Assignment 2

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Absque sudore et labore nullum opus perfectum est

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Problem 1. Convert the following natural-language text into an equivalent set of propositional premises, together with an associated propositional conclusion. Your answers should identify the text that corresponds to each proposition variable you use and the proposition formulæ that correspond to the premises and the conclusion. Determine if the argument is valid.

1. If the Big Bang Theory is correct then either there was a time before anything existed or the world will come to an end. The world will not come to an end. Therefore, if there was no time before anything existed, the Big Bang Theory is incorrect.
2. To win a gold medal, an athlete must be very fit. If s/he does not win a gold medal, then either s/he arrived late for the competition or his/her training was interrupted. If s/he is not very fit, s/he will blame his/her coach. If s/he blames his/her coach, or his/her training is interrupted, then s/he will will not get into the competition. Therefore, if s/he gets into the competition, s/he will not have arrived late.
3. If Hector wins the battle, he will plunder the city. If he does not win the battle, he will either be killed or go into exile. If he plunders the city, then Priam will lose his kingdom. If Priam loses his kingdom or Hector goes into exile, then the war will end. Therefore, if the war does not end, Hector will be killed.
4. If the colonel was out of the room when the murder was committed then he couldn't have been right about the weapon used. Either the butler is lying or he knows who the murderer was. If Lady Barntree was not the murderer then either the colonel was in the room at the time or or the butler is lying. Either the butler knows who the murderer was or the colonel was out of the room at the time of the murder. Therefore, if the colonel was right about the weapon then Lady Barntree was the murderer.

Solution

1. P: Big Bang Theory is correct, Q: There was a time before anything existed, R: The world will come to an end

$$\frac{P \to (Q \oplus R),\ \neg Q,\ \neg R}{\neg P}$$

We know $\neg Q,\ \neg R$, and thus $(Q \oplus R)$ is $False$ from the first proposition. This clearly implies $\neg P$

2. W: Win a gold medal, F: Athlete is fit, L: Athlete arrived late for the competition, I: Training was interrupted, B: Athlete blames the coach, C: Athlete gets into the competition

$$\frac{W \to F,\ \neg W \to (L \oplus I),\ (B \oplus I) \to \neg C, C}{\neg L}$$

3. H: Hector wins the battle, P: City is plundered by Hector, K: Hector gets killed, E: Hector goes into the exile, L: Priam will lose his kingdom, W: War will end

$$\frac{H \to P,\ \neg H \to (E \oplus K),\ P \to L,\ (L \oplus E) \to W, \neg W}{K}$$

The given conclusion is true. This can be established by backtracking the statements.
We know $\neg W$, this implies from the second last proposition that $\neg L$ and $\neg E$. $\neg L$ clearly implies $\neg P$ from the third last proposition. $\neg P$ implies $\neg H$ from the first equation. $\neg H$ now using the second proposition implies $(E \oplus K)$ but we already know $\neg E$, this clearly concludes on K being $True$

4. O: Colonel was out of the room when the murder was committed, R: Colonel is correct about the weapon used in the murder, B: Butler is lying, K: Butler knows who the murderer, L: Lady Barntree was the murderer

$$\frac{O \to \neg R,\ B \oplus K,\ \neg L \to (B \oplus \neg O),\ K \oplus O,\ R}{\neg L}$$

The given conclusion is false. This can be established by backtracking the statements.
We know R, this implies that $\neg O$ from the first proposition. Using $\neg O$ in the second last proposition we can say that K is true. Using K in the second proposition we can say that B is false. Using all this we can say that $(B \oplus \neg O)$ is $True$. But since $\neg L \to (B \oplus \neg O)$ and $(B \oplus \neg O)$ are both true we can not necessarily say that $\neg L$ is $True$

Problem 2. Which of the following are tautologies? Which are contradictions? Which are neither? Justify your answer with truth tables, or sufficient models as is necessary.

