

Multiple-supplier inventory models in supply chain management: A review

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Abstract

This paper reviews inventory models with multiple supply options and discusses their contribution to supply chain management. After discussing strategic aspects of supplier competition and the role of operational flexibility in global sourcing, inventory models which use several suppliers in order to avoid or reduce the effects of shortage situations are outlined. Further, related inventory problems from the fields of reverse logistics and multi-echelon systems are presented. Finally, issues for future research and a synthesis of available supply chain management and multiple supplier inventory models are discussed.

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1. Introduction

Supply chain management is an integrative approach for planning and control of materials and information flows with suppliers and customers as well as between different functions within a company. This bridges the inventory management focus in operations management and the analysis of relationships from industrial organization. This field has received considerable attention during the last years. Nevertheless, most quantitative analysis on supply chain management issues is dominated by the framework of multi-echelon serial systems or distribution systems where relationships between a single vendor and a single buyer or a single vendor and several buyers is

considered. The situation of multiple suppliers and a single/multiple buyers has received less attention. The aim of this paper is to review the literature on inventory models with multiple suppliers and to discuss their potential contribution to supply chain management issues.

There exist several reasons for either having a few (as an extreme, a single supplier) or multiple suppliers. Strategies of Just-In-Time replenishments and Total Quality Management are often going along with the suggestion to reduce the supplier base for each item and to build long-term relationships with suppliers. Current trends in supplier management and sourcing during the past decade show increasing global sourcing, a reduction of the supplier base for a single item, and long term relationships with suppliers instead of spot market replenishments. Having less suppliers reduces the coordination efforts in order to ensure on time deliveries and facilitates the

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provision of a common and high quality level. The prerequisite of business specific assets, costly quality improvements and decreasing production costs with increasing cumulative volume (due to learning curve effects) make it necessary to establish long term relationships in order to be beneficial for both, the vendor and the buyer. Reasons for single sourcing are high costs of product design and supplier development which often makes it unattractive to have several suppliers. Larger purchasing volume often implies that more attractive contract terms can be obtained, e.g. quantity discounts and terms of payment and delivery. The size of the business increases the bargaining power to negotiate on price, lead times, quality, and flexibility. Benefits of long-term relationships can be drawn from dynamic economies of scale and continuous quality improvements. Learning curve effects, i.e. decreasing manufacturing costs when cumulative production increases, can be exploited best if long-term contracts with the highest possible volume are implemented.

Empirical studies show, that the current situation in most business areas is still dominated by dual or multiple sourcing (Goffin et al., 1997; De Toni and Nassimbeni, 1999; Shin et al., 2000). Simplifications in trade regulations and price differentials between suppliers in developing and developed countries offer a broader scope for supplier search and sourcing strategies compared to the situation where only domestic suppliers are considered. However, these price differentials might disappear due to longer transportation lead times, higher variability within replenishment lead times, and problems with supplier reliability. Purchasing managers often fear the dependency on a single supplier associated with several kinds of risk and therefore favour multiple sourcing. The risk of increasing prices in global sourcing due to exchange rate volatility, supply disruptions due to machine breakdowns, labor strikes or political instability, capacity limitations, lead time variability can be diversified if multiple suppliers are available. If a buyer depends on the reliability of a single supplier, agency problems like opportunistic behavior and information asymmetries with respect to true manufacturing costs and dynamic

cost development may appear that can be overcome by enhancing supplier competition in procurement auctions and competitive bidding. Another line of interest related to the problem of managing several suppliers is the choice of supply modes, especially the question whether a product should be transported via air, connected with short lead times but high transportation costs, or by sea shipments connected with long lead times but lower transportation costs. This question especially receives attention within distribution system redesign which often means consolidation of distribution stages and reducing the number of stocking points at each stage. Therefore, multiple supply options can represent several suppliers or simply several modes of shipments from a supplier to the buyer (or a combination of both aspects).

Supply Chain Management issues that are closely related to or included in multiple supplier operations are buyer–vendor relationships and supplier selection. Maloni and Benton (1997) review opportunities for operations research models in supply chain partnerships. Christy and Grout (1994) investigate supply chain relations from a contracting perspective. They identify different contracting issues depending on product and process specificity and review quantitative methods from economics, transaction cost theory, principal agent theory, and game theory in order to model and analyze the respective problems. Models for the coordination of supply and manufacturing as well as manufacturing and distribution are reviewed by Thomas and Griffin (1996).

The main criteria for supplier choice are prices (net prices, discounts, payment conditions), quality, and supplier service (delivery time, lead time variability, reliability, flexibility). A review of the respective literature is given by Weber et al. (1991). Therefore, the optimal choice in an environment with multiple potential suppliers has to trade-off the respective aspects of direct purchasing costs and supplier service. The latter will, in general, cause indirect cost such as costs for holding safety inventory to cover against supply and demand variability. Eventually, this trade-off has to be analyzed with respect to further constraints like supplier capacities or local content regulations.

