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REVIEW ARTICLE

A survey on one-bit compressed sensing: theory and applications

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Abstract In the past few decades, with the growing popularity of compressed sensing (CS) in the signal processing field, the quantization step in CS has received significant attention. Current research generally considers multi-bit quantization. For systems employing quantization with a sufficient number of bits, a sparse signal can be reliably recovered using various CS reconstruction algorithms.

Recently, many researchers have begun studying the one-bit quantization case for CS. As an extreme case of CS, one-bit CS preserves only the sign information of measurements, which reduces storage costs and hardware complexity. By treating one-bit measurements as sign constraints, it has been shown that sparse signals can be recovered using certain reconstruction algorithms with a high probability. Based on the merits of one-bit CS, it has been widely applied to many fields, such as radar, source location, spectrum sensing, and wireless sensing network.

In this paper, the characteristics of one-bit CS and related works are reviewed. First, the framework of one-bit CS is introduced. Next, we summarize existing reconstruction algorithms. Additionally, some extensions and practical applications of one-bit CS are categorized and discussed. Finally, our conclusions and the further research topics are summarized.

Keywords compressed sensing, one-bit quantization, sign information, support, consistency

1 Introduction

Compressed sensing (CS) is a fascinating novel signal processing theory [1–4]. By using linear projection, it is possible for CS to recover sparse signals with few measurements at a high probability. CS merges data sampling and data compression into a single step using the following linear projection operation:

$$\boldsymbol{y} = \boldsymbol{\Phi x}, \tag{1}$$

where $\boldsymbol{\Phi} \in \mathbb{R}^{M\times N}$ is the measurement matrix, $\boldsymbol{y} \in \mathbb{R}^{M}$ is the measurement vector with $M \leqslant N$, and $\boldsymbol{x}$ is a K-sparse signal with only K ($K \ll M$) non-zero coefficients in $\boldsymbol{x}$. The locations of the non-zero coefficients of $\boldsymbol{x}$ are defined as the support of $\boldsymbol{x}$. The sampling method in Eq. (1) allows CS to use a low sub-Nyquist sampling rate for sparse signal processing.

For practical systems, a quantization step is essential prior to signal processing, which inevitably introduces quantization error. The error from quantization is generally regarded as measurement noise with limited energy. Recently, an extreme case of CS called one-bit CS has received significant attention. Each measurement in one-bit CS is quantized into a single bit. The framework for one-bit CS, which is shown in Fig. 1, consists of sparse representation, one-bit quantization, and reconstruction algorithms. One-bit CS preserves only the sign information of random measurements. Therefore, it greatly reduces the complexity of hardware implementation for terminal devices.

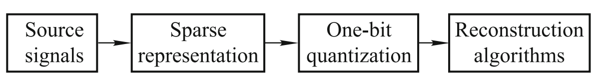

Fig. 1 Theoretical framework of one-bit CS

The quantized measurements in one-bit CS are treated as sign information and do not provide any amplitude information. This means that classical CS recovery algorithms can-

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not exactly recover a sparse signal $\boldsymbol{x}$. Current recovery algorithms for one-bit CS generally exploit the consistency principle, which means that the product of the sparse signal and measurements is always positive. For practical applications, the general guarantee of one-bit CS recovery has been theoretically proved in [5–11]. Additionally, in [12], Boufounos and Baraniuk first proposed a recovery based on fixed point continuation [13]. Subsequently, scholars designed a series of reconstruction algorithms [6, 14–19]. For example, [14, 15] considered noiseless conditions with necessary prior information at a sparse level and [16–19] considered noisy conditions with an inevitable sign-flipping phenomenon for practical systems. Additionally, Jacques et al. [6] proposed a method with two different norm terms for noiseless and noisy conditions. Based on the outcomes of the above studies, researchers have proposed significant extensions of one-bit CS, such as one-bit Hamming CS (HCS) [20] and one-bit distributed CS (DCS) [21], which exploit the special distribution properties of single sparse signals and the correlations between distributed sparse signals, respectively.

Recently, one-bit CS has been widely applied to various applications, such as wireless sensing networks [22–27], cognitive radio [28, 29], radar [30–32], and medicine [33, 34]. By designing effective schemes for the above applications, it has been proved that systems based on one-bit CS enjoy improved speeds for processing vast quantities of sparse data by reducing the cost of communication and the complexity of hardware implementation.

In this paper, we review the outcomes of the studies above and attempt to provide additional reference for future research. The remainder of this paper is organized as follows. In Section 2, the quantization step and performance analysis of one-bit CS are presented. In Section 3, we provide a comprehensive review of one-bit CS reconstruction algorithms. Furthermore, we analyze some extensions of one-bit CS. Next, the main applications of one-bit CS are classified in Section 4. Finally, our conclusions on one-bit CS and future work are discussed in Section 5.

2 Framework of one-bit CS

To facilitate the presentation of one-bit CS, we first present some CS background information in this section. Next, one-bit CS is introduced. Finally, we list some results regarding the theoretical performance of one-bit CS recovery.

2.1 Classical compressed sensing

The classical CS framework has three main parts: sparse representation, measurement matrix, and reconstruction algorithms. The first part is finding a sparse representation of sparse signals using a particular basis, such as a Fourier transform, a wavelet transform, curvelets, and contourlets.

The measurement matrix is the key element for sampling and recovering sparse signals. The measurement matrix $\boldsymbol{\Phi}$ must satisfy a significant constraint called the restricted isometry property (RIP) [35]. $\boldsymbol{\Phi}$ satisfies the RIP of order K if there exists a constant $\delta_K \in (0, 1)$ such that

$$(1-\delta_K)\|\boldsymbol{x}\|_2^2 \leqslant \|\boldsymbol{\Phi x}\|_2^2 \leqslant (1+\delta_K)\|\boldsymbol{x}\|_2^2, \tag{2}$$

holds for all $\boldsymbol{x} \in \mathbb{R}^N$ where $\|\boldsymbol{x}\|_0 \leqslant K$ ($\|\boldsymbol{x}\|_0$ is the number of non-zero elements in $\boldsymbol{x}$ and $\|\boldsymbol{x}\|_2$ is the Euclidean norm of $\boldsymbol{x}$). $\boldsymbol{\Phi}$ acts as an approximate isometry of the set of K-sparse signals. The RIP provides a sufficient condition to guarantee successful recovery using reconstruction algorithms.

The main purpose of CS reconstruction algorithms is to recover $\boldsymbol{x} \in \mathbb{R}^N$ from an incomplete observation $\boldsymbol{y}$. Generally speaking, classical CS reconstruction algorithms are divided into three main types. The first is convex algorithms, which have excellent recovery performance, but high complexity. These algorithms include sparse reconstruction by separable approximation [36] and gradient projection for sparse reconstruction [37]. The second type is greedy algorithms, which have low complexity, but suffer from performance degradation. These algorithms include iterative hard threshold (IHT) [38], orthogonal matching pursuit [39], regularized orthogonal matching pursuit [40], compressive sampling matching pursuit (CoSaMP) [41], and stage-wise orthogonal matching pursuit [42]. The third type is the Bayesian compressed sensing (BCS) algorithms [43], which constitute a trade-off between the first two types.

Quantization is an important step for practical systems, but quantization error is inevitable. Generally, quantization error is modeled as measurement noise. Therefore, the quantization problem can be formulated as follows:

$$\tilde{\boldsymbol{y}} = Q(\boldsymbol{\Phi x}) = \boldsymbol{\Phi x} + \boldsymbol{n}, \tag{3}$$

where $Q(\cdot)$ is the quantizer and $\boldsymbol{n} \in \mathbb{R}^M$ is the measurement noise that is energy-limited to some ϵ depending on the quantization accuracy $\|\boldsymbol{n}\|_2 \leqslant \epsilon$, where $\epsilon \leqslant \sqrt{M\Delta^2/12}$ for a uniform linear quantizer with a quantization interval Δ [12].

