ELECENG 3CL4: Introduction to Control Systems
Lab 2: Closed-loop System Identification

Dr. Tim Davidson and Dr. Shahrukh Athar
Ext. 24818, 26503
davidson@mcmaster.ca, athars3@mcmaster.ca

Objective

To identify the plant (process) model of a marginally-stable DC servomotor.

Assessment

This laboratory is conducted in groups of no more than two students. Students are required to attend their assigned lab section. The assessment of this lab will occur based on your answer to the pre-lab questions, in-lab activities, and a written laboratory report. **Each group is required to submit their answers to the pre-lab questions as a hard copy at the beginning of their designated lab session to the Teaching Assistants (TAs) on duty.** No marks will be awarded to pre-labs submitted 10 minutes after the designated start time of the lab session. Only one submission is necessary per group. **Both group members should clearly state their full name, student numbers, Lab section, and Class section(s) on the title page of the pre-lab answers document.** It is up to you if you want to submit a handwritten document or a printed one. You will earn a maximum of 100 marks from Lab 2 activities. **Lab 2 will constitute 5% of your total grade for this course**. The components of the assessment are:

- Pre-lab Questions (30 marks), which must be completed before the lab and submitted at the beginning of the lab. **This Lab Manual contains 10 Pre-Lab questions.**
- Two experiments (40 marks). See Section 3 of the Lab handout.
- A laboratory report (30 marks). See Section 4 of the Lab handout.

Your performance in the experiments will be evaluated during the lab by the TA's. The marks for each component are clearly indicated in this lab manual.

1 Description of Laboratory Equipment

As explained in Lab 1, in these laboratories we will deal with a closed-loop angular positioning system based around a DC motor. Such systems are often used to position heavy or difficult to move objects using a 'command tool' that is easy to move, in which case they are often called servomechanisms. One example of a servomechanism is that involved in moving

We would like to acknowledge the efforts of Dr. Ayman Negm in the development of this lab.

the control surfaces of an aircraft using a lever in the cockpit. The goal of this lab is to identify the plant (or process) model for subsequent experiments. The term *identify* means you will determine the transfer function and certain associated parameters of the plant (the motor) through a mix of mathematical derivations and experimental measurements.

In our system, the plant (or process) is the DC motor and its associated electronics. The input to the process is a control voltage, denoted $x(t)$, and the output of the process is the angular position of the shaft, denoted by $\theta(t)$, and measured by an optical encoder. For the purposes of our ELECENG 3CL4 labs, we will presume a linear model for the plant. As explained in lectures, we can use information about the structure of the motor and the theory of rotational Newtonian mechanics to derive the following linear time-invariant differential equation model for the operation of the motor:

$$J\frac{d^2\theta(t)}{dt^2} + b\frac{d\theta(t)}{dt} = K_m x(t), \tag{1}$$

where J is the rotational inertia of the motor, b is the coefficient of viscous friction in the motor structure, and K_m is the (internal) gain of the motor. Taking Laplace transforms of both sides of (1) we obtain the transfer function of the plant:

$$s^2 J\Theta(s) + sb\Theta(s) = K_m X(s) \tag{2}$$

$$\implies G(s) = \frac{\Theta(s)}{X(s)} = \frac{A}{s(s\tau_m + 1)}, \tag{3}$$

where $A = K_m/b$ and $\tau_m = J/b$. Note that A is a *gain parameter* while τ_m is the motor's *time constant*. **In this lab we will identify the parameters A and τ_m, as these are not known in advance in typical industrial applications**. We will do this in two ways:

- The first is based on a time-domain analysis of the step response of a proportionally-controlled closed-loop system with an appropriately chosen gain.
- The second is based on an analysis of the frequency response of a proportionally-controlled closed-loop system with an appropriately chosen gain.

