[STAT W4702] Statistical Inference & Modelling Group Project

Babies

12 December 2015

Abstract

Data Set

This project was conducted on the Low Birth Weight dataset collected in 1986 at Baystate Medical Center, Springfield, Massachusetts as a part of a bigger study on the factors influencing newborn infants' health and risk of serious health problems potentially leading to death. This dataset is distributed as a part of MASS library and contains 189 observations and 10 variables, among which but represents the exact amount of newborn infant's weight in grams and is used as the variable of interest we are trying to predict. The other 9 variables stand for different factors related to mothers' physiological parameters, such as age, weight and race, their health-related habits and behavior during pregnancy (smoking habits, presence of uterine irritability and number of physician visits). Also there is a low birth weight indicator low, which is defined as a binary variable showing whether the weight of an infant is below 2500 grams or not. Brief description of each variable is provided in the table below.

The goal of our research is to identify relationship between these variables and infant weight and understand the influence of each of them on the explained variable. The project pursue both inferential and predictive goals as it is equally important to be able to obtain inference about factors affecting newborn's health and to be able to react on the potential health risks in a timely manner, when the model predicts the low birth weight outcome for a certain observation. In order to accomplish this goal we tried to fit multiple linear and non-linear models exploring the rationale that could provide the evidence for certain types of models and finding balance between interpretability and predictive power of the model.

Cleaning and Exploring Dataset

For the purposes of the research the dataset was cleaned in the following way:

- birth weight variable bwt is converted from grams to kilgrams to reduce the order of magnitude for estimated model coefficients and error values;
- factor variable race was assigned with proper labels white, black and other;
- physisian visits were converted to a factor variable ftv with 3 labels 0, 1 and 2+;
- response is defined as an exact amount of infant's weight from bwt;
- all the columns are assigned with meaningful names.

Variable description table and summary statistics of the tidy dataset are provided below.

Variable	Description
baby.grams	weight of newborn infant in kg
mother.age	mother's age in years
mother.weight	mother's weight in pounds at last menstrual period
race	mother's race, factor variable with following labels: white, black or other
smoke	smoking status during pregnancy, binary variable

Variable	Description
prem.labor	binary variable showing whether mother had premature labors before or not
hypertension	binary variable showing whether mother had hypertension or not
uterine	binary variable showing presence of uterine irritability
physician.visits	number of physician visits during the first trimester: 0 , 1 or $2+$

```
##
      baby.grams
                       mother.age
                                      mother.weight
                                                           race
##
           :0.709
                     Min.
                            :14.00
                                      Min.
                                              : 80.0
                                                       white:96
##
    1st Qu.:2.414
                     1st Qu.:19.00
                                      1st Qu.:110.0
                                                       black:26
##
    Median :2.977
                     Median :23.00
                                      Median :121.0
                                                       other:67
                             :23.24
##
    Mean
           :2.945
                                              :129.8
                     Mean
                                      Mean
##
    3rd Qu.:3.487
                     3rd Qu.:26.00
                                      3rd Qu.:140.0
##
           :4.990
                            :45.00
                                              :250.0
    Max.
                     Max.
                                      Max.
                                 hypertension
##
      smoke
                     prem.labor
                                                    uterine
                     FALSE:159
                                  Mode :logical
##
    Mode :logical
                                                   Mode :logical
                     TRUE : 30
##
    FALSE:115
                                  FALSE: 177
                                                   FALSE:161
    TRUE :74
                                  TRUE:12
                                                   TRUE :28
##
##
    NA's :0
                                  NA's :0
                                                   NA's :0
##
##
##
    physician.visits
    0:100
##
##
    1:47
##
    2+: 42
##
##
##
```

Datatset has only 2 quantitative variables apart from infant weights, however, as shown in the table below, they do not demonstrate strong correlation between each other, which suggests that these variables will not be sufficient themselves in explaining birth weight variation. Variable mother.age demonstrate the lowest correlation with baby.grams and will most probably be omitted in the prediction models further on.

