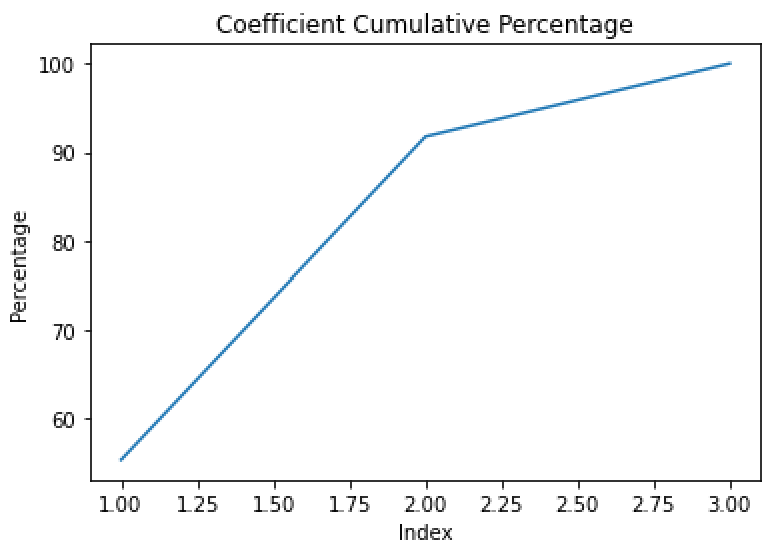
We will first compute fPCA on the aligned functions, by constructing the object and computing the PCA for the number of components, default=3)

[3]:
```
vpca = fs.fdavpca(obj)
vpca.calc_fpca(no=3)
```

We then can plot the principal directions

[4]:
```
vpca.plot()
```





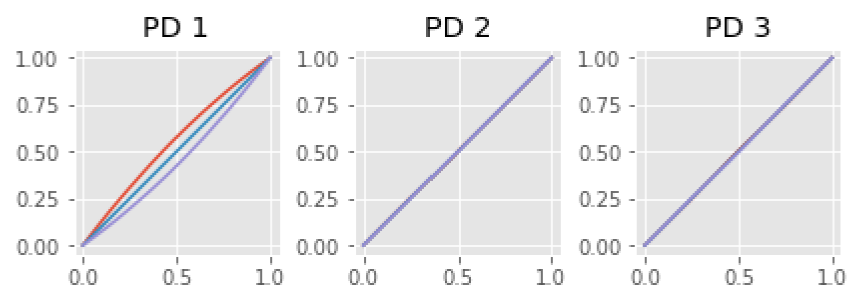
1.2.2 Horizontal fPCA

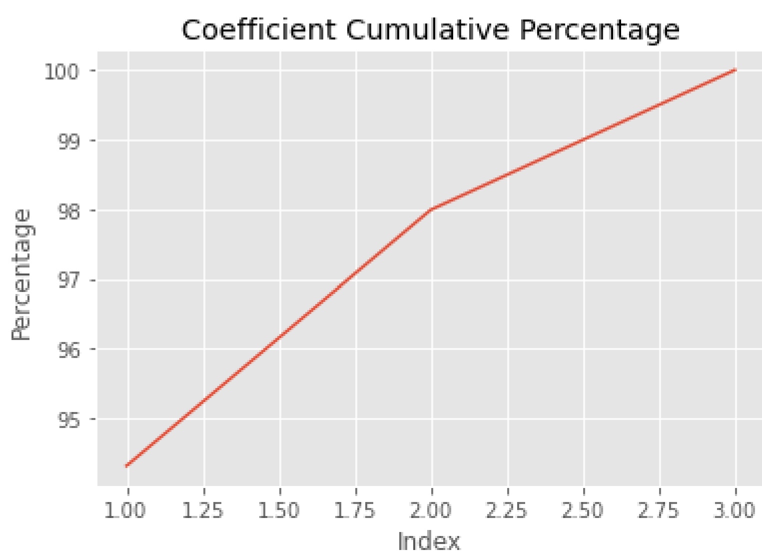
We can then compute PCA on the set of warping functions