The main trade-off that is investigated in most inventory models with several suppliers is between purchase price and supplier lead time and heavily depends on how supplier service is modelled. Therefore, the inventory models in Section 3 are classified according to the underlying assumptions with respect to the supplier lead times.

The paper is organized as follows. Section 2 discusses strategic issues of exchange rate uncertainty in global sourcing and competition in multiple supplier operations. Section 3 presents an overview on multi-supplier inventory research. For classification purposes, two lines of research can be distinguished, the first is characterized by deterministic lead times and the motive of emergency ordering in situations of low inventory, the second involves random lead times and the statistical motive of reducing effective lead time by order splitting. Other inventory models not included in these two categories are presented in a third subsection. Reverse logistics and multi-echelon inventory problems with similar modelling and application aspects are sketched in Section 4, before directions for future research, especially the suitability of multiple supplier inventory models for supply chain management, are discussed.

2. Strategic aspects in multiple and global sourcing

2.1. Strategic interaction

Kraljic (1983) presents a strategic management portfolio approach to identify different strategies for purchasing and supply management. The two key influencing factors for a product are the complexity of the supply market on the one hand and the importance of the item and its purchasing on the other hand. In the field of industrial organization it is analyzed whether required materials should be sourced from (a single or multiple) suppliers or if manufacturing for the part should be vertically integrated (Ellram, 1991). The main criterion for such decisions are the associated transaction costs that are different for the alternatives market and hierarchy. For the sourcing alternative, costs for supplier search, supplier development, supplier switching, contracting,

trading and system operation, and monitoring of supplier performance have to be taken into account. Prabhu et al. (1997) present criteria for the alternatives of single and multiple sourcing as well as for vertical integration. The authors present a framework to analyze the suitability of these strategies under different conditions of industry growth rate and asset specificity. A high industry growth rate is characterized by high uncertainty and varying industry environments. Therefore, the safety motive receives more weight and multiple suppliers are favourable. In order to have more control on operations and to be able to make fast adjustments due to market changes, it may also be favourable to vertically integrate. In situations of low growth rates and therefore stable conditions, low purchasing prices becomes the main driving factor that favours single sourcing. High asset specificity and agency problems favour vertical integration whereas low asset specificity lowers supplier switching costs and therefore multiple suppliers become a profitable alternative.

Another stream of research investigates the impact of competition among suppliers. Dowlatshahi (1999) discusses the bargaining power of buyers and vendors for different market conditions when there is a single buyer or vendor (monopoly situation), some buyers or vendors (oligopoly situation), or several buyers or vendors. Industrial organization literature further analyzes the cost control effect of post-award competition between two/multiple suppliers (Tirole, 1988). Seshadri et al. (1991) analyze the pre-award effect of bidding participation in procurement auctions. If the final contract is made with multiple suppliers, there is a trade-off between a reduced selection risk and increased bidding competition. Bidders might strategically increase their pre-award bids if the decreasing selection risk under multiple sourcing does not compensate for the increase in supplier competition. In extension to the effect of pre- and post-award competition, Klotz and Chatterjee (1995) study the effect of repeated procurement business in an environment with two periods, entry costs, and production learning. In situations where the buyer cannot commit to future supplies, the bidders have a reduced incentive to include expected production

cost decreases (due to learning) into their first period bid and therefore, second period competition becomes important to avoid a monopoly situation. Dual sourcing in the first period is used to compensate the suppliers for their entry cost and to control the way of production learning and therefore to ensure a cost symmetry in the second period. This makes it more likely that both suppliers will again compete for the second period supplies.

Tsay et al. (1999) review the literature on supply chain contracts with respect to different performance measures. These include specification of decision rights, prices, minimum purchase commitments, quantity flexibility, buyback and return policies, allocation rules, lead times, and quality. These contracts mainly refer to the single supplier situation. Nevertheless, when these contracts are regarded as the outcome of negotiations with several suppliers, the supplier choice of the buyer will depend on alternative opportunities of supply and therefore be a driving factor within the negotiation process.

2.2. *Exchange rate volatility*

A different strategic aspect appears within global supply chain management where sourcing cost performance depends on exchange rate volatility. Exchange rate volatility means that purchase prices (if paid in the currency of the supplier) become a random variable. Besides short term financial instruments like forward buying of foreign currencies, futures, or options, the availability of several developed suppliers in different countries offers some operational flexibility in the sense that if the exchange rate develops in a way that purchases from one supplier become too expensive, another global or domestic supplier may be chosen. Depending on the capacity of the suppliers, the entire purchasing volume or part of it may be transferred. Kouvelis (1999) gives an overview of the respective international operations management literature and analyzes operating policies under exchange risk volatility and switch-over costs. In an optimal global sourcing strategy there exists a hysteresis band of inaction for the exchange rate. If the exchange rate lies within this

band it is optimal to maintain the current sourcing strategy whereas if the exchange rate leaves the band, it is optimal to source from a domestic supplier or a supplier located in a different country. The width of the band of hysteresis increases with the size of the switchover cost and the system exchange rate volatility. Under global sourcing it becomes a challenging task to form a portfolio of international suppliers, where the correlation between currencies plays an important role. In order to be able to exploit operating flexibility it is necessary to negotiate flexibility in contracts. The allocation of quantities among several suppliers depends on the respective capacity constraints and exchange rate volatility.