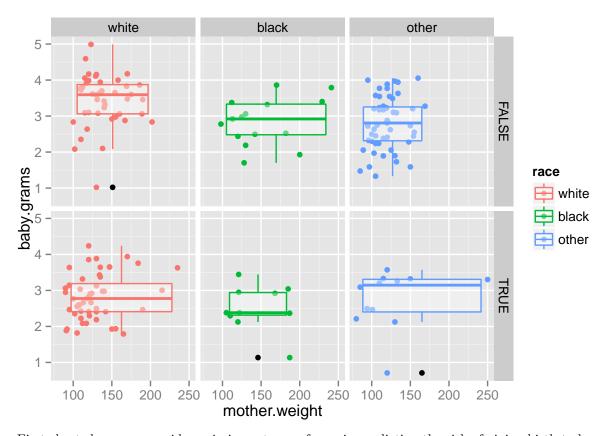
```
## baby.grams mother.age mother.weight

## baby.grams 1.00000000 0.09031781 0.1857333

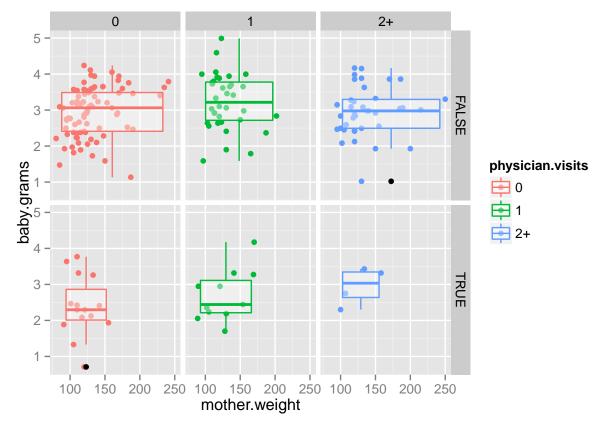
## mother.age 0.09031781 1.00000000 0.1800732

## mother.weight 0.18573328 0.18007315 1.0000000
```

The following charts demonstrate boxplots and splits of the baby.grams data points vs mother.weight across various categorical and binary variables that make part of the working dataset.



First chart shows some evidence in importance of race in predicting the risk of giving birth to low weight baby, as well as smoking habits during pregnancy. Facet scatterplots show that data point corresponding to each of these factors' combinations group around different median values, which can suggest their predictive power on the newborn infant's weight.



The second chart splits all the observations in sample into several groups by number of physician vistis in the first trimester and occurance of premature labor by each subject of the study. For mothers without previous premature births no significant difference is observed with repsect to number of physician visits, whereas women who had premature labors before are exposed to the higher risk of giving birth to low weight baby if they do not pay enough visits to physician during the first trimester of their pregnancy term. However, we need to account for existing outliers in the sample dataset, as there are at three observations of infants that were born with weight less than or equal to 1 kg, which significantly differs from the majority of observations in this dataset.

Exploring Linear Relationships

Our first attempt to find a statistically significant model fit will go through fitting linear model of different factors in dataset vs baby.grams, which is the variable of our interest.

For the purposes of further validation and comparison of results we attribute 75% of the data to training set, saving the rest of the observations for test set.

As dataset consists of only 8 explaining variables, it is computationally acceptable to select the best possible subset of the variables explaining the response of the model.

```
library (leaps)
regfit.full=regsubsets(baby.grams~., bwt.grams.train, nvmax =19)
reg.summary = summary(regfit.full)
par(mfrow =c(2,2))
plot(reg.summary$rss ,xlab=" Number of Variables ",ylab=" RSS", type="l")
plot(reg.summary$adjr2 ,xlab =" Number of Variables ", ylab=" Adjusted RSq",type="l")
max.adjr2=which.max (reg.summary$adjr2)
max.adjr2
```

```
## [1] 6
```

```
points (max.adjr2, reg.summary$adjr2[max.adjr2], col ="red",cex =2, pch =20)

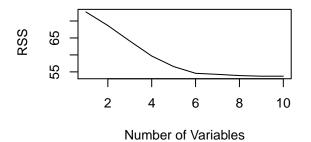
plot(reg.summary$cp ,xlab =" Number of Variables ", ylab="Cp", type='l')
min.cp= which.min (reg.summary$cp )
min.cp
```

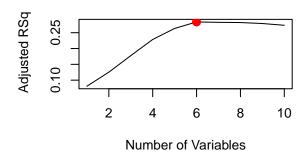
[1] 6

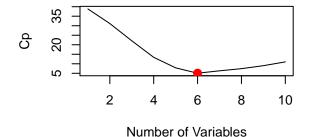
```
points (min.cp, reg.summary$cp[min.cp], col ="red",cex =2, pch =20)
min.bic = which.min(reg.summary$bic)
min.bic
```

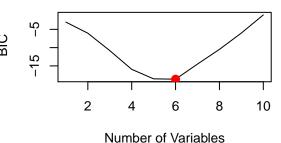
[1] 6

```
plot(reg.summary$bic ,xlab=" Number of Variables ",ylab=" BIC", type='1')
points (min.bic, reg.summary$bic [min.bic], col =" red",cex =2, pch =20)
```

