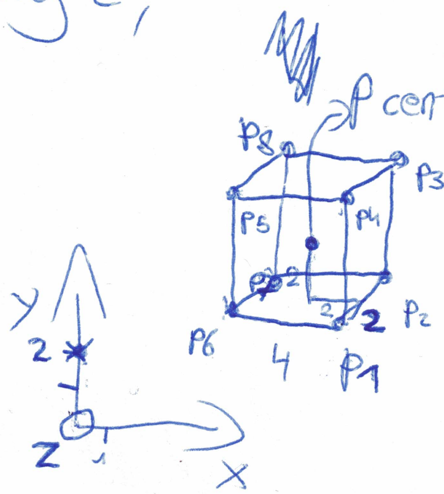
Übung 2,

1a)



$$P_{center} = \begin{array}{c} \begin{pmatrix} 0 \\ 2 \\ -6 \\ 1 \end{pmatrix} \end{array} \begin{array}{c} x \\ y \\ z \\ w \end{array} \qquad p_1 = \begin{pmatrix} 2 \\ 0 \\ -4 \\ 1 \end{pmatrix} \quad p_2 = \begin{pmatrix} 2 \\ 0 \\ -8 \\ 1 \end{pmatrix} \quad p_3 = \begin{pmatrix} 2 \\ 4 \\ -8 \\ 1 \end{pmatrix}$$

$$p_4 = \begin{pmatrix} 2 \\ 4 \\ -4 \\ 1 \end{pmatrix} \quad p_5 = \begin{pmatrix} -2 \\ 4 \\ -4 \\ 1 \end{pmatrix} \quad p_6 = \begin{pmatrix} -2 \\ 0 \\ -4 \\ 1 \end{pmatrix} \quad p_7 = \begin{pmatrix} -2 \\ 0 \\ -8 \\ 1 \end{pmatrix}$$

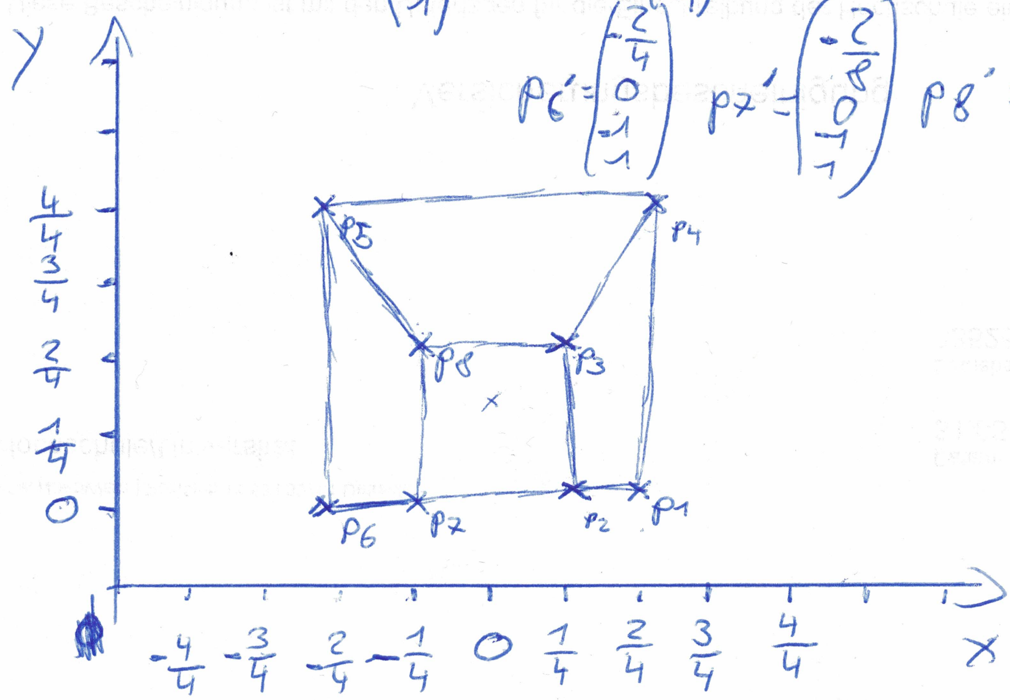
$$p_8 = \begin{pmatrix} -2 \\ 4 \\ -8 \\ 1 \end{pmatrix}$$