[5]:
```
hpca = fs.fdahpca(obj)
hpca.calc_fpca(no=3)
```

We then can plot the principal directions

[6]:
```
hpca.plot()
```





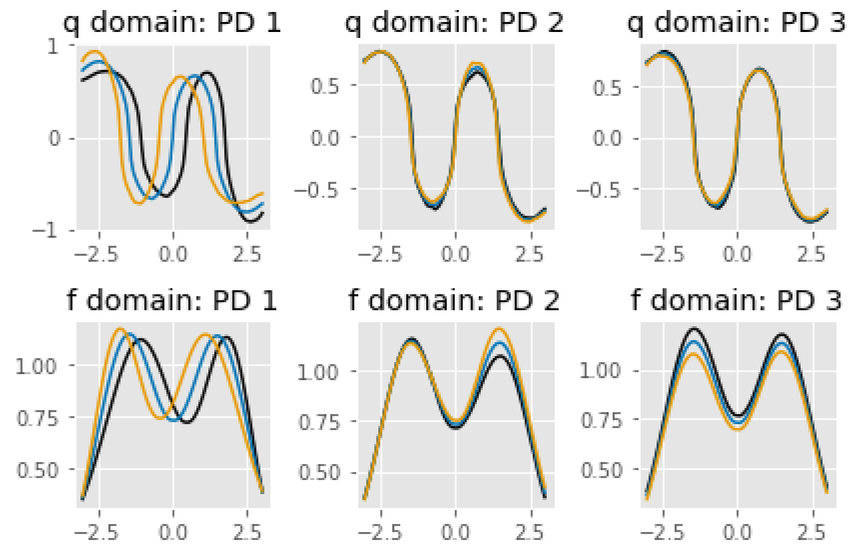
1.2.3 Joint fPCA

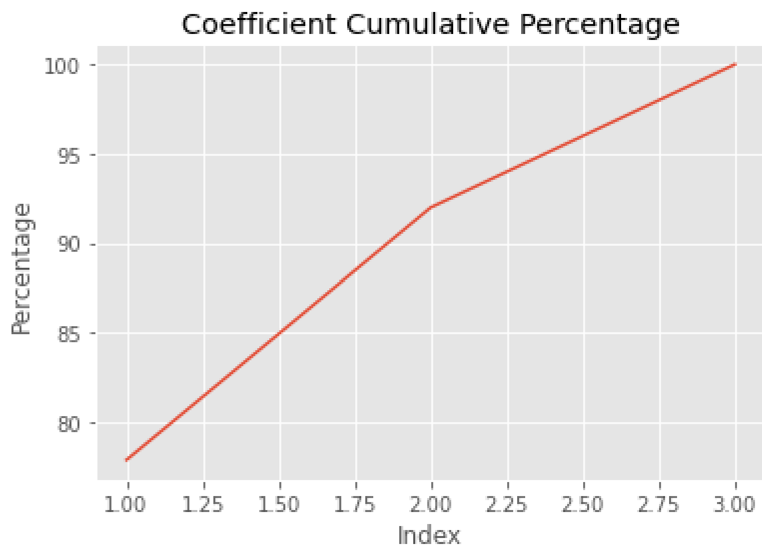
We can also compute the fPCA on jointly on the phase/amplitude space if we feel there is correlation between the variabilities

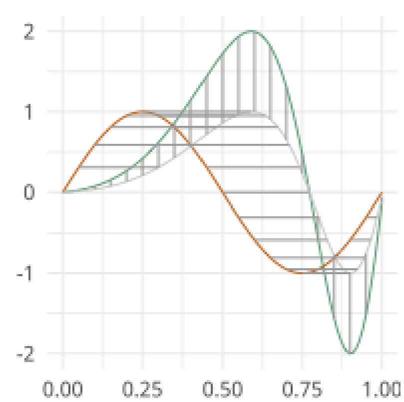
[7]:
```
jpca = fs.fdajpca(obj)
jpca.calc_fpca(no=3)
```