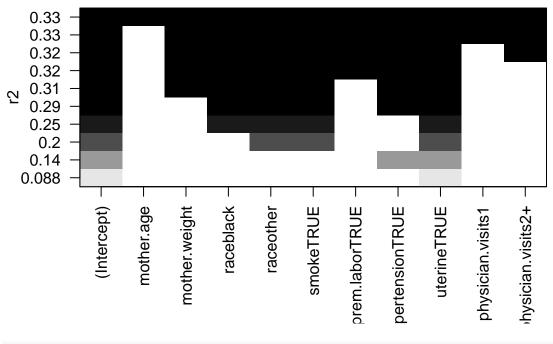




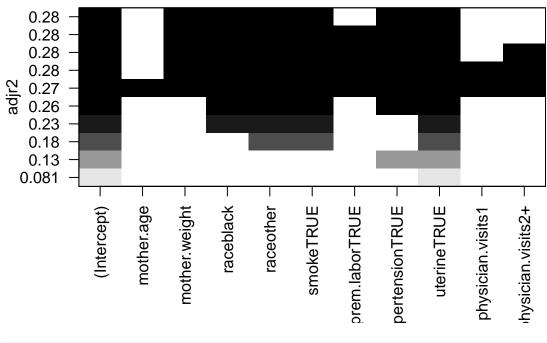




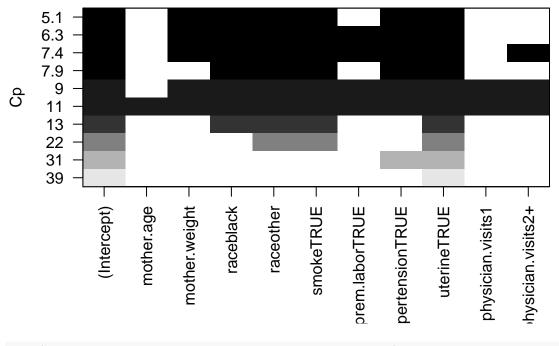
```
par(mfrow = c(1,1))
plot(regfit.full ,scale ="r2", cex.axis = 0.1, las = 1)
```



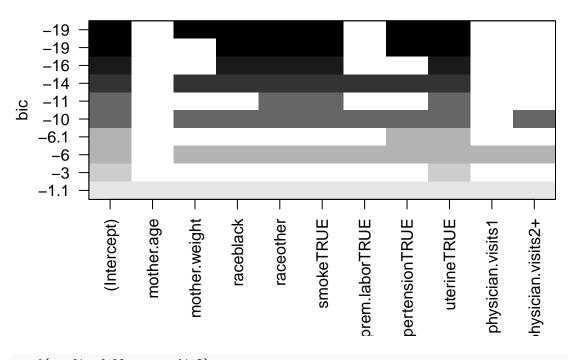
plot(regfit.full ,scale ="adjr2", cex.axis = 0.1, las = 1)



plot(regfit.full ,scale ="Cp", cex.axis = 0.1, las = 1)



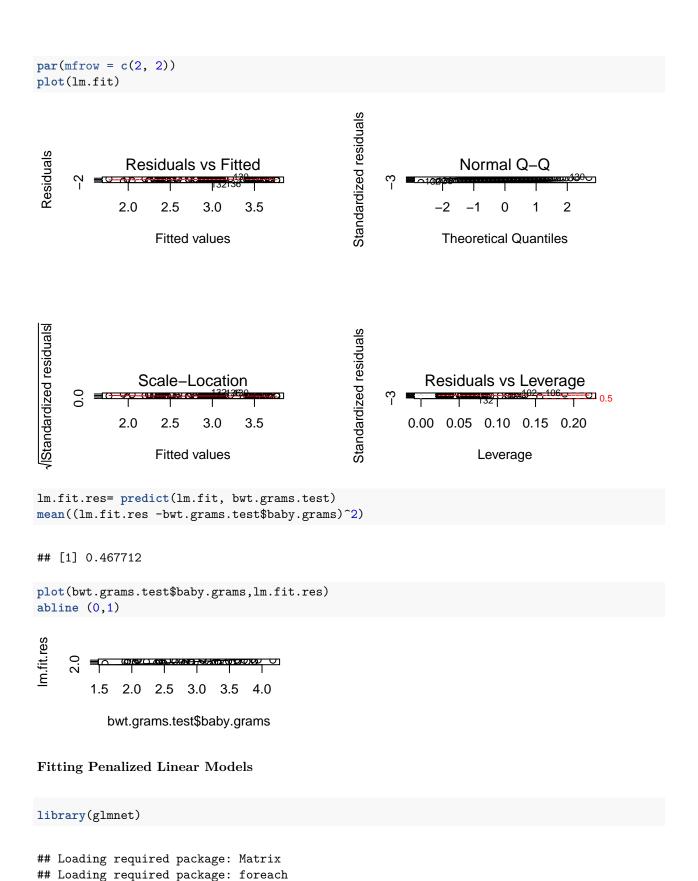
plot(regfit.full ,scale ="bic", cex.axis = 0.1, las = 1)



coef(regfit.full, max.adjr2)