$$P_{std} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$p_1' = p_1 \times P_{std} = \begin{pmatrix} 2 \\ 0 \\ -4 \\ 4 \end{pmatrix} \Rightarrow \text{dehomogen: } \begin{pmatrix} \frac{2}{4} \\ 0 \\ -1 \\ 1 \end{pmatrix}$$

$$p_2' = \begin{pmatrix} \frac{2}{8} \\ 0 \\ -1 \\ 1 \end{pmatrix} \quad p_3' = \begin{pmatrix} \frac{2}{8} \\ \frac{4}{8} \\ -1 \\ 1 \end{pmatrix} \quad p_4' = \begin{pmatrix} \frac{2}{4} \\ 1 \\ -1 \\ 1 \end{pmatrix} \quad p_5' = \begin{pmatrix} -\frac{2}{4} \\ 1 \\ -1 \\ 1 \end{pmatrix}$$

$$p_6' = \begin{pmatrix} -\frac{2}{4} \\ 0 \\ -1 \\ 1 \end{pmatrix} \quad p_7' = \begin{pmatrix} -\frac{2}{8} \\ 0 \\ -1 \\ 1 \end{pmatrix} \quad p_8' = \begin{pmatrix} -\frac{2}{8} \\ \frac{4}{8} \\ -1 \\ 1 \end{pmatrix}$$



b) $M_{lookAt} = \begin{bmatrix} r_x & r_y & r_z & -r^T c \\ u_x & u_y & u_z & -u^T c \\ -d_x & -d_y & -d_z & d^T c \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $u = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ $c = \begin{pmatrix} 6 \\ 2 \\ 0 \\ 1 \end{pmatrix}$

$d = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$ $r \stackrel{?}{=} u \times d = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$

$-u^T c = -2$ $-r^T c = 3\sqrt{2}$

$d^T c = -3\sqrt{2}$

$M_{lookAt} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 3\sqrt{2} \\ 0 & 1 & 0 & -2 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & -3\sqrt{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$

c) 1) $\vec{v} = \begin{pmatrix} 21 \\ 0 \\ 20 \end{pmatrix}$ $|\vec{v}| = ?$

$v \cdot v = |v|^2 = 21^2 + 20^2 = 841$

$|v| = \sqrt{841} = \underline{\underline{29}}$

2) $\vec{v} = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$ $\vec{w} = \begin{pmatrix} 0 \\ 0 \\ 7 \end{pmatrix}$ $\vec{v} \circ \vec{w} = 14$

$|\vec{v}| = \sqrt{v \times v} = \sqrt{2^2 + 2^2} = 2\sqrt{2} = \sqrt{8}$

$|\vec{w}| = \sqrt{7^2} = 7 = \sqrt{w \times w}$

$\cos \varphi = \frac{14}{7\sqrt{8}}$ $\varphi = \cos^{-1}\left(\frac{14}{7\sqrt{8}}\right) \approx 2{,}41 \text{rad}$

$\approx 138{,}08°$

3) $\vec{v} = \begin{pmatrix}6\\6\\6\end{pmatrix}$ $\vec{w} = \begin{pmatrix}0\\4\\2\end{pmatrix}$

$$\vec{v} \times \vec{w} = \begin{bmatrix} 6\cdot 2 - 6\cdot 4 \\ 6\cdot 0 - 6\cdot 2 \\ 6\cdot 4 - 6\cdot 0 \end{bmatrix} = \begin{pmatrix}-12\\-12\\24\end{pmatrix}$$

4) $\vec{w} = \begin{pmatrix}3 - v_1\\2 - v_2\\1 - v_3\end{pmatrix}$ $\vec{v} = \begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}$ $\perp$

$$\vec{v} \circ \vec{w} = \begin{pmatrix}(3 - v_1)\cdot v_1 + \\ (2 - v_2)\cdot v_2 + \\ (1 - v_3)\cdot v_3\end{pmatrix} = 0$$

$$3v_1 - v_1^2 + 2v_2 - v_2^2 + 1v_3 - v_3^2 = 0 \quad \Big| \; v_1 = 0, \; v_2 = 0$$

$$3\cdot 0 + 0 + v_3 - v_3^2 = 0$$

$$v_3 = v_3^2 \Rightarrow v_3 = 1$$

$$(a+b)c = ac + bc$$

$$\vec{v} = \begin{pmatrix}0\\0\\1\end{pmatrix}$$