ELECENG 3CL4: Introduction to Control Systems
Lab 5: Phase Lead-Lag Compensator Design Using Root Locus

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Objective

To design a phase lead-lag compensator for a marginally-stable servomotor using the root locus technique.

Assessment

This laboratory is conducted in groups of no more than two students. Students are required to attend their assigned lab section. The assessment of this lab will occur based on your answers to the pre-lab questions and in-lab activities. **This lab does not require a laboratory report**. **Each group (of two students) must submit an electronic pre-lab report through the Avenue to Learn Dropbox.** Only one student in each group needs to submit the pre-lab report in their Class section's Avenue to Learn Dropbox. The other student should verify the submission. **The pre-lab report is due by 2:00 pm on the day of your lab. Your lab section's Dropbox will close accordingly.** For example, if your lab is on March 31, 2025, then your pre-lab report would be due on March 31, 2025 at 2:00 pm. The pre-lab report should be submitted as a PDF and formatted in single-column, single-spaced, using Times New Roman 12 or equivalent font. Both group members should state their individual contributions to the report in a statement in the beginning of the report. Both group members should clearly state their full name, student numbers, Lab section, and Class section(s) on the title page of the pre-lab report. The report file should be named as: 3CL4_Prelab_A_BB_Student1_Student2.pdf, where *A* is the lab number, *BB* is the lab section that the students belong to, *Student1* is the student number of the first student and *Student2* is the student number of the second student. A 10% penalty will be applied if the above-mentioned title page and/or file naming conventions are not followed. You will earn a maximum of 100 marks from Lab 5 activities. **Lab 5 will constitute 5% of your total grade for this course**. The components of the assessment are:

- Section 1: Pre-lab warm-up design exercise: Root locus based phase lead-lag control design and evaluation (15 marks);
- Section 2: Pre-Lab design of a phase lead-lag compensator for the lab DC servomotor (45 marks);
- Section 3: In-lab experiments to verify that the phase lead-lag compensator designed in Section 2 satisfies the design requirements (40 marks);

Your performance in the experiments will be evaluated during the lab by the TA's. The marks for each component are clearly indicated in this lab manual.

1 Pre-lab Warm-Up Design Exercise (15 marks)

The purpose of this pre-lab design exercise is to prepare you for the actual phase lead-lag controller design in Section 2. The plant (process) model parameters and design requirements in the pre-lab are somewhat different from those in the actual lab, but the design steps are essentially the same.

We would like to acknowledge the efforts of Dr. Ayman Negm in the development of this lab.

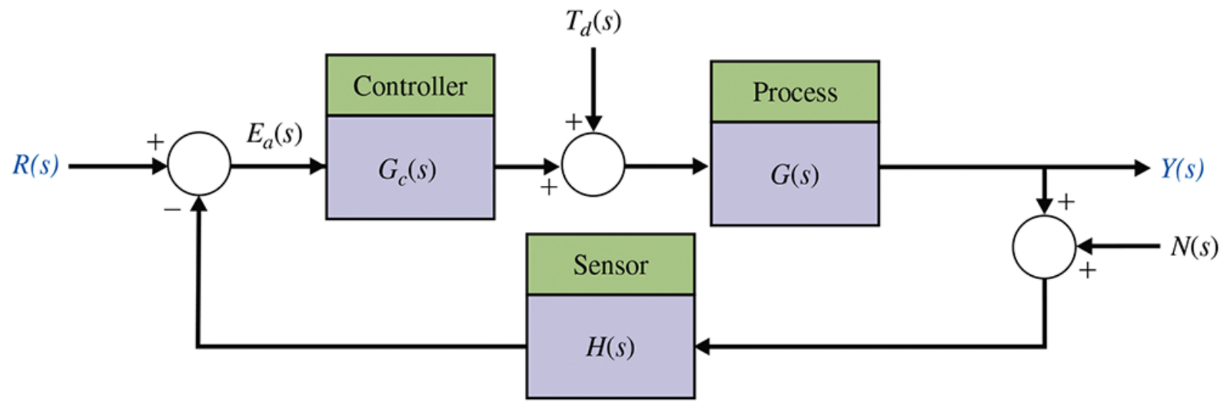

Figure 1: Block diagram of a generic feedback control system. We will focus on the case in which $H(s) = 1$, noise is negligible ($N(s) = 0$) and there is no disturbance ($T_d(s) = 0$) (Figure 4.4 of Dorf and Bishop, *Modern Control Systems*, 14th edition, Pearson, 2022). Note: This figure is in the Laplace domain and $Y(s) = \Theta(s)$.