##	(Intercept)	mother.weight	raceblack	raceother
##	2.955952101	0.004267934	-0.566615552	-0.487289998
##	smokeTRUE	hypertensionTRUE	uterineTRUE	
##	-0.465884551	-0.675297653	-0.572809187	

```
coef(regfit.full, min.cp)
##
       (Intercept)
                   mother.weight
                                      raceblack
                                                     raceother
##
       2.955952101
                     0.004267934
                                   -0.566615552
                                                  -0.487289998
##
                                    uterineTRUE
        smokeTRUE hypertensionTRUE
##
      -0.465884551
                    -0.675297653
                                   -0.572809187
coef(regfit.full, min.bic)
##
       (Intercept)
                   mother.weight
                                      raceblack
                                                     raceother
##
       2.955952101
                     0.004267934
                                   -0.566615552
                                                  -0.487289998
##
        smokeTRUE hypertensionTRUE
                                    uterineTRUE
      -0.465884551
##
                    -0.675297653
                                   -0.572809187
#Linear regression with the predictors selected by best subset
lm.fit = lm( baby.grams~ mother.weight+race+smoke+hypertension+uterine, data=bwt.grams.train)
summary(lm.fit)
##
## Call:
## lm(formula = baby.grams ~ mother.weight + race + smoke + hypertension +
##
      uterine, data = bwt.grams.train)
##
## Residuals:
      Min
                  Median
               1Q
                               3Q
## -1.91697 -0.40046 0.04839 0.36803 1.50909
## Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
                2.955952  0.283542  10.425  < 2e-16 ***
## mother.weight
                0.004268 0.001932 2.209 0.028907 *
                 ## raceblack
## raceother
                 ## smokeTRUE
## uterineTRUE
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.6382 on 134 degrees of freedom
## Multiple R-squared: 0.3147, Adjusted R-squared: 0.284
## F-statistic: 10.25 on 6 and 134 DF, p-value: 2.508e-09
confint(lm.fit)
##
                        2.5 %
                                  97.5 %
                  2.3951550476 3.516749155
## (Intercept)
## mother.weight
                  0.0004458445 0.008090024
                 -0.8937222132 -0.239508891
## raceblack
## raceother
                 -0.7460777241 -0.228502271
## smokeTRUE
                 -0.7036072464 -0.228161856
## hypertensionTRUE -1.1011324705 -0.249462836
## uterineTRUE
                -0.8804804084 -0.265137966
```



Loaded glmnet 2.0-2

```
##
bwt.x.train=model.matrix( baby.grams~., data=bwt.grams.train)[,-1]
bwt.y.train=bwt.grams.train$baby.grams

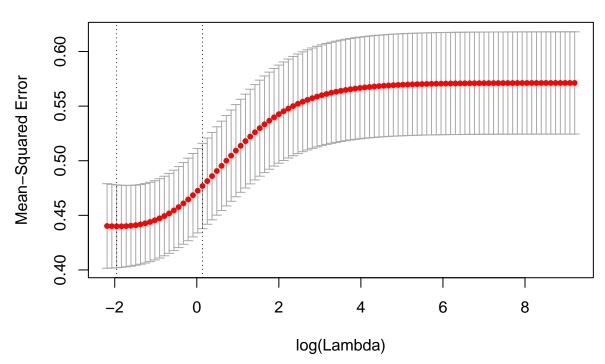
bwt.x.test=model.matrix( baby.grams~., data=bwt.grams.test)[,-1]
bwt.y.test=bwt.grams.test[,1]

grid.bwt =10^seq (-1,4, length =100)

# With alpha =0, glmnet computes the ridge

ridge =cv.glmnet(bwt.x.train,bwt.y.train,alpha =0, lambda =grid.bwt, nfolds=6)
plot(ridge)
```





ridge.opt = glmnet(bwt.x.train,bwt.y.train,alpha =0, lambda =ridge\$lambda.min)
ridge.opt\$beta

```
## 10 x 1 sparse Matrix of class "dgCMatrix"
##
                                s0
## mother.age
                       0.002379072
## mother.weight
                       0.003740105
## raceblack
                      -0.390607098
## raceother
                      -0.320259968
## smokeTRUE
                      -0.311151546
## prem.laborTRUE
                      -0.195046362
## hypertensionTRUE
                      -0.567559864
## uterineTRUE
                      -0.436600708
## physician.visits1
                       0.124887593
## physician.visits2+ -0.068069904
```