3. *Single-stage inventory models*

In the following, single-stage models with deterministic or stochastic lead times that explicitly model several supply modes are reviewed. Another category that does not explicitly mention a second supply mode for emergency orders are lost sales inventory models. Instead of assuming that unsatisfied demands are lost, associated with a penalty, a different interpretation is that these customers are not satisfied from regularly replenished items but instead an emergency supply is initiated (associated with an extra cost).

3.1. *Deterministic lead times*

The focus of research for inventory models with several supply modes, deterministic lead times, and random demands is mainly on two aspects. Earlier contributions use a dynamic programming functional equation framework with periodic review in order to determine the structure of an optimal inventory control policy (Section 3.1.1) whereas more recent contributions investigate the choice of policy parameters for a given inventory control strategy (Section 3.1.2).

3.1.1. *Optimal replenishment policies*

The general assumptions within this category of models are the following. There are multiple suppliers $i = 1, \dots, n$ with different lead times

$\lambda_1 < \dots < \lambda_n$ and different unit purchase prices $c_1 > \dots > c_n$, i.e. the fastest supplier has the highest cost per unit. In addition, a fixed setup cost K_i for each order placement with supplier i might be present. Due to the complexity of this problem most models are restricted to two suppliers, zero setup costs and a lead time difference of one period, often even further reduced to the situation of a single period regular supply lead time and instantaneous emergency supply. Barankin (1961) formulates the inventory control problem for T periods and analyzes the outcome for the single period case. Extensions to the T period case have been analyzed by Daniel (1963), Neuts (1964), Bulinskaya (1964a, b), and Fukuda (1964). For an optimal (stationary) policy (see Veinott, 1966) there exists a pair of an emergency order-up-to-level y^0 and a cumulative order-up-to-level y^1 such that for an initial inventory x at the beginning of a period: (i) if $x < y^0$, order $y^0 - x$ by the emergency mode and $y^1 - y^0$ by the regular mode, (ii) if $y^0 \leq x < y^1$, only a regular order of size $y^1 - x$ is placed, and (iii) if $x \geq y^1$, no order is placed. Extensions to three supply modes (where it is assumed that they are only available in even periods) and a common fixed cost if any type of order is placed in a period are included in Fukuda (1964). Wright (1968) studies an extension to the multi-product situation where a constraint for the total of emergency orders is present. The most general formulation is given by Whittemore and Saunders (1977) who consider an arbitrary lag between the lead times of the two suppliers. This model leads to a complex multi-state dynamic programming problem. Therefore, explicit results on the optimal ordering policy are again only provided for the case of lead times differing by a single period. Zhang (1996) analyzes a model with three suppliers and lead times that differ by one and two periods. The optimal policy is explicitly stated. In addition, simple heuristic ordering policies are discussed and a heuristic framework, based on newsvendor considerations, is developed in order to provide decision support for finding appropriate replenishment policy parameters. This framework of simple expressions for overage and underage costs presents a very promising idea to be applied in other areas where multiple suppliers

and therefore a multiple-state dynamic programming problem appear, especially for the two-supplier model with arbitrary lead time differences. Scheller-Wolf and Tayur (1999) analyze the structure of optimal dual-sourcing policies under the presence of state-dependent minimum and maximum order quantities and unit prices, again for lead times differing by one period.

3.1.2. Parameters for given policies

One suggestion to overcome the problems of finding the complex structure for optimal policies is to analyze simple replenishment policies and to find (near) cost optimal parameters. Within the class of continuous review policies, Moynzadeh and Nahmias (1988) analyze an extension of the (s, Q) policy. In their model, two supply modes with continuous lead times $0 < \lambda_2 < \lambda_1$ are available. An order of each type is associated with fixed ordering costs of K_1 and K_2 and unit purchase cost of c_1 and c_2 , respectively. No specific relation between these cost parameters is required. Inventory control performance is measured by holding costs per item and unit of time and shortage penalties for unsatisfied demand. The objective is to minimize long run average costs. The suggested ordering policy is an (s_1, s_2, Q_1, Q_2) policy based on the available on-hand stock. The reason for connecting the policy to on-hand/net stock rather than to the inventory position is that an exact policy would have to incorporate not only the size but also the exact timing of outstanding orders. Whenever the reorder point s_1 for the regular mode is reached an order of size Q_1 is placed. If, within the replenishment lead time of the regular order, the emergency reorder point s_2 is reached, an order of size Q_2 is placed, but only if this order will arrive before the arrival of the regular order. Johansen and Thorstenson (1998) analyze a similar model where regular replenishments with a long lead time are controlled by a continuous review (s, Q) policy. The lead time of emergency orders is assumed to be much smaller, i.e. a fraction $1/\lambda$ of the regular mode. Between the placement and the receipt of a regular replenishment, the state i of the system (given by the net inventory) is reviewed at certain time points j/λ where $j = \lambda, \lambda - 1, \dots, 1$. A state and time

