Test and Roll Package Vignette

Based off of “Profit-maximizing a/b tests’ ’ by Elea McDonell Feit and Ron Berman.

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1. Introduction

1.1 About Test & Roll

The Test & Roll Package implements the methods in the paper “Profit-maximizing a/b tests” by Elea McDonnell Feit and Ron Berman.
Source: Feit, E. M. & Berman, R. (2018). Profit-maximizing a/b tests. Available at SSRN.

Test & Roll (tactical A/B testing) describes an experimentation process in marketing where a subset of customers are randomly assigned to a treatment and customer response data is first collected in a “test” stage. In the “roll” stage that follows, marketers deploy one treatment to all remaining customers based on the test results.
Marketers often use A/B testing as a tool to compare marketing treatments in a test stage and then deploy the better-performing treatment to the remainder of the consumer population. While these tests have traditionally been analyzed using hypothesis testing, we re-frame them as an explicit trade-off between the opportunity cost of the test (where some customers receive a sub-optimal treatment) and the potential losses associated with deploying a sub-optimal treatment to the remainder of the population.
Profit-maximizing Test & Roll has several advantages over traditional hypothesis testing (see paper for more).
(1) The typical significance levels (alpha) in hypothesis tests which aim to limit Type I errors have little consequence for profit, assuming no deployment costs. If the null is mistakenly rejected, but both treatments yield identical effects, the same profit will be earned regardless of which treatment is deployed. Because of the profit trade-off between test-stage learning and roll-stage earning, conservative sample sizes based on null hypothesis testing lower overall expected profit, by exposing too many people to the less effective treatment in the test.
(2) The population available for testing and deploying is often limited, but the recommended sample size does not take this constraint into account. In online advertising experiments where effects are often small (but profitable), the recommended sample size may be larger than the size of the population itself. Yet when the population is limited, smaller tests that will never reach statistical significance can still have substantial benefit in improving expected profit. With profit-maximizing test and roll, improved performance is achieved because profit-maximizing tests identify the best performing treatment with high probability when treatment effects are large; the lost profit (regret) from errors in treatment selection is small when treatment effects are small.
The Test & Roll package can be used to calculate test sizes as well as resulting profits under a variety of methods (hypothesis testing, profit-maximization, and Thomson Sampling). Note that the profit-maximizing test size is substantially smaller than typically recommended for a hypothesis test, particularly when the response is noisy or when the total population is small.

1.2 Installation

```
library(devtools)
install_github('aronbolun/testandroll.pkg')
library(testandroll.pkg)
#Extra installations for Rmd to knit
install.packages('doParallel', repos='http://cran.us.r-project.org')
```

```
##
## The downloaded binary packages are in
##  /var/folders/49/6l50y03s1rj1g1yd2h6jz2lc0000gn/T//Rtmpzu0AVC/downloaded_packages
```

```
library(doParallel)
registerDoParallel(cores=4) #Determine the the number of cores to use for parallel execution (e.g., 4)
```

1.3 Computing priors from data

Basic method:
Priors represent beliefs about your data before you conduct the experiment.
The testandroll package contains sample data of the proportions of customers who visited a website for longer than 15 minutes. months.means is a vector of length 12 containing the average proportions for each month. To find "μ," simply take the means of the mean responses from the data sets and to find "σ," simply take the standard deviation of the mean responses.

```
months.means
```

```
##   January  February     March     April       May      June      July    August
## 0.6103005 0.4089727 0.5021437 0.6009944 0.6053015 0.5904040 0.6090105 0.5090350
## September   October  November  December
## 0.4078861 0.4967330 0.5061300 0.6154330
```

```
mu <- mean(months.means)
sigma <- sd(months.means)
mu
```

```
## [1] 0.5385287
```

```
sigma
```

```
## [1] 0.077417
```

1.4 Summary of functions

Function parameters:
"n" - sample sizes
"N" - total deployment population
"s" - the known standard deviations of the outcome
"μ" - means of the priors on the response
"σ" - standard deviations of the priors on the response
"K" - number of arms (treatments)
"R" - number of simulation repetitions
"n_{vals}" - possible sample size values