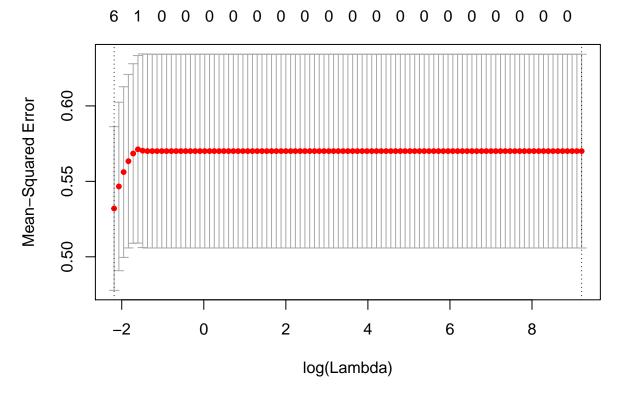
```
ridge.opt.res = predict(ridge.opt, s =ridge$lambda.min, newx=bwt.x.test)
mean((ridge.opt.res -bwt.y.test)^2)
```

[1] 0.4173598

```
# With alpha =1, glmmet computes the lasso
lasso =cv.glmnet(bwt.x.train,bwt.y.train,alpha =1, lambda =grid.bwt, nfolds=6)
lasso$lambda.min
```

[1] 0.1123324

plot(lasso)



lasso.opt = glmnet(bwt.x.train,bwt.y.train,alpha =1, lambda =lasso\$lambda.min)
lasso.opt\$beta

```
## 10 x 1 sparse Matrix of class "dgCMatrix"
##
                                 s0
## mother.age
                       0.0007578126
## mother.weight
## raceblack
## raceother
                      -0.0657574959
## smokeTRUE
                      -0.1162215546
## prem.laborTRUE
                      -0.0383714410
## hypertensionTRUE
                      -0.1709866881
## uterineTRUE
                      -0.2835236922
## physician.visits1
## physician.visits2+
```

```
lasso.opt.res = predict(lasso.opt, s =lasso$lambda.min, newx=bwt.x.test)
mean((lasso.opt.res -bwt.y.test)^2)
```

[1] 0.3968205

Testing for Non-linear Relationships

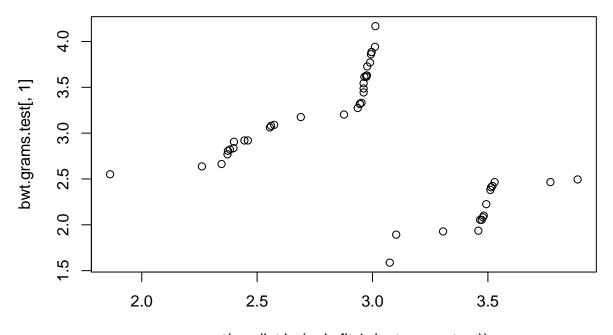
Fitting Polynomial Regression

```
#Create train and test
set.seed(1)
train <- sample(1:nrow(bwt.grams), floor(0.75*nrow(bwt.grams)))
bwt.grams.train <- bwt.grams[train,]
bwt.grams.test <- bwt.grams[-train,]

#Polynomial fit for best subset
poly.fit.1 = lm(baby.grams ~ hypertension + uterine + smoke + race + poly(mother.weight, 2), data = bwt
mean((predict.lm(poly.fit.1, bwt.grams.test) - bwt.grams.test[,1])^2)</pre>
```

[1] 0.4813745

```
plot(sort(predict.lm(poly.fit.1, bwt.grams.test)), bwt.grams.test[,1])
```



sort(predict.lm(poly.fit.1, bwt.grams.test))

```
anova(poly.fit.1)
```

Analysis of Variance Table
##

```
## Response: baby.grams
##
                           Df Sum Sq Mean Sq F value
                                                        Pr(>F)
## hypertension
                               2.827 2.8270 6.9365
                                                      0.009446 **
## uterine
                                      8.1454 19.9863 1.654e-05 ***
                               8.145
## smoke
                               3.026
                                      3.0264
                                             7.4259
                                                      0.007294 **
## race
                            2
                              9.074
                                     4.5371 11.1328 3.386e-05 ***
## poly(mother.weight, 2)
                            2
                              2.363
                                      1.1813 2.8986 0.058590 .
## Residuals
                          133 54.204
                                     0.4075
## ---
                  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
```