In the following labs we will use the model in (3), with the parameters identified in this laboratory, to design more sophisticated controllers for the servomechanism. Thus, it is recommended that you use the same workstation in subsequent labs for which you identify parameters in this lab since these parameters may vary from workstation to workstation. If you use another workstation in the future labs then you should identify its parameters first, i.e., repeat Lab 2 before doing the next lab on it (to avoid doing so, just use the same workstation in future labs as you use in Lab 2). Also notice that the transfer function of our plant in (3) is second-order. As we saw in class, the transfer function of a DC motor is typically third-order. However, as we also discussed in class, we can approximate the third-order DC motor system with a second-order one to simplify our analysis and also to

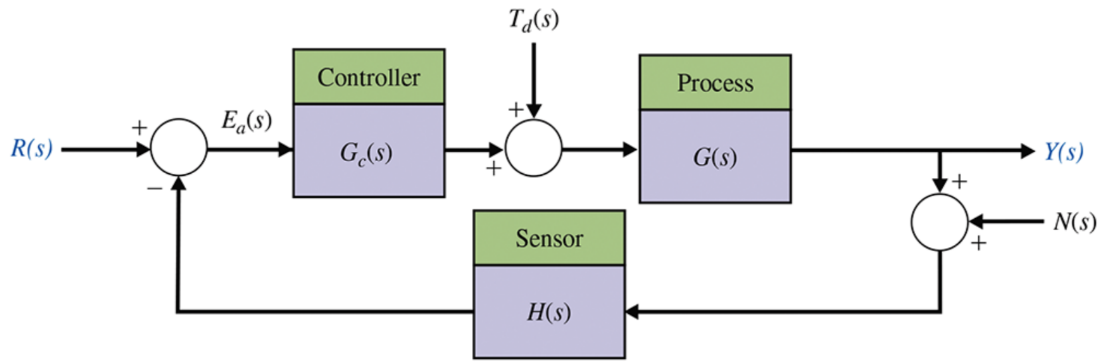

Figure 1: Feedback system with $Y(s) = \Theta(s)$. We will consider the case in which $H(s) = 1$ and $G_c(s) = K$. We neglect the effects of the disturbance $T_d(s)$ and the noise $N(s)$ in the identification of the plant model. (Figure 4.4 of Dorf and Bishop, *Modern Control Systems*, 14th edition, Pearson, 2022.). Note: This figure is in the Laplace domain.

enable using the understanding of second-order systems that we have developed. We are doing such an approximation here as well. Equation (1) is a simplified description of our plant leading to a second-order transfer function in (3), i.e., a second-order approximation.

The labs in this course (i.e., this and the next three labs) are based around an understanding of the model $G(s)$ in (3). **To help develop that understanding, please answer the following pre-lab questions.** Also note that in the pre-lab questions that follow, you will develop or prove some mathematical equations that will then help you identify the parameters A and τ_m for your plant.

Pre-lab Question 1 (2 marks) *Provide a complete derivation of the step response of the model $G(s)$.*

Pre-lab Question 2 (2 marks) *Is that step response bounded? Justify your answer.*

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2 Closed Loop System Identification

Since it has a pole at $s = 0$, the system $G(s)$ in (3) is only 'marginally stable', and this can make it quite difficult to identify A and τ_m directly. In this section we will show how to set up a stable closed loop system, that can be used to identify A and τ_m.

We will construct a stable closed loop by using a simple proportional controller, $G_c(s) = K$, in the configuration in Figure 1, with $H(s) = 1$. We will also neglect the effects of the disturbance $T_d(s)$ and the noise $N(s)$ in the identification of the plant model.

Pre-lab Question 3 (5 marks) *Provide detailed derivations that show when $H(s) = 1$,*

(i) the closed loop transfer function from $R(s)$ to $Y(s) = \Theta(s)$ in Figure 1 can be written in the generic form as in (4a)*; and*

(ii) when $G(s) = \frac{A}{s(s\tau_m+1)}$ and $G_c(s) = K$ the generic form can be re-written in the form as in (4b)*.*

Here,

$$T(s) = \frac{\Theta(s)}{R(s)} = \frac{G(s)G_c(s)}{1 + G(s)G_c(s)} \tag{4a}$$

$$= \frac{KA/\tau_m}{s^2 + (1/\tau_m)s + KA/\tau_m} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{4b}$$

where $\omega_n = \sqrt{\frac{KA}{\tau_m}}$ and $\zeta = \frac{1}{2\omega_n\tau_m}$.