We then can plot the principal directions

[8]:
```
jpca.plot()
```


q domain: PD 1
q domain: PD 2
q domain: PD 3
f domain: PD 1
f domain: PD 2
f domain: PD 3


Coefficient Cumulative Percentage
Percentage
Index



1.3 Elastic Curve Alignment

Otherwise known as time warping in the literature is at the center of elastic functional data analysis. Here our goal is to separate out the horizontal and vertical variability of the open/closed curves

[1]:
```
import fdasrsf as fs
import numpy as np
```

Load in our example data

[2]:
```
data = np.load('../../bin/MPEG7.npz',allow_pickle=True)
Xdata = data['Xdata']
curve = Xdata[0,1]
n,M = curve.shape
K = Xdata.shape[1]

beta = np.zeros((n,M,K))
for i in range(0,K):
    beta[:,:,i] = Xdata[0,i]
```

We will then construct the `fdacurve` object

[3]:
```
obj = fs.fdacurve(beta,N=M)
```

We then will compute karcher mean of the curves

[4]:
```
obj.karcher_mean()
```

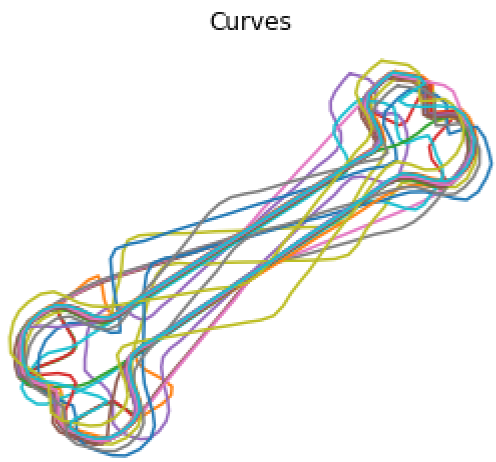
```
Computing Karcher Mean of 20 curves in SRVF space..
updating step: 1
updating step: 2
updating step: 3
updating step: 4
updating step: 5
updating step: 6
updating step: 7
```

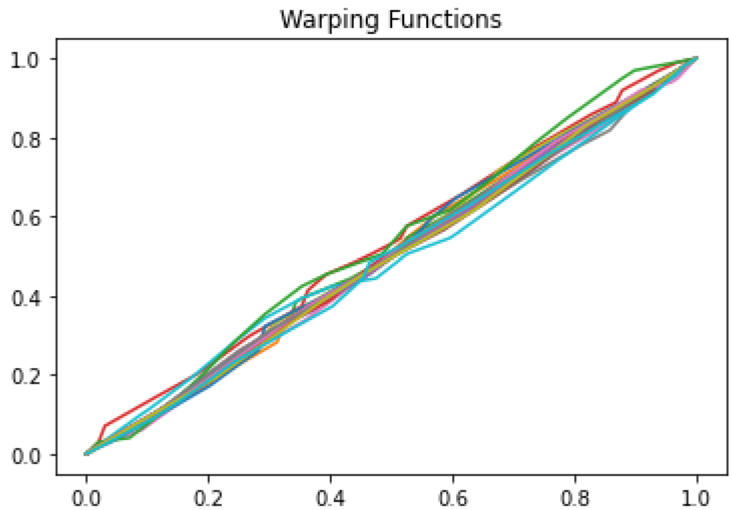
We then can align the curves to the karcher mean