When we fit a polynomial model on the predictors obtained from best subset, we observe a Mean Squared Error of 0.4813745. The smaller the Mean Squared Error, the closer the fit is to the data. But, as he value of MSE is high, it suggests that this model does not provide a good fit for the data. The plot also shows that there are irregularities in the prediction and that the polynomial model of degree 2 obtained by using predictors suggested by the best subset is not sufficient. When we perform Analysis of Variance (ANOVA) on the polynomial fit, we see that, the *p-values* for the all the predictors - except mother.weight are less that 0.5 and thus, the NULL hypothesis that these variables affect the baby weight at birth can be rejected.

Different models were tried by increasing the degree of the polynomial but still using the predictors suggested by the best subset and the following results were obtained:

```
poly.fit.2 = lm(baby.grams ~ hypertension + uterine + smoke + race + poly(mother.weight, 3), data = bwt
mean((predict.lm(poly.fit.2, bwt.grams.test) - bwt.grams.test[,1])^2)
## [1] 0.4640868
poly.fit.3 = lm(baby.grams ~ hypertension + uterine + smoke + race + poly(mother.weight, 4), data = bwt
mean((predict.lm(poly.fit.3, bwt.grams.test) - bwt.grams.test[,1])^2)
## [1] 0.4619314
anova(poly.fit.1, poly.fit.2, poly.fit.3)
## Analysis of Variance Table
##
## Model 1: baby.grams ~ hypertension + uterine + smoke + race + poly(mother.weight,
##
       2)
## Model 2: baby.grams ~ hypertension + uterine + smoke + race + poly(mother.weight,
##
## Model 3: baby.grams ~ hypertension + uterine + smoke + race + poly(mother.weight,
##
       4)
##
    Res.Df
               RSS Df Sum of Sq
                                     F Pr(>F)
## 1
        133 54.204
```

We note that as the degree of the polynomial increases, the MSE decreases, but the drop is not significant, suggesting that these predictors are not sufficient enough to predict the correct baby weight. Performing the ANOVA test to compare how the three models perform with respect to each other, we observe high p-values which state that the none of the models are good enough.

0.37915 0.9239 0.3382

0.06393 0.1558 0.6937

2

3

132 53.825

131 53.761

1

1

When we remove the predictors with very low *p-values*, which were suggested by the best subset - namely smoke, race and add other predictors which were rejected by the best-subset, namely - mother.age, prem.labor and physician.visits, we see that the Mean Squared Error starts to decrease. A low MSE denotes a better fit. Thus, the predictors which were rejected by the best subset selection, were actually significant in predicting the correct birthweight.

```
poly.fit.4 = lm(baby.grams ~ hypertension + uterine + poly(mother.age,2) + poly(mother.weight,3), data
mean((predict.lm(poly.fit.4, bwt.grams.test) - bwt.grams.test[,1])^2)

## [1] 0.3890751

poly.fit.5 = lm(baby.grams ~ hypertension + uterine + smoke + prem.labor + poly(mother.age,2) + poly(mothe
```

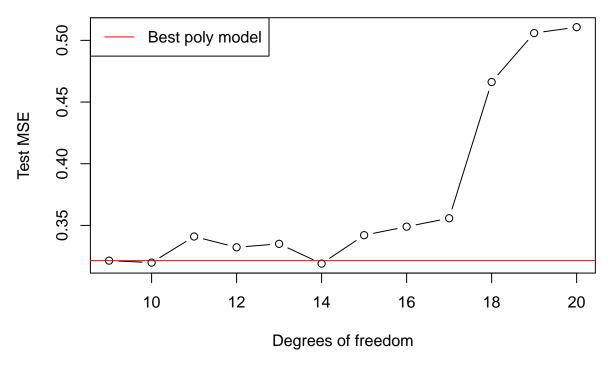
Fitting Natural Splines

```
library(splines)
poly.fit = lm(baby.grams ~ hypertension + uterine + smoke + prem.labor + poly(mother.age, 2) + poly(mother.age, pred = predict.lm(poly.fit, bwt.grams.test)
mse = mean((pred - bwt.grams.test[,1])^2)
mse
```