1. $(P \wedge Q) \rightarrow (P \rightarrow Q)$
2. $(P \wedge Q) \leftrightarrow (P \rightarrow Q)$
3. $(\neg P \vee Q) \rightarrow (P \rightarrow \neg Q)$
4. $(((P \rightarrow Q) \rightarrow P) \rightarrow Q)$
5. $(P \rightarrow (Q \rightarrow (P \rightarrow Q)))$
6. $((P \wedge \neg Q) \rightarrow \neg R) \leftrightarrow ((P \wedge R) \rightarrow Q)$
7. $(((P \vee Q) \vee R) \vee S) \leftrightarrow (P \vee (Q \vee (R \vee S)))$
8. $(((P \rightarrow Q) \rightarrow R) \rightarrow S) \leftrightarrow (P \rightarrow (Q \rightarrow (R \rightarrow S)))$
9. $(P \rightarrow (\neg R \rightarrow \neg S)) \vee ((S \rightarrow (P \vee \neg T)) \vee (\neg Q \rightarrow R))$

Solution

Table 1: Part 1. Clearly a Tautology

P	Q	$P \rightarrow Q$	$P \wedge Q$	$(P \wedge Q) \rightarrow (P \rightarrow Q)$
F	F	T	F	T
F	T	T	F	T
T	F	F	F	T
T	T	T	T	T

Table 2: Part 2. Clearly satisfiable

P	Q	$(P \wedge Q) \leftrightarrow (P \rightarrow Q)$
F	F	F
F	T	F
T	F	T
T	T	T

Table 3: Part 3. Clearly satisfiable

P	Q	$(\neg P \vee Q) \rightarrow (P \rightarrow \neg Q)$
F	F	T
F	T	T
T	F	T
T	T	F

Table 4: Part 4. Clearly satisfiable

P	Q	$(((P \rightarrow Q) \rightarrow P) \rightarrow Q)$
F	F	T
F	T	T
T	F	F
T	T	T

Table 5: Part 5. Clearly a Tautology

P	Q	$(P \rightarrow (Q \rightarrow (P \rightarrow Q)))$
F	F	T
F	T	T
T	F	T
T	T	T

Table 6: Part 6. Clearly a Tautology

P	Q	R	$((P \wedge \neg Q) \rightarrow \neg R) \leftrightarrow ((P \wedge R) \rightarrow Q)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

Table 7: Part 7. Clearly a Tautology

P	Q	R	$(((P \vee Q) \vee R) \vee S) \leftrightarrow (P \vee (Q \vee (R \vee S)))$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

Table 8: Part 8. Clearly satisfiable

P	Q	R	S	$(((P \rightarrow Q) \rightarrow R) \rightarrow S) \leftrightarrow (P \rightarrow (Q \rightarrow (R \rightarrow S)))$
T	T	T	T	T
T	T	T	F	T
T	T	F	T	T
T	T	F	F	T
T	F	T	T	T
T	F	T	F	F
T	F	F	T	T
T	F	F	F	F
F	T	T	T	T
F	T	T	F	F
F	T	F	T	T
F	T	F	F	T
F	F	T	T	T
F	F	T	F	F
F	F	F	T	T
F	F	F	F	T

Table 9: Part 9. Clearly a Tautology

P	Q	R	S	T	$(P \to (\neg R \to \neg S)) \vee ((S \to (P \vee \neg T)) \vee (\neg Q \to R))$
T	T	T	T	T	T
T	T	T	T	F	T
T	T	T	F	T	T
T	T	T	F	F	T
T	T	F	T	T	T
T	T	F	T	F	T
T	T	F	F	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	T	T	F	T
T	F	T	F	T	T
T	F	T	F	F	T
T	F	F	T	T	T
T	F	F	T	F	T
T	F	F	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	T	T	F	T
F	T	T	F	T	T
F	T	T	F	F	T
F	T	F	T	T	T
F	T	F	T	F	T
F	T	F	F	T	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	T	T	F	T
F	F	T	F	T	T
F	F	T	F	F	T
F	F	F	T	T	T
F	F	F	T	F	T
F	F	F	F	T	T
F	F	F	F	F	T

Problem 3. For the propositional formulæ in Question 2, use Semantic Tableaux to show whether each is a tautology, contradiction, or neither.