Pre-lab Question 4 (3 marks) *Derive expressions that would enable you to determine A and τ_m as functions of ζ and ω_n.*

2.1 Closed-loop System Identification from the Step Response (Time Domain)

If the input $r(t)$ is a unit step function, then the output of the system in Figure 1 is the step response, and can be computed using

$$\theta_{\text{step}}(t) = \mathscr{L}^{-1}\left\{\frac{T(s)}{s}\right\}, \tag{5}$$

where $w(t) = \mathscr{L}^{-1}\{W(s)\}$ denotes the inverse Laplace Transform of $W(s)$ written as a function of t. Now assume that K is chosen so that $T(s)$ (4b) is under-damped. That is, K is chosen such that $[s^2 + (1/\tau_m)s + KA/\tau_m]$ has complex roots. Equivalently, K is chosen such that $0 < \zeta < 1$. We have discussed the general solution of second-order underdamped systems in class and you have seen it in previous courses as well. In this case, the step response for $t \geq 0$ is,

$$\theta_{\text{step}}(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n\sqrt{1-\zeta^2}\, t + \phi\right), \tag{6}$$

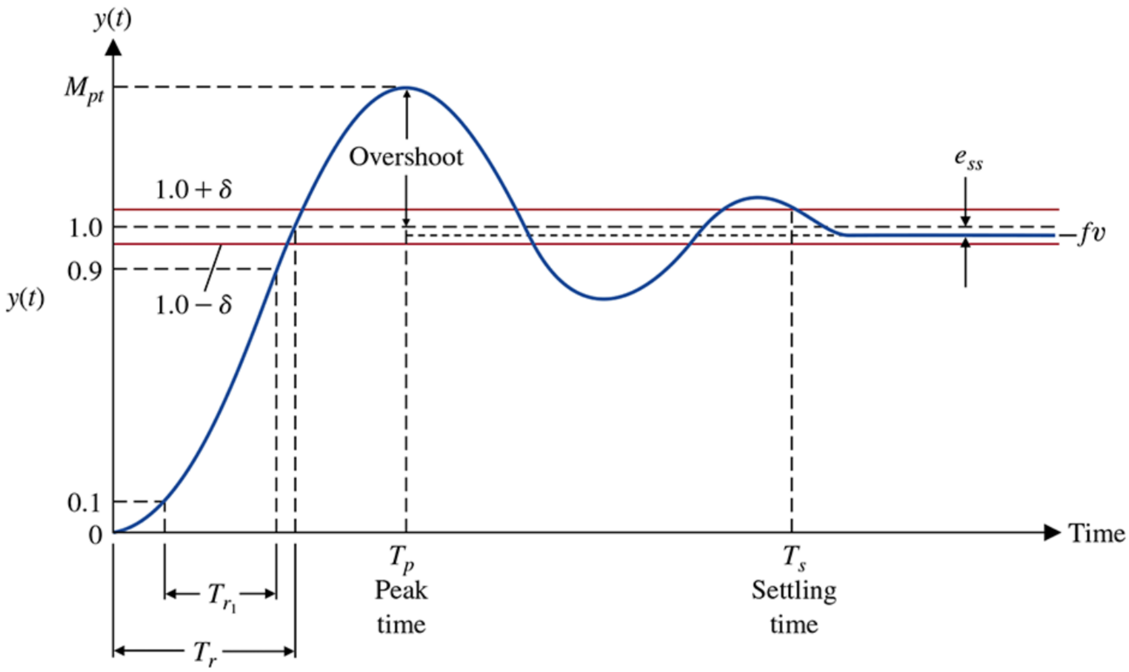

Figure 2: Generic step response of an under-damped second-order system (Figure 5.6 of Dorf and Bishop, *Modern Control Systems*, 14th edition, Pearson, 2022).

where $\phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right) = \cos^{-1}(\zeta)$. A plot of a generic step response from a standard under-damped second-order system is given in Figure 2.

Pre-lab Question 5 (3 marks) *Consider the generic step response from Figure 2. Show that the percent overshoot is*

$$P.O. = 100 \exp \frac{-\zeta\pi}{\sqrt{1-\zeta^2}} \tag{7}$$

Pre-lab Question 6 (3 marks) *Show that the peak time is determined by:*

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \tag{8}$$

Pre-lab Question 7 (3 marks) *Derive expressions that would enable you to calculate ζ and ω_n as functions of P.O. and T_p.*

2.2 Closed-Loop System Identification using Frequency Response

If the closed loop is appropriately under-damped, the values of A and τ_m can be identified from the 'peak' of the frequency response of $T(s)$ in (4b) in the following way. Observe that:

$$|T(j\omega)|^2 = \frac{\omega_n^4}{|\omega_n^2 - \omega^2 + j2\zeta\omega_n\omega|^2}. \tag{9}$$

Sketches of $|T(j\omega)|$ on a log-log scale for different values of ζ are provided in Figure 3.

