ECE 250 - Fall 2018

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Week 1 Notes

1 Introduction

Key details given on Course Outline, Project submissions and Policy 71

2 Mathematical background

2.1 Floor and Ceiling Functions

Floor: The *floor* function maps any real number x onto the greatest integer less than or equal to x
- Consider it to be rounding towards *negative infinity* Example: floor(0.5) == 0, floor(3.2) == 3

Ceiling: The *ceilling* function maps any real number x onto the least integer integer greater than or equal to x
- Consider it to be rounding towards *positive infinity* Example: ceil(0.5) == 1, ceil(3.2) == 4

```
// Both of these functions are implemented in the cmath library
# include <cmath>

double floor(double);
double ceil(double);

/*
They're double because double has a greater range (just under 2^1024)
over long which can represent upto (2^63-1)
*/
```

2.2 L'Hôpital's rule

If you are trying to determine: $\lim\limits_{x \to c} \frac{f(x)}{g(x)}$ but both $\lim\limits_{x \to c} f(x) = \infty$ and $\lim\limits_{x \to c} g(x) = \infty$

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

This rule can be repeated as necessary

2.3 Logarithms and Exponentials

- If $n = e^m$, we define $m = ln(n)$. It is always true that $e^{ln(n)} = n$; however, $ln(e^n) =$ n requires that n is real
- Exponentials grow faster than any non-constant polynomial

$$\lim_{n\to\infty} \frac{e^n}{n^d} = \infty$$

  for any $d > 0$
- Logarithms grow slower than any polynomial

$$\lim_{n\to\infty} \frac{ln(n)}{n^d} = 0$$

  for any $d > 0$
- All logarithms are linear multiples of each other

$$log_b(n) = \frac{ln(n)}{ln(b)}$$

- ```
  // the base-2 logarithm log2(n) is written as lg(n)

  double log(double); //ln(n)
  double log10(double); //log10(n)
  ```
- $m^{log_b(n)} = n^{log_b(m)}$
  ```

2.4 Series

2.4.1 Arithmetic Series

Each term in an arithmetic series is increased by a constant value (usually 1):

$$0 + 1 + \ldots + n = \sum_{k=0}^{n} k = \frac{n(n+1)}{2}$$

$$0^2 + 1^2 + \ldots + n^2 = \sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$0^3 + 1^3 + \ldots + n^3 = \sum_{k=0}^{n} k^3 = (\frac{n(n+1)}{4})^2$$

To generalise:

$$\sum_{k=0}^{n} k^d \approx \int_0^n x^d dx = \frac{n^{d+1}}{d+1}$$

Absolute Error:

$$\frac{n^d}{2} \leq \sum_{k=0}^{n} k^d - \frac{n^{d+1}}{d+1} < n^d$$

The relative error of approximation of the equation goes to zero as n tends to ∞

2.4.2 Geometric Series

The next series we will look at is the geometric series with common ratio r:

$$\sum_{k=0}^{n} r^k = \frac{1 - r^{n+1}}{1 - r}$$

and if $|r| < 1$ then it is also true that

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1 - r}$$

2.5 Recurrence Relations

- A recurrence relationship is a means of defining a sequence based on previous values in the sequence.
- Such definitions of sequences are said to be *recursive*

Define an initial value: e.g., $x_1 = 1$
Defining x_n in terms of previous values: For example,

$$x_n = x_{n-1} + 2$$

$$x_n = 2x_{n-1} + n$$

$$x_n = x_{n-1} + x_{n-2}$$

2.6 Weighted Average

Given n objects $x_1, x_2, x_3, \ldots, x_n$, the average is

$$\frac{x_1 + x_2 + x_3 \ldots + x_n}{n}$$

Given a sequence of coefficients $c_1, c_2, c_3, \ldots, c_n$ where

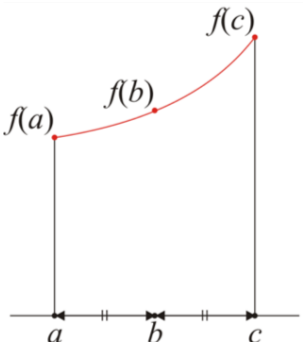
$$c_1 + c_2 + \ldots + c_n = 1$$

then we refer to:

$$\frac{c_1x_1 + c_2x_2 + c_3x_3 \ldots + c_nx_n}{n}$$

as a weighted average
For an average, $c_1 = c_2 = \ldots = c_n = 1$
Examples:

- Simpson's method approximates an integral by sampling the function at three points: f(a), f(b), f(c)
- The average value of the function is approximated by



2.7 Combinations

Given n distinct items, in how many ways can you choose k of these? The number of ways such items can be chosen is written:

$$^nC_k = \frac{n!}{(k!)(n-k)!}$$

This is also a recursive definition: $^nC_k = {}^{n-1}C_k + {}^{n-1}C_{k-1}$

- These are also the co-efficients of Pascal's Triangle
- They are also the coefficients that we use to expand $(x+y)^n$

3 A Brief Introduction to C++

3.1 Operators