For the model in Eq. (3), there are various classical and robust reconstruction CS algorithms for sparse signal recovery, such as basis pursuit denoising (BPDN) [44] and least absolute shrinkage and selectionator operator analysis [45]. BPDN recovers sparse signals by solving:

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \|\boldsymbol{x}\|_1 \ \text{ s.t. } \ \|\tilde{\boldsymbol{y}} - \boldsymbol{\Phi x}\|_2 \leqslant \epsilon. \tag{4}$$

In this case, the reconstruction error norm is bounded by $\|\hat{\boldsymbol{x}}-\boldsymbol{x}\|_2 \leqslant C\epsilon$, where the constant C depends on the properties of the measurement system $\boldsymbol{\Phi}$, but not on the signal [46].

2.2 One-bit compressed sensing

One-bit CS is an extreme case of quantized CS that acquires measurements such that

$$\boldsymbol{y} = \mathrm{sign}(\boldsymbol{\Phi x}), \tag{5}$$

where sign(·) denotes an operator that performs an element-wise sign function on the vector. The sign function returns 1 for positive numbers and −1 for non-positive numbers. $\boldsymbol{y} = [y_1, y_2, \ldots\ y_i, \ldots, y_M]^{\mathrm{T}}$ are the original measurements and y_i is the i-th quantized measurement. Note from Eq. (5) that one-bit CS only preserves sign information, which greatly reduces the cost of storage and subsequent signal processing.

Recovery algorithms for one-bit CS are based on the consistency property, meaning the quantized measurements y_i have the same sign information as the corresponding original measurements $\mathrm{sign}(\boldsymbol{\Phi}\hat{\boldsymbol{x}})_i$, where $\hat{\boldsymbol{x}}$ is the recovered signal. Mathematically, consistency means that the product of the sparse signals lies on the unit sphere and that measurements are always non-negative:

$$y_i \cdot \mathrm{sign}(\langle \boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{e}_i, \boldsymbol{x}\rangle) \geqslant 0, \tag{6}$$

where $\langle\cdot,\cdot\rangle$ is the inner product of two vectors and $\boldsymbol{e}_i$ is a unit vector with one for its i-th entry.

Based on the consistency property, the first reconstruction algorithm was proposed by Boufounos and Baraniuk [12]. Because amplitude information is lost in the sign measurement, one must find a way around this missing information. In doing so, Boufounos and Baraniuk set a unit ℓ_2-sphere energy constraint for the recovered signal in [12]:

$$\|\boldsymbol{x}\|_2 = \sqrt{\sum_i x_i^2} = 1. \tag{7}$$

The proposed energy constraint significantly reduces the optimization search space and improves reconstruction performance.

2.3 Theoretical performance of one-bit compressed sensing

First, for classical CS, Laska and Baraniuk [5] derived an upper bound for reconstruction error based on the number of quantization bits under noisy conditions. Some scholars then directly defined certain performance bounds to demonstrate the potential of one-bit CS, such as [6–11]. First, a lower bound for the worst case error was derived in [6] under noiseless conditions.

Theorem 1 [6] Defining $\sum_K^* := \{\boldsymbol{x} \in \mathbb{R}^N : \|\boldsymbol{x}\|_0 \leqslant K$ and $\|\boldsymbol{x}\|_2 = 1\}$ and $\boldsymbol{x} \in \sum_K^*$, the estimation $\hat{\boldsymbol{x}} \in \sum_K^*$ of $\boldsymbol{x}$ obtained from $\boldsymbol{y} = \mathrm{sign}(\boldsymbol{\Phi x})$ in any reconstruction decoder has a lower bound for worst case error of:

$$\|\hat{\boldsymbol{x}} - \boldsymbol{x}\| \geqslant C\frac{K}{M + K^{3/2}}, \tag{8}$$

where C is a positive constant and $\boldsymbol{\Phi} \in \mathbb{R}^{M\times N}$ is the measurement matrix.

Most one-bit algorithms prefer to assume that the measurement matrix follows a Gaussian distribution. In this situation, [6] provides a uniform reconstruction result with a high probability, which is shown below in Theorem 2.

Theorem 2 [6] Fix $0 \leqslant \beta \leqslant 1$ and $\varepsilon > 0$. If the entries in the measurement matrix follow a Gaussian distribution and the number of measurements is:

$$M \geqslant \frac{2}{\varepsilon}\left(2K\log(N) + 4K\log\left(\frac{17}{\varepsilon}\right) + \log\left(\frac{1}{\beta}\right)\right), \tag{9}$$

then, for all $\boldsymbol{x}, \hat{\boldsymbol{x}} \in \sum_K^*$, and we get:

$$\|\hat{\boldsymbol{x}} - \boldsymbol{x}\|_2 > \varepsilon \Longrightarrow \mathrm{sign}(\boldsymbol{\Phi x}) \neq \mathrm{sign}(\boldsymbol{\Phi}\hat{\boldsymbol{x}}), \tag{10}$$

or, equivalently:

$$\|\hat{\boldsymbol{x}} - \boldsymbol{x}\|_2 \leqslant C\frac{K}{M}\log\left(\frac{MN}{K}\right), \tag{11}$$

with a probability greater than $1-\beta$.

Note that Theorems 1 and 2 discuss performance bounds based on the norm constraint $\|\boldsymbol{x}\|_2 = 1$ under noiseless conditions. Both the lower bound in Eq. (8) and upper bound in Eq. (11) become approximate functions of $\frac{K}{M}$ as the number of measurements M increases.

Additionally, one can see that the error decay rate in Therorem 2 is approximately a linear function. By adjusting the measurement procedure with adaptive thresholds for the quantization step, the work [7] improves reconstruction error as the exponential decay rate.

Based on this result, various error bounds have been proposed for one-bit CS [8–10]. Knudson et al. [8] found an error bound for a Gaussian measurement matrix with a norm estimation of $\boldsymbol{x}$. Additionally, Plan et al. [9] and Ai et al. [10] considered noisy models based on Gaussian and sub-Gaussian measurement matrices, respectively. Furthermore, for the threshold one-bit model in [11], the authors calculated a bound that does not depend on the energy constraint $\|\boldsymbol{x}\|_2 = 1$.

3 Reconstruction algorithms

In this section, recovery algorithms for one-bit CS are discussed. Generally speaking, based on different types of optimization models, most reconstruction algorithms for one-bit CS can be divided into three categories. The first is regularizer-class algorithms that add a regularization term to the optimization model to measure sign violations, such as [12, 14, 17]. The second is penalty-class algorithms that simply minimize a penalty function to recover a sparse signal, such as [6, 10, 15, 16]. The third is Bayesian compressed sensing frameworks, such as [18, 19].

3.1 Regularizer-class algorithms

Generally speaking, the basic idea of regularizer-class algorithms is to add an additional term $f(\boldsymbol{a})$ to the cost function of a classical CS recovery problem. This term $f(\boldsymbol{a})$ is used to enforce consistency between the sparse signals and measurements. Therefore, the solution model is given in the form:

$$\hat{\boldsymbol{x}} = \min_{\boldsymbol{x}} \|\boldsymbol{x}\|_1 + \lambda f(\boldsymbol{a}) \ \text{ s.t. } \ \|\boldsymbol{x}\|_2 = 1, \tag{12}$$

where $\boldsymbol{a}$ is a vector and λ is a real value. For various algorithms (e.g., those in [12, 14, 17, 47–50]), $f(\cdot)$, $\boldsymbol{a}$, and λ have different definitions.