Pre-lab Question 8 **(3 marks)** *By differentiating the denominator of* (9) *with respect to* ω *and setting the derivative to zero, show that for* $\zeta \leq 1/\sqrt{2}$ *the denominator reaches a minimum, and hence* $|T(j\omega)|^2$ *reaches a maximum, when* $\omega = \omega_p$*, where*

$$\omega_p = \omega_n\sqrt{1 - 2\zeta^2}. \tag{10}$$

Hint: Recall that for a complex number $a + jb$*, the square of the magnitude is* $|a + jb|^2 = a^2 + b^2$*.*

Pre-lab Question 9 **(3 marks)** *Show that the value of the peak in the frequency response is*

$$M_p^2 = \max_{\omega} |T(j\omega)|^2 = |T(j\omega_p)|^2 = \frac{1}{4\zeta^2(1-\zeta^2)}. \tag{11}$$

Pre-lab Question 10 **(3 marks)** *Derive expressions that would enable you to calculate* ζ *and* ω_n *as functions of* ω_p *and* M_p

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3 Perform Closed Loop System Identification

We will perform the required closed loop system identification, i.e., **find A and τ_m in the plant's (process's) transfer function**, using two approaches, both of which will utilize the work that was done in the Pre-Lab:

- **Time domain identification**: We will observe the step response of the system using a proportional feedback gain of $K = 4$. We will experimentally measure peak time (T_p) and percentage overshoot ($P.O.$), as defined in Figure 2 and its associated discussion in Section 2.1, based on which we can calculate the unknown plant parameters.
- **Frequency domain identification**: We will observe the frequency response of the system using a proportional feedback gain of $K = 4$. We will experimentally measure the peak gain (M_p) and the frequency at which this peak occurs (ω_p), as discussed in Section 2.2, based on which we can calculate the unknown plant parameters.

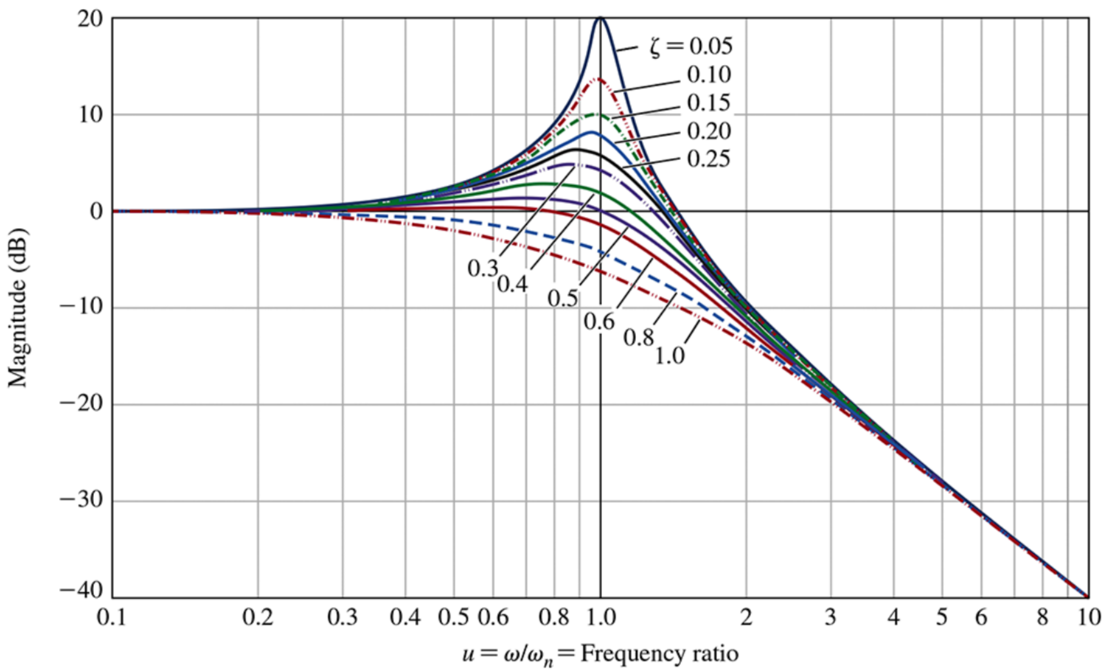

Figure 3: Bode plots of $|T(j\omega)|$ in (9) for various values of ζ with a normalized frequency axis (Figure 8.10a of Dorf and Bishop, *Modern Control Systems*, 14th edition, Pearson, 2022).