[1] 0.3214657

Now that we have tried a lot of different polynomial regressions we can wonder if it is possible to improve our best polynomial model by introducing splines. Here we added in the regression formula several basis functions for the variable mother.weight. Between each knots we fit a 9 - degree - polynomial. We tried different values for the number of degrees of freedom so as to find the best parameter. Here is the resulting plot:

Evolution of the test MSE with the number of degrees of freedom



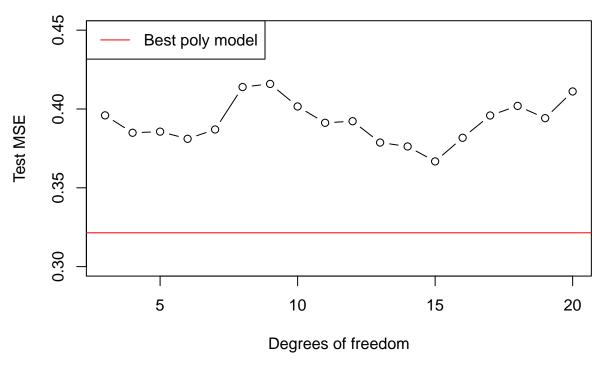
The minimum MSE is obtained when we have 14 degrees of freedom. With the R built-in function bs(), R automatically puts knots on the quantile values of the variable. Here for 14 degrees of freedom our knots are: $q_{16.7}$, $q_{33.3}$, q_{50} , $q_{66.7}$ and $q_{83.3}$. Thus between each quantile R fits a degree 9 polynomial on the mothers' weights. It also makes sure that the 1st, 2nd, ... and 8th derivatives are continuous at each knots. Thus the relation between the number of degrees of freedom d and the number of knots K is the following:

$$d = K + 9$$

We can see that this formula is verified in our case (14 = 5 + 9).

Natural splines are fitted in order to account for more flexibility in the model in attempt to find a better fit.

Evolution of the test MSE withs the number of degrees of freedom



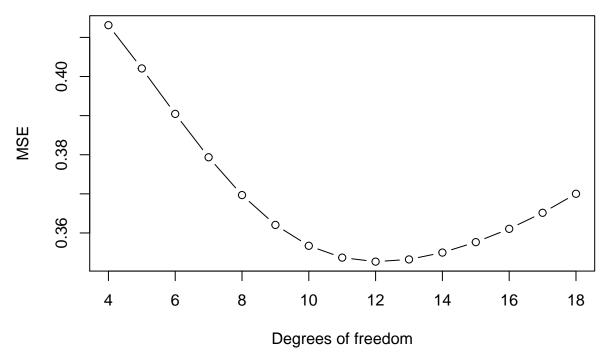
When we try with natural splines we have worse results than with the normal splines model. It is mainly due to the fact that R can only fit cubic natural splines, there is no degree argument in the R built-in function.

Now we can try to see if there is an improvement if we use smoothed splines. We have to use the General Additive Models R library to perform this analysis.

```
library(gam)
```

```
## Loaded gam 1.12
```

Evolution of the MSE



We can notice that the results are still not better than with our optimal model with degree 9 splines. The smoothing effect does not bring more predictive power to the final model. To conclude this part on splines we managed to find a model that outperforms slightly our best polynomial model. This was expected as splines models are more flexible than polynomial models. Nonetheless the improvement in test MSE is quite low and we can wonder if the splines model is really better than the polynomial model. Indeed, fitting a degree nine polynomial between each splines brings a lot of flexibility to the model but the increase of variance can be huge too. If we have had more observations we could have answered to this question by testing our models on a big test set. Nevertheless we can run a ANOVA test to verify if the difference between our best polynomial model and our best splines model is really significant:

```
## Analysis of Variance Table
##
## Model 1: baby.grams ~ hypertension + uterine + smoke + prem.labor + poly(mother.age,
       2) + poly(mother.weight, 9)
##
## Model 2: baby.grams ~ hypertension + uterine + smoke + prem.labor + ns(mother.weight,
##
       df = 14) + poly(mother.age, 2)
##
              RSS Df Sum of Sq
        125 56.829
## 1
        120 52.185
                         4.6437 2.1356 0.06576 .
## 2
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

We can see that the resulting p-value is a little above 0.066. Thus depending on the level of the test we want, we may reject or accept H_0 . Nevertheless we can say that the difference of performance between those two tests is not obvious. Thus we will maybe prefer to keep the less complex model ie the polynomial model.

mother.weight

1st Qu.:110.0

: 80.0

white:96

black:26

Min.

Building Classification Model

Testing for Classification Threshold

mother.age

1st Qu.:19.00

Min.

:14.00

uterine, family = binomial, data = bwt[train,])

3Q

1.153607

0.008422

0.653759

0.561766

0.511265

0.777850

Estimate Std. Error z value Pr(>|z|)

0.8262

Median

-0.4335

-0.143646

-0.019410

1.671922

1.543006

1.395900

1Q

hypertensionTRUE 2.023435

Fitting Logistic Regression

:0.0000

below.2500

1st Qu.:0.0000

##

##

##

##

##

##

##

Deviance Residuals:

Min

Coefficients:

mother.weight

(Intercept)

raceblack

raceother

smokeTRUE

-1.8144 -0.7984

Min.