Specifically, in [12], Boufounos and Baraniuk proposed a renormalized fixed point iteration (RFPI) algorithm with the definition $\boldsymbol{a} = [a_1, \ldots, a_M] = \mathbf{Y\Phi}\boldsymbol{x}$ and a cost function in the form of:

$$f(\boldsymbol{a}) = \|(\mathbf{Y\Phi}\boldsymbol{x})\_\|_2^2, \tag{13}$$

where $(t)_- = (t - |t|)/2$ is a function that only retrains negative values and $\mathbf{Y} = diag(\mathbf{y})$ denotes a matrix with the measurement signs in the main diagonal. In order to solve the problem in Eq. (13), [12] designed a gradient descent method. In each iteration, the estimation $\boldsymbol{x}$ is updated by exploiting the gradient of the cost function $f(\mathbf{Y\Phi}\boldsymbol{x})$ computed as $f(\mathbf{Y\Phi}\boldsymbol{x})' = -(\mathbf{Y\Phi})^{\mathrm{T}}(\mathbf{Y\Phi}\boldsymbol{x})_-$. Finally, RFPI updates estimated signals with a normalization step on the unit sphere as a significant constraint for one-bit CS recovery. A formal description of this algorithm is provided in Algorithm 1.

In [14], Laska et al. introduced a restricted step shrinkage (RSS) algorithm with the definition $\boldsymbol{a} = \boldsymbol{x}$ and a cost function $f(\boldsymbol{x})$ as follows:

$$f(\boldsymbol{x}) = \sum_{i=1}^{M} \rho((\mathbf{\Phi}\boldsymbol{x} - b), \lambda, \mu), \tag{14}$$

where ρ is a function in the form of:

$$\rho(t, \lambda, \mu) = \begin{cases} -\lambda t + \dfrac{1}{2}\mu t^2, \text{ if } t - \frac{\lambda}{\mu} \geqslant 0; \\ -\dfrac{1}{2\mu}\lambda^2, \quad \text{otherwise,} \end{cases} \tag{15}$$

where λ and μ are Lagrangian multipliers with different update methods. It is clear that $f(\boldsymbol{x})$ is a nonlinear and differentiable function. The procedure for the RSS algorithm is comprised of RSS-outer and RSS-inner operations. RSS-outer recovers sparse signals by using an augmented Lagrangian framework. The solution model illustrated in Eq. (12), which is a core intermediate step of RSS-outer, is solved by RSS-inner using a restricted step subroutine. Compared to the RFPI algorithm, the simulation results in [14] demonstrate that the RSS algorithm has three important characteristics: provable convergence, orders of magnitude speedup, and improved consistency and feasibility performance. The formal description of this algorithm is provided in Algorithm 2.

Algorithm 1 Renormalized fixed point iteration (RFPI) algorithm

Input: descent step size δ ,
measurement matrix $\mathbf{\Phi}$, sign measurement vector $\boldsymbol{y}$.
Output: the reconstructed signal $\hat{\boldsymbol{x}}_\ell$.
Initialize: $\ell = 0$, $\hat{\boldsymbol{x}}_0$ s.t. $\|\hat{\boldsymbol{x}}_0\|_2 = 1$.
while not converged **do**
One-sided quadratic gradient:
$\partial f_\ell = -(\mathbf{Y\Phi})^{\mathrm{T}}(\mathbf{Y\Phi}\hat{\boldsymbol{x}}_{\ell-1})_-$.
Gradient projection on sphere surface:
$\widetilde{f_\ell} = \partial f_\ell - \langle \partial f_\ell, \hat{\boldsymbol{x}}_{\ell-1}\rangle$.
One-sided quadratic gradient descent:
$\widetilde{\boldsymbol{x}}_\ell = \hat{\boldsymbol{x}}_{\ell-1} - \delta \widetilde{f_\ell}$.
Shrinkage ℓ_1 gradient descent:
$\boldsymbol{u}_\ell = \operatorname{sign}(\widetilde{\boldsymbol{x}}_\ell(i)) \max\left\{|\widetilde{\boldsymbol{x}}_\ell(i)| - \frac{\delta}{\lambda}, 0\right\}$, where i is the ith location of $\widetilde{\boldsymbol{x}}_\ell(i)$.
Normalization:
$\hat{\boldsymbol{x}}_\ell = \frac{\boldsymbol{u}_\ell}{\|\boldsymbol{u}_\ell\|_2}$.
end while

The above algorithms deal with noiseless cases. When a system suffers from noise, some of the signs may be flipped to the opposite state. In order to remedy sign flips, Movahed proposed a noise-adaptive renormalized fixed point iteration (NARFPI) algorithm that embeds detection technology into RFPI in [17]. Interestingly, NARFPI only needs to obtain prior knowledge of the total number of sign flips, but not prior knowledge of the sparsity level of the signal.

For the case where $\lambda = 0$, an ℓ_1 minimization algorithm was proposed in [50], which does not require knowledge of the sparsity level under noiseless conditions. In order to demonstrate the effectiveness of this convex optimization for the accurate recovery of a K-sparse signal $\boldsymbol{x}$, [50] provided a constant for the error bound.

Algorithm 2 Restricted step shrinkage (RSS) algorithm

Input: step size μ_0 and $\beta > 0$, measurement matrix $\mathbf{\Phi}$.

Output: the reconstructed signal $\hat{\boldsymbol{x}}_\ell$.

Initialize: $\ell = 0$, initial solution $\hat{\boldsymbol{x}}_0$, Lagrangian multiplier $\lambda_0 = 0$.

while (not converged) **do** (via RSS-outer)

  Initialize: $t = 0$, initial solution $\hat{\boldsymbol{x}}_0$, step size τ_0,

  set $0 < \theta_1 \leqslant \theta_2 < 1$ and $0 < \gamma_1 \leqslant \gamma_2 < 1 < \gamma_3$.

  while (not converged) **do** (via RSS-inner)

   Compute step:

   1. $z_t = \frac{\mathcal{S}_t(\tau_t \boldsymbol{x}_\ell - \mu_\ell \boldsymbol{g}_\ell, 1)}{\|\mathcal{S}_t(\tau_t \boldsymbol{x}_\ell - \mu_\ell \boldsymbol{g}_\ell, 1)\|_2}$, where $\mathcal{S}_t$ is a shrinkage operator defined as $\mathcal{S}_t(\alpha, Q) = \text{sign}(\alpha) \odot \max\{|\alpha| - Q, 0\}$, $\boldsymbol{g}$ is the gradient of $f(\boldsymbol{x})$.

   2. $r_t = \frac{\xi(\boldsymbol{x}_\ell) - \xi(z_t)}{\delta(\boldsymbol{x}_\ell, z_t)}$,

   where $\xi(\boldsymbol{x}) = \|\boldsymbol{x}\|_1 + f(\boldsymbol{x}_\ell)$, $f(\boldsymbol{x}) = \sum_{i=1}^{M} \rho((\mathbf{\Phi}\boldsymbol{x} - b), \lambda_\ell, \mu_\ell)$, $\delta(\boldsymbol{x}, z) = \|\boldsymbol{x}\|_1 - \|z\|_1 - \mu_\ell(\boldsymbol{g}_\ell)^{\mathrm{T}}(z_t - \boldsymbol{x}_\ell)$.