Function purposes:
Usage for 2-arm tests
test_size_nht() - computes test sizes under hypothesis testing
test_size_nn() - computes profit-maximizing test sizes under test & roll
profit_nn() - computes profit under profit-maximizing tests

profit_perfect_nn() - computes profit with perfect information
error_rate_nn() - computes rate of incorrect deployments

Usage for K-arm tests (with simulation)
test_size_nn_sim() - computes profit-maximizing test sizes for a multi-armed test and roll
one_rep_test_size() - utility function used in test_size_nn_sim() to simulate one set of potential outcomes
profit_nn_sim() - computes the per-customer profit for test & roll with K arms
one_rep_profit() - utility function used in function 'profit_nn_sim()' to simulate one set of potential outcomes
test_eval_nn() - provides summary statistics of a test and roll plan

Section 4) Visualizing prior effects on profits and mean response
plot_prior_mean_resp_nn() - plot prior densities against mean response (profit per customer)
plot_prior_resp_nn() - plot prior densities against response (profit per customer)
plot_prior_effect_nn() - plot prior densities against treatment effect (difference in profit per customer)

2. Usage for 2-arm tests

2.1 Computing profit-maximizing test sizes (2-arm tests)

test_size_nht() computes test sizes under hypothesis testing.

```
test_size_nht(s=c(0.5,0.10), d=0.2, conf=0.95, power=0.8, N=NULL)
```

```
## [1] 58.86660 11.77332
```

```
test_size_nht(s=0.5, d=0.2, conf=0.95, power=0.8, N=NULL)
```

```
## [1] 98.111
```

Takes in parameters:
"s" - vector of length 1 (symmetric) or 2 (asymmetric) indicating response standard deviation(s)
"d" - minimum detectable difference between treatments
"conf" - 1 - type I error rate
"power" - 1 - type II error rate
"N" - finite deployment population, if NULL no finite population correction is used
and returns a vector containing the recommended sample sizes

For the symmetric case $s_1 = s_2 = s$, the recommended sample size is calculated using $n_1 = n_2 \approx (z_{1-\alpha/2} + z_\beta)^2(\frac{2s^2}{d^2})$ where α is 1-conf and β is the power. For the asymmetric case $s_1 \neq s_2$, the recommended sample sizes are calculated using $n_1 = (z_{1-\alpha/2} + z_\beta)^2(\frac{{s_1}^2 + s_1 s_2}{d^2})$ and $n_2 = (z_{1-\alpha/2} + z_\beta)^2(\frac{s_1 s_2 + {s_2}^2}{d^2})$. With finite population correction, $n_1 = \frac{(z_{1-\alpha/2}+z_\beta)^2(N)(s_1^2+s_1 s_2)}{d^2(N-1)+(z_{1-\alpha/2}+z_\beta)^2(s_1+s_2)^2}$ and $n_1 = \frac{(z_{1-\alpha/2}+z_\beta)^2(N)(s_1 s_2+s_2^2)}{d^2(N-1)+(z_{1-\alpha/2}+z_\beta)^2(s_1+s_2)^2}$

test_size_nn() computes the profit-maximizing test size for test and roll with 2 arms

```
test_size_nn(N=10000, s=.1, mu=c(.7,.7), sigma=c(.05,.05))
```

```
## [1] 97.04499 97.04499
```

```
test_size_nn(N=10000, s=c(.1,.2), mu=c(.7,.7), sigma=c(.05,.05))
```

```
## [1]  97.38121 187.41676
```

Takes in parameters:
"N" - size of deployment population
"s" - vector of length 2 containing the standard deviations of the outcome
"μ" - vector of length 2 containing the means of the prior on the mean response
"σ" vector of length 2 containing the standard deviations of the prior on the mean response
and returns a vector containing the sample sizes

For the symmetric case $s_1 = s_2 = s$, the recommended sample size is calculated using $n_1 = n_1 = \frac{\sqrt{9\sigma^4 + 4n\sigma^2\sigma_0^2} - \frac{3}{4}\sigma^2}{4\sigma_0^2}$. For the asymmetric case, the recommended sample size is calculated using the optim() function to find the sample sizes which maximize profit_nn().