dependent emergency order of size $u(j) - i$ is placed, if $i \leq s(j)$. A simpler version which neglects the time impact is to place an emergency order of size $u - i$ whenever $i \leq s$. The authors propose a policy iteration algorithm to solve this Markovian decision problem. One finding is that the availability of an emergency mode may lead to significant cost reductions. However, most of this cost reduction potential can already be exploited by the simple emergency policy and the more complex, state and time dependent emergency policy only provides small additional cost reductions.

For environments without or with negligible setup costs for order placements and unit sized (Poisson) demand, one-for-one ordering policies are an appropriate basis for two supply mode extensions. Moynadeh and Schmidt (1991) analyze such an extension which takes into account all available information, i.e. net inventory and the timing of all outstanding orders. Regular orders have a lead time of λ_1 whereas emergency orders only require λ_2 periods to be delivered. The cost for an emergency order exceeds the cost for a regular order. Their policy is based on the state of the system given by the time t , the net inventory k , and the times x_1, \dots, x_{S-k} until the respective outstanding orders arrive. The decision variables are given by the order-up-to-level S and a level S_e which triggers emergency orders. When a demand occurs, the replenishment decision is as follows: (i) if $k \geq S_e + 1$, place a regular order (ii) if $k \leq S_e$ and $x_{S_e-k+1} \leq \lambda_2$, place a regular order (iii) if $k \leq S_e$ and $x_{S_e-k+1} > \lambda_2$, place an emergency order. This implies that if the emergency trigger level is reached and the next incoming order (where net inventory reaches the trigger level) will arrive before the emergency order would, a regular order is placed whereas if the emergency order would arrive earlier, it is preferred over the regular order. For $S = S_e$, only emergency orders are placed whereas for $S_e = -\infty$, only regular orders take place.

Chiang and Gutierrez (1996) analyze a periodic review inventory system with two supply modes, a long review period (compared to the replenishment lead times), identical unit purchasing cost and a fixed cost for emergency orders. The policy used is

an order-up-to-level policy. They show that there exists an indifference level which guides the type of replenishment order to be placed. If the inventory level is below the indifference level, an emergency order is placed that raises the inventory position to the order-up-to-level, otherwise a regular order is placed that raises the inventory position to the order-up-to-level. In a numerical study, the authors compare the benefits of this emergency policy with the two alternatives of using a single mode (regular or emergency) exclusively. In a second paper, Chiang and Gutierrez (1998) analyze a similar model where, in contrast to the first one, the emergency supply decision can be taken continuously (between regular review epochs and the arrival of the next order) instead of periodically, and there are no fixed costs but different variable costs. They show the optimality of an emergency order-up-to-level policy, where the emergency order-up-to-level depends on the number of periods until the regular replenishment order will arrive. Tagaras and Vlachos (2001) analyze a periodic review order-up-to-level type policy with a regular and an emergency replenishment mode. The regular mode is used to raise the inventory to a base-stock level whereas the faster emergency mode might be used within the replenishment cycle in order to avoid stockouts. In Vlachos and Tagaras (2001) the emergency mode is capacitated.

Rosenshine and Obee (1976) investigate a different kind of two-supply mode inventory system. One supplier delivers a constant quantity Q in every period. If the inventory level drops below a level s , an emergency order with zero lead time is released that increases the inventory level to S . The supply chain management issue behind this kind of inventory model is that the regular supplier might offer a price discount due to the contracted delivery quantities which reduces the supplier's demand uncertainty and offers a longer period business with this customer. Janssen and de Kok (1999) analyze a similar periodic review model. One supplier delivers Q units every review period and a second supplier is used in order to increase the inventory position to the level S . Both supply modes have identical lead times. This policy is a combination of a constant supply push and a

flexible pull inventory system. The cost structure includes fixed ordering costs for each supplier, purchase costs per unit for each supplier, and a holding cost rate. Stockouts are limited by a fill rate service level constraint.

The contribution of inventory models with emergency orders to global supply chain management issues is that global suppliers offer a cheaper price, but because of the regional distance, require longer lead times (except for air transportation modes). Therefore, these suppliers are contracted with some stable amount of replenishments and in situations of large requirements, a more expensive domestic supplier is used. The models assist to allocate the purchasing volume among these supply options.

3.2. Stochastic lead times

The advantage of placing orders with multiple suppliers when lead times are random is that (mean and variances of) effective lead times of arriving orders are reduced. Besides this statistical argument reviewed in Section 3.2.1 further reductions in shortages and inventories make the splitting of total requirements among several suppliers profitable from a cost minimization point of view which is discussed in Section 3.2.2. These models almost exclusively use continuous review (s, Q) policies and determine the optimal number of suppliers, the reorder point, the total order quantity and its allocation among the suppliers.