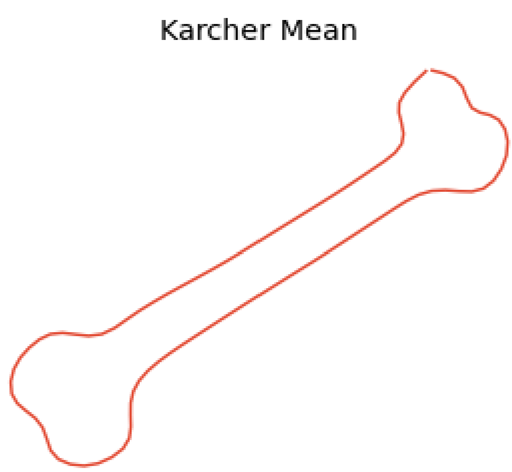
[5]:
```
obj.srvf_align(rotation=False)
```

Plot the results

[6]:
```
obj.plot()
```


Curves


Warping Functions
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


Karcher Mean

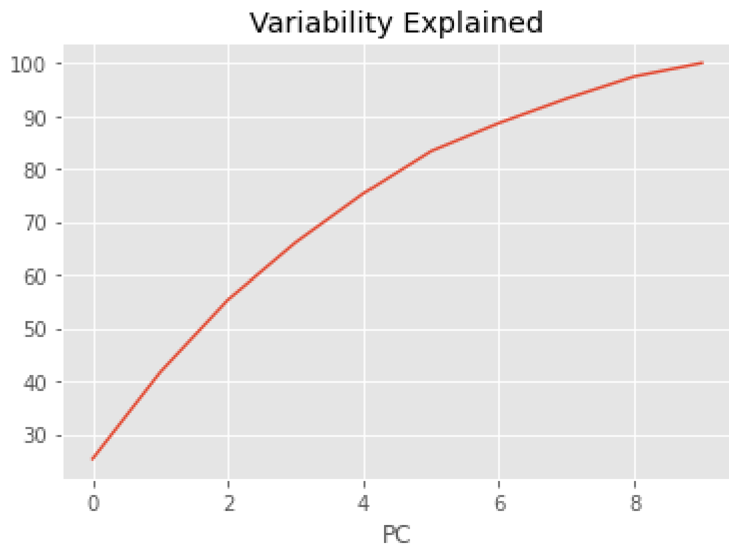
1.3.1 Shape PCA

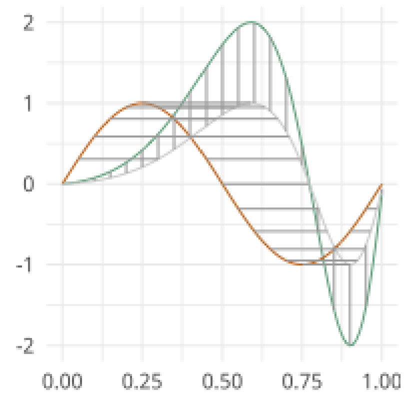
We then can compute the Karcher covariance and compute the shape pca