1. $(P \wedge Q) \to (P \to Q)$
2. $(P \wedge Q) \leftrightarrow (P \to Q)$
3. $(\neg P \vee Q) \to (P \to \neg Q)$
4. $(((P \to Q) \to P) \to Q)$
5. $(P \to (Q \to (P \to Q)))$
6. $((P \wedge \neg Q) \to \neg R) \leftrightarrow ((P \wedge R) \to Q)$
7. $(((P \vee Q) \vee R) \vee S) \leftrightarrow (P \vee (Q \vee (R \vee S)))$
8. $(((P \to Q) \to R) \to S) \leftrightarrow (P \to (Q \to (R \to S)))$
9. $(P \to (\neg R \to \neg S)) \vee ((S \to (P \vee \neg T)) \vee (\neg Q \to R))$

Solution

1. $((P \wedge Q) \to (P \to Q)) : F$

   $(P \wedge Q), \neg(P \to Q)$

   $P : T, Q : T$

   $P : T, \neg Q : T$

   x

   Therefore this statement is a Tautology

2. $(P \wedge Q) \leftrightarrow (P \to Q) : F$

   $(P \wedge Q) : T, (P \to Q) : F$ $(P \wedge Q) : T, (P \to Q) : F$

   $P : T, Q : T$ $P : F$ $Q : T$

   $P : T, Q : F$ $Q : F$ $P : F, Q : F$

   x   O   x

   Therefore this statement is satisfiable

3. $(\neg P \vee Q) \to (P \to \neg Q) : F$

   $(\neg P \vee Q) : T, (P \to \neg Q) : F$

   $P : F$ $Q : T$

   $P : T$ $Q : T$ $P : T$ $Q : T$

   x   O   x   O

   Therefore this statement is satisfiable

4. $(((P \to Q) \to P) \to Q) : F$ $(((P \to Q) \to P) \to Q) : T$

   $(P \to Q) \to P) : T$ $Q : F$ $(P \to Q) \to P) : F$ $Q : T$

   $(P \to Q) : F$ $P : T$ $(P \to Q) : T$ $P : F$

   O   O

   $P : T$ $Q : F$ $P : F$ $Q : T$

   O   O   O   O

   Since the proposition can be both True and False it is Satisfiable

5. $(P \to (Q \to (P \to Q))) : F$

   $P : T,\ (Q \to (P \to Q)) : F$

   $Q : T,\ (P \to Q) : F$

   $P : T,\ Q : F$

   x

Therefore this statement is a Tautology

6. $((P \wedge \neg Q) \rightarrow \neg R) \leftrightarrow ((P \wedge R) \rightarrow Q)$

- $((P \wedge \neg Q) \rightarrow \neg R) : F,\ ((P \wedge R) \rightarrow Q) : T$
  - $(P \wedge \neg Q) : T,\ \neg R : F$
    - $P : T,\ Q : F,\ R : T$
      - $(P \wedge R) : F$
        - $P : F$ x
        - $Q : F$ x
      - $Q : T$ x
- $((P \wedge \neg Q) \rightarrow \neg R) : T,\ ((P \wedge R) \rightarrow Q) : F$
  - $P \wedge R : T,\ Q : F$
    - $P : T, R : T$
      - $\neg R : T$
        - $R : F$ x
      - $(P \wedge \neg Q) : F$
        - $P : F$
        - $\neg Q : F$
          - $Q : T$ x

Therefore this statement is a Tautology

7. $(((P \vee Q) \vee R) \vee S) \leftrightarrow (P \vee (Q \vee (R \vee S))) : F$

- $(((P \vee Q) \vee R) \vee S) : T,\ (P \vee (Q \vee (R \vee S))) : F$
  - $\neg P : T,\ \neg(Q \vee (R \vee S)) : T$
    - $\neg Q : T,\ \neg(R \vee S) : T$
      - $\neg R : T, \neg S : T$
        - $((P \vee Q) \vee R)$
          - $(P \vee Q)$
            - P x
            - Q x
          - R x
        - S x
- $\neg(((P \vee Q) \vee R) \vee S) : T, (P \vee (Q \vee (R \vee S))) : T$
  - $((P \vee Q) \vee R) : F, \neg S : T$
    - $\neg(P \vee Q) : T, \neg R : F$
      - $\neg P : T, \neg Q : T$
        - $P : T$ x
        - $(Q \vee (R \vee S)) : T$
          - $Q : T$ x
          - $R \vee S : T$
            - $R : T$ x
            - $S : T$ x

Therefore this statement is a Tautology

8. $(((P \rightarrow Q) \rightarrow R) \rightarrow S) \leftrightarrow (P \rightarrow (Q \rightarrow (R \rightarrow S))) : F$

