Corrigé du TP 4 D'Algèbre linéaire

Exercice 1

```
> restart;
> with(linalg):
Warning, the protected names norm and trace have been redefined and
unprotected
```

1)

a)

```
> u:=vector([4,5,-3]);v:=vector([7,-2,3]);
  matadd(u,v,alpha,beta);
```

$$u := [4, 5, -3]$$

$$v := [7, -2, 3]$$

$$[4\,\alpha + 7\,\beta,\ 5\,\alpha - 2\,\beta,\ -3\,\alpha + 3\,\beta]$$

b)

```
> A:=matrix(3,3,[1,2,-3,5,0,2,1,-1,1]);B:=matrix(3,3,[4,-
  6,9,0,-7,10,5,8,11]);
  matadd(A,B,alpha,beta);
>
```

$$A := \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

$$B := \begin{bmatrix} 4 & -6 & 9 \\ 0 & -7 & 10 \\ 5 & 8 & 11 \end{bmatrix}$$

$$\begin{bmatrix} \alpha + 4\,\beta & 2\,\alpha - 6\,\beta & -3\,\alpha + 9\,\beta \\ 5\,\alpha & -7\,\beta & 2\,\alpha + 10\,\beta \\ \alpha + 5\,\beta & -\alpha + 8\,\beta & \alpha + 11\,\beta \end{bmatrix}$$

2)

```
> e1:=vector([3,1,-4]);e2:=vector([-2,5,3]);e3:=vector([4,7,
  -5]);u:=vector([-1,11,2]);
  v:=matadd(matadd(matadd(e1,e2,x,y),e3,1,z),u,1,-1);#On a
  effectué la combinaison linéaire v=x*e1+y*e2+z*e3-u
```

$$e1 := [3, 1, -4]$$

$$e2 := [-2, 5, 3]$$

$$e3 := [4, 7, -5]$$

$$u := [-1, 11, 2]$$

$$v := [3\,x - 2\,y + 4\,z + 1,\ x + 5\,y + 7\,z - 11,\ -4\,x + 3\,y - 5\,z - 2]$$

```
> solve({v[1]=0,v[2]=0,v[3]=0},{x,y,z});#Résolution du
  système linaire
```

$$\{z = z,\ x = 1 - 2\,z,\ y = 2 - z\}$$

```
> #Ce résultat montre que le système admet une infinité de solutions.
  #Par exemple pour z=0 on x=1 et y=2. On a bien u=e1+2*e3;
```

Exercice 2

```
> restart;with(linalg):
Warning, the protected names norm and trace have been redefined and unprotected
```

1)

```
> u1:=vector([1,1,0]);u2:=vector([1,0,1]);u3:=vector([3,2,-5]);
  A:=stackmatrix(u1,u2,u3);det(A);
```

$$u1 := [1, 1, 0]$$

$$u2 := [1, 0, 1]$$

$$u3 := [3, 2, -5]$$

$$A := \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 3 & 2 & -5 \end{bmatrix}$$

$$6$$

```
> #det(u1,u2,u3)<>0, donc les vecteurs sont linéairement indépendants (libres)
```

2)

a)

```
> v1:=vector([1+theta,1,1,1]);v2:=vector([1,1+theta,1,1]);v3:=vector([1,1,1+theta,1]);
  v4:=vector([1,1,1,1+theta]);;
  A:=stackmatrix(v1,v2,v3,v4);det(A);
```

$$v1 := [1+\theta, 1, 1, 1]$$

$$v2 := [1, 1+\theta, 1, 1]$$

$$v3 := [1, 1, 1+\theta, 1]$$

$$v4 := [1, 1, 1, 1+\theta]$$

$$A := \begin{bmatrix} 1+\theta & 1 & 1 & 1 \\ 1 & 1+\theta & 1 & 1 \\ 1 & 1 & 1+\theta & 1 \\ 1 & 1 & 1 & 1+\theta \end{bmatrix}$$

$$4\,\theta^3 + \theta^4$$

b)

```
> solve(%=0,theta);
```

$$-4, 0, 0, 0$$

```
> #les vecteurs sont liés si et seulement si theta=0 ou -4
```

Exercice 3

```
> restart;with(linalg):
Warning, the protected names norm and trace have been redefined and
unprotected
```