   Accept or reject the trial point:

    If $r_t \geqslant \theta_1$, $\boldsymbol{x}_{\ell+1} = z_t$

    else $\boldsymbol{x}_{\ell+1} = \boldsymbol{x}_\ell$

    End

   Update step-size τ

$$\tau_{t+1} = \begin{cases} [\gamma_1 \tau_t, \gamma_2 \tau_t], & \text{if } r_s \geqslant \theta_2; \\ [\gamma_2 \tau_t, \tau_t], & \text{if } r_s \in [\theta_1, \theta_2); \\ [\gamma_3 \tau_t, \tau_{\max}], & \text{if } r_s \leqslant \theta_1. \end{cases} \tag{16}$$

  end while

Update multiplier and μ

$$\lambda_{\ell+1} = \max\{\lambda_\ell - \mu_\ell(\mathbf{\Phi}\boldsymbol{x}_{\ell+1} - \boldsymbol{b}), 0\},$$

$$\mu_{\ell+1} = \beta \mu_\ell.$$

end while

Theorem 3 [50] Let $\mathbf{\Phi} \in \mathbb{R}^{M \times N}$ be a random matrix with independent standard normal entries. Set

$$\delta = C\Big(\frac{K}{M} \log(2N/K) \log(2N/M + 2M/N)\Big)^{1/5}, \tag{17}$$

Then, with a probability of at least $1 - C\exp(-g\delta M)$ for all K-sparse signals $\boldsymbol{x}$ satisfying $\|\boldsymbol{x}\|_1/\|\boldsymbol{x}\|_2 \leqslant \sqrt{K}$, the estimation $\hat{\boldsymbol{x}}$ of $\boldsymbol{x}$ obtained from $\boldsymbol{y} = \text{sign}(\mathbf{\Phi}\boldsymbol{x})$ satisfies:

$$\left\| \frac{\hat{\boldsymbol{x}}}{\|\hat{\boldsymbol{x}}\|_2} - \frac{\boldsymbol{x}}{\|\boldsymbol{x}\|_2} \right\|_2 \leqslant \delta, \tag{18}$$

for the following minimization program

$$\min \|\boldsymbol{x}\|_1, \quad \text{s.t. } \boldsymbol{y} = \text{sign}(\mathbf{\Phi}\boldsymbol{x}) \text{ and } \|\mathbf{\Phi}\boldsymbol{x}\|_1 = M, \tag{19}$$

where C and g are positive absolute constants.

Note that Theorem 3 does not consider the general energy constraint of $\|\boldsymbol{x}\|_2 = 1$. To avoid obtaining a solution of zero, the work [50] set a linear constraint $\|\mathbf{\Phi}\boldsymbol{x}\|_1 = M$. In addition to the error bound above, Yaniv Plan and Roman Vershynin proved that the proposed algorithm can accurately recover a K-sparse vector $\boldsymbol{x} \in \mathbb{R}^N$ from the signs of $\boldsymbol{O}(K \log^2(N/K))$ random linear measurements of $\boldsymbol{x}$ in [50].

3.2 Penalty-class algorithms

In order to enforce consistency, penalty-class algorithms such as [6, 10, 15, 16, 51–54] penalize sign violations by minimizing the p norm of the sign violations as follows:

$$\hat{\boldsymbol{x}} = \min_{\boldsymbol{x}} \|(\mathbf{Y\Phi}\boldsymbol{x})_-\|_p^q, \\ \text{s.t. } \|\boldsymbol{x}\|_2 = 1, \ \ \boldsymbol{x}^c = 0 \text{ and } \|\boldsymbol{x}\|_0 = K, \tag{20}$$

where $\boldsymbol{x}^c$ are the coefficients outside the support set. The cost function in Eq. (20) provides a measure of inconsistency. For the ideal case where reconstruction is exactly consistent, the penalty is zero.

In [15], Boufounos proposed a greedy algorithm called matching sign pursuit (MSP) that considers $p = 2$ and $q = 2$. The procedure for MSP is very similar to CoSaMP. It computes a proxy $\boldsymbol{s}_\ell = \mathbf{\Phi}^{\mathrm{T}}\mathbf{Y}\hat{\boldsymbol{x}}_{\ell-1}$ in the ℓ-th iteration and selects the $2K$ components with the largest magnitudes, where $\hat{\boldsymbol{x}}_{\ell-1}$ is the previous signal estimation. Then, MSP attempts to identify violated constraints during the refinement step by iterating a simple gradient descent method. A formal description of this algorithm is provided in Algorithm 3. Compared to ℓ_1 minimization on the unit ℓ_2 sphere, the main characters of MSP are that it always returns a sparse solution through reconstruction and facilitates the use of the ℓ_1 norm as a signal magnitude constraint instead of the ℓ_2 norm.

However, in [6], using a one-sided linear objective for the enforcement of consistency, the binary iterative hard threshold (BIHT) algorithm was proposed by Jacques et al. This algorithm is a simple modification of the IHT algorithm in [38]. In order to impose consistency in Eq. (20), the BIHT algorithm flexibly considered the ℓ_1 term ($p = q = 1$) for noiseless conditions and ℓ_2 term ($p = q = 2$) for noisy conditions. In the ℓ-th iteration, BIHT computes an estimate as follows:

$$\hat{\boldsymbol{x}}_\ell = \begin{cases} \eta_K(\hat{\boldsymbol{x}}_{\ell-1} + \dfrac{\lambda}{2}\mathbf{\Phi}^{\mathrm{T}}(\boldsymbol{y} - \text{sign}(\mathbf{\Phi}\hat{\boldsymbol{x}}_{\ell-1}))) \text{ for } \ell_1 \text{ term}, \\ \eta_K(\hat{\boldsymbol{x}}_{\ell-1} + \dfrac{\lambda}{2}(\mathbf{Y\Phi})^{\mathrm{T}}(\mathbf{Y\Phi}\hat{\boldsymbol{x}}_{\ell-1})_-) \quad \text{for } \ell_2 \text{ term}, \end{cases} \tag{21}$$

where $\eta_K(\cdot)$ is a hard threshold function that selects the K largest coefficients and λ is a constant used to control the gradient descent step size. A formal description of this algorithm is provided in Algorithm 4.

The simulation results of BIHT are presented from two viewpoints in [6]. First, compared to MSP and RSS, the simulation results demonstrate that BIHT has a lower bound on the best achievable reconstruction error for noise-free conditions. Second, Jacques et al. proposed a binary ϵ-stable embedding (BϵSE) property to characterize the robustness of the measurement process for flipped signs. The authors then considered the reconstruction robustness of measurement errors and noise. It was shown in [16] that the performance of BIHT is better than MSP [15] and RSS [14] in terms of reconstruction error and consistency.

Algorithm 3 Matching sign pursuit (MSP) algorithm

Input: sparsity level K , measurement matrix $\mathbf{\Phi}$, sign measurement $\boldsymbol{y}$.
Output: the reconstructed signal $\hat{\boldsymbol{x}}_\ell$.
Initialize: $\ell = 0, \Lambda_0 = \emptyset, \hat{\boldsymbol{x}}_0$ s.t. $\|\hat{\boldsymbol{x}}_0\|_2 = 1$.
while not converged **do**
Compute measurement estimates:
$\boldsymbol{y}_\ell = \mathrm{diag}(\boldsymbol{y})\mathbf{\Phi}\hat{\boldsymbol{x}}_{\ell-1}$.
Identify sign violations:
$\boldsymbol{r}_\ell = (\boldsymbol{y}_\ell)_-$,
where the function $(\cdot)_-$ preserves all negative elements and sets all positive elements to 0.
Compute correction signal proxy:
$\boldsymbol{s}_\ell = \mathbf{\Phi}^{\mathrm{T}}\mathrm{diag}(\boldsymbol{y})\boldsymbol{r}_\ell$.
Identify correction support:
$\Lambda_\ell = \mathrm{supp}(\boldsymbol{s}_\ell|_{2K}) \cup \mathrm{supp}(\hat{\boldsymbol{x}}_{\ell-1})$,
where $\boldsymbol{s}_\ell|_{2K}$ indicates selecting the $2K$ components with the largest magnitudes of $\boldsymbol{s}_\ell$.
Perform consistent reconstruction over chosen support:
$\boldsymbol{u}_\ell|_{\Lambda_\ell} = \arg\min_{\boldsymbol{x}} \|(\mathrm{diag}(\boldsymbol{y})\mathbf{\Phi}\boldsymbol{x})_-\|_2^2$ s.t. $\|\boldsymbol{x}\|_2 = 1$ and $\boldsymbol{x}|_{\Lambda_\ell^c}$.
Optimization is only over the support Λ_ℓ. This (also non-convex) optimization is performed by iterating a gradient descent step followed by a normalization step until convergence.
Truncation and Normalization:
$\hat{\boldsymbol{x}}_\ell = \frac{\boldsymbol{u}_{\ell|K}}{\|\boldsymbol{u}_{\ell|K}\|_2}$,
where $\boldsymbol{u}_{\ell|K}$ indicates selecting the K components with the largest magnitudes of $\boldsymbol{u}_\ell|_{\Lambda_\ell}$.
end while