- $(((P \rightarrow Q) \rightarrow R) \rightarrow S) : T, P \rightarrow (Q \rightarrow (R \rightarrow S))) : F$
  - $P : T, (Q \rightarrow (R \rightarrow S)) : F$
    - $Q : T, (R \rightarrow S) : F$
      - $R : T, S : F$
        - $S : T$ x
        - $((P \rightarrow Q) \rightarrow R) : F$
          - $(P \rightarrow Q), R : F$ x
- $(((P \rightarrow Q) \rightarrow R) \rightarrow S) : F, P \rightarrow (Q \rightarrow (R \rightarrow S))) : T$
  - $S : F, ((P \rightarrow Q) \rightarrow R) : T$
    - $(P \rightarrow Q) : T$
      - $P : F$ O

Thus the statement is Satisfiable

9. $((P \to (\neg R \to \neg S)) \vee ((S \to (P \vee \neg T)) \vee (\neg Q \to R))) : False$

$$
\begin{array}{c}
((P \to (\neg R \to \neg S)) \vee ((S \to (P \vee \neg T)) \vee (\neg Q \to R))) : False \\
| \\
\neg(P \to (\neg R \to \neg S)) : True, \neg((S \to (P \vee \neg T)) \vee (\neg Q \to R)) : True \\
| \\
P : True, \neg(\neg R \to \neg S) : True \\
| \\
\neg(S \to (P \vee \neg T)) : True, \neg(\neg Q \to R) : True \\
| \\
S, \neg(P \vee \neg T) : True \\
| \\
\neg P : True, \neg\neg T : True \\
\text{x}
\end{array}
$$

Therefore this statement is a Tautology

Problem 4. For the propositional formulæ in Question 2, for any case where the formula was a tautology, prove that it is a tautology with a Kalish-Montegue derivations.

1. $(P \wedge Q) \to (P \to Q)$
2. $(P \wedge Q) \leftrightarrow (P \to Q)$
3. $(\neg P \vee Q) \to (P \to \neg Q)$
4. $(((P \to Q) \to P) \to Q)$
5. $(P \to (Q \to (P \to Q)))$
6. $((P \wedge \neg Q) \to \neg R) \leftrightarrow ((P \wedge R) \to Q)$
7. $(((P \vee Q) \vee R) \vee S) \leftrightarrow (P \vee (Q \vee (R \vee S)))$
8. $(((P \to Q) \to R) \to S) \leftrightarrow (P \to (Q \to (R \to S)))$
9. $(P \to (\neg R \to \neg S)) \vee ((S \to (P \vee \neg T)) \vee (\neg Q \to R))$

Solution

1.

1.	~~Show~~ $(P \wedge Q) \to (P \to Q)$	2–7 Conditionalization
2.	$(P \wedge Q)$	1 Ass CD
3.	~~Show~~ $(P \to Q)$	4–5 Conditionalization
4.	P	2 SIMP
5.	Q	2 SIMP

5.

1.	~~Show~~ $(P \to (Q \to (P \to Q)))$	2–8 Conditionalization
2.	P	1 Ass CD
3.	Q	1 Ass CD
4.	~~Show~~ $(Q \to (P \to Q))$	6–7 Conditionalization
5.	Q	3 Repeat
6.	~~Show~~ $(P \to Q)$	6–7 Conditionalization
7.	P	2 Repeat
8.	Q	3 Repeat

Citing De-Morgan's Law proff from question 5

1.	~~Show~~ $((P \wedge \neg Q) \to \neg R) \leftrightarrow ((P \wedge R) \to Q)$	2–20 Conditionalization
2.	~~Show~~ $((P \wedge \neg Q) \to \neg R) \to ((P \wedge R) \to Q)$	1 Subderivation
3.	$((P \wedge \neg Q) \to \neg R)$	2 Ass CD
4.	~~Show~~ $((P \wedge R) \to Q)$	7–8 Sub-Derivation
5.	$P \wedge R$	4 Conditional Derivation
6.	P	5 Simplification
7.	R	5 Simplification
8.	$\neg\neg R$	7 Double Negation
9.	$\neg(P \wedge \neg Q)$	3,7 Modus Tollens
10.	$\neg P \vee Q$	9 de-Morgan's law
11.	Q	6,10 Modus Tollendo Ponens

6.

