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Theory and Methodology

Inventory control under speculation: Myopic heuristics and exact procedures

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Abstract

We consider a periodic review inventory problem in which the purchasing cost exhibits a noticeable increase (deterministic or stochastic) in the second period and remains at the higher value for the remainder of the problem. (This simplification clarifies the nature of the myopic heuristic, but is not necessary for use of the heuristic in practice.) This results in a strategy that holds inventories due to speculation. We develop solution procedures to find the optimal inventory levels for both stationary and non-stationary demands. We establish that the problem with stochastic speculation behaves exactly like a problem with deterministic speculation with the same mean increase in price. We propose, based on the case of deterministic demands, simple myopic heuristics and study their effectiveness. We observe that these heuristics perform very well for exponential demands. However, for the case of uniform demands these heuristics are most effective when the increase in price is large compared to the holding cost.

Keywords: Inventory control; Speculation; Optimization; Heuristics

1. Introduction

We study the problem of determining inventory levels in the presence of stochastic demands with the principal objective being the minimization of purchasing, holding, and penalty costs. Traditionally these problems have been studied under the assumption of no speculation, i.e. the sum of purchasing cost and the holding cost in a period is greater than the purchasing cost in the next period, thus preventing (in the absence of setup costs) carrying of inventories due to speculation. However, many situations exist in which information is available that the purchasing cost will rise dramatically in the future. Such an increase in price could be due to various reasons such as controlled supply (e.g. OPEC oil supply [14]), environmental regulations (e.g. Freon for air-conditioners [15]), increased demand (e.g. toys in holiday season [16]) etc. Speculation of a price increase will result in

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forward buying of goods to take advantage of the current low price. Once these goods are purchased, significant costs are incurred in holding the inventories. In this paper we study this trade-off between savings due to forward buying and costs of holding in the presence of random demands.

Forward buying is a popular strategy used by retailers to combat an expected price increase at the supplier. Pache [8] examines this strategy as practiced in the French food retailing industry. A number of articles (e.g. [10,2,7]) addressing this problem have been published in the literature. Most of these articles assume that the demand rate is known (e.g. [10,2]). Our objective in this paper is to study this problem in the presence of random demands. When the purchasing cost remains constant over time this problem is the famous newsvendor problem with optimal inventory level equal to the newsvendor solution. A speculation that price is going to increase and remain at the higher value in the future, may result in forward buying and carrying inventory higher than the newsvendor solution. The amount of this speculative inventory will depend on increase in price, distribution of demand, and the holding and penalty costs. Our main objective of this paper is to understand the relations between these quantities in the presence of speculation.

We consider a periodic review model with stationary demand (with mean μ) from a distribution with the cumulative distribution function (c.d.f) $\Psi(\cdot)$ and the probability distribution function (p.d.f) $\psi(\cdot)$. The holding and penalty costs in every period are h and p, respectively per unit per period. The purchasing cost in the first period is c_0 per unit and jumps to c_1 per unit in the second period. We assume that after the initial jump, the purchasing cost remains at the higher value for the remainder of the problem. The capacity in each period is infinity and there are no setup or salvage costs. Under these assumptions the speculation is deterministic and the demands are stationary. We will relax these two assumptions in the later parts of the paper.

Inventory control in the presence of stochastic demands has been studied by many researchers. These models were first addressed by Arrow et al. [1]. Karlin [5] and Veinott [12] analyzed the problem in the presence of non-stationarities. Kalymon [4] and Sethi and Cheng [9] studied the non-stationary inventory control problem in the presence of setup costs. However, these articles above did not provide efficient solution procedures for computing the optimal parameters in the presence of non-stationarities. For the special case of cyclic changes, Karlin [6] and Zipkin [13] provided an efficient solution procedure. Song and Zipkin [11] presented a solution procedure for the problem of continuous time non-stationary inventory control in the presence of Poisson demands. The models addressed in this paper, while being non-stationary, are not cyclic as in Refs. [6,13], and not continuous in time as in Ref. [11]. Thus their solution procedures are not directly applicable here. We present an efficient new approach, using recursive derivative estimation, to solve this problem. Details on the efficiency of this approach are reported in Ref. [3].