2.2 Computing profits and errors under different testing methods (2-arm tests)

profit_nn() computes the per-customer profit for test & roll with 2 arms

```
profit_nn(n=100, N=10000, s=.1, mu=c(.7,.5), sigma=c(.2,.2))
```

```
## [1] 0.8104432
```

```
profit_nn(n=c(100,200), N=10000, s=.1, mu=c(.7,.5), sigma=c(.2,.2))
```

```
## [1] 0.8093503
```

Takes in parameters:
"n" - vector of length 2 containing the sample sizes
"N" - size of deployment population
"s" - vector of length 2 containing the known standard deviations of the outcome
"μ" - vector of length 2 containing the means of the prior on the mean response
"σ" - vector of length 2 containing the standard deviations of the prior on the mean response
"log_n" - whether or not log(n) is an input rather than n (to avoid negative solutions), 'TRUE' or 'FALSE'
and returns a numeric that is the per-customer profit for N customers

For the symmetric case, the deploy stage profit is calculated using $(N - n_1 - n_2)\left[\mu + \frac{\sqrt{2}\sigma^2}{\sqrt{\pi}\sqrt{2\sigma^2 + \frac{n_1+n_2}{n_1 n_2}s^2}}\right]$ and the test stage profit is calculated with $\mu_1(n_1 + n_2)$. For the asymmetric case, the deploy stage profit is calculated using $(N - n_1 - n_2)\left[\mu_1 + e\Phi\left(\frac{e}{v}\right) + v\phi\left(\frac{e}{v}\right)\right]$ where $e = (\mu_2 - \mu_1)$ and $v = \sqrt{\frac{\sigma_1^4}{\sigma_1^2 + s_1^2/n_1} + \frac{\sigma_2^4}{\sigma_2^2 + s_2^2/n_2}}$. The test stage profit is $\mu_1 n_1 + \mu_2 n_2$

profit_perfect_nn() computes the profit with perfect information

```
profit_perfect_nn(mu=.7, sigma=.02)
```

```
## [1] 0.7112838
```

Takes in parameters:
"μ" - means of the prior on the mean response
"σ" - standard deviations of the prior on the mean response
and returns a numeric that is the per-customer profit with perfect information

For the symmetric case, the perfect information profit is calculated using $\left(\mu + \frac{\sigma}{\sqrt{\pi}}\right) N$. The asymmetric case has not yet been implemented.

error_rate_nn() computes the rate of incorrect deployments

```
error_rate_nn(n=100, s=.5, sigma=.2)
```

```
## [1] 0.07797913
```

```
error_rate_nn(n=c(100,200), s=.5, sigma=.2)
```

```
## [1] 0.0678686
```

Takes in parameters:
"n" - vector of length 2 containing the sample sizes
"s" - vector of length 1 (symmetrical) containing the standard deviations of the outcome
"σ" - vector of length 1 (symmetrical) containing the standard deviations of the prior on the mean response
and returns a numeric that is the error rate
With symmetric normal priors, the error rate is calculated using $\frac{1}{4} - \frac{1}{2\pi} \arctan\left(\frac{\sqrt{2}\sigma}{s}\sqrt{\frac{n_1 n_2}{n_1+n_2}}\right)$. The asymmetric case has not yet been implemented.

3. Usage for K-arm tests (with simulation)

3.1 Computing profit-maximizing test sizes (K-arm tests) with simulation

test_size_nn_sim() Computes the profit-maximizing test size and profits for a multi-armed test & roll

```
test_size_nn_sim(N=1000, s=.1, mu=.1, sigma=.05, K=2, R=1000)
```

```
## $n
## [1] 32 32
##
## [[2]]
## [1] 0.1232724
```

Takes in parameters:
"N" - deployment population
"s" - standard deviations of the response (length 1(symmetric) or K)
"μ" - vector of length K containing means of the priors on the mean response
"σ" - vector of length K containing the standard deviations of the priors on the mean response
"K" - number of arms (treatments)
"R" - number of simulation replications
and returns a list with the sample sizes and expected profit per customer

For the symmetric case, the function creates a vector containing all possible values of n (floor(N/K)-1). Then, one_rep_test_size() is used to calculate the profits for each of the values of n. The values of n that maximize the profit are then chosen as the sample sizes for that particular repetition. The asymmetric case has not yet been implemented.

one__rep__test__size() (utility function for test__size__nn__sim()) simulates one set of potential outcomes

```
one_rep_test_size(1:(floor(10/2)-1), N=10, s=10, mu= 20, sigma=10, K=2)
```

```
##      n   profit
## [1,] 1 148.5384
## [2,] 2 153.0009
## [3,] 3 136.3781
## [4,] 4 146.4552
```