Consider the closed loop feedback control system in Figure 1 where $H(s) = 1$, $N(s) = 0$, $T_d(s) = 0$, and $G(s) = \frac{4.7}{s(s+3.2)}$. In the pre-lab exercise of Lab 4, for this $G(s)$, you designed a phase-lead compensator of the form $G_c(s) = k_c\frac{s+z}{s+p}$, $p > z$, so that for a unit step input the percentage overshoot of the output $y(t)$ was 10% and its 2% settling time was 1 second. In this Section 1 pre-lab exercise, you will modify your controller from Pre-lab 4 (Section 1) to satisfy the following additional design requirement:

- The velocity error constant, k_v, is increased by a factor of **10** compared to its value with the phase-lead compensator alone.

To achieve this objective, you should modify your phase-lead controller from Pre-lab 4 (Section 1) by adding a phase-lag compensator ($z_l > p_l$) as follows:

$$\bar{G}_c(s) = G_c(s) \cdot \frac{s + z_l}{s + p_l} = k_c \frac{s + z}{s + p} \cdot \frac{s + z_l}{s + p_l} \tag{1}$$

Here $G_c(s)$ is the phase-lead controller from the pre-lab exercise of Lab 4 Section 1, and $\frac{s+z_l}{s+p_l}$ ($z_l > p_l$) is an additional lag compensator to be designed. The combined new controller $\bar{G}_c(s)$ is a phase lead-lag compensator. The phase lead component provides for the transient response requirements whereas the phase lag part helps achieve the steady-state tracking objective. The design approach taken here first deals with the transient response goals with the phase lead controller (that you have already done in Pre-lab 4) and then augments the phase lag controller for the steady-state error objective.

To this end, do the following pre-lab exercises:

Pre-Lab Question 1 **(2 marks)** Retrieve the parameters of the phase lead compensator, k_c, z, and p from the pre-lab exercise of Lab 4 Section 1. Also state the velocity error constant, k_v, that you computed for the lead compensated system in Lab 4 Section 1.

Pre-Lab Question 2 **(7 marks)** Select the values of z_l and p_l for the phase lag compensator to satisfy the new requirement on k_v, without significantly impacting the overshoot and settling time values achieved by the design of the phase lead compensator. Justify your design choices using the principles of root locus.

Pre-Lab Question 3 **(6 marks)** Use MATLAB (*rlocus* command) to plot the root locus of the following two closed-loop systems: 1) The phase lead compensated system developed in Pre-Lab 4 (Section 1); 2) The newly designed phase lead-lag compensated system. How does the root locus of only the lead compensated system compare with that of the lead-lag compensated system? Is this what you expected? Justify the outcomes and provide both root locus plots in your pre-lab report.

Your pre-lab report must include all relevant computations and plots.

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2 Pre-Lab Design of a Phase Lead-Lag Compensator for the Lab DC Servomotor ($35 + 10 = 45$ marks)

Note: This section is part of the Pre-Lab for Lab 5. However, it also has an in-lab component where you must show your work in Section 2 to the TAs at the beginning of the lab so that the TAs can verify the correctness of your work in this section. Since your work in Section 2 is the basis of the in-lab experiments in Section 3, this will ensure that any mistakes are caught early. The pre-lab component of this section has 35 marks while the in-lab verification component has 10 marks.

In Lab 4 (Section 2.1), you designed a phase lead compensator for the **lab DC servomotor** to achieve the following two transient response objectives for the step response of the closed-loop control system:

1. A 10% percentage overshoot in the step response.
2. A 2% settling time of 0.3 seconds in the step response.

While maintaining these objectives (i.e., the work done already in Lab 4 Section 2.1), the goal here is to additionally:

3. Increase the velocity error constant, k_v, by a factor of 10 compared to its value under phase lead compensation alone.