The rest of this paper is organized as follows. In Section 2 we will present the simple case of deterministic demands, solution of which we will use to develop an effective heuristic for the case of random demands. In Section 3 we present analysis for the case of stationary demands. This section also contains the solution procedure for this problem. In Section 4 we study the problem when the increase in price is stochastic and establish that it has the same solution as the problem when the increase in price is deterministic and is equal to the mean. In Section 5 we study the situation of non-stationary demands and present a solution procedure for it. Section 6 describes the heuristics motivated by the case of deterministic demands. We report our computational results in Section 7 and conclude in Section 8.

Here we briefly describe our results. The optimal solution procedures developed in Sections 3 and 5 are very efficient in solving these problems. For stationary exponential demand we established that the heuristic solution is exactly equal to the optimal solution. Such was not the case for stationary uniform demands and our simulation results indicate that the heuristic is very effective when the price increase was higher than the holding cost per period. For non-stationary demands the heuristic was very effective (percentage deviation being at most 3.94%) for both uniform and exponential demands.

2. Deterministic demands

In this section we study the effect of speculation when the demands during the various periods are known. Let $D_1, D_2, \ldots$ be the known demands during periods $1, 2, 3, \ldots$, respectively. We want to know how much inventory we should obtain in period 1. If we purchase a unit in the first period to be sold in the tth period, the money we spent on this unit is $c_0 + (t-1)h$. It is advisable to do this if $c_0 + (t-1)h \leqslant c_1$. Using this logic the optimal strategy is to obtain z units in the first period where

$$z = \sum_{i=1}^{k+1} D_i, \quad \text{where} \quad k = \left\lfloor \frac{c_1 - c_0}{h} \right\rfloor.$$

This simple strategy which is optimal for deterministic demands will be useful in developing effective heuristics for the situation of random demands which we analyze in the remainder of this paper.

3. Stationary stochastic demands

In this section we will analyze the problem with stationary demands (i.e.demand distribution is identical in every period). It is well known from earlier literature (see Refs. [5,12]) that under these assumptions, the optimal policy in the first period for both finite horizon and infinite horizon is order upto. Let $L(y)$ be the one period expected holding and penalty cost when the inventory level is y. Let $V_n(x)$ be the optimal cost of an n-period problem, with initial inventory of x units, with purchasing cost c_1 in all periods. The objective of the $(n+1)$-period problem is to find $W(x)$ where

$$W(x) = \min_{y \geqslant x}\{c_0(y-x) + L(y) + E_\xi[V_n(y-\xi)]\}.$$

$W(x)$ is minimized at z that satisfies the relation

$$\Psi(z) = \frac{p - c_0 - E_\xi[V_n'(z-\xi)]}{p+h}.$$

So, if the current inventory level is below z, it is optimal to order upto z.

Let $V_n, \hat{V}_n, y_n, \hat{y}_n, z, \hat{z}$ be the corresponding definitions under two values of the speculated costs c_1, $\hat{c}_1$, respectively. It is reasonable to believe that when the price increase is larger the optimal inventory level will be higher. This is established in Property 2 below. Assume that $\hat{c}_1 \geqslant c_1$. First, we establish the relationship between V_n and $\hat{V}_n$.

Property 1. *For all values of* x, $\hat{V}_n'(x)$, $V_n'(x)$ *satisfy the following properties:*
1. $\hat{V}_n'(x) \leqslant V_n'(x)$,
2. $\hat{c}_1 + \hat{V}_n'(x) \geqslant c_1 + V_n'(x)$.

Proof. Under the assumption of no salvage cost (i.e.$V_0(x) = 0$), (1) and (2) trivially hold for the base case $n = 0$. Assume they are true for $n-1$. From property (2) for $n-1$, for any value y, we have

$$p - \hat{c}_1 - E_\xi[\hat{V}_{n-1}'(y-\xi)] \leqslant p - c_1 - E_\xi[V_{n-1}'(y-\xi)]$$

thus implying that $\hat{y}_n \leqslant y_n$. The functions $V_n'(x)$ and $\hat{V}_n'(x)$ are as defined below:

$$V_n'(x) = \begin{cases} -c_1 & \text{if } x < y_n, \\ L'(x) + E_\xi[V_{n-1}'(x-\xi)] & \text{if } x \geqslant y_n, \end{cases}$$

$$\hat{V}_n'(x) = \begin{cases} -\hat{c}_1 & \text{if } x < \hat{y}_n, \\ L'(x) + E_\xi[\hat{V}_{n-1}'(x-\xi)] & \text{if } x \geqslant \hat{y}_n. \end{cases}$$

By considering three different ranges for x (i.e. $x \leqslant \hat{y}_n$, $\hat{y}_n < x < y_n$, and $x \geqslant y_n$), it can be established that the properties (1) and (2) must hold for n. By induction, these properties must hold for all n. □

Property 2. *If* $\hat{c}_1 \geqslant c_1$, *then* $\hat{z} \geqslant z$.