Takes in parameters:
"n_{vals}" - potential values for n
"N" - deployment population
"s" - standard deviations of the outcome (vector length 1 or K) "μ" - means of the priors on the mean response (vector length 1 or K) "σ" - standard deviations of the priors on the mean response (vector length 1 or K)
"K" - number of arms (treatments)
and returns a 2-column matrix with values of n in the first column and profits in the second column

The function picks a true mean for each arm and then simulates N observations for each of the K arms based off of that mean. Then, the arm with highest posterior mean is chosen and the profit is calculated for each potential value of n (n__vals).

3.2 Computing profits and errors under different testing methods (K-arm tests) with simulation

profit__nn__sim() computes the per-customer profit for test & roll with K arms

```
profit_nn_sim(n=c(100,200,300), N=1000, s=c(.1,.2,.3), mu=c(.1,.2,.3), sigma=c(.01,.03,.05), K=3, TS=FA
```

```
## $profit
##            perfect_info test_roll thom_samp
## exp_profit    0.2793900 0.2397323        NA
## 5%            0.2102495 0.1822540        NA
## 95%           0.3395843 0.2824626        NA
##
## $regret
##            perfect_info  test_roll thom_samp
## exp_regret            0 0.14051870        NA
## 5%                    0 0.09817709        NA
## 95%                   0 0.18375252        NA
##
## $error_rate
## [1] 0
##
## $profit_draws
##    perfect_info test_roll thom_samp error
## 1      281.3933  250.3088        NA     0
## 2      273.1305  237.2272        NA     0
## 3      300.2746  255.2819        NA     0
## 4      302.9303  248.6393        NA     0
## 5      361.5860  294.1147        NA     0
## 6      312.6934  268.2211        NA     0
## 7      238.9081  217.8550        NA     0
## 8      261.6437  232.6182        NA     0
## 9      274.5385  239.9305        NA     0
## 10     186.8015  153.1259        NA     0
##
```

```
## $regret_draws
##    perfect_info  test_roll thom_samp
## 1             0 0.11046650        NA
## 2             0 0.13145079        NA
## 3             0 0.14983841        NA
## 4             0 0.17921924        NA
## 5             0 0.18659817        NA
## 6             0 0.14222316        NA
## 7             0 0.08812211        NA
## 8             0 0.11093535        NA
## 9             0 0.12605880        NA
## 10            0 0.18027451        NA
```

Takes in parameters:
"n" - sample sizes for test & roll (vector of length 1 or K)
"N" - deployment population
"s" - standard deviations of the outcome (vector of length 1 or K)
"μ" - means of the priors on the mean response (vector of length 1 or K)
"σ" - standard deviations of the priors on the mean response (vector of length 1 or K)
"K" - number of arms (2 by default)
"TS" - whether or not to run Thomson sampling, 'TRUE' or 'FALSE' (FALSE by default)
"R" - number of simulation replications (1000 by default)
and returns a list containing the profit, regret, and error rates

The function uses one_rep_profit() to simulate each repetition. Then, the summary values are combined into a list. test_size_nn() or test_size_nn_sim() can be used to find the profit-maximizing sample sizes (n) to input into the function.

one_rep_profit() (utility function for profit_nn_sim()) simulates one set of potential outcomes, draws a true mean for each arm and generates N observations from each arm

```
one_rep_profit(n=c(100,100), N=1000, s=c(.1,.1), mu=c(.1,.1), sigma=c(.05,.05), K=2, TS=FALSE)
```

```
## perfect_info    test_roll    thom_samp        error
##     92.65536     89.00617           NA      0.00000
```

```
one_rep_profit(n=c(100,200,300), N=1000, s=c(.1,.2,.3), mu=c(.1,.2,.3), sigma=c(.01,.03,.05), K=3, TS=F
```

```
## perfect_info    test_roll    thom_samp        error
##     314.2459     272.9759           NA       0.0000
```