Thus, in this section, you are required to design a phase lead-lag compensator **for the lab DC servomotor** that satisfies the above three design objectives simultaneously. You should start with the phase lead compensator that you designed for the lab DC servomotor in Lab 4 Section 2.1.

This part of the pre-lab also has MATLAB-based components. *The following MATLAB commands may be used to complete the steps below*: **step**, **feedback**, **tf**, **zero**, **pole**, **lsim**. In the MATLAB command line, type *doc step* to open MATLAB documentation on how to use this command. You can similarly refer to MATLAB help to see how all of these commands are to be used. Example of workflow: You can pass the numerator and denominator coefficients of the process or controller transfer functions to the **tf** command and it will return the process or controller transfer function. You can then pass these transfer functions to the **feedback** command to build a closed-loop transfer function. Next you can pass this closed-loop system to the **step** command to obtain the step response or to the **lsim** command to obtain the ramp response. Remember, if your input $r(t)$ is a ramp, then $r(t) = At$, where A is the slope of the ramp (for unit ramp $A = 1$). Thus, your **lsim** command would look something like this for the ramp case: *lsim(sys, u, t)*, where *sys* is the closed-loop system, *u* is the ramp signal, and *t* is time. See MATLAB help for more details.

For this part of the pre-lab, you must perform the following tasks and use MATLAB when required:

Pre-Lab Question 4 **(2 marks)** Retrieve the parameters of the phase lead compensator, k_c, z, and p, that you designed **for the lab DC motor servomechanism** in Lab 4 (Section 2.1). Also state the velocity error constant, k_v, that you computed for the lead compensated system in Lab 4 Section 2.1.

Pre-Lab Question 5 **(8 marks)** Follow the procedure in the pre-lab warm-up exercise (Section 1) to augment the phase lead compensator designed in Lab 4 (Section 2.1) **for the lab DC motor servomechanism** with a phase lag compensator in order to satisfy the additional design requirement on k_v. You have now designed a phase lead-lag compensator for your lab DC motor.

Pre-Lab Question 6 **(3 marks)** Determine the open-loop transfer function, $\bar{G}_c(s)G(s)$, where $\bar{G}_c(s)$ is the phase lead-lag compensator that you designed in Pre-Lab Question 5 above (refer to Eq. 1) and $G(s)$ is the transfer function of the process, i.e., your lab's DC motor.

Pre-Lab Question 7 **(6 marks)** Determine the closed-loop transfer function and its poles and zeros for the phase lead-lag compensator based open-loop system that you determined in Pre-Lab Question 6. Compare these (closed-loop transfer function, poles and zeros) with those of the system with only the phase lead compensator (built in Lab 4 Section 2.1). Discuss the impact of adding the phase lag compensator to the lead system developed in Lab 4 (Section 2.1). You can do so by comparing the closed-loop transfer functions of the two systems, their respective poles and zeros, and specifically by comparing the locations of the dominant poles. Using MATLAB in this question will significantly reduce your workload.

Pre-Lab Question 8 **(8 marks)** Use MATLAB to plot and then compare the the step responses of the closed-loop systems with the phase lead and phase lead-lag compensators. Does your phase lead-lag compensator satisfy the percentage overshoot and 2% settling time design requirements? In case any deviations are observed, explain why you think these deviations happened.

Pre-Lab Question 9 **(8 marks)** Use MATLAB to plot and then compare the ramp responses of the closed-loop systems with the phase lead and phase lead-lag compensators. Use a ramp input signal of the form $r(t) = 6t$, where time t should cover approximately six time constants of the slowest closed loop pole in the system. Does your phase lead-lag compensator based system satisfy the velocity error constant, k_v, design requirement compared to the k_v for the phase lead compensator based system?

Your pre-lab report must include all relevant computations and plots.