3.2.1. Statistical aspects

Assume the total order is split into n portions which are simultaneously placed at different suppliers with stochastic lead times L_1, \dots, L_n . Then, the effective lead times of the incoming orders are distributed according to the ordered set of these lead times, i.e. $L_{(1)} \leq \dots \leq L_{(n)}$. All these approaches (for identically distributed supplier lead times) have in common that they prove that the mean and the variance of effective lead times (time until the first order arrives, inter-arrival times between later orders) are smaller than the mean and variance of the individual suppliers lead times. Sculli and Wu (1981) analyze the two supplier situation with normally distributed lead

times and use numerical integration to derive tables for the mean and standard deviation of the effective lead times until the first/second order arrives. Closed form expressions have been published by Fong (1992). Sculli and Shum (1990) extend these findings to the case of n suppliers with non-identical allocation portions of supply and give expressions for the mean and variance of the effective lead times. Pan et al. (1991) derive general expressions for n suppliers with independently, but identically, distributed lead times and give explicit formulas for the cases of uniform, exponential, and normal lead times in a two supplier model. Guo and Ganeshan (1995) analyze the same problem setting under uniform or exponential lead times with respect to the optimal number of suppliers. They suggest to increase the number of suppliers until the mean or the variance of the effective lead time $L_{(1)}$ reaches a required level. Kelle and Silver (1990a) assume Weibull lead times and prove the reduction in mean and variance for the effective lead time of the first incoming order. In addition to the pure order statistics based results on effective lead times they determine the required reorder point, given a stockout probability service level constraint for the time period until the first order arrives. The proposed methodology applies for deterministic demand with a constant rate and for (stationary) normal or Poisson demand. The analysis of the continuous review (s, Q) inventory system with order splitting holds under the assumptions that (1) orders do not cross, i.e. all orders/portions have arrived when the reorder point is reached and new orders are placed (2) the stockout risk is mainly present in the period until the first order arrives and negligible for all further inter-arrival times for split orders. The authors provide bounds on the total order quantity Q such that these assumptions are satisfied. In Kelle and Silver (1990b), the determination of a reorder point under multiple sourcing is presented for the case where the expected shortage (before the arrival of the first order) instead of the stockout probability is restricted by a service level constraint. Similar statistical considerations for effective lead times and stockout probabilities (respective expected shortages) are made by Fong et al. (2000) for a dual sourcing

inventory system with normally distributed demands and lead times being distributed according to mixtures of Erlang distributions.

3.2.2. *Economic criteria*

In order to assess the benefits of order splitting in an economic context it is necessary to consider models where the total cost for ordering (order releases, order receipts), purchase prices, inventory holding, and stockout penalties are minimized. Ramasesh et al. (1991) present such a cost minimization approach for two potential suppliers where the order quantity is evenly split between the suppliers. Demand is assumed to be constant, shortages are backordered and a penalty cost per item and time unit is incurred. Both suppliers have identical lead time distributions, either being uniformly or exponentially distributed. From approximations for the (non-linear) total cost functions, the optimal values for the reorder point s and the order quantity Q are derived by numerical search. In Ramasesh et al. (1993) these findings are extended to the case of suppliers with non-identical lead time distributions, purchase prices, and order splitting portions. This approach is especially useful in order to analyze the trade off between different lead time characteristics (mean, variability) and purchase price differentials. Chiang and Benton (1994) investigate a model with normally distributed demands and shifted exponentially distributed lead times. Both suppliers are identical with respect to their lead time characteristics and purchase prices. In contrast to the majority of the other cost models they assume a shortage penalty that is independent of the stockout duration. As an alternative to avoid shortage cost measurement, a service level constraint approach is suggested. Lau and Lau (1994) analyze the effect of offsetting lead times and purchase prices in an environment with deterministic demand. They provide an extensive numerical study with beta distributed lead times to point out under which parameter configurations it is advantageous to use a single or two supply sources. The same trade-off between supplier reliability and purchase price differences is analyzed for normally distributed demands by Ganehan et al. (1999).

All the models discussed so far point out the advantage of using multiple suppliers and the resulting reductions in inventory and shortage cost. Lau and Zhao (1993) and Zhao and Lau (1992) analyze a model with stochastic demand and lead times where their main finding is that the major part of the inventory cost reduction is due to cycle rather than safety stock reduction. This directly leads to the criticism by Hill (1996). Instead of splitting an order and placing the portions at several suppliers at the same time, an alternative would be to place and stagger the portions of the total order at a single supplier. In comparison to single and multiple sourcing environments this strategy requires a higher reorder point but reduces the average on-hand inventory considerably. In addition, such a strategy might reduce lead times when these depend on the order size and several consecutive batches can be manufactured in shorter time compared to a single large batch. Chiang and Chiang (1996) present a single supplier order splitting model with deterministic lead times and normally distributed demands where the inter-arrival times between the split orders are optimized. Sedarage et al. (1999) provide a cost approach for an (s, Q) policy with non-identical suppliers, random demands, and lead times where they simultaneously determine the optimal number of suppliers, the reorder level, the order quantity, and the splitting portions. One finding of their analysis is that it might be favourable to include a supplier which has a worse purchase price and a larger mean and standard deviation with respect to his lead time performance. Further extensions within the line of continuous review (s, Q) policies under random lead times include the analysis of lost sales models (Mohebbi and Posner, 1998).