12.	~~Show~~ $((P \wedge R) \to Q) \to ((P \wedge \neg Q) \to \neg R)$	1 Subderivation
13.	$((P \wedge R) \to Q)$	12 Ass CD
14.	~~Show~~ $((P \wedge \neg Q) \to \neg R)$	7–8 Sub-Derivation
15.	$P \wedge \neg Q$	12 Conditional Derivation
16.	P	5 Simplification
17.	$\neg Q$	5 Simplification
18.	$\neg(P \wedge R)$	13,5 Modus Tollens
19.	$\neg P \vee \neg R$	18 de-Morgan's law
20.	$\neg R$	16,19 Modus Tollendo Ponens
21.	$((P \wedge \neg Q) \to \neg R) \leftrightarrow ((P \wedge R) \to Q)$	2,12 Conditional to Biconditional, DD

7.

1. ~~Show~~ $(((P \vee Q) \vee R) \vee S) \leftrightarrow (P \vee (Q \vee (R \vee S)))$ — 2–20 Conditionalization
2. ~~Show~~ $(((P \vee Q) \vee R) \vee S) \rightarrow (P \vee (Q \vee (R \vee S)))$ — 1 Subderivation
3. $(((P \vee Q) \vee R) \vee S)$ — 2 Ass CD
4. ~~Show~~ $(P \vee (Q \vee (R \vee S)))$ — 7–8 Sub-Derivation
5. $\boxed{P \wedge R}$ — 4 Conditional Derivation
6. ~~Show~~ $(P \vee (Q \vee (R \vee S))) \rightarrow (((P \vee Q) \vee R) \vee S)$ — 1 Subderivation
7. $((P \wedge R) \rightarrow Q)$ — 12 Ass CD
8. ~~Show~~ $((P \wedge \neg Q) \rightarrow \neg R)$ — 7–8 Sub-Derivation
9. $\boxed{P \wedge \neg Q}$ — 12 Conditional Derivation

9.

1. ~~Show~~ $(P \rightarrow (\neg R \rightarrow \neg S)) \vee ((S \rightarrow (P \vee \neg T)) \vee (\neg Q \rightarrow R))$ — 2–20 Conditionalization
2. ~~Show~~ $(((P \vee Q) \vee R) \vee S) \rightarrow (P \vee (Q \vee (R \vee S)))$ — 1 Subderivation
3. $(((P \vee Q) \vee R) \vee S)$ — 2 Ass CD
4. ~~Show~~ $(P \vee (Q \vee (R \vee S)))$ — 7–8 Sub-Derivation
5. $\boxed{P \wedge R}$ — 4 Conditional Derivation
6. ~~Show~~ $(P \vee (Q \vee (R \vee S))) \rightarrow (((P \vee Q) \vee R) \vee S)$ — 1 Subderivation
7. $((P \wedge R) \rightarrow Q)$ — 12 Ass CD
8. ~~Show~~ $((P \wedge \neg Q) \rightarrow \neg R)$ — 7–8 Sub-Derivation
9. $\boxed{P \wedge \neg Q}$ — 12 Conditional Derivation

Problem 5. If a set of premises is inconsistent, then attempting to prove things with these premises is necessarily useless. For example, given the (clearly inconsistent) premises:

1. P
2. $\neg P$

The proof for *any* statement Q is then:

1. ~~Show~~ Q
2. P Premise 1, ID
3. $\neg P$ Premise 2

One technique for determining if a set of premises is inconsistent is to determine if their conjunction is a contradiction (*i.e.*, if there are N premises, identified as P_i for $i = 1$ to N, then the premises are inconsistent if $\forall^N_{T_r} E_P(P_1 \wedge P_2 \wedge \ldots \wedge P_N) \rightarrow F$).

1. Considering all possible models to show that a set of premises is inconsistent will take $O(2^N)$ time, where N is the number of propositional variables. Proving inconsistency instead might be preferable. However, as noted above, proving things with a set of inconsistent premises is necessarily useless. How (precisely) can you prove a set of premises, P_i for $i = 1$ to N, is inconsistent using (a) Semantic Tableaux (b) Kalish-Montegue derivations?
2. It is possible that a set of premises is collectively a tautology (*i.e.*, if there are N premises, identified as P_i for $i = 1$ to N, then $\models (P_1 \wedge P_2 \wedge \ldots \wedge P_N)$). Does this cause a problem for proving things? Are there any other implications of having a set of premises whose conjection is a tautology?
3. Consider the following set of premises: "Sales of houses fall off if interest rates rise. Auctioneers are not happy if sales of houses fall off. Interest rates are rising. Auctioneers are happy."
   (a) Formalize these premises into a set of propositional formulæ.
   (b) Demonstrate that they are inconsistent using truth tables
   (c) Prove that they are inconsistent using a Kalish-Montegue derivation