Note: To obtain marks for the nine pre-lab questions in Sections 1 and 2 of this lab, you must provide answers to all these questions in your pre-lab report where you must ensure that required computations, plots, comparisons, and analysis of results are provided as required. Furthermore, you should call the Teaching Assistant to have your Section 2 work evaluated at the beginning of the lab session. You must show a TA your worked solution for Section 2, including the computed position for the closed-loop poles and zeros, the final form of the phase lead-lag compensator, and the computed value of k_v. You also need to demonstrate whether the closed-loop transfer function (with lead-lag compensation) fulfills the three design requirements.

3 In-Lab Experiments: Implementation of the Phase Lead-Lag Compensator (40 marks)

In this section, you will implement the phase lead-lag compensator and experimentally obtain the closed-loop system responses to step and ramp inputs. To this end, follow the procedure outlined below:

3.1 Step Response (20 marks)

i) Download the *Simulink starter file* from Avenue (Content > Laboratory > Simulink Starter File > EE3CL4_Lab.slx). You should create a separate folder for this lab on the computer. Place the starter file in the Lab 5 folder and name it appropriately (e.g., Lab5_Exp1). Alternatively, you can use the Simulink file developed in Lab 4 as a base for this lab.

ii) Open MATLAB, then Simulink, and open your Lab 5 Simulink file that was saved above.

iii) Find the *HIL Write Analog*, *HIL Read Encoder*, *Pulse Generator*, *Sum*, *Gain*, *Scope*, *To Workspace* and *Transfer Fcn* blocks from the Simulink library and add them to the model.

iv) Follow Figure 1 to build the feedback control system in Simulink environment. As mentioned in the figure caption, $H(s) = 1$ (replaced by a wire) and $N(s) = 0$ (noise is negligible). For this section, we will also not consider the disturbance $T_d(s)$.

v) Model the *reference input* ($r(t)$ in time domain, which is shown as $R(s)$ in Laplace domain in Figure 1) by using a *Pulse Generator* block and set its *Amplitude*, *Period* and *Pulse Width* to 1.5, 6 and 50, respectively. This would set up the reference input $r(t)$ as a pulse train with amplitude of 1.5 radians or 85.94°, period of 6 seconds, and duty cycle of 50%.

vi) Open the *Transfer Fcn* block and enter the parameters of the transfer function of the phase lead-lag compensator that you have designed in Section 2.

vii) We will run the motor only for 6 second intervals in this experiment. To do this, go to the *Modeling* tab in the Simulink window and then click on *Model Settings*. In the *Solver* tab and in the *Simulation time* area, set the *Stop time* to be 6 seconds. Click *Apply* and then OK. Save your Simulink model.

viii) When you are done building your model by following Figure 1, it should look like the target model shown in Figure 2. **STOP and VERIFY the following (Failure to do so may lead to damaging your motor):**

- Have you converted *counts* to *radians* at the output of the *Read Encoder* block? (Hint: You have already done this in Labs 1, 2, 3, 4).
- Make sure that the closed-loop is set as a negative feedback loop.

ix) Turn ON the Qube-Servo 3 motor block using its ON/OFF button. Align the engraved line on the red circular load with the 0^o mark at the top of the Qube-Servo 3.

x) In the Simulink window, go to the *Hardware* tab and press *Build* (found in the *Build, Deploy & Start* drop-down menu) to generate the Simulink model and then press the *Monitor and Tune* button to initiate the experiment.

xi) Send your data to the MATLAB workspace by using the *To Workspace* block and save it for at least one complete step response period. You will need to use the *save* command in MATLAB to do so. Ask the TAs for help, if required.

xii) Verify that the control system design objectives of percentage overshoot and 2% settling time are satisfied for the step response (use the cursors in the Simulink scope or work in MATLAB workspace).