Proof. Recall that z and $\hat{z}$ are solution to the equations

$$\Psi(z) = \frac{p - c_0 - E_\xi[V_n'(z-\xi)]}{p+h},$$

$$\Psi(\hat{z}) = \frac{p - c_0 - E_\xi[\hat{V}_n'(\hat{z}-\xi)]}{p+h}.$$

From Property 1, we know that $\hat{V}_n(x) \leqslant V_n(x)$ implying that

$$p - c_0 - E_\xi[\hat{V}'_n(y - \xi)] \geqslant p - c_0 - E_\xi[V'_n(y - \xi)].$$

So, $\hat{z}$ must be bigger than z. □

Let $V'_*(x)$ be the derivative of the infinite horizon (discounted or undiscounted) cost of an inventory control problem with purchasing cost c_1. For the infinite horizon problem with speculation, the order upto level in the first period is z that satisfies the relation

$$\Psi(z) = \frac{p - c_0 - E_\xi[V'_*(z - \xi)]}{p + h}.$$

Solution to the above equation requires the evaluation of $E_\xi[V'_*(y - \xi)]$. In Section 7 we will establish that it can be expressed in closed form for stationary exponential demand. However, that is not the case for general demand distributions and special algorithms must be developed to compute it efficiently.

To compute the optimal order upto level in the first period, we must first compute $V'_*(\cdot)$. Due to our assumption of infinite capacity we know that $V'_*(0) = -c_1$. Let π_j be the probability that the demand in a period is j units. Then, $V'_*(x)$ can be recursively defined as (notice that $V'_*(x)$ appears on both sides of the equation)

$$V'_*(x) = L'(x) + \sum_{j=0}^{x} \pi_j V'_*(x - j) - c_1 \sum_{j=x+1}^{\infty} \pi_j.$$

Suppose we know the derivatives of the cost function for all inventory levels upto $x - 1$, then the derivative at an inventory level x can be computed using the relation

$$V'_*(x) = \frac{1}{1 - \pi_0}\left[L'(x) + \sum_{j=1}^{x} \pi_j V'_*(x - j) - c_1 \sum_{j=x+1}^{\infty} \pi_j\right]. \tag{1}$$

Starting with the boundary condition $V'_*(0) = -c_1$ using the recursive Eq. (1) above we can compute the derivative for all values of x.

3.1. Optimal solution procedure

The solution procedure recursively estimates the derivatives $W'(x)$ and $V'_*(x)$ for all values of x. First, we discretize the demand distribution. Let π_j be the probability that the demand in a period is j units. Remember that $L'(y) = (h + p)\Psi(y) - p$.

Step 1. Set $W'(0) = -c_0$ and $V'_*(0) = -c_1$.

Step 2. Repeat the following recursions for values of x starting at 1 and increasing by 1:

$$V'_*(x) = \frac{1}{1 - \pi_0}\left[L'(x) + \sum_{j=1}^{x} \pi_j V'_*(x - j) - c_1 \sum_{j=x+1}^{\infty} \pi_j\right],$$

$$W'(x) = L'(x) + \sum_{j=0}^{x} \pi_j V'_*(x - j) - c_0 \sum_{j=x+1}^{\infty} \pi_j.$$

At any point during the recursion, if $W'(x)$ is less than $-c_0$, we set it equal to $-c_0$. Similarly, if $V'_*(x)$ is less than $-c_1$, we set it equal to $-c_1$.

Step 3. Find the order upto level, $z = \inf\{x | W'(x) > -c_0\}$.

The stopping condition in step 2 is when the order upto level, z, has been determined. If desired, these recursions can be continued to compute the derivatives at inventory levels above the optimal order upto level.

The intuition behind the solution procedure is as follows. Clearly, due to our assumption of infinite capacity, step 1 is true. The recursions in step 2 follow from the dynamic programming formulation above. The key step is that when the derivative, $W'(x)$ is less than $-c_0$, we know that the optimal inventory level is above x and so we set the derivative $W'(x)$ equal to $-c_0$. Similar reasoning applies for $V'_*(x)$. Once the derivative of the cost function at all inventory levels are known, the optimal solution is one at which the derivative is zero.