```
Assignment              =

Arithmetic              +     -     *     /     %
                        +=    -=    *=    /=    %=

Autoincrement           ++
Autodecrement           --

Logical                 &&        ||      !

Relational              ==    !=    <     <=    >=    >

Comments                /*  Block Comments
                            over multiple lines */
                        // to end of line

Bitwise                 &     |     ^     ~
                        &=    |=    ^=

Bit shifting            <<    >>
                        <<=   >>=
```

3.2 Control Statements

```
/* Conditionals */

if(statement){
    something;
}
else if(statement2){
    something_else;
}
else{
    finally_something;
}

/* Loops */

// for loop
for(int i = 0; i < n; i++){
    statement[i] = i;
}

// while loop
while(statement){
    something;
}

// do while
do{
    something;
}while(statement)
```

3.3 Printing

Printing to console in C++ is done by overloading the << operator.

If the left-hand argument of << is an object of type << ostream (output stream) and the right-hand argument is a double, int, string, or even a custom defined class (provided you have defined a method inside the class to handle this,) an appropriate function which prints the object is called

```
cout << "The square of 3 is " << sqr(3) << endl;
// prints string + int + end of line identifier
// the line above is equivalent to:
((cout << "The square of 3 is ") << sqr(3)) << endl;
// or
operator<<(operator<<(operator<<(cout, "The square of 3 is "), sqr(3)), endl);
```

3.4 Arrays

```
const int ARRAY_CAPACITY = 10; // const prevents reassignment
int array[ARRAY_CAPACITY];
 /* for any number to be used as a size of an array it has to be a const */
array[0] = 1; // setting first element to be 1

for (int i = 0; i < ARRAY_CAPACITY; ++i) {
    // arrays go from 0 to ARRAY_CAPACITY-1
    array[i] = 2*array[i-1] + 1;
}
```

- The **capacity** of an array is the entries it can hold
- The **size** of an array is the number of useful entries

3.5 Functions

```
// A function with a global name
int sqr(int n){
    return n*n;
}

int main(){
    cout << "The square of 3 is " << sqr(3) << endl;
    return 0;
}
```

3.6 Pre-processing statements

Any command starting with a # in the first column is not a C/C++ statement, but rather a preprocessor statement

The preprocessor performs very basic text-based (or lexical) substitutions and the output of which is sent to the compiler

The preprocessor just copy pastes the code from the source file to the file where it is called

```
// Guard statements ensure that we don't import the same file twice

#ifndef SINGLE_LIST_H
#define SINGLE_LIST_H

template <typename Type>

class Single_list {
        ///...
};
#endif
```

3.7 Compilation

In C/C++, the file is the base unit of compilation. Any .cpp file may be compiled into object code Only files containing an int main() function can be compiled into an executable

