ECE 250 - Fall 2018

Aditya Arora

Week 1 Notes

1 Introduction

Key details given on Course Outline, Project submissions and Policy 71

2 Mathematical background

2.1 Floor and Ceiling Functions

Floor: The *floor* function maps any real number x onto the greatest integer less than or equal to x
- Consider it to be rounding towards *negative infinity* Example: floor(0.5) == 0, floor(3.2) == 3

Ceiling: The *ceilling* function maps any real number x onto the least integer integer greater than or equal to x
- Consider it to be rounding towards *positive infinity* Example: ceil(0.5) == 1, ceil(3.2) == 4

```
// Both of these functions are implemented in the cmath library
#include <cmath>

double floor(double);
double ceil(double);

/*
They're doible because double has a greater range (just under 2^1024)
over long which can represent upto (2^63-1)
*/
```

2.2 L'Hôpital's rule

If you are trying to determine: $\lim_{x \to c} \frac{f(x)}{g(x)}$ but both $\lim_{x \to c} f(x) = \infty$ and $\lim_{x \to c} g(x) = \infty$

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

This rule can be repeated as necessary

2.3 Logarithms and Exponentials

- If $n = e^m$, we define $m = ln(n)$. It is always true that $e^{ln(n)} = n$; however, $ln(e^n) =$ n requires that n is real

- Exponentials grow faster than any non-constant polynomial

$$\lim_{n\to\infty} \frac{e^n}{n^d} = \infty$$

  for any $d > 0$

- Logarithms grow slower than any polynomial

$$\lim_{n\to\infty} \frac{ln(n)}{n^d} = 0$$

  for any $d > 0$

- All logarithms are linear multiples of each other

$$log_b(n) = \frac{ln(n)}{ln(b)}$$

-
```
// the base-2 logarithm log2(n) is written as lg(n)

double log(double); //ln(n)
double log10(double); //log10(n)
```

- $m^{log_b(n)} = n^{log_b(m)}$

2.4 Series

2.4.1 Arithmetic Series

Each term in an arithmetic series is increased by a constant value (usually 1):

$$0 + 1 + \ldots + n = \sum_{k=0}^{n} k = \frac{n(n+1)}{2}$$

$$0^2 + 1^2 + \ldots + n^2 = \sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$0^3 + 1^3 + \ldots + n^3 = \sum_{k=0}^{n} k^3 = (\frac{n(n+1)}{4})^2$$

To generalise:

$$\sum_{k=0}^{n} k^d \approx \int_0^n x^d dx = \frac{n^{d+1}}{d+1}$$

The relative error of approximation of the equation goes to zero as n tends to ∞

2.4.2 Geometric Series

The next series we will look at is the geometric series with common ratio r:

$$\sum_{k=0}^{n} r^k = \frac{1 - r^{n+1}}{1 - r}$$

and if $|r| < 1$ then it is also true that

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1 - r}$$

2.5 Recurrence Relations

- A recurrence relationship is a means of defining a sequence based on previous values in the sequence.
- Such definitions of sequences are said to be *recursive*

Define an initial value: e.g., $x_1 = 1$
Defining x_n in terms of previous values: For example,

$$x_n = x_{n-1} + 2$$

$$x_n = 2x_{n-1} + n$$

$$x_n = x_{n-1} + x_{n-2}$$

-

2.6 Weighted Average

Given n objects $x_1, x_2, x_3, \ldots, x_n$, the average is

$$\frac{x_1 + x_2 + x_3 \ldots + x_n}{n}$$

Given a sequence of coefficients $c_1, c_2, c_3, \ldots, c_n$ where

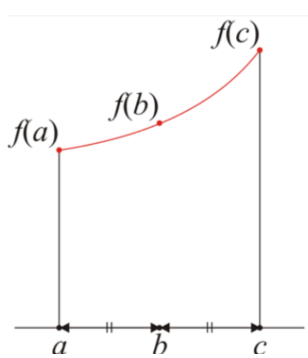
$$c_1 + c_2 + \ldots + c_n = 1$$

then we refer to:

$$\frac{c_1x_1 + c_2x_2 + c_3x_3 \ldots + c_nx_n}{n}$$

as a weighted average
For an average, $c_1 = c_2 = \ldots = c_n = 1$
Examples:

- Simpson's method approximates an integral by sampling the function at three points: f(a), f(b), f(c)
- The average value of the function is approximated by



2.7 Combinations

Given n distinct items, in how many ways can you choose k of these? The number of ways such items can be chosen is written:

$$^{n}C_{k} = \frac{n!}{(k!)(n-k)!}$$

This is also a recursive definition: $^{n}C_{k} = {}^{n-1}C_{k} + {}^{n-1}C_{k-1}$

- These are also the co-efficients of Pascal's Triangle
- They are also the coefficients that we use to expand $(x+y)^{n}$