3.1 Experiment 1: Time Domain Identification (20 marks)

The following steps need to be followed in order to measure the percentage overshoot ($P.O.$) and peak time (T_p) of the closed-loop system from which the motor transfer function parameters A and τ_m will be calculated.

i) Download the *Simulink starter file* from Avenue (Content > Laboratory > Simulink Starter File > EE3CL4_Lab.slx). You should create a separate folder for each part of this lab on the computer. For now, place the starter file in the Experiment 1 folder and name it appropriately. You will build your Lab 2 model using the same base file.

ii) Open MATLAB, then Simulink, and open your Lab 2 Simulink file that was saved above.

iii) Find the *HIL Write Analog*, *HIL Read Encoder*, *Pulse Generator*, *Sum*, *Gain*, *Scope* and *To Workspace* blocks from the respective Simulink libraries and add them to the model. Follow Figure 1 to build a feedback control system with proportional gain in the Simulink environment. Refer to the Lab 1 manual if you need more information on where to find these blocks. The only new block mentioned here (that was not discussed in Lab 1) is the *To Workspace* block which can be found in the Library: Simulink > Sinks. This block is used to export data from your Simulink model to the MATLAB workspace where you can do further processing on it such as generate plots or do statistical analysis. Open the *To Workspace* block and set its *Save format* to "*Structure With Time*".

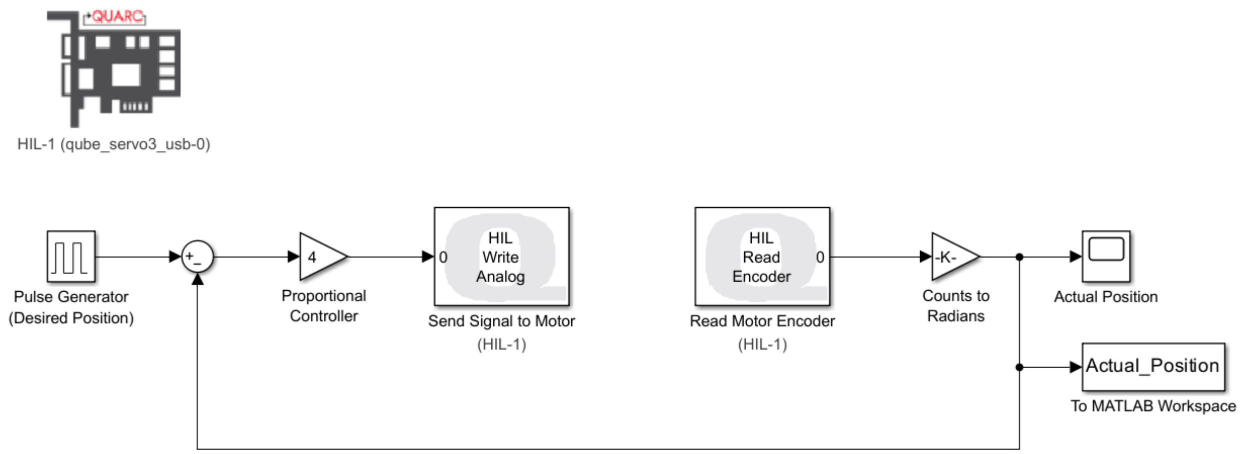

Figure 4: Target Simulink Model for Experiment 1.

iv) When you are done building your model by following Figure 1, it should look like the target model shown in Figure 4. **STOP and Note:** Remember from Lab 1 that the output of the *Read Encoder* block is in *Counts*. You must convert *Counts to Radians* by using a *Gain* block at the output of the *Read Encoder* block as shown in Figure 4. Failure to do so may lead to damaging your motor.

v) Open the *Pulse Generator* block and set the *Amplitude*, *Period* and *Pulse Width* to 1, 6 and 50, respectively. This would set up the reference input $r(t)$ as a pulse train with amplitude of 1 radian or 57.3°, period of 6 seconds, and duty cycle of 50%.

vi) Open the *Gain* block representing the *Proportional Controller* (see Figure 4) and set its *Gain* to 4, i.e., $K = 4$. This gain value will lead to an under-damped system (second-order) which is what we want. This *Gain* block is your controller for this lab, i.e., we are using a *proportional* controller.

vii) Open the *Gain* block at the output of the *Read Encoder* block which is meant to convert *Counts to Radians* (see Figure 4). Set its gain so that it receives *Counts* at its input and outputs position in *Radians* (Hint: You have already done this in Lab 1; Double-check with the TAs if you are not sure).