Takes in parameters:
"n" - sample sizes for test & roll (vector of length 1 or K)
"N" - deployment population
"s" - standard deviations of the outcome (vector of length 1 or K)
"μ" - means of the priors on the mean response (vector of length 1 or K)
"σ" - standard deviations of the priors on the mean response (vector of length 1 or K)
"K" - number of arms (2 by default)
"TS" - whether or not to run Thomson sampling, 'TRUE' or 'FALSE' (FALSE by default)
and returns the profits and error rates under perfect information, test & roll, and Thomson sampling under one simulation The function draws a true mean for each arm based off sigma and simulates N observations for each of the K arms. The profit from perfect information is calculated by picking the arms with the highest true mean. Test and roll profit is calculated by summing the profit from the first n observations in each arm (test stage) with the profit from the selected arm (roll stage with remainder of population). For Thomson Sampling, mu and sigma are updated with every K observations (1 from each arm).

test_eval_nn() provides summary of a test & roll plan

```r
test_eval_nn(n=c(100,100), N=1000, s=.1, mu=.1, sigma=.05)
```

```
##    n1  n2 profit_per_cust   profit profit_test profit_deploy profit_rand
## 1 100 100       0.1221293 122.1293          20      102.1293         100
##   profit_perfect profit_gain     regret error_rate tie_rate
## 1       128.2095   0.7844645 0.04742351 0.06283296        0
```

```r
test_eval_nn(n=c(100,200), N=1000, s=c(.1,.2), mu=c(.1,.2), sigma=c(.05,.1))
```

```
##    n1  n2 profit_per_cust   profit profit_test profit_deploy profit_rand
## 1 100 200       0.1976957 197.6957          50      147.6957         150
##   profit_perfect profit_gain     regret error_rate tie_rate
## 1       209.3098   0.8041781 0.05548791     0.0344        0
```

Takes in parameters: "n" - sample sizes for test & roll (vector of length 2) "N" - deployment population
"s" - standard deviations of the outcome (vector of length 2)
"μ" - means of the priors on the mean response (vector of length 2)
"σ" - standard deviations of the priors on the mean response (vector of length 2)
and returns a data frame containing summary statistics such as profit per customer, profits from test phase, error rates, etc.

For the symmetric case, the total profit is calculated using profit_nn()*N so profit_per_cust becomes total profit/N. profit_test is calculated using

$$\mu * (n_1 + n_2)$$

. profit_deploy is total profit - profit_test. profit_rand (from random selection) is calculated using

$$mu * N$$

. profit_perfect is calculated using the profit_perfect_nn() function. profit_gain is simply total profit - profit_rand. The regret is 1 - total profit/profit_perfect and represents the % lost potential profit. The error_rate is calculated as part of profit_nn_sim(). For the asymmetric case, the only results that differ are profit_test becoming