Algorithm 4 Binary iterative hard threshold (BIHT) algorithm

Input: sparsity level L ,
measurement matrix $\mathbf{\Phi}$, sign measurement vector $\boldsymbol{y}$.
Output: the reconstructed signal $\hat{\boldsymbol{x}}_\ell$.
Initialize: $\ell = 0, \Lambda^0 = \emptyset, \hat{\boldsymbol{x}}_0$ s.t. $\|\hat{\boldsymbol{x}}_0\|_2 = 1$.
while not converged **do**
One-sided quadratic gradient:
$\partial f_\ell = \frac{1}{2}(\mathbf{Y\Phi})^{\mathrm{T}}(\mathbf{Y\Phi}\hat{\boldsymbol{x}}_{\ell-1})_-)$, for ℓ_2 term;
$\partial f_\ell = \frac{1}{2}\mathbf{\Phi}^{\mathrm{T}}(\boldsymbol{y} - \mathrm{sign}(\mathbf{\Phi}\hat{\boldsymbol{x}}_{\ell-1}))$, for ℓ_1 term.
Compute and update:
$\boldsymbol{u}_\ell = \eta_K(\hat{\boldsymbol{x}}_{\ell-1} + \partial f_\ell)$.
Normalization:
$\hat{\boldsymbol{x}}_\ell = \frac{\boldsymbol{u}_\ell}{\|\boldsymbol{u}_\ell\|_2}$.
end while

Because it is very difficult to detect impulse noise and sign flips, the sign-flipped measurements will degrade reconstruction performance. Yan et al. [16] utilized the concept from [55] to adaptively find sign flips. As a modified version of BIHT, the adaptive outlier pursuit (AOP) algorithm was proposed. By considering the noisy signal model $\boldsymbol{y} = \mathrm{sign}(\mathbf{\Phi}\boldsymbol{x} + \boldsymbol{n})$, AOP considers $p = q = 1$ and changes the solution model (Eq. (20)) into:

$$\begin{aligned} \min_{\boldsymbol{x},\mathbf{\Lambda}} \quad & \|\mathbf{\Lambda}[\mathbf{Y} \odot (\mathbf{\Phi}\boldsymbol{x})]_-\|_1, \\ \text{s.t.} \quad & \|\boldsymbol{x}\|_2 = 1, \ \ \|\boldsymbol{x}\|_0 = K, \\ & \sum_{i=1}^{M}(1 - \Lambda_i) \leqslant L, \\ & \Lambda_i \in \{0, 1\} \ \ i = 1, 2, \ldots, M, \end{aligned} \tag{22}$$

where $\mathbf{\Lambda} = [\Lambda_1, \Lambda_2, \ldots, \Lambda_M]$ denotes the correct data (i.e., $\Lambda_i = 1$ if y_i is correct and zero if it is incorrect). L is a proper integer such that at most L elements of the total measurements are incorrectly detected (have sign flips). $\boldsymbol{n} = [n_1, n_2, \ldots, n_M]^{\mathrm{T}}$ is the noise vector. Because it is difficult to find $(\boldsymbol{x}, \Lambda)$ together, [16] used an alternative minimization method to separate the energy minimization over $\boldsymbol{x}$ and Λ into two steps.

In conclusion, the above two basic penalty-class algorithms demonstrate that the core concept of this type of algorithm is to measure consistency using a penalty function. However, there are some variants based on other penalty schemes. For example, based on BIHT, North and Needell [51] embeded partial support set prior information into BIHT algorithm and Fu et al. [52] designed an algorithm that can adaptively detect the locations of sign flips. By exploiting the core concept of support vector machines, X. Huang et al. designed a mixed loss function and proposed a pinball loss iterative hard thresholding method for one-bit CS recovery in [54].

3.3 Bayesian compressed sensing frameworks

In [18, 19], robust one-bit Bayesian compressed sensing (BCS) frameworks for addressing sign-flip errors were proposed by Yang et al. and Li et al. Specifically, sign-flip errors were modeled as a result of corrupted unquantized observations in a sparse noise vector. In [19], a Gaussian-inverse-Gamma hierarchical prior was assigned to the noise vector to encourage sparsity. For $\mathbf{y} = \mathrm{sign}(\mathbf{\Phi x} + \mathbf{n})$, the Gaussian-inverse-Gamma hierarchical priors were assigned to both $\mathbf{x}$ and $\mathbf{n}$ as follows:

$$\begin{aligned} p(\mathbf{x}|\boldsymbol{\alpha}) &= \prod_{i=1}^{N} \mathcal{N}\left(x_i|0, \alpha_i^{-1}\right), \\ p(\boldsymbol{\alpha}) &= \prod_{i=1}^{N} \mathrm{Gamma}(\alpha_i|a, b), \\ p(\mathbf{n}|\boldsymbol{\beta}) &= \prod_{i=1}^{N} \mathcal{N}\left(n_i|0, \beta_i^{-1}\right), \\ p(\boldsymbol{\beta}) &= \prod_{i=1}^{N} \mathrm{Gamma}(\beta_i|c, d), \end{aligned} \tag{23}$$

where $\mathcal{N}\left(x|\mu,\sigma^2\right)$ represents a Gaussian distribution with mean μ and variance σ^2. In order to provide non-informative hyperpriors over $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, the Gamma distribution parameters a, b, c, and d are set to be very small values. A broad hyperprior allows the posterior means of α_i and β_i to become arbitrarily large, which corresponds to the zero values of x_i and n_i [56].

Based on this model, Li et al. [19] developed a variational expectation-maximization (EM) algorithm that simultaneously identifies bit-flip errors and recovers sparse signals. Thus, for obtaining a sparse solution, the main objective is to find an approximation of the posterior distribution by maximizing $L(q,\delta)$ as follows:

$$L(q,\delta) = \int q(\boldsymbol{\theta}) \ln \frac{G(\mathbf{y},\boldsymbol{\theta},\delta)}{q(\boldsymbol{\theta})} d\boldsymbol{\theta},$$

where $G(\mathbf{y},\boldsymbol{\theta},\delta) = F(\mathbf{y},\mathbf{x},\mathbf{n},\delta)\, p(\mathbf{x}|\boldsymbol{\alpha})\, p(\boldsymbol{\alpha})\, p(\mathbf{n}|\boldsymbol{\beta})\, p(\boldsymbol{\beta})$, $F(\mathbf{y},\mathbf{x},\mathbf{n},\delta) = \prod_{i=1}^{M} \sigma(\delta_i) \exp\left(\frac{z_i-\delta_i}{2} - \lambda(\delta_i)\left(z_i^2-\delta_i^2\right)\right)$, and $q(\boldsymbol{\theta}) = q_{\mathbf{x}}(\mathbf{x})\, q_{\alpha}(\boldsymbol{\alpha})\, q_{\mathbf{n}}(\mathbf{n})\, q_{\beta}(\boldsymbol{\beta})$. The work [19] used the variational EM algorithm to maximize $L(q,\delta)$ by updating $q_{\mathbf{x}}(\mathbf{x})$, $q_{\alpha}(\boldsymbol{\alpha})$, $q_{\mathbf{n}}(\mathbf{n})$, and $q_{\beta}(\boldsymbol{\beta})$ in the E-step and update the parameters $\boldsymbol{\delta}$ in the M-step.

In contrast to requiring prior knowledge of the total number of sign flips in AOP and NAFRPI, such information is not required for this proposed algorithm.

3.4 Others

In addition to the algorithms mentioned above, there are some one-bit CS recovery algorithms that are not as well-known [7, 8, 57]. Based on convex programming and hard thresholding, Baraniuk et al. [7] proposed two algorithms to achieve an exponential decay rate for reconstruction error by exploiting an adaptive quantization step. Considering a scenario with no prior information regarding the norm of $\boldsymbol{x}$, Knudson et al. proposed two one-bit CS algorithms in [8]. Chen and Banerjee [57] proposed a closed-form solution by using a K-support norm.