1)

```
> u1:=vector([1,0,1]);u2:=vector([1,1,0]);u3:=vector([1,0,1]
);u4:=vector([0,0,2]);
Base:=basis([u1,u2,u3,u4]);'dim'=nops(Base);
```

$$u1 := [1, 0, 1]$$

$$u2 := [1, 1, 0]$$

$$u3 := [1, 0, 1]$$

$$u4 := [0, 0, 2]$$

$$Base := [u1, u2, u4]$$

$$dim = 3$$

2)

```
> v1:=vector([1,1]);v2:=vector([1,-1]);
B:=basis([v1,v2]);'dim'=nops(Base);
```

$$v1 := [1, 1]$$

$$v2 := [1, -1]$$

$$B := [v1, v2]$$

$$dim = 3$$

```
> #Donc la dimension du s.e.v.engendré par B est égale à 2
qui est la dim de
#l'espace entier càd R 2. Il est égal donc au plan R 2.
```

3)

```
> e1:=vector([1,0,0]);e2:=vector([0,1,0]);e3:=vector([0,0,1]
);
Ba:=basis([evalm(e1-e2),evalm(e2-e3),evalm(e3+e1)]);'dim'=
nops(Ba);
```

$$e1 := [1, 0, 0]$$

$$e2 := [0, 1, 0]$$

$$e3 := [0, 0, 1]$$

$$Ba := [[1, -1, 0], [0, 1, -1], [1, 0, 1]]$$

$$dim = 3$$

```
> # Le s.e.v. engendré par
{evalm(e1-e2),evalm(e2-e3),evalm(e3+e1)} est de dimension
3.
# Il se confond donc avec l'espace R 3.
> #autre méthode
A:=stackmatrix(evalm(e1-e2),evalm(e2-e3),evalm(e3+e1));det
```

```
(A);
```

$$A := \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$2$$

```
> #Donc la famille {evalm(e1-e2),evalm(e2-e3),evalm(e3+e1)} de 3 éléments est libre
  # dans un epace de dimension 3. C'est donc une base de R 3.
```

Exercice 4

```
> restart;with(linalg):
Warning, the protected names norm and trace have been redefined and unprotected
```

1)

```
> w1:=vector([1,2,-1,3]);w2:=vector([2,3,-3,2]);w3:=vector([0,1,1,4]);w4:=vector([1,0,-3,-5]);
  Base:=basis([w1,w2,w3,w4]);'dim'=nops(Base);
```

$$w1 := [1, 2, -1, 3]$$

$$w2 := [2, 3, -3, 2]$$

$$w3 := [0, 1, 1, 4]$$

$$w4 := [1, 0, -3, -5]$$

$$Base := [w1, w2]$$

$$dim = 2$$

```
> # Le rang de la fimille est égal à 2.
```

2)

```
> A:=stackmatrix(w1,w2,w3,w4);
```

$$A := \begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 3 & -3 & 2 \\ 0 & 1 & 1 & 4 \\ 1 & 0 & -3 & -5 \end{bmatrix}$$

```
> rank(A);
```

$$2$$

3)

```
> L:=seq(evalm(col(A,j)),j=1..4);
  basis([L]);nops(%);
```

$$L := [1, 2, 0, 1], [2, 3, 1, 0], [-1, -3, 1, -3], [3, 2, 4, -5]$$

$$[[1, 2, 0, 1], [2, 3, 1, 0]]$$

$$2$$

```
> #Conclusion : voir le cours d'algèbre linéaire
```

Exercice 5

```
> restart;with(linalg):
Warning, the protected names norm and trace have been redefined and
unprotected
```

1)

```
> mat:=proc(n::integer)
  global M;
  local i,j;
  M:=matrix(n,n);
  for i from 1 to n do;
      for j from 1 to n do;
          if j=i then if i=n then M[i,j]:=1;
                      else M[i,j]:=1-(1/2)^(n-i) fi;
          elif j=i-1 then M[i,j]:=(1/2)^(n-i)
          else M[i,j]:=0 fi;
  od;
  od;
  return evalm(M);
  end;
```

```
mat := proc(n::integer)
local i, j;
global M;
    M := matrix(n, n);
    for i to n do for j to n do
            if j = i then
                if i = n then M[i, j] := 1 else M[i, j] := 1 − (1 / 2)^(n − i) end if
            elif j = i − 1 then M[i, j] := (1 / 2)^(n − i)
            else M[i, j] := 0
            end if
        end do
    end do;
    return evalm(M)
end proc
```

```
> C:=mat(15);
```

