TD n° 3 — Probabilités

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1 Exercice 3

1.1 Préliminaires

1.1.1 Densité du couple

R Loi uniforme sur $\{1, 5\} \times \{a, b\}$

$$f(x) = \frac{1}{b-a} \text{ si } a \leq x \leq b. \text{ sinon } 0$$

$$\begin{aligned} f(x,y) &= \frac{1}{(b-a)^2} \text{ si } a \leq x \leq b \\ &= O \text{ sinon} \end{aligned}$$

1.1.2

$a = 0$ et $b = 1$

$$Z = X + Y$$

$$F(z) = \int\int_{x+y \leq z} f(x,y)dxdy = \int\int \frac{1}{(b-a)^2}dxdy = x + y \leq z$$

$$F(z) = 0 \text{ si } z < O$$

$$F(z) = \int_0^z \int_0^{z-x} \frac{1}{(1-0)^2}dxdy = \int_0^z [y]_0^{z-x}dx = \int_0^z z - xdx = [xz - \frac{x^2}{2}]_0^z$$

$$F(z) = z^2 - \frac{z^2}{2} = \frac{z^2}{2}$$

$f(x,y) = cxy$ si $0 < x < 4$ et$1 < y < 5$ sinon 0.

1.2 Calculer C

$$\begin{aligned} \int_0^4 \int_1^5 cxydxdy = 1 &\Leftrightarrow c[\frac{y2}{2}]_1^5[\frac{x^2}{2}]_0^4 = 1 \\ c[\frac{y^2}{2}]_1^5[\frac{x^2}{2}]_0^4 = 1 &\Leftrightarrow C(\frac{25}{2} - \frac{1}{2})(\frac{16}{2}) = 1 \\ &\Leftrightarrow \frac{c \times 24 \times 16}{2 \times 2} = 1 \\ &\Leftrightarrow C = \frac{1}{96} \end{aligned}$$

1.3 $p(1 < x < 2, 2 < y < 3)$

$$\begin{aligned}
\int_1^2 \int_2^3 \frac{1}{96} xydxdy &= \frac{1}{96} \int_1^2 [x\frac{y^2}{2}]_2^3 dx \\
&= \frac{1}{96} \int_1^2 (\frac{9}{2}x - 2x)dx \\
&= \frac{1}{96} \int_1^2 \frac{5}{2} xdx \\
&= \frac{1}{96} \times [\frac{5x^2}{4}]_1^2 \\
&= \frac{1}{96} \times (5 - \frac{5}{4}) = \frac{1}{96} \times \frac{15}{4} = \frac{5}{128} \\
C[\frac{y^2}{2}][\frac{x^2}{2}]_0^4 &= 1 \Leftrightarrow C(\frac{25}{2} - \frac{1}{2})(\frac{16}{2} = 1 \\
&\Leftrightarrow \frac{c \times 24 \times 16}{2 \times 2} = 1 \\
C &= \frac{1}{96}
\end{aligned}$$

1.4 Lois marginales

$$\begin{aligned}
f_1(x) &= \int_1^5 \frac{1}{96} xydy = [\frac{xy^2}{96 \times 2}]_1^5 = \frac{25}{96} \times 2 - \frac{x}{96 \times 2} \\
&= \frac{24x}{96 \times 2} = \frac{1}{8}x \\
f_2(y) &= \int_0^4 \frac{1}{96} xydx = [\frac{yx^2}{96 \times 2}]_0^4 = \frac{816y}{96 \times 2} = \frac{y}{12}.
\end{aligned}$$

1.5 $F(x, y)$, $(F(x)$, $(F - y)$

$$\begin{aligned}
F(x, y) &= \int_0^x \int_1^y \frac{1}{96} xydxdy \\
F(x) &= \int_0^x \frac{1}{8} xdx \\
F(y) &= \int_1^y \frac{1}{12} ydy \\
F(x, y) &= F(x) \times F(y) \text{ donc x et y sont indépendants}
\end{aligned}$$