3.5 Extended reconstruction algorithms for special scenarios

Based on one-bit CS, there are some extended reconstruction algorithms designed to achieve better performance in special scenarios.

3.5.1 Hamming compressed sensing

Because quantization is a coarse and irreversible description of the original signal, the basic idea of recovering quantization values is similar to that of recovering a box constraint. Therefore, it is possible for one-bit Hamming compressed sensing (HCS) to recover the quantization of a dense signal from a small number of sampling bits [20, 58]. The core idea of one-bit HCS is that each dimension of the signal can be recovered from a corresponding Bernoulli distribution.

Using the method in [20] as an example, the authors developed a one-bit HCS method to recover the quantized signal from its one-bit measurements very quickly and without a signal sparsity constraint. For a normalized signal $\boldsymbol{x} \in \mathbb{R}^N$ with $\|\boldsymbol{x}\|_2 = 1$ and a normalized Gaussian random matrix $\boldsymbol{\Phi}$, Zhou and Tao [20] proved that there is a bijection between each dimension of the signal and a Bernoulli distribution $\boldsymbol{P}(x_i)$ through the following Theorem.

Theorem 4 (Theorem 1 of [20]) For $\boldsymbol{x} \in \sum_K^*$ and a normalized Gaussian random vector $\boldsymbol{\phi}$ that is drawn uniformly from the unit ℓ_2 sphere in $\mathbb{R}^N$, given the ith dimension of the signal $\boldsymbol{x}_i$ and corresponding coordinate unit vector $\boldsymbol{e}_i$ with its ith entry being 1, there exists a bijection $P : \mathbb{R} \longrightarrow \mathbb{P}$ from $\boldsymbol{x}_i$ to the Bernoulli distribution of the binary random variable $u_i = \mathrm{sign}(\langle \boldsymbol{x},\boldsymbol{\phi}\rangle) \cdot \mathrm{sign}(\langle \boldsymbol{e}_i,\boldsymbol{\phi}\rangle)$:

$$P(x_i) = \begin{cases} \Pr(u_i = -1) = \dfrac{1}{\pi} \arg\cos(\boldsymbol{x}_i), \\ \Pr(u_i = 1) = 1 - \dfrac{1}{\pi} \arg\cos(\boldsymbol{x}_i). \end{cases} \tag{24}$$

By defining the interval Δ and boundaries x_{inf}, x_{sup} for quantization in the signal domain, one-bit HCS obtains q corresponding boundaries:

$$P_i = \begin{cases} P_i^- = \Pr(-1) = \dfrac{1}{\pi} \arg\cos(x_{inf}) - i\Delta, \\ P_i^+ = \Pr(1) = 1 - \Pr(-1), \end{cases} \tag{25}$$

where the assumption is that $-1 < x_{inf} \leqslant x_i \leqslant x_{sup} < 1$ and $\Delta = \frac{1}{q-1}\left[\frac{1}{\pi}\arccos\left(x_{inf}\right) - \frac{1}{\pi}\arccos\left(x_{sup}\right)\right]$.

As shown in Fig. 2, the Bernoulli distribution $P(x_i)$ can be estimated as $\hat{P}(x_i)$ from one-bit measurements $\boldsymbol{y}$ as follows:

$$\hat{P}(x_i) = \begin{cases} \hat{P}(x_i)^- = |j : [\mathbf{y}_j \cdot (\mathrm{sign}(\boldsymbol{\Phi}_i))_j] = -1|/M, \\ \hat{P}(x_i)^+ = 1 - \hat{P}(x_i)^-, \end{cases} \tag{26}$$

where $\boldsymbol{\Phi}_i$ is the ith column of the measurement matrix $\boldsymbol{\Phi}$ and $\boldsymbol{y}_j$ is the jth element of $\boldsymbol{y}$.

Then, in the Bernoulli distribution domain, the distance between P_j and $\hat{P}(x_i)$ can be obtained by computing the Kullback-Leibler (KL) divergence as follows:

$$D_{KL}\left(P_j \| \hat{P}(x_i)\right) = P_j^- \log \frac{P_j^-}{\hat{P}(x_i)^-} + P_j^+ \log \frac{P_j^+}{\hat{P}(x_i)^+}. \tag{27}$$

HCS searches for the nearest neighbor of $\hat{P}(x_i)$ among the boundaries P_j by minimizing the distance between P_j and

$\hat{P}(x_i)$. Next, one-bit HCS obtains an interval index q^* as follows:

$$q^* = 1 + \arg\min_j D_{KL}\left(P_j \| \hat{P}(x_i)\right), \tag{28}$$

HCS then recovers the ith element of $\boldsymbol{x}$ that satisfies $S_{q^*-1} \leqslant x_i \leqslant S_{q^*}$ with S from Eq. (29) such that:

$$S_{q^*} = \begin{cases} x_{inf}, & q^* = 0; \\ \cos\left(\frac{\pi}{1+f(P^-_{q^*})}\right), & q^* = 1, 2, \ldots, q-1; \\ x_{sup}, & q^* = q, \end{cases} \tag{29}$$

where

$$f\left(P^-_{q^*}\right) = \left(\frac{\left(P^-_{q^*}\right)^{\left(P^-_{q^*}\right)}\left(1 - P^-_{q^*}\right)^{\left(1-P^-_{q^*}\right)}}{\left(P^-_{q^*-1}\right)^{\left(P^-_{q^*-1}\right)}\left(1 - P^-_{q^*-1}\right)^{\left(1-P^-_{q^*-1}\right)}}\right).$$

S_{q^*} is clearly a function of $P^-_{q^*-1}$ and $P^-_{q^*}$,

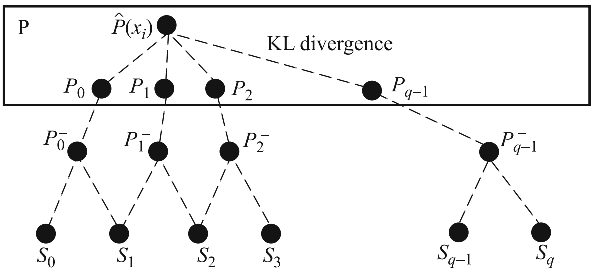

Fig. 2 Quantized recovery in one-bit HCS ($\hat{P}(x_i)$ from Theorem 1 is the estimate of the Bernoulli distribution $P(x_i)$ that can be obtained from Eq. (26). P_i from Eq. (25) is the nearest neighbor of $\hat{P}(x_i)$ among the q boundaries in the Bernoulli distribution domain. S_i from Eq. (29) is a mapping of P_{i-1} and P_i in the signal domain. The one-bit HCS quantization of x_i is recovered as the interval between the two boundaries S_{i-1} and S_i corresponding to the nearest neighbor. The disks represent the boundary values and the dashed lines represent a function mapping among P_i, $\hat{P}(x_i)$, and S_i from Eqs. (25), (26), and (29))

The core procedure of HCS is summarized in Fig. 3.

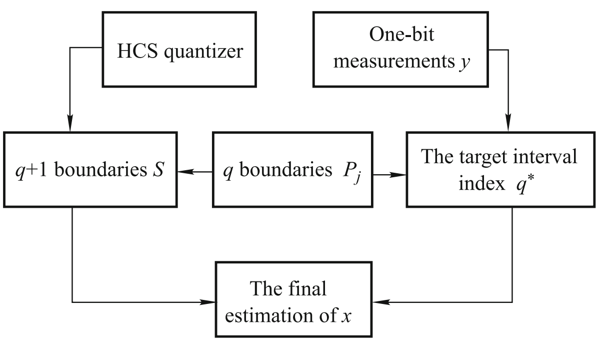

Fig. 3 Core procedure of one-bit HCS

Compared to CS and one-bit CS, the main advantages of one-bit HCS are as follows. First, one-bit HCS makes the recovery processing of a quantized signal for digital systems much more simple. Second, for the one-bit recovery of N-dimensional signals, one-bit HCS only requires the computation of KL divergences. This means that one-bit HCS requires considerably less recovery time and substantially fewer measurements. Therefore, one-bit HCS is more efficient than CS and one-bit CS.