4. Stochastic speculation

In all our previous analysis we have assumed that the analyst is aware of the expected price increase, i.e.the speculation was deterministic. In this section we will study the situation when the analyst expects a price increase but is not sure how much it would be. The price in the second period is random, but once it has been determined remains at its new value. Let us assume the price in the second period takes a value C_1^i with probability α_i for $i \in \{1, 2, \ldots, k\}$ for some finite k. Let $V_*^i(x)$ be the infinite horizon average cost corresponding to the price C_1^i. Then the optimal order upto level, z, for the first period satisfies the relation

$$\Psi(z) = \frac{p - c_0 - \sum_{i=1}^{k} [\alpha_i E_\xi [V_*'^i(z - \xi)]]}{p + h}.$$

Recall from our previous section that for $i, j,\ i \neq j,\ V_*'^i(0) = -C_1^i,\ V_*'^j(0) = -C_1^j$. By computing the derivatives $V_*'^i(x), V_*'^j(x)$ using the recursive Eq. (1) above it is easily established that $C_1^i + V_*'^i(x) = C_1^j + V_*'^j(x)$ for all values of x. We will use this result in establishing the next property.

Property 3. *The solution to a problem under stochastic speculation is equal to the solution under deterministic speculation with the increase in price equal to the average increase under stochastic speculation.*

Proof. Let $\{C_1^i, \alpha_i\}, \{\hat{C}_1^i, \hat{\alpha}_i\}$ be two stochastic speculations with same mean say $\bar{C}_1$. That is,

$$\bar{C}_1 = \sum_{i=1}^{k} \alpha_i C_1^i = \sum_{i=1}^{\hat{k}} \hat{\alpha}_i \hat{C}_1^i.$$

Using the relation $C_1^i + V_*'^i(x) = C_1^j + V_*'^j(x)$, it is easily established that

$$\sum_{i=1}^{k} [\alpha_i E_\xi [V_*'^i(z - \xi)]] = \sum_{i=1}^{\hat{k}} [\hat{\alpha}_i E_\xi [\hat{V}_*'^i(z - \xi)]]$$

which implies that the order upto level z that satisfies the relation

$$\Psi(z) = \frac{p - c_0 - \sum_{i=1}^{k} [\alpha_i E_\xi [V_*'^i(z - \xi)]]}{p + h}$$

also satisfies the relation

$$\Psi(z) = \frac{p - c_0 - \sum_{i=1}^{\hat{k}} [\hat{\alpha}_i E_\xi \hat{V}_*'^i(z - \xi)]]}{p + h}.$$

Thus the optimal order upto level in the first period is the same under both the speculations. Since a deterministic speculation with $C_1 = \bar{C}_1$ is one such stochastic speculation, the order upto level under any stochastic speculation must be equal to the order upto level under deterministic speculation with the mean increase in price being the same. □

Furthermore the cost from a situation of stochastic speculation will be exactly to equal to the cost from a situation of deterministic speculation with the same average increase in price. So in the rest of this paper and in the computational study we will concentrate only on the case of deterministic speculation.

5. Non-stationary stochastic demands

In the previous section we assumed that the distribution of demand did not change from one period to the next. When the price of a good changes, it is reasonable to expect that the demand also changes. However as more periods go by with price remaining at the new level the demand will have a tendency to stabilize and remain at a steady value after some time. Since the price changes only once we assume that its effect on the demand lasts only for l periods where l is finite. Let $\Psi_2, \Psi_3, \ldots, \Psi_l$ be the sequence of demand distributions in periods two through l. The distribution of demand in periods $l + 1$ and beyond is assumed to be equal to Ψ_l. Under these conditions, the optimal order policy in the first period is still order upto. We will modify the recursive solution procedure in the previous section to accommodate the non-stationary demands.

Let $W'(x)$ be the derivative of the long term cost in period 1 at inventory level x. Similarly, let $V_i'(x), i \in \{2, 3, \ldots, l\}$, be the derivative of the long term cost in period i at inventory x. Let π_{ij} be the probability that demand in period i is j units.

Step 1. Set $W'(0) = -c_0$ and $V_i'(0) = -c_1$ $\forall i \in \{2, 3, \ldots, l\}$.