$$C := \begin{bmatrix} \frac{16383}{16384}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\ \frac{1}{8192}, \frac{8191}{8192}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\ 0, \frac{1}{4096}, \frac{4095}{4096}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\ 0, 0, \frac{1}{2048}, \frac{2047}{2048}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\ 0, 0, 0, \frac{1}{1024}, \frac{1023}{1024}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, \frac{1}{512}, \frac{511}{512}, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0, \frac{1}{256}, \frac{255}{256}, 0, 0, 0, 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0, 0, \frac{1}{128}, \frac{127}{128}, 0, 0, 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0, 0, 0, \frac{1}{64}, \frac{63}{64}, 0, 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{32}, \frac{31}{32}, 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{16}, \frac{15}{16}, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{8}, \frac{7}{8}, 0, 0, 0 \\ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{4}, \frac{3}{4}, 0, 0 \\ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, \frac{1}{2}, 0 \\ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1 \end{bmatrix}$$

2)

```
> F1:=[seq(col(C,i),i=1..6)];F2:=[seq(col(C,i),i=12..15)];
```

$$F1 := \left[\left[\frac{16383}{16384}, \frac{1}{8192}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right],\right.$$
$$\left[0, \frac{8191}{8192}, \frac{1}{4096}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right],$$
$$\left[0, 0, \frac{4095}{4096}, \frac{1}{2048}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right],$$
$$\left[0, 0, 0, \frac{2047}{2048}, \frac{1}{1024}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right],$$
$$\left[0, 0, 0, 0, \frac{1023}{1024}, \frac{1}{512}, 0, 0, 0, 0, 0, 0, 0, 0, 0\right],$$

$$\left[0, 0, 0, 0, 0, \frac{511}{512}, \frac{1}{256}, 0, 0, 0, 0, 0, 0, 0, 0\right]\Bigg]$$

$$F2 := \Bigg[\left[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{7}{8}, \frac{1}{4}, 0, 0\right], \left[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{3}{4}, \frac{1}{2}, 0\right],$$

$$\left[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 1\right], [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]\Bigg]$$

```
> B1:=intbasis(F1,F2);B2:=sumbasis(F1,F2);
```

$$B1 := \{ \}$$

$$B2 := \Bigg[\left[\frac{16383}{16384}, \frac{1}{8192}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right],$$

$$\left[0, \frac{8191}{8192}, \frac{1}{4096}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right],$$

$$\left[0, 0, \frac{4095}{4096}, \frac{1}{2048}, 0, 0, 0, 0, 0, 0, 0, 0, 0\right],$$

$$\left[0, 0, 0, \frac{2047}{2048}, \frac{1}{1024}, 0, 0, 0, 0, 0, 0, 0, 0\right],$$

$$\left[0, 0, 0, 0, \frac{1023}{1024}, \frac{1}{512}, 0, 0, 0, 0, 0, 0, 0\right],$$

$$\left[0, 0, 0, 0, 0, \frac{511}{512}, \frac{1}{256}, 0, 0, 0, 0, 0, 0, 0, 0\right], \left[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{7}{8}, \frac{1}{4}, 0, 0\right],$$

$$\left[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{3}{4}, \frac{1}{2}, 0\right], \left[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 1\right],$$

$$[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]\Bigg]$$

```
> nops(B1);nops(B2);
```

$$0$$

$$10$$

Exercice 6

```
> restart;with(linalg):
Warning, the protected names norm and trace have been redefined and
unprotected
```

1)

```
> A:=matrix(3,3,[-1,1,1,1,0,1,-1,2,3]):
  B:=matrix(3,3,[1,2,-3,3,6,-9,-1,-2,3]):
  C:=matrix(1,2,[2,1]): Delta:=matrix(2,2,[1,2,5,0]):
> kernel(A,'k');k;kernel(B,'r');r;kernel(C,'s');s;kernel(Del
  ta,'t');t;
>
```

$$\{[-1, -2, 1]\}$$

$$1$$
$$\{[-2, 1, 0], [3, 0, 1]\}$$
$$2$$
$$\{[1, -2]\}$$
$$1$$
$$\{\ \}$$
$$0$$

```
> #Donne la base et la dimension de chaque noyau0
```