viii) We will run the motor only for 6 second intervals in this experiment. To do this, go to the *Modeling* tab in the Simulink window and then click on *Model Settings*. In the *Solver* tab and in the *Simulation time* area, set the *Stop time* to be 6 seconds. Click *Apply* and then OK. Save your Simulink model.

ix) Turn ON the Qube-Servo 3 motor block using its ON/OFF button. Align the engraved line on the red circular load with the 0^o mark at the top of the Qube-Servo 3.

x) In the Simulink window, go to the *Hardware* tab and press *Build* (found in the *Build, Deploy & Start* drop-down menu) to generate the Simulink model and then press the *Monitor and Tune* button to initiate the experiment.

xi) Next, you need to make your measurements. You can do this by either sending your data to the MATLAB workspace by using the *To Workspace* block or by using cursors in the Simulink *scope* window. **Important Note:** In any case, save the data for at least one complete step response period to the MATLAB workspace so that you can copy it for later use to generate required plots in your lab report.

xii) **System Identification in Time Domain: Measurement Procedure**

- Refer to Section 2.1 of the Pre-Lab. We need to experimentally measure $P.O.$ and T_p in this part of the lab.
- We have set up our model such that an entire period of the step response (6 seconds) will be visible on your Simulink *scope* screen. The initial part of your step response should look like Figure 2 which is what we are interested in.
- Measure the height of the first overshoot peak which is the largest overshoot peak. This will give you the peak value of the step response denoted as M_{pt} in Figure 2.

  M_pt = 1.61

- Next measure the steady-state value of the step response which occurs after the settling time and is denoted as fv in Figure 2. We called it y_{ss} in the Pre-Lab hints document.

  y_ss = 0.994

- Once you have both M_{pt} and fv (or y_{ss}), you can find percent overshoot ($P.O.$) as given in Equation (2) of the Pre-Lab hints document.

  P.O = 62.0%

- Next measure the peak time T_p, i.e., the time at which the maximum peak of the step response occurs (where M_{pt} occurs). T_p is the time difference between the time when the first peak occurs and the point where the input square wave went from low to high (at which point the step response waveform will start going from low to high). This is important because if you just note down the time at which the peak occurs then this will only be valid if you are making your measurement for the very first period of the waveform which starts at the origin. However, if you are making this measurement for a subsequent period then you must measure T_p relative to where that particular waveform started going from low to high.

  T_p = 0.094s

- Now that you have both the $P.O.$ and T_p experimental measurements, go ahead and plug these values into the ζ and ω_n expressions that you obtained in terms of $P.O.$ and T_p in Pre-Lab Question 7. This will provide you with the ζ (damping ratio) and ω_n (natural frequency) values for your process (DC motor).
- Finally, use the ζ and ω_n values, obtained above, in the expressions derived in Pre-Lab Question 4 to obtain the required A and τ_m values for your system. Congratulations, you have identified the required system parameters in time domain!

  A = 28.1
  t_m = 0.098s

xiii) Turn OFF the Qube-Servo 3 motor.

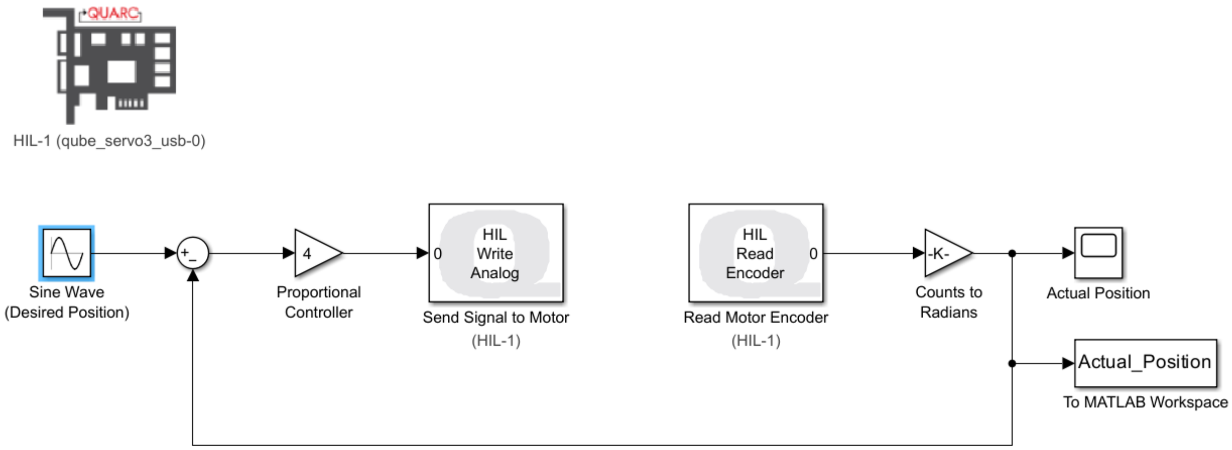

Figure 5: Target Simulink Model for Experiment 2.