Step 2. Repeat the following recursions for values of x starting at 1 and increasing by 1:

$$V_l'(x) = \frac{1}{1-\pi_{l0}}\left[L_l'(x) + \sum_{j=1}^{x}\pi_{lj}V_l'(x-j) - c_1\sum_{j=x+1}^{\infty}\pi_{lj}\right],$$

$$V_k'(x) = L_k'(x) + \sum_{j=0}^{x}\pi_{kj}V_{k+1}'(x-j) - c_1\sum_{j=x+1}^{\infty}\pi_{kj}, \quad k \in [l-1, 2],$$

$$W'(x) = L_1'(x) + \sum_{j=0}^{x}\pi_{1j}V_2'(x-j) - c_0\sum_{j=x+1}^{\infty}\pi_{1j}.$$

At any point during the recursion, if $W'(x)$ is less than $-c_0$, we set it equal to $-c_0$ indicating that x is below the optimal solution. Similarly for $i \geqslant 2$ if $V_i'(x)$ is less than $-c_1$, we set it equal to $-c_1$.

Step 3. Find the order upto levels,

$$z_1 = \inf\{x | W'(x) > -c_0\}$$

and

$$z_i = \inf\{x | V_i'(x) > -c_1\}, \; i \in \{2, 3, \ldots, l\}.$$

This algorithm recursively computes the derivative of the cost function at all inventory levels in all periods. Once the derivatives are known, the optimal upto levels can be recovered by finding the points at which the derivatives are zero which is done in step 3.

6. Heuristic solution

For the infinite horizon problem with the purchasing cost of c_1, the optimal order upto level in every period is $y^m = \Psi^{-1}(p/(p+h))$. Since the purchasing cost in the first period is c_0 which is smaller than c_1, clearly z is going to be bigger than y^m. Using the intuition developed in Section 2, we would like this excess inventory to last for $c_1 - c_0/h$ periods on the average. So our heuristic order upto level, z^h, for the case of stationary demands with mean μ is

$$z^h = y^m + \frac{c_1 - c_0}{h}\mu.$$

For non-stationary demands, we will use a similar heuristic which computes the expected demand in $c_1 - c_0/h$ periods, keeping in mind it may be fractional, and adds it to the myopic level y^m.

We used the solution procedures presented in the previous sections to compute the optimal order upto levels and compared them to the heuristic order upto levels computed as defined above. Via simulation, we also compared the costs of using these two policies. The results of our computational study are presented in the next section.

7. Computational results

In all our computational experiments, we used the following values for our parameters

$$p = 5, \quad c_0 = 1,$$
$$h \in \{0.1, 0.5, 1.0\}, \quad c_1 \in \{1.5, 2, 2.5, 3, 3.5, 4, 4.5\}.$$

We used both exponential and uniform distributions to model the demands. We implemented the optimal solution procedures described above for both the stationary and non-stationary demands. Our main objective of the computational study is to see how the heuristic performs and understand when it is an effective alternative to the optimal solution and when it is not.

7.1. Stationary demands

In this section we will present our experimental results for the stationary demand situation.

7.1.1. Exponential demands

First we consider the situation when the demand is exponential with mean 100 units. The optimal and heuristic order upto levels for various values of c_1 are given below (Table 1). Remember that the myopic solution y^m is 179 units.

Notice that the heuristic solution was exactly equal to the optimal solution for all values of c_1. In fact for the exponential demand situation, the heuristic solution is always equal to optimal solution. We establish this result formally below.

Property 4. *For a stationary inventory control problem with exponentially distributed demand, the derivative of the infinite horizon cost, $V_*'(x)$ is:*

$$V_*'(x) = \begin{cases} -c_1 & if\ x < y_m, \\ -c_1 + \dfrac{x - y_m}{\mu} h & if\ x \geqslant y_m. \end{cases}$$

Proof. Since it is a stationary inventory control problem, we know that in every period the optimal order upto level is y_m the newsvendor solution. So, if the inventory level x is below it then we order $y_m - x$ units and thus the derivative of the cost with respect to x is $-c_1$.