2)

a)

```
> E1:=[x+2*y=0];kernel(genmatrix(E1,{x,y}),'k');k;
```

$$E1 := [x + 2\, y = 0]$$
$$\{[-2, 1]\}$$
$$1$$

```
> E2:=[x+y=0];kernel(genmatrix(E2,{x,y}),'k');k;
```

$$E2 := [x + y = 0]$$
$$\{[-1, 1]\}$$
$$1$$

```
> F1:=[2*x+3*y-z=0];kernel(genmatrix(F1,{x,y,z}),'k');k;
```

$$F1 := [2\, x + 3\, y - z = 0]$$
$$\{[3, 0, 1], [2, 1, 0]\}$$
$$2$$

```
> F2:=[x+y-z=0,3*x-y+z];kernel(genmatrix(F2,{x,y,z}),'k')
  ;k;
```

$$F2 := [x + y - z = 0, 3\, x - y + z]$$
$$\{[1, 0, 1]\}$$
$$1$$

b)

```
> G1:=[x1+x2+x3+x4=0];kernel(genmatrix(G1,{x1,x2,x3,x4}),
  'k');k;
```

$$G1 := [x1 + x2 + x3 + x4 = 0]$$
$$\{[-1, 0, 0, 1], [-1, 1, 0, 0], [-1, 0, 1, 0]\}$$
$$3$$

```
> G2:=[x1+x2=0,x3+x4=0];kernel(genmatrix(G2,{x1,x2,x3,x4}
  ),'k');k;
```

$$G2 := [x1 + x2 = 0, x3 + x4 = 0]$$
$$\{[0, 0, -1, 1], [-1, 1, 0, 0]\}$$
$$2$$

Exercice 7

```
> restart;with(linalg):
```
Warning, the protected names norm and trace have been redefined and unprotected

1)

```
> A:=matrix(2,2,[1,2,2,1]):B:=matrix(3,3,[1,0,-1,1,2,1,2,2,3]):
> charpoly(A,lambda);charpoly(B,lambda);
```

$$\lambda^2 - 2\lambda - 3$$

$$\lambda^3 - 6\lambda^2 + 11\lambda - 6$$

2)

Pour les valeurs propres

```
> SA:=solve(charpoly(A,lambda),lambda);SB:=solve(charpoly(B,lambda),lambda);
```

$$SA := 3, -1$$

$$SB := 1, 2, 3$$

```
> #autre méthode
  eigenvals(A);eigenvals(B);
```

$$3, -1$$

$$1, 2, 3$$

Pour les sous espaces propres

```
> eigenvects(A);eigenvects(B);
```

$$[-1, 1, \{[-1, 1]\}], [3, 1, \{[1, 1]\}]$$

$$[2, 1, \{[-2, 1, 2]\}], [3, 1, \{[-1, 1, 2]\}], [1, 1, \{[-1, 1, 0]\}]$$

3)

```
> evalb(det(A)=SA[1]*SA[2]);evalb(det(B)=SB[1]*SB[2]*SB[3]);
```

true

true

```
> evalb(trace(A)=SA[1]+SA[2]);evalb(trace(B)=SB[1]+SB[2]+SB[3]);
```

true

true

Exercice 8

```
> restart;with(linalg):
```
Warning, the protected names norm and trace have been redefined and unprotected

1)solve

```
> sys:={x+2*y+z-1,2*x+3*y-z+3,-x+4*y+4*z-3};
```

$$sys := \{x + 2\,y + z - 1, 2\,x + 3\,y - z + 3, -x + 4\,y + 4\,z - 3\}$$

```
> solu:=solve(sys,{x,y,z});
```

$$solu := \{z = 2, y = -1, x = 1\}$$

```
> #La solution sous forme de vecteur
  vector(subs(solu,[x,y,z]));
```

$$[1, -1, 2]$$

2)linsolve *après* genmatrix

```
> S:=genmatrix(sys,[x,y,z],flag);#Génère la matrice
  augmentée du système
  A:=submatrix(S,1..3,1..3);b:=col(S,4);
```

$$S := \begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & 3 & -1 & -3 \\ -1 & 4 & 4 & 3 \end{bmatrix}$$

$$A := \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ -1 & 4 & 4 \end{bmatrix}$$

$$b := [1, -3, 3]$$

```
> linsolve(A,b);
```

$$[1, -1, 2]$$

3)gausselim *puis* backsub

```
> backsub(gausselim(S));
```

$$[1, -1, 2]$$

Exercice 9

```
> restart;with(linalg):
Warning, the protected names norm and trace have been redefined and
unprotected
```