3.5.2 Distributed one-bit compressed sensing

Distributed compressed sensing (DCS) [59, 60] is an extension of CS that takes advantage of both inter- and intra-signal correlations. It is widely used as a powerful method for multi-signal sensing and compression in many fields.

Considering a type of joint sparsity model where each signal contains a common component and innovation component, we proposed a joint one-bit reconstruction algorithm based on BIHT by iteratively deriving the sign information of each component for DCS in [21, 61]. The J signal nodes are denoted as $\{\mathbf{x}_j = \mathbf{x}_{j,co} + \mathbf{x}_{j,in}, \mathbf{x}_j \in \mathbb{R}^N\}$, $j \in \{1, 2, \ldots, J\}$, where $\mathbf{x}_{j,co}$ and $\mathbf{x}_{j,in}$ are the K_{co}-sparse common component and K_{in}-sparse innovation component, respectively.

The algorithm proposed in [21] consists of two main parts that correspond to the estimations of $\mathbf{x}_{j,co}$ and $\mathbf{x}_{j,in}$, respectively. On one hand, assume that $\mathbf{x}_{j,co}$ is available. Then, the estimation of $\mathbf{x}_{j,in}$ can be accomplished using the following solution model:

$$\begin{aligned} \hat{\mathbf{x}}_{j,in} = \arg\min_{\mathbf{x}_{j,in}} & \left\|\left(\mathbf{y}_j \odot \mathbf{\Phi}\left(\mathbf{x}_{j,co} + \mathbf{x}_{j,in}\right)\right)_-\right\|_1, \\ \text{s.t. } & \left\|\mathbf{x}_{j,in}\right\|_0 = K_{in}, \ \left\|\mathbf{x}_{j,in}\right\|_2 = 1. \end{aligned} \tag{30}$$

On the other hand, assume $\mathbf{x}_{j,in}$ is available. Then, the estimation of $\mathbf{x}_{j,co}$ can be accomplished using the following solution model:

$$\begin{aligned} \hat{\mathbf{x}}_{j,co} = \arg\min_{z_{j,co}} & \left\|\left(\sum_{j=1}^{J} \mathbf{y}_j \odot \left(J\mathbf{\Phi}\mathbf{x}_{j,co} + J\sum_{j=1}^{J} \mathbf{\Phi}\mathbf{x}_{j,in}\right)\right)_-\right\|_1. \\ \text{s.t. } & \left\|\mathbf{x}_{j,co}\right\|_0 = K_{co}, \ \left\|\mathbf{x}_{j,co}\right\|_2 = 1, \end{aligned} \tag{31}$$

To illustrate the performance of the proposed scheme, we present the results of its application to compressed spectrum sensing [62]. The simulation results in Fig. 4 demonstrate that under conditions with the same numbers of quantization bits, one-bit DCS can recover signals more efficiently than multi-bit DCS algorithms. The performance of the proposed one-bit DCS scheme is superior to that of the sensing scheme based on multi-bit quantization at low signal-to-noise ratios.

4 Applications

Because of the many merits of one-bit CS, it has been applied to many fields. In this section, we describe its applications to wireless sensor networks (WSNs), cognitive radio, and biosignal processing.

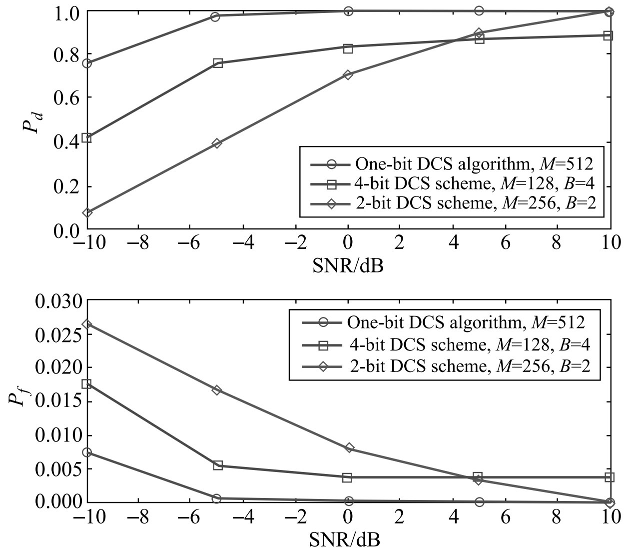

Fig. 4 Simulation results of one-bit DCS from [21] (We define the detection probability P_d as the correct proportion of the recovery set and the alarm probability P_f as the incorrect proportion of the recovery set. M is the number of measurements and B is the number of quantization bits). (a) P_d; (b) P_f

4.1 Wireless sensor networks

Recently, many one-bit CS frameworks for WSNs have been proposed. WSNs, which are spatially distributed networks consisting of many inexpensive and small wireless sensors, represent an efficient framework to monitor physical environments. Bandwidth and energy are severely constrained for sensors in WSNs, meaning one-bit CS, which requires very little data, is very attractive for WSNs.

In terms of data gathering in WSNs, each sensor node sends an extra norm value of $\boldsymbol{x}$ to the base station to recover missing signal amplitudes. Xiong et al. [22] applied one-bit CS to data gathering and designed a blind one-bit CS algorithm called blind BIHT. To implement source localization, which is a significant operation in WSNs, one-bit CS can be applied to very limited bandwidth and energy budgets by utilizing as few data bits as possible [23, 24]. Based on one-bit CS, Shen et al. [23] studied the problem of source localization in WSNs. Because the locations of non-zero components have practical physical implications, the support set of sparse signals is of the utmost concern in source localization.

Later, by employing a data processing protocol, Chen and Wu [25] applied one-bit CS to noisy WSNs suffering from sign flips and proposed an amplitude-aided signal reconstruction scheme. In addition to the above applications, one-bit CS was applied to other scenarios (e.g., environment monitoring and sparse event detection [26, 27]).

Note that most studies on WSNs, expect [25], considered noiseless conditions. Furthermore, all of the above schemes are based on single nodes and do not exploit the correlations between the nodes in WSNs.

4.2 Cognitive radio

The available electromagnetic radio spectrum is a limited and expensive natural resource. Because of the rapidly increasing number of wireless devices and applications, the radio spectrum becomes increasingly crowded every day. Current static frequency allocation schemes cannot satisfy the requirements of an increasing number of high data rate devices. By introducing opportunistic usage of the spectrum that is not heavily occupied by licensed users, cognitive radio (CR) aims to facilitate an efficient method of exploiting the available spectrum. In CR, it is a key problem to identify the occupied spectrum. One-bit CS techniques can be utilized in CR to acquire a sensed spectrum from extracted sign data.

In [28], Lee et al. analyzed the trade-off between calculation cost and compression performance. By using one-bit CS technology, they proved that the communication costs for spectrum sensing in a networked system can be minimized. Additionally, they also proposed a block reconstruction algorithm for one-bit CS that uses the block sparsity of signals.

CR devices typically need to monitor a wide range within the available spectrum. According to the Nyquist theory, the sampling rates of devices are a major challenge for hardware implementations. One-bit CS has significant advantages in terms of reducing sample load. Based on CS, a modulated wideband converter (MWC) can sample sparse multiband signals at a sub-Nyquist rate. Fu et al. [29] proposed a sub-Nyquist one-bit sampling system for sparse multiband signals by using low-pass filtering, sampling, and a one-bit quantizer. The primary design goals were efficient hardware implementation and a low bit-budget.

4.3 Radar

In typical synthetic aperture radar (SAR) systems, the received signals are encoded on board and decoded at a ground station. To save on the costs of data transmission, SAR raw data are typically encoded with a low number of bits. The general process for such a system closely follows that of one-bit CS.