```
Median :23.00
##
   Median :0.0000
                                      Median :121.0
                                                      other:67
           :0.3122
                             :23.24
                                             :129.8
   Mean
                     Mean
                                      Mean
                     3rd Qu.:26.00
##
   3rd Qu.:1.0000
                                      3rd Qu.:140.0
                                             :250.0
##
   Max.
           :1.0000
                     Max.
                             :45.00
                                      Max.
##
      smoke
                    prem.labor hypertension
                                                  uterine
                    FALSE:159
                                Mode :logical
##
   Mode :logical
                                                 Mode :logical
                    TRUE: 30
   FALSE: 115
                                FALSE: 177
                                                 FALSE: 161
##
   TRUE : 74
                                TRUE:12
##
                                                 TRUE:28
##
   NA's :0
                                NA's :0
                                                 NA's :0
##
##
##
   physician.visits
##
   0:100
##
   1:47
##
   2+: 42
##
##
##
#Logistic regression with the predictors selected by best subset
log.fit = glm( below.2500~ mother.weight+race+smoke+hypertension+uterine, family = binomial, data=bwt[t
summary(log.fit)
##
## Call:
## glm(formula = below.2500 ~ mother.weight + race + smoke + hypertension +
```

Max

-0.125

2.557

0.90090

0.01055 *

-2.305 0.02119 *

2.485 0.01296 *

3.018 0.00254 **

2.601 0.00929 **

2.1800

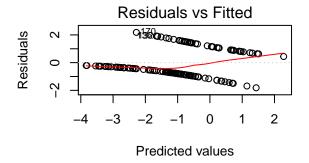
```
## uterineTRUE
                     1.041207
                                0.545658
                                           1.908 0.05637 .
##
                  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
##
##
   (Dispersion parameter for binomial family taken to be 1)
##
##
       Null deviance: 175.05 on 140 degrees of freedom
## Residual deviance: 142.72 on 134 degrees of freedom
## AIC: 156.72
##
## Number of Fisher Scoring iterations: 5
```

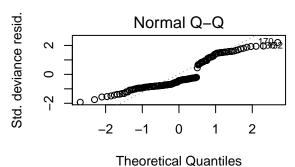
confint(log.fit)

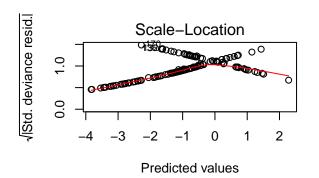
Waiting for profiling to be done...

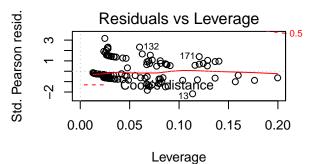
```
##
                                       97.5 %
                           2.5 %
##
  (Intercept)
                    -2.32618747
                                  2.220864159
  mother.weight
                    -0.03731747 -0.004067736
## raceblack
                      0.41072752
                                  3.005448309
## raceother
                      0.33601380
                                  2.561460286
## smokeTRUE
                      0.58304495
                                  2.610272893
## hypertensionTRUE
                     0.54469293
                                  3.659423231
## uterineTRUE
                     -0.02878761
                                  2.132040177
```

```
par(mfrow = c(2, 2))
plot(log.fit)
```









```
pred.train <- predict(log.fit, type = "response")</pre>
low.train <- sapply(pred.train, function(x) {ifelse(x > 0.5, 1, 0)})
table(low.train, bwt$below.2500[train])
##
## low.train 0 1
          0 87 25
##
##
          1 10 19
mean(low.train == bwt$below.2500[train])
## [1] 0.751773
pred.test <- predict(log.fit, newdata = bwt[-train, -1], type = "response")</pre>
low.test <- sapply(pred.test, function(x) {ifelse(x > 0.5, 1, 0)})
table(low.test, bwt$below.2500[-train])
##
## low.test 0 1
##
      0 29 11
##
         1 4 4
mean(low.test == bwt$below.2500[-train])
## [1] 0.6875
```

Results and Conclusion