```
int main () {
    // does some stuff
    return 0;
}
```

The operating system is expecting a return value from `int main()` which is usually 0. Functions called in `int main()` can originate from other files which can be `#include`(d)

3.8 Namespaces

Namespaces can help us differentiate between scopes of variables, functions and classes.

Variables defined:
- In functions are *local* variables
- In classes are *member* variables
- Elsewhere are *global* variables

Functions defined:
- In classes are *member* functions
- Elsewhere are *global* functions

A namespace adds an extra disambiguation between similar names

```
namespace a{
    int n = 3;
    void test(){
        int a = 3, b = 5;
        cout << a + b;
    }
}
// To access variables and functions inside a namespace there are 2 methods:

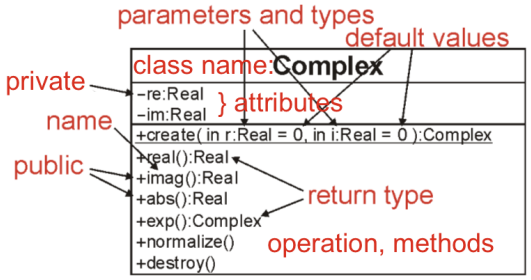
// first method using scope operator (::)
cout << a::n;

// second method using keyword using
using namespace a; /* this maintains the namespace in the
                      following lines until you change it */
test(); // no need for scope operator

/*
The default namespace is std,
all variables and functions in the
standard library are in the std namespace
*/
using namespace std;
```

3.9 Classes

A way to summarize the properties of a class is through UML Class Diagrams.
UML, the Unified Modeling Language is a collection of best practices used in designing/modeling (among other things) software systems




```
// Class declaration, does not include the implementation
#ifndef _COMPLEX_H
#define _COMPLEX_H
#include <cmath>

class Complex {

    private:
        double re, im;

    public:
        Complex(double = 0.0, double = 0.0);

        // Accessors
        double real() const;
        double imag() const;
        double abs() const;
        Complex exp() const;

        // Mutators
        void normalize();
};
```

```
// Class Implementation:
Complex::Complex(double r, double i):re(r),im(i){}
// For built in data-types the above constructor is equal to
Complex::Complex(double r, double i):re(0), im(0){
    re = r;
    im = i;
}

// return the real component
double Complex::real() const {
    return re;
}

// return the imaginary component
double Complex::imag() const {
    return im;
}

// return the absolute value
double Complex::abs() const {
    return std::sqrt(re*re + im*im);
}

// Return the exponential of the complex value
Complex Complex::exp() const {
    double exp_re = std::exp(re);
    return Complex(exp_re*std::cos(im), exp_re*std::sin(im));
}

// Normalize the complex number (giving it unit absolute value, |z| = 1)
void Complex::normalize() {
    if (re == 0 && im == 0) {
        return;
    }

    double absval = abs(); // calls the abs function declared above
    re /= absval;
    im /= absval;
}

#endif
```

3.9.1 Visibility

```
class ClassY;                   // declare that ClassY is a class
class ClassX {
    private:
        int privy;              // the variable privy is private

    friend class ClassY;        // ClassY is a "friend" of ClassX
};

class ClassY {                  // define ClassY
    private:
        ClassX value;           // Y stores one instance of X

    public:
        void set_x() {
            value.privy = 42;   // a member function of ClassY can
        }                       // access and modify the private
};
```

3.9.2 Accessors and Mutators

Accessors: we are accessing and using the class members
Mutators: we are changing—mutating—the class members

Good programming practice is to enforce that a routine specified to be an accessor cannot be accidentally changed to a mutator This is done with the const keyword after the parameter list **double** abs() **const**;

3.9.3 Templates

To build a general class which extends to all data types we use a different mechanism, using a tool called templates
- A function has parameters which are of a specific type
- A template is like a function, however, the parameters themselves are types

```
#include<iostream>
using namespace std;

template <typename Type>
Type sqr(Type x) {
    return x*x;
}

int main() {
    cout << "3 squared is " << sqr<int>(3) << endl;
cout << "Pi squared is " << sqr<double>(3.141592653589793) << endl;
return 0;
}
```