Algorithme de Gauss

a)Procédure pivotcherchega

```
> pivotcherchega:=proc(A::matrix,k::integer)
  global B,ind;
  local i,j,n,p;
  n:=rowdim(A);
  B:=matrix(n,n);
  B:=copy(A);
   p:=B[k,k]:
   ind:=k:
   if (p=0) then
     for i from k+1 to n while p=0 do
```

```
        p:=B[i,k]: ind:=i:
      od:
      if (p=0) then print( "erreur: la matrice n'est pas
   inversible"); break;
   fi;
    fi;
   return (ind);
   end:
```

b)Procédure pivotelimga

```
> pivotelimga:=proc(A::matrix,Y::matrix,k::integer)
  global B,b;
  local i,j,n,p,ech,coef;
  n:=rowdim(A);
  B:=matrix(n,n);
  b:=matrix(n,1);
  B:=copy(A);
  b:=copy(Y);
   for i from k+1 to n do
     coef:=B[i,k]/B[k,k]:
     for j from k to n do
       B[i,j]:=B[i,j]-coef*B[k,j]:
     end do:
     b[i,1]:=b[i,1]-coef*b[k,1]:
   od:
  return [evalm(B),evalm(b)];
  end:
```

c)Procédure triangsolvega

```
> triangsolvega:=proc(A::matrix,Y::matrix)
  global B,b,X;
  local i,j,n,p,somme;
  n:=rowdim(A);
  B:=copy(A);
  b:=copy(Y);
  X:=matrix(n,1);
  X[n,1]:=b[n,1]/B[n,n]:
  for i from 1 to n-1 do
  somme:=0:
  for j from n by -1 to n-i+1 do
  somme:=expand(somme+B[n-i,j]*X[j,1]):
  end do:
  X[n-i,1]:=eval((b[n-i,1]-somme)/B[n-i,n-i]):
  end do:
  #Impression de la solution et du résidu.
  return evalm(X);
```

```
end:
```

d)Procédure gausssolvega

```
> gausssolvega:=proc(A::matrix,Y::matrix)
global C,k,d,F,f,TT;
local j,n,ech,ind;
n:=rowdim(A);
d=matrix(n,1);
C:=matrix(n,n);
C:=copy(A);
d:=copy(Y);
for k from 1 to n-1 do
ind:=pivotcherchega(C,k);#si nécessaire échange des
lignes pour la matrice
     for j from k to n do
           ech:=C[ind,j]:
           C[ind,j]:=C[k,j]:
           C[k,j]:=ech:od:
   ech:=d[ind,1]:# échange des lignes pour le second
membre
   d[ind,1]:=d[k,1]:
   d[k,1]:=ech:
C:=evalm(C);
d:=evalm(d);
TT:=pivotelimga(C,d,k);
C:=evalm(TT[1]);
d:=evalm(TT[2]);
od;
triangsolvega(C,d);
end:
> 
```

Exemple

```
> J:=hilbert(6);w:=matrix(6,1,i->i^2);
```

$$J := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} \\ \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} \\ \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} \end{bmatrix}$$

$$w := \begin{bmatrix} 1 \\ 4 \\ 9 \\ 16 \\ 25 \\ 36 \end{bmatrix}$$

```
> linsolve(J,w);gausssolvega(J,w);
```

$$\begin{bmatrix} -3996 \\ 132510 \\ -1004640 \\ 2857680 \\ -3389400 \\ 1416492 \end{bmatrix}$$

$$\begin{bmatrix} -3996 \\ 132510 \\ -1004640 \\ 2857680 \\ -3389400 \\ 1416492 \end{bmatrix}$$

```
> U:=matrix(6,1,(i,j)->i^2):
  L:=matrix(6,6,(i,j)->i+j):

> gausssolvega(L,U);det(L);
```