One major contribution of stochastic lead time inventory models with multiple suppliers to supply chain management is the decision support with respect to the trade-off between lead time characteristics (mean, variability), required buffer inventories, and purchase price differential between several available suppliers. In a global sourcing context with cheaper international suppliers which are on the other hand characterized by longer and more variable replenishment lead

times, and faster, more reliable, but expensive domestic suppliers, these models serve as a basis for determining if a single domestic or global supplier should be used or if it is beneficial to combine domestic and global suppliers and how the portions should be allocated. One major shortcoming of the discussed models is that they exclusively assume independent lead times. In situations where the world wide level of demands for a product increases/decreases, all suppliers might face higher/smaller lead times and therefore, their lead times will be correlated.

3.3. *Miscellaneous aspects*

Parlar and Perry (1996) present a model where the availability of a single or multiple suppliers is uncertain because of disruptions like equipment breakdowns, labor strikes, weather and agricultural yield etc. In this kind of model several suppliers offer the possibility of supplier substitution in addition to holding safety inventories. A state of such a system is characterized by the individual states (ON, OFF) for each potential supplier. The transition between these states follows a stochastic process and the total number of states is 2^n . A suggested ordering policy is to define state dependent reorder points s_i and order quantities Q_i . As a simplification, the authors analyze a policy with a single reorder point and state dependent order quantities.

Anupindi and Akella (1993) investigate multiple supplier models under different kinds of supply uncertainty related to lead time and yield variability. In a first model, an order is delivered immediately or in the next period (with respective probabilities). In a second model, a random fraction of an order is delivered and the remaining part cancelled, whereas in a third model, the missing fraction is delivered in the next period. The authors show, for the single period case, that a policy with two parameters, indicating the use of a single or both supply modes is optimal.

Another kind of model, which not necessarily implies several suppliers but different modes of supply uses order expedition in cases where inventories become low. Allen and D'Esopo (1968) analyze an (s, Q) policy under random

demands with constant lead time λ_1 . Whenever the on-hand stock reduces to an emergency level E , an outstanding order is expedited such that it arrives after $\lambda_2 < \lambda_1$ periods. As a consequence, the effective lead time becomes a random variable. Ben-Daya and Raouf (1994) consider an (s, Q) inventory system where the lead time can be reduced at an added cost and therefore, the lead time and the order quantity become decision variables whereas the reorder point is fixed. A similar model is studied by Ouyang et al. (1996). Lead time reduction possibilities under a periodic review (s, S) policy are investigated by Gross and Soriano (1972). In their model, investment into reductions of the mean and lead time variability are taken into account.

Multiple modes from a physical distribution, demand management perspective together with a supply side are discussed by Hollier et al. (1995). They analyze a continuous review (s, S) policy with random, lumpy demands. The applied policy is to serve customer orders of sizes smaller than a maximum issue quantity w from stock whereas larger customer orders are satisfied by special deliveries. If a special delivery is ordered, additional units in order to replenish the inventory to the reorder level S are ordered too. The aim is to find reasonable control parameters w, s, S .

Under pure deterministic conditions with respect to lead times and demand, multiple supplier issues occur under situations of constraints with respect to capacity, average quality performance of average purchase prices. Mixed integer programming models for supply chain design are reviewed by Vidal and Goetschalckx (1997). Syam and Shetty (1996) presents an algorithm for the capacitated multiple supplier inventory grouping problem. This problem includes the decisions upon sourcing of multiple products from several capacitated suppliers. Products are jointly replenished and packaged, quantity discount schedules may exist, products are transported (eventually via freight consolidation points). This problem leads to a mixed integer programming formulation for minimizing total costs for ordering, purchasing, inventory holding, and transportation. Benton (1991) analyzed quantity discounts in a multi-product, multi-supplier environment with storage

space and/or investment constraints. The main question is which product should be sourced from which supplier. Hong and Hayya (1992) analyze a deterministic EOQ-type model where total requirements are split among several suppliers. Suppliers are characterized by different purchase prices and quality levels. The objective is to minimize aggregate ordering and inventory holding cost subject to average purchase price and average quality level constraints. The non-linear programming model provides the optimal number of suppliers and the respective allocation of supply. Rosenblatt et al. (1998) investigate an EOQ-type model with capacitated suppliers. Other mathematical programming models use multi-criteria goal programming (Chaudry et al., 1991) or supplier selection and quantity allocation (Jayaraman et al., 1999).