At this point, you should call the Teaching Assistant to have your Experiment 1 evaluated. You must demonstrate the closed-loop step response and the experimentally determined values of A and τ_m to your TAs in order to obtain the marks for this part.

3.2 Experiment 2: Frequency Domain Identification (20 marks)

The following steps need to be followed in order to measure the peak frequency ω_p and peak gain M_p of the closed-loop system from which the motor transfer function parameters A and τ_m will be calculated.

i) Continue with the model that you developed for the time domain case. Replace the pulse train input $r(t)$ with a sinusoidal input. This can be done by replacing the *Pulse Generator* block with a *Sine Wave* block (Simulink Library: Simulink > Sources). Set the amplitude of the sine wave to 1 which would set up the reference input $r(t)$ as a sine wave with amplitude of 1 radian or 57.3°. Set the phase of the sine wave to zero and its frequency initially to $2\pi \times 1$ rad/sec. When done, your model should look like the target model shown in Figure 5.

ii) We will keep the *Gain* of the *Proportional Controller* gain block (see Figure 5) set at 4 (i.e., $K = 4$) as that leads to a sufficiently underdamped system as required to do these calculations. Remember, this *Gain* block is your *proportional* controller for this lab.

iii) Ensure that the *Gain* block at the output of the *Read Encoder* block is doing the *Counts to Radians* conversion (see Figure 5) as was done in Experiment 1.

iv) We will run the motor for as long as required in this experiment. To do this, go to the *Modeling* tab in the Simulink window and then click on *Model Settings*. In the *Solver* tab and in the *Simulation time* area, set the *Stop time* to be *inf*. Click *Apply* and then OK. Save your Simulink model.

v) Turn ON the Qube-Servo 3 motor block using its ON/OFF button. Align the engraved line on the red circular load with the 0^o mark at the top of the Qube-Servo 3.

vi) In the Simulink window, go to the *Hardware* tab and press *Build* (found in the *Build, Deploy & Start* drop-down menu) to generate the Simulink model and then press the *Monitor and Tune* button to initiate the experiment.

vii) Next, you need to make your measurements. You can do this by either sending your data to the MATLAB workspace by using the *To Workspace* block or by using cursors in the Simulink *scope* window. We strongly recommend that you use the latter option (Simulink scope and cursors) as that will substantially reduce the time required to perform this experiment. Ask the TAs for help, if required.

viii) **System Identification in the Frequency Domain: Measurement Procedure**

- Refer to Section 2.2 of the Pre-Lab. We need to experimentally measure M_p and ω_p in this part of the lab.
- Observe that the signal at the output (running with a frequency of $2\pi \times 1$ rad/sec) is around the same size as the signal at the input (though it is a bit distorted, i.e., not quite sinusoidal). Gain-wise this is what we would expect from the magnitude of the frequency response in Figure 3 (Recall that 0 dB corresponds to a gain of 1). Note down the amplitude value of the output signal.
- Without stopping or pausing the simulation, change the input frequency to $2\pi \times 2$ rad/sec. After the transients have settled, observe that the gain of the system is now slightly greater than one and that the non-linear effects have somewhat diminished. [Remember: by *gain of the system* we mean the amplitude of the output sine wave divided by the amplitude of the input sine wave (which is 1)]. Note down the amplitude value of the output sine wave.
- Without stopping or pausing the simulation, change the input frequency to $2\pi \times 3$ rad/sec. After the transients have settled, observe that the gain of the system is now further greater than one and that the non-linear effects have further diminished. Note down the amplitude value of the output sine wave.
- Repeat the above-mentioned process at input frequencies of $2\pi \times 4$, $2\pi \times 5$, $2\pi \times 6$, and $2\pi \times 7$ rad/sec. After the transients have settled, note down the amplitude value of the output sine wave in each case.
- What we can note above is that as we increased the frequency of the input sine wave from $2\pi \times 1$ rad/sec to $2\pi \times 7$ rad/sec, the gain of the system first went up and then down. Although we are in the time domain, we are actually conducting our analysis in the frequency domain because the only variation that is occurring between the various runs of the experiment is the frequency of the input sine wave. Thus, the system gain that we see here, which is the the amplitude of the output sine wave divided by the amplitude of the input sine wave, is representative of the frequency response (magnitude) of the system.