Although one-bit quantization also has several disadvantages, such as ghost targets in high signal-to-noise ratio (SNR) situations [30] and image degradation in low SNR situations [31], there are several advantages to applying one-bit quantization to SAR systems. First, it is easy to produce a high speed one-bit hardware quantizer using a simple comparator. Second, it is robust to nonlinear distortion, saturation, and other errors. Finally, it is possible to facilitate real-time processing by using one-bit quantized data.

The work [32] concentrated on studying algorithms for one-bit SAR imaging without low-SNR and oversampling limitations. The authors preserved only the sign information of the I and Q channel signals and proposed a one-bit CS approach for SAR imaging of sparse scenes using one-bit quantized data. Within their framework of maximum a posteriori estimation, the SAR image reconstruction problem was formulated as a sparse optimization problem and solved by using a first-order primal-dual algorithm.

Current schemes do not consider sparse structures as prior information. Images can be compressed effectively using a wavelet tool and the wavelet coefficients of images can be efficiently organized as a tree structure, which can be considered as prior information to improve overall performance [63, 64].

4.4 Bio-signal processing

Technology for continuously monitoring personal health is growing rapidly because of the integration, cost, and form factor advantages of nanometer CMOS technology. The amount of data transmitted to smart-phones is increasing as additional sensors are added to body area networks. Thus, the energy consumed by RF transmission, which is proportional to the data rate, is also increasing. Reducing the number of sampling bits has the noteworthy advantages in terms of hardware complexity, storage capacity, power consumption, and channel bandwidth. Furthermore, one-bit CS technology can significantly reduce the number of sampling bits. Therefore, in order to design practical hardware implementations, one-bit CS is an attractive option for bio-signal processing.

It is well known that electrocardiogram (ECG), electromyogram (EMG), and electroencephalogram (EEG) bio-signals are sparse signals. ECG is time sparse and EMG is sparse in both the time and frequency domains. Under signal-to-quantization noise ratios (SQNR) > 60 dB, Allstot et al. [65] proved that compression factors of up to 16 times are achievable using uniform or Gaussian 6-bit random coefficients for ECG and EMG signals. In [33], one-bit random coefficients were shown to provide compression factors up to 16 times with similar SQNR performance. One-bit CS reduces hardware stress and saves energy compared to 6-bit signal processing.

However, the works [33] and [65] focused on the bits of the measurements matrix. In [34], Haboba et al. presented a novel sampling architecture called an analog-to-information converter (AIC). The proposed architecture is illustrated in Fig. 5. Based on the Sigma-Delta (Σ/Δ) modulator studied in [66], the proposed AIC produces a stream of one-bit compressed measurements, where compression refers to the total number of bits used to represent the signal. As shown in Fig. 5(a), the input signal $\boldsymbol{x}(t)$ is first multiplied to obtain a discrete sampling sequence $\boldsymbol{y}[n]$ using the antipodal signal $\boldsymbol{p}(t)$ created from a pulse amplitude modulated (PAM) sequence. Then, $\boldsymbol{y}[n]$ is modulated using a Σ/Δ modulator to output a single bit sequence $\boldsymbol{z}[n]$. Figure 5(b) shows the decoding and recovery scheme. The downsampled measurement $\boldsymbol{z}_d[n]$ is obtained from the one-bit measurements $\boldsymbol{z}[n]$ using a low-pass filter and downsampling operator. Finally, the sparse signal $\boldsymbol{x}[n/M]$ is recovered from $\boldsymbol{z}_d[n]$ using a classical greedy pursuit algorithm.

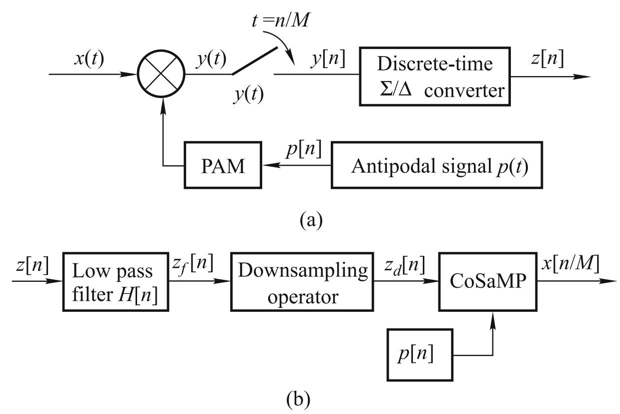

Fig. 5 Proposed one-bit CS architecture for bio-signal processing in [34]. (a) One-bit coding scheme; (b) one-bit decoding and reconstruction scheme

4.5 Others

In addition to the main applications introduced above, one-bit CS has been used in many other application fields. One-bit CS has also been applied to source and channel coding [67], edge detection [68], MIMO channel [69], capacitive touch sensing [70], direction-of-arrival estimation [71], and sparse classification [72].

5 Conclusion and future research

5.1 Conclusion

In this study, we investigated the theories and applications of one-bit CS. The quantized measurements of one-bit CS are treated as sign information and do not provide any amplitude information. In order to recover signals from sign measurements, most one-bit algorithms employ the energy constraint of an ℓ_2 norm unit sphere. One-bit CS is most suitable for applications where the locations of non-zeros have physical significance in sparse signals. It is reasonable to conclude that one-bit CS is a topic that will remain an important research field in coming years and see significant further progress and development. This survey provided a snapshot of significant studies in the area of one-bit CS.

5.2 Future research

The following subsections summarize some possible future research directions.

5.2.1 New reconstruction algorithms

By taking advantage of the sign information of measurements and the energy constraint of an ℓ_2 norm unit sphere, scholars have proposed a series of one-bit CS reconstruction algorithms. Many simulation results from existing algorithms, such as BIHT, RFPI, AOP, and one-bit BCS require a very large number of measurements ($M \geqslant N$) to achieve excellent performance. Additionally, for traditional CS, the relationship between the number of measurements M, sparsity K, and the signal dimension N is a significant research topic [73–76]. Therefore, inspired by studies on CS, it is of great interest to study similar relationships in one-bit CS and propose new algorithms that can derive excellent reconstruction results with ($M << N$). During such research, the missing amplitude information of sparse signals will be a major challenge to overcome.

Furthermore, there are some conventional problems, such as sign flipping under noisy conditions and perturbation of measurement matrices that have not been optimally solved. Therefore, new approaches are needed to solve these problems.

Although most algorithms have excellent performance, they do not make full use of certain special sparse structures or prior information of certain signals (e.g., tree structures and block sparse structures [77–81]).

5.2.2 Analysis of necessary and sufficient conditions

For classical CS, based on RIP from Eq. (2), scholars have obtained numerous results regarding the necessary and sufficient conditions for different cases [82–87]. However, similar studies on one-bit CS are very scarce. With the continuous development of one-bit CS, the relationship between the RIP constant δ_K and sparsity K or number of measurements M to ensure exact recovery from one-bit measurements is an important research topic. From the mathematical point of view, one major obstacle is the sign operation, which may be overcome by approximating such an operation using various nonlinear functions.

5.2.3 Application prospects

Because of its low complexity and low computational cost, one-bit CS has been efficiently applied to many applications. Implementing one-bit CS in fields such as big data analysis, target location, and cloud equipment monitoring will become a significant research goal in the future. One-bit CS is especially suitable for WSNs. With the development of one-bit DCS and WSN, the utilization of the temporal and spatial correlations between sensors will also be an important research topic.

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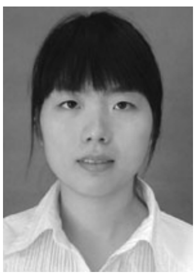

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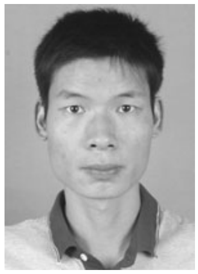

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