[7]:
```
obj.karcher_cov()
obj.shape_pca()
```

Plot the principal directions

[8]:
```
obj.plot_pca()
```

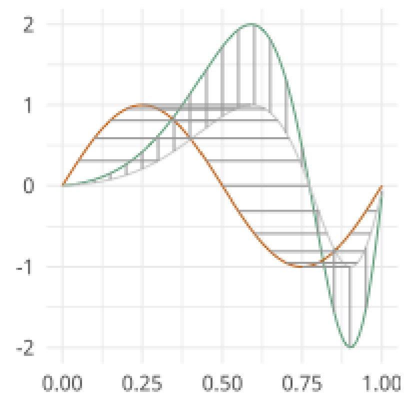



2
1
0
-1
-2
0.00
0.25
0.50
0.75
1.00

CHAPTER

TWO

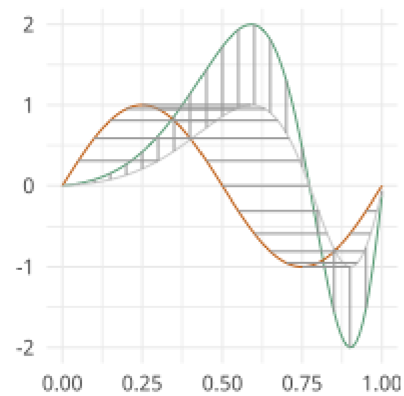
FUNCTIONAL ALIGNMENT



CHAPTER

THREE

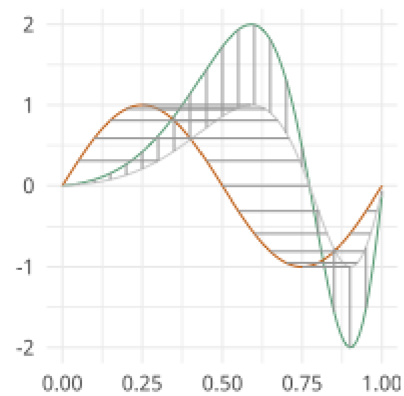
FUNCTIONAL PRINCIPAL COMPONENT ANALYSIS



CHAPTER

FOUR

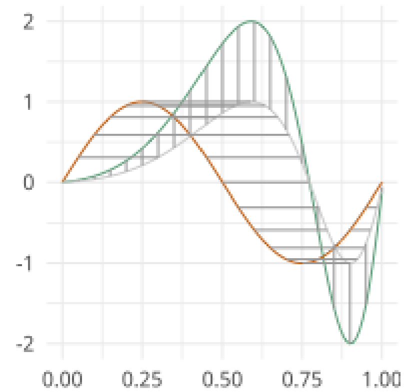
ELASTIC FUNCTIONAL BOXPLOTS



CHAPTER

FIVE

FUNCTIONAL PRINCIPAL LEAST SQUARES



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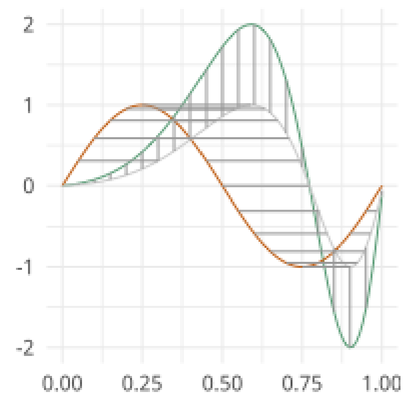
SIX

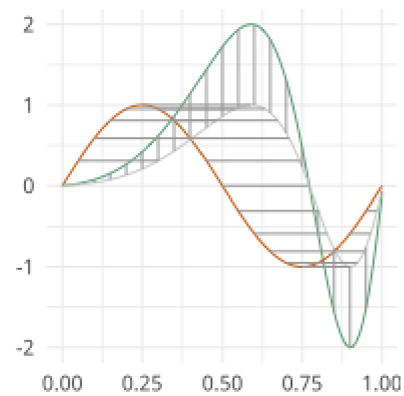
ELASTIC REGRESSION



CHAPTER

SEVEN

ELASTIC PRINCIPAL COMPONENT REGRESSION



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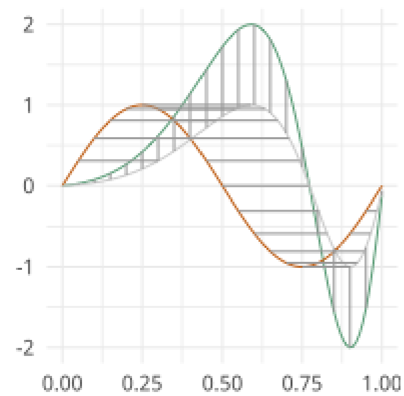
EIGHT

ELASTIC GLM REGRESSION



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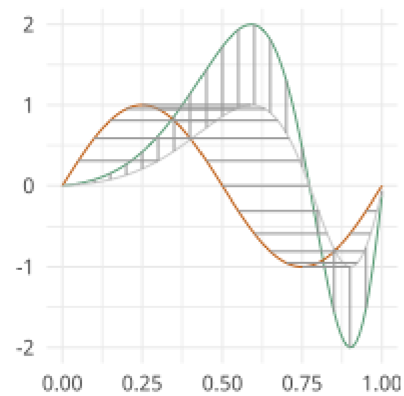
NINE

