Q1

$$\cfrac{\cfrac{\cfrac{\cfrac{Inf(0,1)\quad Inf(1,2)}{Inf(0,1)\wedge Inf(1,2)}(I\wedge)\qquad (Inf(0,1)\wedge Inf(1,2))\to Inf(0,2)}{Inf(0,2)}(E\to)\qquad Inf(2,3)}{Inf(0,2)\wedge Inf(2,3)}(I\wedge)\qquad (Inf(0,2)\wedge Inf(2,3))\to Inf(0,3)}{Inf(0,3)}(E\to)$$

Q2 a. $H \vdash Inf(0,1)$ car $Inf(0,1) \in H$.

b. HI : $\boxed{H \vdash Inf(0,n)}$ et IFD : $\boxed{H \vdash Inf(0,n+1)}$

$H \vdash Inf(0,n) \Rightarrow$ il existe une preuve en D.N de $Inf(0,n)$ sous les hyp. de H, de plus $Inf(n,n+1) \in H$ ainsi que $(Inf(0,n)\wedge Inf(n,n+1)) \to Inf(0,n+1)$

Preuve de $H \vdash Inf(0,n)$

$$\overbrace{\qquad\qquad}^{H}$$
$$\vdots$$

$$\cfrac{\cfrac{Inf(0,n)\quad Inf(n,n+1)}{Inf(0,n)\wedge Inf(n,n+1)}(I\wedge)\qquad (Inf(0,n)\wedge Inf(n,n+1))\to Inf(0,n+1)}{Inf(0,n+1)}(E\to)$$

Preuve de ~~Inf~~ $\vdash Inf(0,n+1)$.

B. <u>Q3</u> Il faut vérifier que $\forall B \in H.\ v(B) = 1$

a. $B \in \{Inf(i,i+1) \mid i \text{ entier}\}$ alors $v(Inf(i,i+1)) = 1$ car $i < i+1$.

b. $B \in \{(Inf(i,j) \wedge Inf(j,k)) \to Inf(i,k) \mid i,j,k : \text{entiers}\}$.

Seul cas où $v(B)$ serait faux ($v(B) = 0$) serait,

$v(Inf(i,j)) = 1 = v(Inf(j,k))$ et $v(Inf(i,k)) = 0$

mais alors $i<j$ et $j<k$ donc $i<k$ et donc $v(Inf(i,k)) = 1$.

<u>Q4</u>

éléments de H

1. $Inf(0,1)$
2. $Inf(1,2)$
3. $(Inf(0,1) \wedge Inf(1,2)) \to Inf(0,2)$

$\vdots$

4 $\neg Inf(0,2)$

$\to : 3$ | $\to : 3$

- 5. $\neg(Inf(0,1) \wedge Inf(1,2))$
  - $\neg\wedge : 5$ — 6. $\neg Inf(0,1)$; $\perp (1,6)$; $\times$
  - $\neg\wedge : 5$ — 7. $\neg Inf(1,2)$; $\perp (2,7)$; $\times$
- 6. $Inf(0,2)$; 7. $\perp (4,6)$; $\times$

tableau fermé !

<u>Q5</u>

a.

$$\dfrac{\dfrac{\dfrac{r \to (p \to q) \qquad \dfrac{\cancel{r \wedge p}^{1}}{r}E_\wedge}{p \to q}E_\to \qquad \dfrac{\cancel{r \wedge p}^{1}}{p}E_\wedge}{q}E_\to}{(r \wedge p) \to q}I_\to(1)$$

b.

$$\dfrac{\dfrac{\dfrac{(a \vee b) \leftrightarrow c}{(a \vee b) \to c}E_\wedge \qquad \dfrac{\cancel{a}^{1}}{a \vee b}I_\vee}{c}E_\to}{a \to c}I_\to(1)$$

c.

par l'absurde

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{p \to (q \to r) \qquad \dfrac{\cancel{p \wedge \neg r}^{1}}{p}E_\wedge}{q \to r}E_\to \qquad \cancel{q}^{2}}{r}E_\to \qquad \dfrac{\cancel{p \wedge \neg r}^{1}}{\neg r}E_\wedge}{\perp}I_\perp}{\neg q}I_\neg(2)}{(p \wedge \neg r) \to \neg q}I_\to(1)$$

Q5

d.

$$\dfrac{\dfrac{p \to (q \vee (r \to s)) \qquad \dfrac{\cancel{p \wedge r}^{(1)}}{p}\,E\wedge}{q \vee (r \to s)}\,E\to \qquad \dfrac{\cancel{q}^{(2)}}{q \vee s}\,I\vee \qquad \dfrac{\dfrac{\cancel{r \to s}^{(3)} \qquad \dfrac{\cancel{p \wedge r}^{(1)}}{r}\,E\wedge}{s}\,E\to}{q \vee s}\,I\vee}{\dfrac{q \vee s}{(p \wedge r) \to (q \vee s)}\,I\to(1)}\,E\vee(2,3)$$

e

$$\dfrac{\dfrac{\neg(p \vee (q \to r)) \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\cancel{\neg q}^{(1)} \qquad \cancel{q}^{(2)}}{\bot}\,I\bot}{r}\,E\bot}{q \to r}\,I\to(2)}{}}{p \vee (q \to r)}\,I\vee}{\bot}\,I\bot}{\dfrac{\neg\neg q}{q}\,E\neg}\,I\neg(1)$$

D. Q6

a. $I(Ana) \wedge A(Ana, Bob)$

b. $\exists x . (M(x) \wedge A(Ana, x))$

c. $\forall x (M(x) \to \neg A(Bob, x)) \wedge A(Bob, Bob)$

d. $\forall x (M(x) \vee I(x))$

e. $\forall x \left(M(x) \to \exists y \left(I(y) \wedge A(x,y) \wedge x \neq y\right)\right)$

f. $\exists x (I(x) \wedge \forall y (M(y) \to A(x,y)))$

g. $\forall x . ((I(x) \wedge \exists y . (M(y) \wedge A(x,y))) \to M(x))$

h. $\forall x . \left(\left(M(x) \wedge \exists y . (M(y) \wedge \exists z . (I(z) \wedge A(y,z)) \wedge A(x,y))\right) \to \forall z . (I(z) \to A(x,z))\right)$

i. $\forall x . ((I(x) \to R(x)) \wedge (Mx \to \neg R(x)))$

Q7

Il faut montrer que:

$a, \ldots, i \vdash \neg \exists x (Mx \wedge Ix)$

$$\dfrac{\dfrac{\cancel{\exists x (Mx \wedge Ix)}^{(0)} \qquad \dfrac{\dfrac{\dfrac{\cancel{Mx \wedge Ix}^{(1)}}{Mx}\,E\wedge \qquad \dfrac{i}{Mx \to \neg Rx}\,E\forall, E\wedge}{\neg Rx}\,E\to \qquad \dfrac{\dfrac{\cancel{Mx \wedge Ix}^{(1)}}{Ix}\,E\wedge \qquad \dfrac{i}{Ix \to Rx}\,E\forall, E\wedge}{Rx}\,E\to}{\bot}\,I\bot}{\bot}\,E\exists(1)}{\neg \exists x (Mx \wedge Ix)}\,I\neg(0)$$