If the inventory level x is above the optimal level y_m, then the derivative at x is a solution to the differential equation

$$V_*'(x) = L'(x) + \int_0^{x-y_m} V_*'(x-t)\psi(t)\,\mathrm{d}t + \int_{x-y_m}^{\infty} [-c_1]\psi(t)\,\mathrm{d}t.$$

Since the demand is distributed exponentially, $\psi(t) = \frac{1}{\mu}\mathrm{e}^{-t/\mu}$ and

$$L'(x) = (h+p)[1 - \mathrm{e}^{-x/\mu}] - p = h - (h + p)\,\mathrm{e}^{-x/\mu} = h - (h+p)\,\mathrm{e}^{-y_m/\mu}\,\mathrm{e}^{-x-y_m/\mu}.$$

Since $y_m = -\mu\ \ln(h/(h+p))$ we get

$$L'(x) = h[1 - \mathrm{e}^{-x-y_m/\mu}].$$

Incorporating these observations into the differential equation above, we get

$$V_*'(x) = h[1 - \mathrm{e}^{-x-y_m/\mu}] + \int_0^{x-y_m} V_*'(x - t)\frac{1}{\mu}\mathrm{e}^{-t/\mu}\,\mathrm{d}t - c_1\,\mathrm{e}^{-x-y_m/\mu}.$$

Table 1
Optimal and heuristic solutions for exponential demand, $h = 1$

c_1	Opt. sol.	Heu. sol.
1.5	229	229
2.0	279	279
2.5	329	329
3.0	379	379
3.5	429	429
4.0	479	479
4.5	529	529

Proof follows from the fact that

$$V_*'(x) = -c_1 + \frac{x - y_m}{\mu} h$$

satisfies this differential equation. □

Using this derivative the optimal order upto level in the first period must satisfy the relation

$$c_0 - c_1 + \frac{x - y_m}{\mu} h = 0$$

resulting in the order upto level $y_m + c_1 - c_0/h\mu$ thus establishing that the heuristic solution described above is in fact equal to the optimal solution for exponential demand.

7.1.2. Uniform demands

Next we consider the situation when the demand is uniformly distributed between 0 and 200. Unlike the case of exponential demands, for the uniform demands the heuristic solution was not equal to the optimal solution. The results of our study are presented in Tables 2–4.

Columns 2 and 3 of these tables contain the optimal and heuristic solutions while columns 4 and 5 contain optimal and heuristic costs computed using simulation. These costs were computed using 100,000 simulation runs. In each run we start with the appropriate inventory level and run the simulation until the inventory level falls below the steady state inventory level. Using only these periods we compute the average cost per period. This helps us capture difference in the costs of the optimal and heuristic levels before they get absorbed by the long run costs. Column 6 contains the percentage difference between the optimal and heuristic costs.

Some observations from these results are:

1. When the holding cost is small compared to c_0 then the cost of the heuristic solution is very close to the cost of the optimal solution. This is noticed in the last column of Table 4 in which the percentage difference is at most 1.42%.

2. For larger values of the holding cost, the heuristic is not as effective. The heuristic performs particularly poorly when $c_1 \leqslant c_0 + h$. This can be observed from the first two rows in Table 2, and the first row in Table 3. In these rows the per-

centage difference ranges between 6.92% to 8.94%. However, when $c_1 > c_0 + h$, as is the case in rest of the rows in Tables 2 and 3, the heuristic performs much better with the percentage difference ranging from 0.98% to 4.09%.

7.2. Non-stationary demands

In this section we report our computational results for the non-stationary demand situation as detailed in Section 5. The experimental setup in this section is similar to the one in Section 7.1 except for the demands. We assumed that the first period had a mean demand of 100 units. The change in price from the first period to the second will affect the demands. There are many ways in which non-stationary demands could be generated. We have chosen the following setup for our testing. Under this setup the demand initially drops with increase in price, but slowly increases and stabilizes at its original demand. We assumed that this effect lasts only for five periods and the demands in those five periods have means of 50, 62.5, 75, 87.5, and 100, respectively. After the sixth period the demand remains stationary at mean 100. First we present the results for the case of exponential demands.