Coarse = 5 Hz
Fine = 5.2 Hz

- While we did a *coarse-grain* search until now, to further isolate the value and location of the peak in the frequency domain we need to perform a *fine-grain* search. Of the observations made above, identify the frequency for which the amplitude of the output sine wave was the maximum. Next, test a set of frequencies around this initially identified frequency, but this time, with a much smaller difference between successive frequencies and measure the amplitude of the output sine wave each time. For example, if your initial peak amplitude occurred at $2\pi \times x$ rad/sec, then you can identify a region of let's say $2\pi \times [x - 0.5 : 0.1 : x + 0.5]$ rad/sec around this initial frequency and test your system for each of the 11 frequency values. In each measurement, note down the amplitude of the resulting output sine wave. Your goal is to pinpoint the frequency at which the amplitude of the output sine wave becomes maximum. For this you must ensure that the amplitude falls on either side of this target frequency value. If this is not the case, redefine your test region.

Amplitude = 3.387
M_p = 3.387
w_p = 5.2 Hz

- As a result of the above-mentioned *fine-grain* search, the frequency at which the amplitude of the output sine wave becomes maximum is ω_p (Section 2.2). Furthermore, $M_p = \frac{\text{Amplitude of resulting output sine wave}}{1}$, where M_p is the peak value of the frequency response (Section 2.2).
- Now that you have both the M_p and ω_p experimental measurements, go ahead and plug these values into the ζ and ω_n expressions that you obtained in terms of M_p and ω_p in Pre-Lab Question 10. This will provide you with the ζ (damping ratio) and ω_n (natural frequency) values for your process (DC motor).

A = 27.8
t_m = 0.100s

- Finally, use the ζ and ω_n values, obtained above, in the expressions derived in Pre-Lab Question 4 to obtain the required A and τ_m values for your system. Congratulations, you have identified the required system parameters in frequency domain!

ix) Before you leave, make sure that you send data for the frequency (only one case) where you received the maximum output sine wave amplitude to MATLAB by using the *To Workspace* block. This data will be needed to provide relevant plot(s) in the lab report.

x) Turn OFF the Qube-Servo 3 motor.

At this point, you should call the Teaching Assistant to have your Experiment 2 evaluated. You must demonstrate the closed-loop frequency response and the experimentally determined values of A and τ_m to your TAs in order to obtain the marks for this part.

4 Laboratory report (30 marks)

Each group (of two students) must submit an electronic report through the Avenue to Learn Dropbox. Only one student in each group needs to submit the report in their Class section's Avenue to Learn Dropbox. The other student should verify the submission. The report is due by 11:59 pm seven days from the day of your lab. For example, if your lab is on February 1, 2025, then your report would be due on February 8, 2025 at 11:59 pm.

The report should be submitted as a PDF and formatted in single-column, single-spaced, using Times New Roman 12 or equivalent font. The group members should clearly state their individual contributions to the report in a statement in the beginning of the report. Both group members should clearly state their full name, student numbers, Lab section, and Class section(s) on the title page of the report. The report file should be named as: 3CL4_Lab_A_BB_Student1_Student2.pdf, where *A* is the lab number, *BB* is the lab section that the students belong to, *Student1* is the student number of the first student and *Student2* is the student number of the second student. A 10% penalty will be applied if the above-mentioned title page and/or file naming conventions are not followed. **The laboratory report must include the following items:**

- A brief description of the objective of this lab.
- A discussion of the time domain approach for the identification of the motor transfer function parameters. You should include a relevant data plot and your calculations for A and τ_m in time domain. You should also provide the motor transfer function in the form of Equation (3) but with the measured plant parameters (A and τ_m) plugged in.
- A discussion of the frequency domain approach for the identification of the motor transfer function parameters. You should include a relevant data plot and your calculations for A and τ_m in frequency domain. You should also provide the motor transfer function in the form of Equation (3) but with the measured plant parameters (A and τ_m) plugged in.