"erreur: la matrice n'est pas inversible"

```
Error, (in gausssolvega) break or next outside do loop
```

$$0$$

```
> linsolve(L,U);
```

Résolution numérique

1)Méthode du pivot partiel

a)

i)

```
> eqn:=[1.0*10^(-20)*x+1.0*y-1.0,1.0*x+1.0*y-2.0];
S:=genmatrix(eqn,[x,y],flag);G:=submatrix(S,1..2,1..2);
g:=submatrix(S,1..2,3..3);gausssolvega(G,g);
#on trouve une solution inacceptable !!!
```

$$eqn := [0.1000000000\ 10^{-19}\ x + 1.0\ y - 1.0,\ 1.0\ x + 1.0\ y - 2.0]$$

$$S := \begin{bmatrix} 0.1000000000\ 10^{-19} & 1.0 & 1.0 \\ 1.0 & 1.0 & 2.0 \end{bmatrix}$$

$$G := \begin{bmatrix} 0.1000000000\ 10^{-19} & 1.0 \\ 1.0 & 1.0 \end{bmatrix}$$

$$g := \begin{bmatrix} 1.0 \\ 2.0 \end{bmatrix}$$

$$\begin{bmatrix} 0. \\ 1.000000000 \end{bmatrix}$$

ii)

```
> eqs:=[1.0*x+1.0*y-2.0,1.0*10^(-20)*x+1.0*y-1.0];
T:=genmatrix(eqs,[x,y],flag);F:=submatrix(T,1..2,1..2);
f:=submatrix(T,1..2,3..3);gausssolvega(F,f);
```

$$eqs := [1.0\ x + 1.0\ y - 2.0,\ 0.1000000000\ 10^{-19}\ x + 1.0\ y - 1.0]$$

$$T := \begin{bmatrix} 1.0 & 1.0 & 2.0 \\ 0.1000000000\ 10^{-19} & 1.0 & 1.0 \end{bmatrix}$$

$$F := \begin{bmatrix} 1.0 & 1.0 \\ 0.1000000000\ 10^{-19} & 1.0 \end{bmatrix}$$

$$f := \begin{bmatrix} 2.0 \\ 1.0 \end{bmatrix}$$

$$\begin{bmatrix} 1.000000000 \\ 1.000000000 \end{bmatrix}$$

linsolve donne

```
> linsolve(G,g);#donne la même solution que ii)
```

$$\begin{bmatrix} 1.000000000 \\ 1.000000000 \end{bmatrix}$$

b)

pivotcherchepp

```
> pivotcherchepp:=proc(A::matrix,k::integer)
global B;
local i,j,n,p,ind;
n:=rowdim(A);
B:=matrix(n,n);
B:=copy(A);
 p:=B[k,k]:
```

```
 ind:=k:
   for i from k+1 to n  do
    if abs(p)<abs(B[i,k]) then p:=B[i,k]:
ind:=i:fi:
   end do:
   if (p=0) then print( "erreur: la matrice n'est
pas inversible"); break;
 fi;
return (ind);
end:
```

```
> gausssolvepp:=proc(A::matrix,Y::matrix)
global C,k,d,F,f,TT;
local j,n,ech,ind;
n:=rowdim(A);
d=matrix(n,1);
C:=matrix(n,n);
C:=copy(A);
d:=copy(Y);
for k from 1 to n-1 do
ind:=pivotcherchepp(C,k);#si nécessaire échange
des lignes pour la matrice
     for j from k to n do
           ech:=C[ind,j]:
           C[ind,j]:=C[k,j]:
           C[k,j]:=ech:od:
   ech:=d[ind,1]:# échange des lignes pour le
second membre
   d[ind,1]:=d[k,1]:
   d[k,1]:=ech:
C:=evalm(C);
d:=evalm(d);
TT:=pivotelimga(C,d,k);
C:=evalm(TT[1]);
d:=evalm(TT[2]);
od;
triangsolvega(C,d);
end:
```

```
> gausssolvepp(F,f);gausssolvepp(G,g);
```

$$\begin{bmatrix} 1.000000000 \\ 1.000000000 \end{bmatrix}$$

$$\begin{bmatrix} 1.000000000 \\ 1.000000000 \end{bmatrix}$$

```
> #On trouve la même solution pour les 2 systèmes
équivalents.
```