Table 2
Optimal and heuristic solutions for uniform demand, $h = 1.0$

c_1	Opt. sol.	Heu. sol.	Opt. cost	Heu. cost	% Diff.
1.5	183	217	231.64	247.51	6.92
2.0	198	267	275.71	295.66	7.27
2.5	266	317	317.02	325.53	2.68
3.0	323	367	341.22	352.29	3.23
3.5	368	417	366.59	380.68	3.83
4.0	417	467	391.72	407.15	4.09
4.5	470	517	417.33	433.39	3.84

Table 3
Optimal and heuristic solutions for uniform demand, $h = 0.5$

c_1	Opt. sol.	Heu. sol.	Opt. cost	Heu. cost	% Diff.
1.5	200	282	195.44	212.92	8.94
2.0	331	382	235.09	237.40	0.98
2.5	425	482	256.76	265.13	3.26
3.0	527	582	280.96	290.39	3.36
3.5	627	682	305.60	315.60	3.27
4.0	728	782	330.04	340.72	3.24
4.5	828	882	354.63	365.97	3.20

Table 4
Optimal and heuristic solutions for uniform demand, $h = 0.1$

c_1	Opt. sol.	Heu. sol.	Opt. cost	Heu. cost	% Diff.
1.5	634	697	147.17	148.99	1.24
2.0	1137	1197	169.63	172.04	1.42
2.5	1639	1697	193.77	196.35	1.33
3.0	2142	2197	218.45	220.95	1.14
3.5	2644	2697	243.23	245.69	1.01
4.0	3147	3197	268.00	270.59	0.97
4.5	3649	3697	293.15	295.38	0.76

7.2.1. Exponential demands

Tables 5–7 present our computational results for the non-stationary exponential demands for the three values of the holding cost. Columns 2 and 3 contain the optimal and heuristic levels for the first period. Since the demands are stochastically increasing in periods 2–5, the optimal and heuristic order upto levels in those periods were exactly equal to the corresponding newsvendor solution. Columns 4 and 5 contain the optimal and heuristic costs. The simulations were conducted as described in the previous section with the only difference being that each run consisted of at least five periods to capture the non-stationarity of the demands. Column 6 in each of these tables contains the percentage difference between the optimal and heuristic costs.

Notice that, unlike the case of stationary demands, the optimal and heuristic levels were different for non-stationary demands. However as c_1 increases the difference between these two levels keeps decreasing. As reported in column 6 of these tables, the performance of the heuristic solution is very good with the maximum deviation being 0.77%.

Table 5
Optimal and heuristic solutions for non-stationary exponential demands, $h = 1.0$

c_1	Opt. sol.	Heu. sol.	Opt. cost	Heu. cost	% Diff.
1.5	182	203	260.63	261.30	0.26
2.0	211	228	298.68	299.04	0.12
2.5	244	259	334.11	334.64	0.16
3.0	279	290	366.79	366.90	0.03
3.5	318	328	396.46	396.89	0.11
4.0	360	366	424.01	424.08	0.02
4.5	404	410	448.87	449.50	0.14

Table 6
Optimal and heuristic solutions for non-stationary exponential demands, $h = 0.5$

c_1	Opt. sol.	Heu. sol.	Opt. cost	Heu. cost	% Diff.
1.5	258	288	214.21	214.74	0.25
2.0	324	350	249.23	249.98	0.30
2.5	406	425	279.65	279.77	0.04
3.0	502	513	305.55	306.20	0.21
3.5	603	613	331.29	332.66	0.41
4.0	706	713	359.03	360.16	0.31
4.5	807	813	387.09	388.38	0.33

Table 7
Optimal and heuristic solutions for non-stationary exponential demands, $h = 0.1$

c_1	Opt. sol.	Heu. sol.	Opt. cost	Heu. cost	% Diff.
1.5	743	765	147.21	148.35	0.77
2.0	1259	1265	185.26	185.69	0.23
2.5	1757	1765	213.87	214.29	0.20
3.0	2255	2265	240.11	240.54	0.18
3.5	2752	2765	265.66	266.21	0.21
4.0	3250	3265	290.85	291.75	0.31
4.5	3747	3765	316.31	317.12	0.26

7.2.2. Uniform demands

In this section we report our computational results for non-stationary uniform demands. The means in each of the periods are as described in the previous section. Tables 8–10 contain these results in the format described above.

Notice that difference in optimal and heuristic costs was at most only 3.94%. Compared to the case of stationary demands, the heuristic performs a lot better in the case of non-stationary demands.