$$\mu_1 * n_1 + \mu_1 * n_2$$

and profit_rand becoming

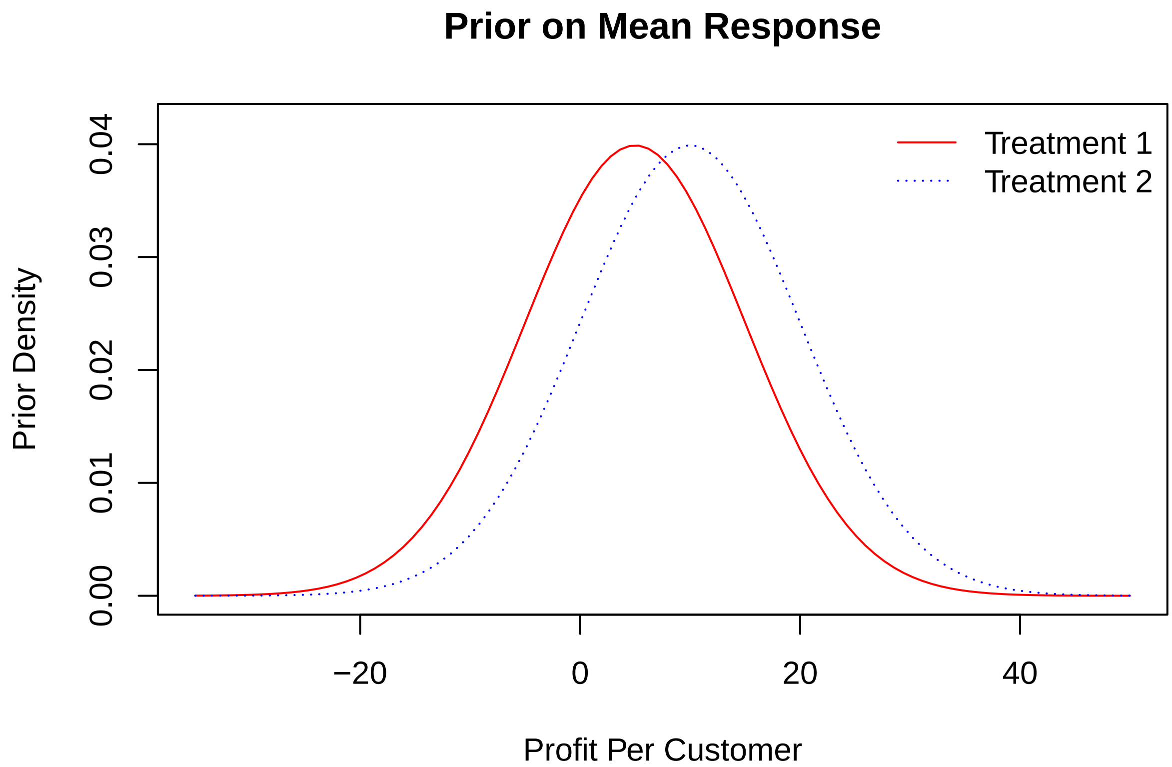
$$(\mu_1 + \mu_2) * .5 * N$$

.

4. Visualizing prior effects on profits and mean response

plot_prior_mean_resp_nn() plots prior densities against mean response (profit per customer)

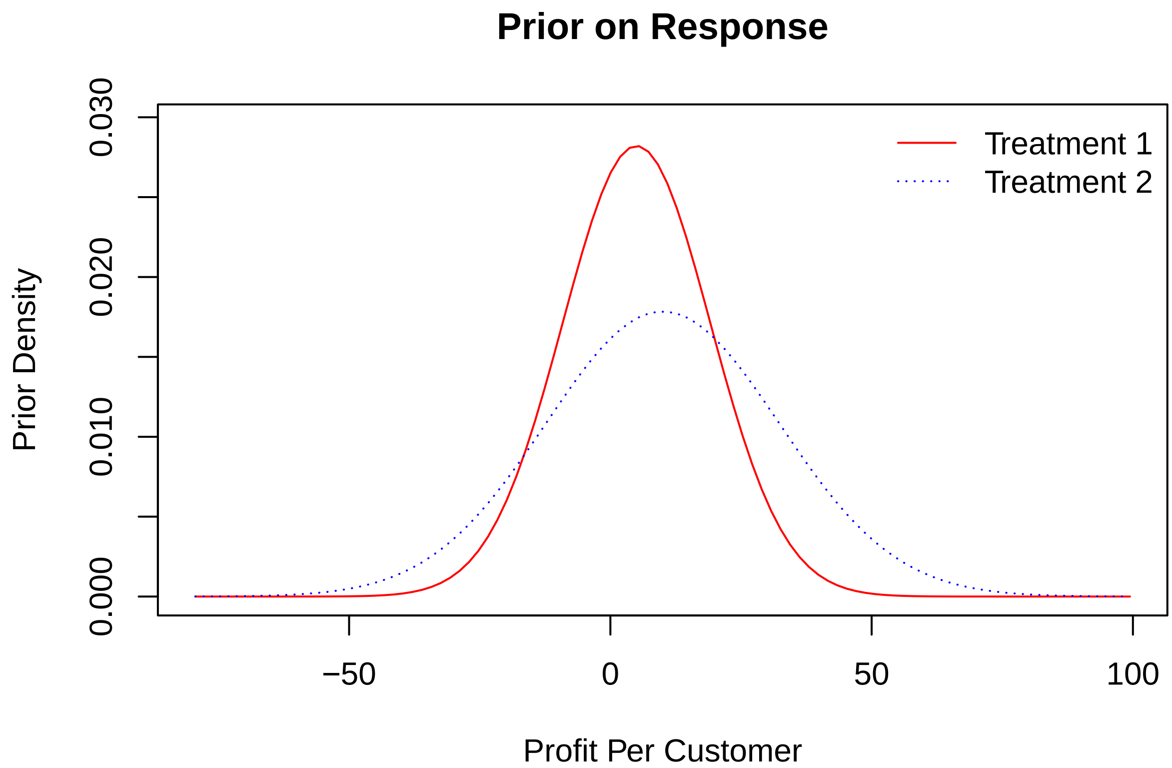
```
plot_prior_mean_resp_nn(mu=c(5,10), sigma=c(10,10))
```