```
#ifndef _COMPLEX_H
#define _COMPLEX_H
#include <cmath>
template <typename Type>

class Complex {

    private:
        Type re, im;

    public:

        Complex(Type const & = Type(), Type const & = Type());

        // Accessors
        Type real() const;
        Type imag() const;
        Type abs() const;
        Complex exp() const;

        // Mutators
        void normalize();
};

// Constructor
template <typename Type>
/*The modifier template <typename Type> applies only
to the following statement, so each time we define a
function, we must restate that Type is a templated symbol:*/

Complex<Type>::Complex(Type const &r, Type const &i):re(r), im(i) {
    // empty constructor
}

// return the real component
template <typename Type>
Type Complex<Type>::real() const {
    return re;
}

// return the imaginary component
template <typename Type>
Type Complex<Type>::imag() const {
    return im;
}

// return the absolute value
template <typename Type>
Type Complex<Type>::abs() const {
    return std::sqrt(re*re + im*im);
}

// return the exponential of the complex value
template <typename Type>
Complex<Type> Complex<Type>::exp() const {
    Type exp_re = std::exp(re);
    return Complex<Type>(exp_re*std::cos(im), exp_re*std::sin(im));
}
```

```
// normalize the complex number (giving it unit norm, |z| = 1)
template <typename Type>
void Complex<Type>:noramlize() {
    if (re == 0 && im == 0) {
        return;
    }

    Type absval = abs();
    re /= absval;
    im /= absval;
}

#endif
```

```
// using the template files
#include <iostream>
#include "Complex.h"

using namespace std;

int main() {
    Complex<double> z(3.7, 4.2);
    Complex<float>  w(3.7, 4.2);
    cout.precision(20); // Print up to 20 digits

    cout << "|z| = " << z.abs() << endl;
    cout << "|w| = " << w.abs() << endl;

    z.normalize();
    w.normalize();

    cout << "After normalization, |z| = " << z.abs() << endl;
    cout << "After normalization, |w| = " << w.abs() << endl;

    return 0;
}
```

3.10 Pointer

- Every variable (barring optimization) is stored somewhere in memory
- That address is an integer, so we can store the address the in a variable
We could simply have an 'address' type: `address ptr;` however, the compiler does not know what it is an address of (is it the address of an int, a double, etc.) Instead, we have to indicate what it is pointing to:

```
int *ptr;   // a pointer to an integer, the address of the integer variable 'ptr'
int m = 5;    // m is an int storing 5
ptr = &m;     // assign to ptr the address of m
cout << ptr << endl; // prints 0xffffd352, a 32-bit number
// note: the computer uses 32-bit addresses
// access/modify what is stored at that memory location by using the * operator
cout << *ptr << endl; // prints 5
*ptr = 3;    // store 3 at that memory location
cout << m << endl;  // prints 3
```

```
// pointers to objects must, similarly be dereferenced:
Complex z( 3, 4 );
Complex *pz;
pz = &z;
cout << z.abs() << endl;
cout << (*pz).abs() << endl;
// shorthand pz->abs() == (*pz).abs()
cout << pz->abs() << endl;
```

3.11 Memory Allocation

Memory allocation in C++ and C# is done through the `new` operator
This is an explicit request to the operating system for memory This is a very expensive operation
The OS must:
- Find the appropriate amount of memory,
- Indicate that it has been allocated, and
- Return the address of the first memory location C++ requires the user to explicitly de-allocate memory, For each new, there must be a corresponding delete

Note:
- managed C++ has garbage collection
- other tools are also available for C++ to perform automatic garbage collection

Inside a function, memory allocation of declared variables is dealt with by the compiler automatically Memory for a single instance of a class (one object) is allocated using the new operator, e.g.,

```
Complex<double> *pz = new Complex<double>( 3, 4 );
cout << "The address pz is " << pz << endl; // The address pz is 0x00ef3b40
// new operator returns the address of the first byte of the memory allocated
delete pz; // de-allocates the memory
```