8. Conclusions

In this paper we studied the problem of inventory control under speculation of a large increase in purchasing cost. After establishing the structure of the optimal policy and some of its properties we developed a solution procedure that computes the optimal order upto levels. This procedure works for both stationary and non-stationary demands and is computationally very efficient. We established that the problem with stochastic speculation behaves exactly like a problem with deterministic speculation. We proposed a simple heuristic and studied its effectiveness under various system parameters. We established that the heuristic gives the optimal solution for stationary exponential demands. Our computational results show that heuristic performs very well in the presence of non-stationary demands with the worst performance being 0.77% for exponential and 3.94% for uni-

Table 8
Optimal and heuristic solutions for non-stationary uniform demands, $h = 1.0$

c_1	Opt. sol.	Heu. sol.	Opt. cost	Heu. cost	% Diff.
1.5	169	192	187.29	189.10	0.97
2.0	182	217	226.23	229.72	1.54
2.5	195	248	264.09	268.75	1.76
3.0	225	279	300.52	303.67	1.05
3.5	265	316	333.62	336.79	0.95
4.0	303	354	363.73	366.30	0.71
4.5	348	397	390.38	393.25	0.74

Table 9
Optimal and heuristic solutions for non-stationary uniform demands, $h = 0.5$

c_1	Opt. sol.	Heu. sol.	Opt. cost	Heu. cost	% Diff.
1.5	190	232	156.33	158.94	1.67
2.0	230	295	194.36	196.66	1.18
2.5	308	370	226.74	228.46	0.76
3.0	399	457	252.53	254.94	0.95
3.5	500	557	272.90	277.93	1.84
4.0	602	657	292.99	302.29	3.17
4.5	703	757	317.42	329.94	3.94

Table 10
Optimal and heuristic solutions for non-stationary uniform demands, $h = 0.1$

c_1	Opt. sol.	Heu. sol.	Opt. cost	Heu. cost	% Diff.
1.5	506	568	119.81	121.82	1.67
2.0	1012	1068	153.44	156.63	2.08
2.5	1514	1568	183.24	186.05	1.53
3.0	2017	2068	210.64	213.41	1.32
3.5	2519	2568	237.04	239.69	1.11
4.0	3022	3068	263.04	265.54	0.95
4.5	3524	3568	288.78	291.08	0.80

form. For the case of stationary uniform demands our results show that heuristic is very effective (with the worst performance being 4.09%) when the increase in cost is large compared to the holding cost. When the increase in cost was smaller than the holding cost, the heuristic was not as effective, as much as 8.94% away from the optimal solution.

References

[1] K.J. Arrow, S. Karlin, H. Scarf, Studies in Applied Probability and Management Science, Stanford Univ. Press, Stanford, CA, 1963.

[2] M. Bastian, A perfect lot-tree procedure for the discounted dynamic, lot-size problem with speculation, Naval Research Logistics 39 (5) (1992) 651–658.

[3] S. Gavirneni, Inventories in supply chains under cooperation, Ph.D.Thesis, Carnegie Mellon University, Pittsburgh, PA, September 1997.

[4] B.A. Kalymon, Stochastic prices in a single-item inventory purchasing model, Operations Research 19 (6) (1971) 1435–1458.

[5] S. Karlin, Dynamic inventory policy with varying stochastic demands, Management Science 6 (3) (1960) 231–258.

[6] S. Karlin, Optimal policy for dynamic inventory process with stochastic demands subject to seasonal variations, J. SIAM 8 (1960) 611–629.

[7] B. Lev, H.J. Weiss, Inventory models with cost changes, Operations Research 39 (1) (1990) 53–63.

[8] G. Pache, Speculative inventories in the food retailing industry: A comment on French practices, International Journal of Retail and Distribution Management 23 (12) (1995) 36–42.

[9] S. Sethi, F. Cheng, Optimality of (s, S) policies in inventory models with Markovian demands, Operations Research, to appear.

[10] R.J. Tersine, Economic replenishment strategies for announced price increases, European Journal of Operations Research 92 (2) (1996) 266–280.

[11] J. Song, P. Zipkin, Inventory control in a fluctuating demand environment, Operation Research 41 (2) (1993) 351–370.

[12] A.F. Veinott, Optimal policy in a dynamic, single product, nonstationary inventory model with several demand classes, Operations Research 13 (3) (1965) 761–778.

[13] P. Zipkin, Critical number polices for inventory models with periodic data, Management Science 35 (1) (1989) 71–80.

[14] Oil deal pushes prices up, CNNfn, http://cnnfn.com/markets/9803/oil/index.html.

[15] Coolant costs burn drivers, CNNfn, http://cnnfn.com/hotstories/economy/9610/08/ freon_prices_pkg/index.html.

[16] Nintendo 64 is red hot, CNNfn http://cnnfn.com/digitaljam/9612/23/nintendo_pkg/index-txt.html.