4. Related inventory problems

4.1. Multi-stage inventory models

Multi-echelon inventory systems with multiple-supply mode characteristics can be classified into models that include some of the features of single-stage models with multiple suppliers and models that allow for transshipments between inventory stocking points.

Ganeshan (1999) studies a continuous review, one warehouse, multiple retailer distribution system. The multiple-supplier aspect is incorporated by splitting the warehouse order into n equal portions in order to reduce effective lead times. Muckstadt and Thomas (1980) analyze a similar system where all inventory points apply $(S-1, S)$ policies. If a retailer has zero on-hand inventory and an item is demanded, an emergency order is placed at the warehouse which implies a shorter resupply time compared to a regular order. The same applies for the replenishment policy at the warehouse. In Aggarwal and Moinzadeh (1994) a production/distribution system is analyzed where the retailers replenish demands (generated by independent Poisson processes) according to an $(S-1, S)$ strategy. Orders to the production facility are announced to be regular if the number

of outstanding orders does not exceed a level k . Otherwise, the order is placed as an emergency order. The production facility does not keep any inventory. Retailer orders are kept in two pipelines, regular and emergency orders. All emergency orders have to be processed before a regular order can be manufactured. Besides manufacturing priorities there are no time or cost advantages/disadvantages of regular orders over emergency orders. The objective is to find appropriate maximum inventory levels S and emergency order trigger levels k for each retailer. Moinzadeh and Aggarwal (1997) study a multi-echelon system with the emergency ordering mechanism similar to the one developed by Moinzadeh and Schmidt (1991) for the single-stage inventory model.

Minner et al. (1999) consider a periodic review, two echelon system where, as an alternative to rationing a depot shortage, outstanding orders can be speeded up with a certain probability. The multi-echelon system extension of the maximum issue quantity is discussed by Dekker et al. (1998) as the break quantity rule, that is the quantity that if a customer order size is larger than the break quantity, the request is satisfied directly from the warehouse and otherwise shipped by the associated retailer. The objective is to find appropriate order-up-to-levels and the break quantity in order to minimize system operating costs. In so called no-delay multi-echelon inventory models (Minner, 2000), safety stocks at every stocking point are provided to cover against reasonable demand variability whereas extraordinary large orders are excluded from the analysis by assuming some kind of operating flexibility. This modelling approach implicitly assumes the presence of two supply alternatives, a regular one for demands not exceeding a predetermined level of variability and an emergency mode to deal with excessive variations.

A different category of models that shares some aspects of multiple supply modes are multi-echelon models with lateral transshipments. A retailer regularly replenishes its material from the warehouse. Under random demands, the inventory status of the retailers may be out of balance, that is some retailer may have a lot of inventory whereas others are out of stock. Besides waiting for the

next regular warehouse shipment or placing emergency orders at the warehouse, transshipments from other, nearby, retailers with sufficient inventory might immediately (or after a short lead time) cover the out of stock situation. As a consequence, retailers face two sources of demand (customers, other retailers) and two sources of supply (warehouse, other retailers). Lee (1987) presents a multi-echelon system with identical retailers and transshipments for repairable items and reviews the earlier literature on transshipment problems. In the model pooling groups are introduced. Whenever a retailer is out of stock, the demanded item is sourced from another retailer within the same pooling group. Different priority rules for choosing the supplying retailer (random, retailer with maximum stock) are analyzed. Optimal stocking levels are determined subject to service level constraints (for the warehouse and retailers) on immediate and after transshipment fill rates. Axsäter (1990) presents an improved approach which also applies to non-identical retailers. Alfredsson and Verrijdt (1999) present a model with emergency flexibility which includes transshipments from other retailers. If these are not available, direct shipments from the warehouse, and if even this is not possible, direct shipments from the external supplier are allowed. Finally, if all retailers are periodically supplied, it might be possible to rebalance inventories every review period. Such models are provided by Jönsson and Silver (1987) and Diks and de Kok (1996).

4.2. *Reverse logistics inventory models*

Increasing environmental consciousness of customers and product take back regulations of governments offer the possibility to satisfy customer demands from recovered used products instead of exclusively manufacturing new products and disposing of all returned products. Inventory models for these product recovery systems share several features with two-supplier inventory models in that product returns represent a second mode of supply in addition to the production alternative. This mode of supply may offer cost advantages over production if the sum of produc-

tion and disposal unit costs exceeds the unit cost for remanufacturing. Another criterion concerns the relation of manufacturing and recovery lead times, i.e. remanufacturing may become advantageous due to lead time advantages, e.g. spare parts demand are remanufactured from returns instead of initiating a resupply with a substantial lead time. On the other hand, the product recovery problem is more complex than the situation assumed for most multiple-supplier inventory models. First, the recovery supply mode is capacitated in the sense that control over returned products is limited and that only available returns can be recovered. Second, returns will, in general, be a random variable and therefore, the size of availability is random. For a more comprehensive review of quantitative (including inventory) models see Fleischmann et al. (1997).