Solution

(a) S: Sales of houses fall off, I: Interest rates rise, A: Auctioneers are happy
Propositional formulæ: $I \rightarrow S,\ S \rightarrow \neg A,\ I,\ A$
(They are in the same order as the given statements)

(b) According to the problem, the premises are inconsistent if $\forall^N_{T_r} E_P(P_1 \wedge P_2 \wedge \ldots \wedge P_N) \rightarrow F$) for N premises identified as P_i for $i = 1$ to N
Therefore using the same property in our premises:
Conjunction of Premises $= (I \rightarrow S) \wedge (S \rightarrow \neg A) \wedge I \wedge A$

Table 10: The conjunction is clearly always false

I	S	A	$(I \rightarrow S) \wedge (S \rightarrow \neg A) \wedge I \wedge A$
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

(c)

1.	~~*Show*~~ $\neg((I \to S) \wedge (S \to \neg A) \wedge I \wedge A)$	2–8 Conditionalization
2.	I	3rd Premise
3.	$(I \to S)$	1st Premise
4.	$(S \to \neg A)$	2nd Premise
5.	A	4th Premise
6.	$\neg(\neg A)$	5 Double Negation
7.	$\neg S$	3,6 Modus Tollens
8.	$\neg I$	2,7 Modus Tollens

Problem 6. Convert the following propositions to Conjunctive Normal Form (CNF) with at most 3 literals per clause

1. $(P \wedge Q) \rightarrow (P \rightarrow Q)$
2. $(P \wedge Q) \leftrightarrow (P \rightarrow Q)$
3. $(\neg P \vee Q) \rightarrow (P \rightarrow \neg Q)$
4. $(((P \rightarrow Q) \rightarrow P) \rightarrow Q)$
5. $(P \rightarrow (Q \rightarrow (P \rightarrow Q)))$
6. $((P \wedge \neg Q) \rightarrow \neg R) \leftrightarrow ((P \wedge R) \rightarrow Q)$
7. $(((P \vee Q) \vee R) \vee S) \leftrightarrow (P \vee (Q \vee (R \vee S)))$
8. $(((P \rightarrow Q) \rightarrow R) \rightarrow S) \leftrightarrow (P \rightarrow (Q \rightarrow (R \rightarrow S)))$
9. $(P \rightarrow (\neg R \rightarrow \neg S)) \vee ((S \rightarrow (P \vee \neg T)) \vee (\neg Q \rightarrow R))$

Solution

1. True
2. P
3. $\neg P \vee \neg Q$
4. $\neg P \vee Q$
5. True
6. True
7. True
8. $(\neg P \vee Q \vee S) \wedge (P \vee \neg R \vee S)$
9. True

Problem 7. Define:

$$\bigvee_{i=1}^{N} P_i = P_1 \vee P_2 \ldots \vee P_N$$

and

$$\bigwedge_{i=1}^{N} P_i = P_1 \wedge P_2 \ldots \wedge P_N$$

Show that

$$\bigvee_{i=1}^{N} P_i \leftrightarrow \neg \bigwedge_{i=1}^{N} (\neg P_i)$$

and

$$\bigwedge_{i=1}^{N} P_i \leftrightarrow \neg \bigvee_{i=1}^{N} (\neg P_i)$$

are true for $N \in \mathbb{N}$

Hint: in addition to basic Kalish-Montegue derivation, you will need to add induction.

Solution

1.	~~Show~~ $(((P \to Q) \land (\neg R \to \neg Q)) \to (P \to R))$	2–13 Conditionalization
2.	$((P \to Q) \land (\neg R \to \neg Q))$	Supposition
3.	~~Show~~ $(P \to R)$	4–13 Conditionalization
4.	P	Supposition
5.	$((P \to Q) \land (\neg R \to \neg Q))$	2 Repeat
6.	$(P \to Q)$	5 Simplification
7.	Q	4, 6 Modus Ponens
8.	$(\neg R \to \neg Q)$	5 Simplification
9.	~~Show~~ R	10–13 Reductio ad Absurdum
10.	$\neg R$	Supposition
11.	$(\neg R \to \neg Q)$	8 Repeat
12.	$\neg Q$	10, 11 Modus Ponens
13.	Q	7 Repeat