2)Conditionnement d'un système linéaire

a)

i)

```
> equan:=[10*a+7*b+8*c+7*d-32,7*a+5*b+6*c+5*d-23,8*a
  +6*b+10*c+9*d-33,7*a+5*b+9*c+10*d-31];
  SY:=genmatrix(equan,[a,b,c,d],flag):CD:=submatrix(
  SY,1..4,1..4):
  cd:=submatrix(SY,1..4,5..5):gausssolvega(CD,cd);
```

$$equan := [10\,a + 7\,b + 8\,c + 7\,d - 32,\ 7\,a + 5\,b + 6\,c + 5\,d - 23,\ 8\,a + 6\,b + 10\,c + 9\,d - 33,\ 7\,a + 5\,b + 9\,c + 10\,d - 31]$$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

ii)Le système légèrement modifiés

```
> equans:=[10*a+7*b+8*c+7*d-32-1/(10),7*a+5*b+6*c+5*
  d-23+1/(10),8*a+6*b+10*c+9*d-33-1/(10),7*a+5*b+9*c
  +10*d-31+1/(10)]:
  SYS:=genmatrix(equans,[a,b,c,d],flag):CDS:=submatr
  ix(SYS,1..4,1..4):
  cds:=submatrix(SYS,1..4,5..5):gausssolvega(CDS,cds
  );
```

$$\begin{bmatrix} \frac{46}{5} \\ \frac{-63}{5} \\ \frac{9}{2} \\ \frac{-11}{10} \end{bmatrix}$$

```
> #La solution du système perturbé est très loin de
  celle du système initial
```

b)La matrice de Hilbert

```
> K:=hilbert(10);Y:=matrix(10,1,(i,j)->1);
```

$$K := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} \\ \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} \\ \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} \\ \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} \\ \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} \\ \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} \\ \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} & \frac{1}{19} \end{bmatrix}$$

$$Y := \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

```
> gausssolvega(K,Y);gausssolvega(map(evalf,K),map(evalf
  ,Y));gausssolvepp(map(evalf,K),map(evalf,Y));
```

$$\begin{bmatrix} -10 \\ 990 \\ -23760 \\ 240240 \\ -1261260 \\ 3783780 \\ -6726720 \\ 7001280 \\ -3938220 \\ 923780 \end{bmatrix}$$

$$\begin{bmatrix} 19.37768750 \\ -1328.443633 \\ 22041.88590 \\ -150370.9151 \\ 501865.3834 \\ -834627.5123 \\ 534033.3112 \\ 246764.7138 \\ -511454.9667 \\ 193150.5367 \end{bmatrix}$$

$$\begin{bmatrix} 6.235750600 \\ -477.7088652 \\ 8646.079598 \\ -63064.27868 \\ 220524.5856 \\ -372040.1959 \\ 208393.6138 \\ 201085.1352 \\ -322894.6596 \\ 119904.7026 \end{bmatrix}$$

```
> linsolve(K,Y);
```

$$\begin{bmatrix} -10 \\ 990 \\ -23760 \\ 240240 \\ -1261260 \\ 3783780 \\ -6726720 \\ 7001280 \\ -3938220 \\ 923780 \end{bmatrix}$$

```
> Digits := 20:
> gausssolvega(K,Y);gausssolvega(map(evalf,K),map(evalf
  ,Y));gausssolvepp(map(evalf,K),map(evalf,Y));
```

$$\begin{bmatrix} -10 \\ 990 \\ -23760 \\ 240240 \\ -1261260 \\ 3783780 \\ -6726720 \\ 7001280 \\ -3938220 \\ 923780 \end{bmatrix}$$

$$\left[\begin{array}{c} -9.9999997336410615200 \\ 989.99997693635009884 \\ -23759.999508137570649 \\ 240239.99552505184526 \\ -0.12612599786452908461\ 10^{7} \\ 0.37837799412792821366\ 10^{7} \\ -0.67267199036415208262\ 10^{7} \\ 0.70012799068730860195\ 10^{7} \\ -0.39382199511078529304\ 10^{7} \\ 923779.98924799278539 \end{array}\right]$$

$$\left[\begin{array}{c} -9.9999997725253241600 \\ 989.99998031653145952 \\ -23759.999580495129731 \\ 240239.99618560801499 \\ -0.12612599818071618438\ 10^{7} \\ 0.37837799499975393962\ 10^{7} \\ -0.67267199179829112471\ 10^{7} \\ 0.70012799207639918514\ 10^{7} \\ -0.39382199584151007290\ 10^{7} \\ 923779.99085782792215 \end{array}\right]$$

>