It takes in as parameters:
"μ" - means of the prior on the mean response
"σ" - standard deviations of the priors on the response
and returns a plot of the prior density against the mean response

plot_prior_resp_nn() plot prior densities against response (profit per customer)

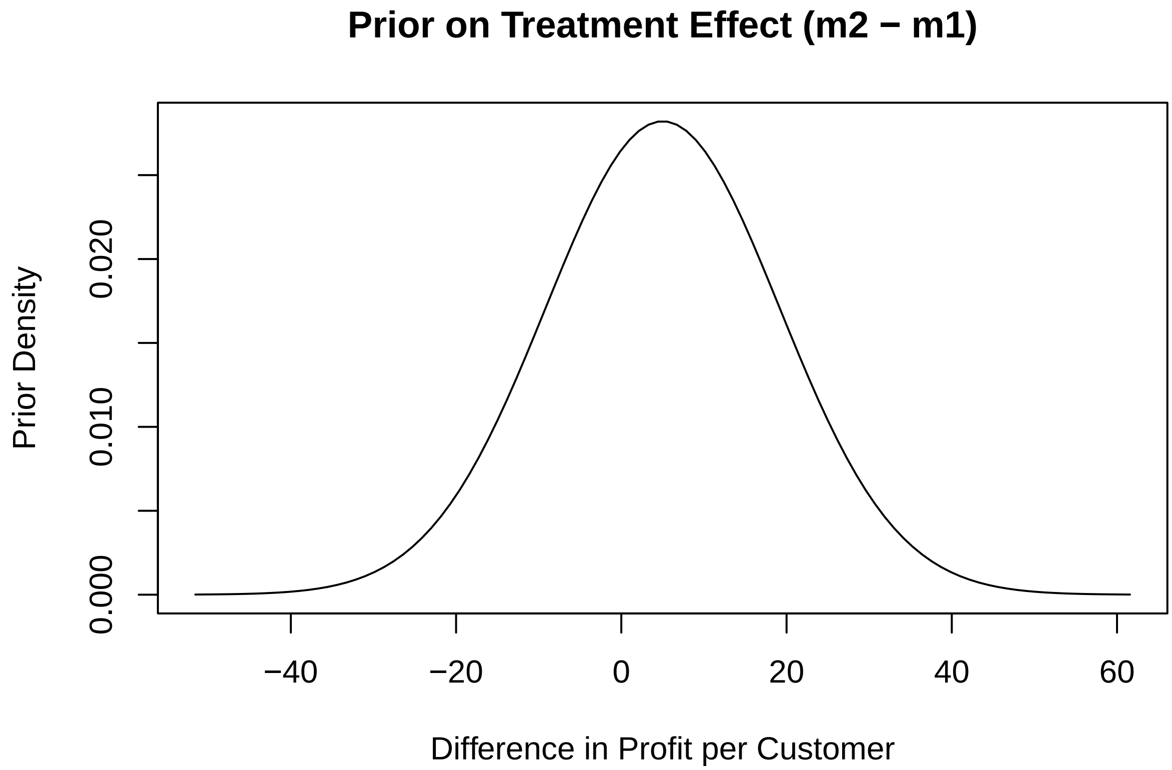
```
plot_prior_resp_nn(s=c(10,20), mu=c(5,10), sigma=c(10,10))
```



It takes in as parameters:
"s" - the known standard deviations of the outcome
"μ" - means of the prior on the mean response
"σ" - standard deviations of the priors on the response
and returns a plot of the prior density against the response

plot_prior_effect_nn() plots prior densities against treatment effect (difference in profit per customer)

```
plot_prior_effect_nn(mu=c(5,10), sigma=c(10,10), abs=FALSE)
```



It takes in as parameters:
"μ" - means of the prior on the mean response
"σ" - standard deviations of the priors on the response
"abs" - whether or not to take the absolute difference, 'TRUE' or 'FALSE'
and returns a plot of the prior density against the treatment effect