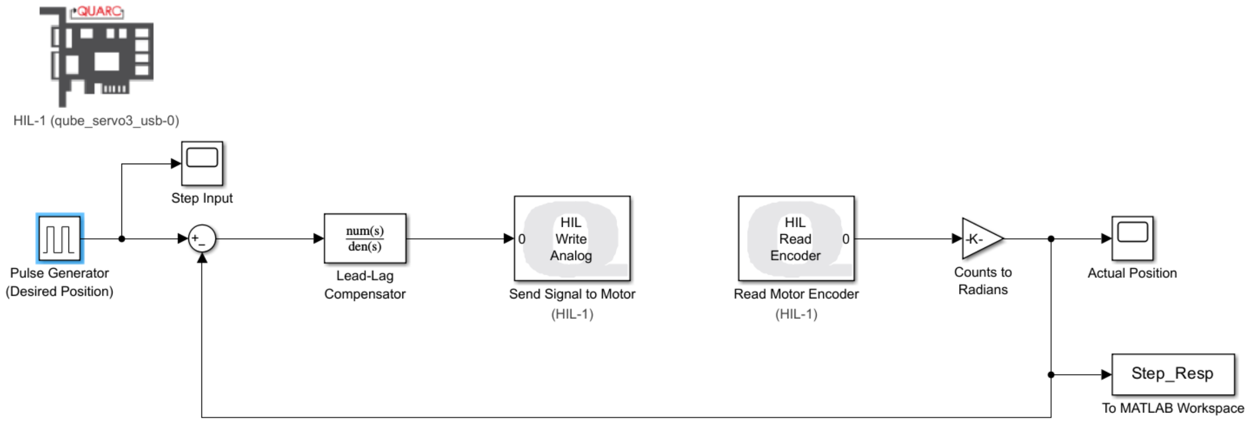

Figure 2: Target Simulink model for the step response experiment with Lead-Lag compensator.

xiii) Turn OFF the Qube-Servo 3 motor.

At this point, you should call the Teaching Assistant to have your Section 3.1 Step Response work evaluated. **Note**: To obtain marks for this section, you must show a TA the step response and that your lead-lag compensated model fulfills the percentage overshoot and 2% settling time design requirements.

3.2 Ramp Response (20 marks)

i) Continue working with the Simulink model developed in Section 3.1 for step response.

ii) To obtain the *Ramp response*, change the reference input to a triangle wave. This can be done by replacing the *Pulse Generator* block with the *Smooth Signal Generator* block. Open the *Smooth Signal Generator* block and select the "Triangle" wave as the signal type. Set the *Amplitude*, *Frequency* and *Initial phase* to 360 radians, 1/120 Hz, and $\pi/2$ radians, respectively. For this experiment, we will run the motor for 120 second intervals. Go to the *Modeling* tab in the Simulink window and then click on *Model Settings*. In the *Solver* tab and in the *Simulation time* area, set the *Stop time* to be 120 seconds. Click *Apply* and then OK. Save your Simulink model.

iii) Press *Build* to generate the Simulink model and then press the *Monitor and Tune* button to initiate the experiment. Send your data to MATLAB workspace and save it.

iv) Measure the steady-state error, e_{SS}, and the velocity error constant, k_v, for the lead-lag compensated system and note them down.

- Follow the procedure specified in the Lab 4 manual (Section 3.2 Point (ix) on Page 6) to do so. You should measure e_{SS} after the ramp response has sufficiently settled, i.e., e_{SS} does not appear to reduce any more. For example, you can measure e_{SS} at around the 85 second mark.

v) Now replace the lead-lag compensator with the lead compensator that you developed in Lab 4. Save your Simulink model and again press *Build* to generate the Simulink model and then press the *Monitor and Tune* button to initiate the experiment. Send your data to MATLAB workspace and save it.

vi) Measure the steady-state error, e_{SS}, and the velocity error constant, k_v, for the lead compensated system and note them down.

vii) Compare the e_{SS} and k_v values measured for the lead-lag compensated system with those measured for the lead compensated system. Does your lead-lag compensated system increase the velocity error constant, k_v, by a factor of 10 compared to the lead compensated system, as required? By extension, the steady-state error of the lead-lag compensated system should be 10 times lower than that of the lead compensated system. If there is a deviation from the design requirement, can you explain it?

viii) Turn OFF the Qube-Servo 3 motor.

At this point, you should call the Teaching Assistant to have your Section 3.2 Ramp Response work evaluated. **Note**: To obtain marks for this section, you must show a TA the ramp response and your measured k_v and e_{SS} values for both the lead compensated and lead-lag compensated systems, and you should be able to comment on whether the velocity error constant design requirement has been met or not.