Richter (1996) studies the problem of lot-size coordination for production and remanufacturing in a model with deterministic and constant demands and returns in an EOQ framework. Teunter (2001) extends this work to a situation where serviceable inventories sourced from production and recovery are subject to different holding cost rates. This is motivated by the difference in capital tied up. To our knowledge, this problem has not been addressed before for inventory models with different supply modes that have offsetting purchase prices. Nevertheless, this implies that a priority rule for inventory depletion would be necessary and it is still an open question if this problem can be overcome by finding an appropriate mechanism for determining the holding costs. Additionally, both the approach of Richter (1996) and the one of Teunter (2001) assume that all lot sizes for production as well as those for remanufacturing have identical size. It can be shown (Minner, 2001) that this is, in general, not the optimal strategy and that costs might be reduced by a strategy with non-increasing consecutive remanufacturing lot sizes.

The assumption of constant and stationary demand and return parameters is common with most reverse logistics inventory models. But product life cycles, cyclic demand patterns, and dependencies of return and past sales motivate the consideration of dynamic models. Minner and

Kleber (2001) present an optimal control formulation for a simple deterministic inventory system and a linear cost structure.

Models with stochastic demands and return flows share modelling similarities with multiple-supplier models under deterministic lead times. Simpson (1978) and Inderfurth (1997) analyze the optimal replenishment policy in a model without fixed costs and zero (or identical) lead times in a periodic review model. For a lead time difference of a single period, the optimal policy can still be determined but is already more complex. For models with an arbitrary lead time difference, the same problems as in the two-supplier model of Whittemore and Saunders (1977) appear. Parameters for given inventory control policies are optimized in a continuous review system by van der Laan et al. (1999). They analyze extensions of (s, Q) policies, i.e., an (s_m, Q_m, Q_r) push policy where a manufacturing lot of size Q_m is ordered whenever the reorder point s_m is reached and a recovery lot of size Q_r is released whenever Q_r items have been collected. An alternative policy is an (s_m, Q_m, s_r, S_r) pull policy, where remanufacturing orders to increase the serviceable inventory position to S_r are only placed if the reorder point s_r is reached. Kiesmüller and Minner (2001) present newsvendor-type expressions for finding approximate parameters of periodic review order policies with two order-up-to-levels that also allows for the investigation of arbitrary differences of the lead times.

5. Directions for future research

The reviewed multiple source inventory models can serve as a building block and an assisting tool for several aspects in designing and negotiating supply contracts. Especially under competition between several potential suppliers, the availability of several supply options and even the possibility to combine sources increases the negotiation power of the buyer. The presented models can assist in the evaluation of several offers. For an optimal replenishment policy, each offer of a potential supplier cannot be assessed alone but has to be evaluated together with other offers

when risk may be diversified by using several modes of supply. The availability of a second developed supplier and the associated costs connected with transferring supply volume to this supplier determine the buyer's band of negotiation on supply contracts with the current major supplier. Competitive models in supply chain management with respect to inventories so far have addressed channel competition between suppliers and buyers (Cachon, 1999). The competition between potential suppliers for the supply volume has only been addressed from an industrial organization perspective without any inventory considerations due to demand and lead time uncertainties. In order to compete for a single sourcing agreement, the supplier has to design a contract offer with substantial flexibility that makes it unfavourable for the buyer to include a second supplier.

In specific situations, there might be a trade off between supplier inventory required in order to fulfill buyer purchasing service level requirements and additional sourcing opportunities available for the supplier. Therefore, the supplier might offer a lower unit price if the buyer contracts a constant supply quantity as analyzed in the standing order inventory systems by Rosenshine and Obee (1976) and Janssen and de Kok (1999). The reduction effect within effective lead times can be used in order to postpone part of the requirements replenishment (in the sense of time phased release of split orders) in order to gather more information on demand characteristics for innovative and fashion products. An idea for further research would be to combine the models of multiple supply modes with quick and accurate response manufacturing (Fisher and Raman, 1996) and distribution strategies.

Information systems for replenishments are mainly based on the concept of Material Requirements Planning and therefore no special aids for selecting from alternative supply modes and for quantity allocation are provided. Nevertheless, the increasing potential of modern information and communication technology offers possibilities to implement more elaborate replenishment decision rules that include several kinds of available information, especially on sizes and arrival times

of outstanding regular and emergency orders as proposed by the models of Moinszadeh and Schmidt (1991) and Moinszadeh and Aggarwal (1997). In addition, close buyer-supplier relationships with information sharing offer an application potential for time phased order splitting models (with a single or multiple supplier) and respective order tracking. The emerging importance of E-business, especially E-procurement possibilities with the use of Internet technology further reduces transaction costs for supplier search and order placement with several suppliers and therefore makes multiple-supplier models even more attractive compared to the single sourcing alternative. This market is especially characterized by spot market offers and continuously changing suppliers will ex ante yield high uncertainty with respect to reliability and lead times, fields where especially multiple supplier replenishment and inventory strategies outperform single sourcing policies.

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