ELASTIC FUNCTIONAL TOLERANCE BOUNDS



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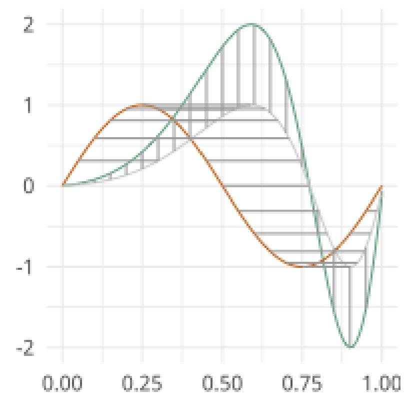
TEN

ELASTIC FUNCTIONAL CLUSTERING



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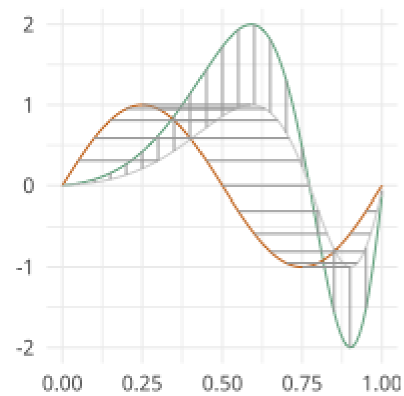
ELEVEN

ELASTIC IMAGE WARPING



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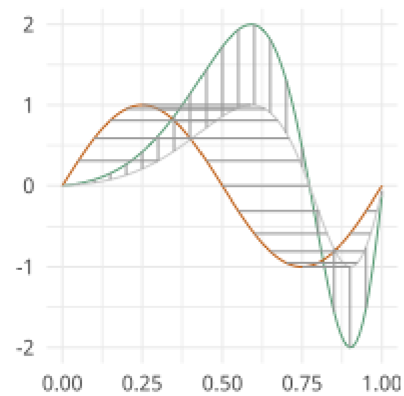
TWELVE

CURVE REGISTRATION



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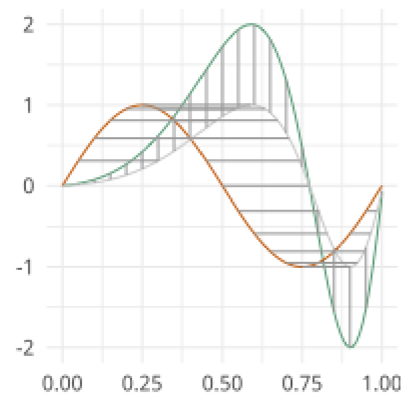
SRVF GEODESIC COMPUTATION



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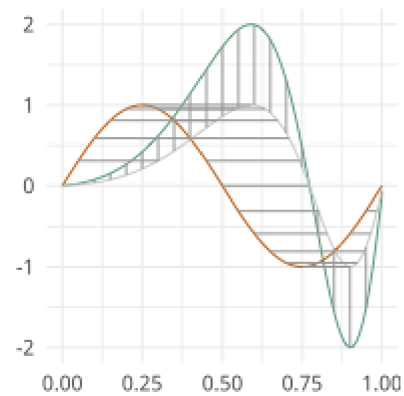
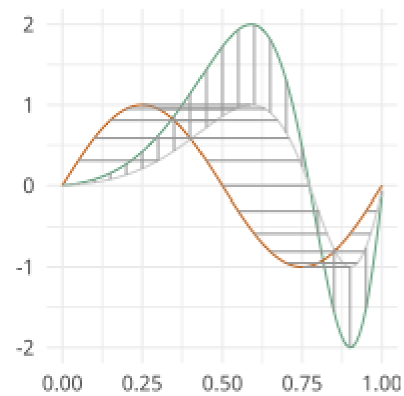
UTILITY FUNCTIONS



CHAPTER

FIFTEEN

CURVE FUNCTIONS



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REFERENCES

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