3CL4 Pre Lab 5

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Question 1

From lab 4 prelab, the parameters were:

$$K_c = 11.11$$
$$z = -4$$
$$p = -9.33$$
$$K_v = 6.99$$

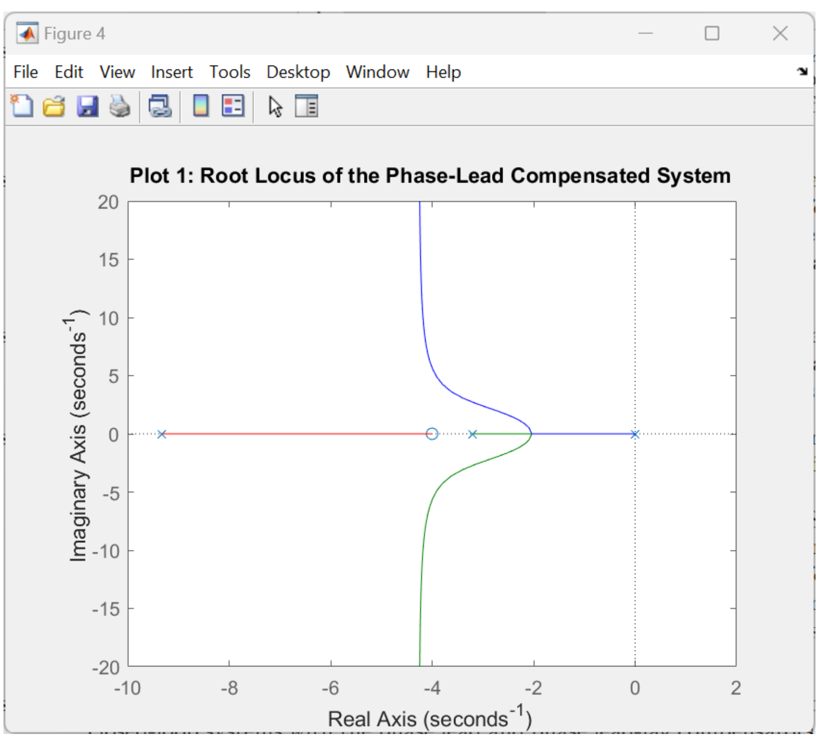
New system parameters:

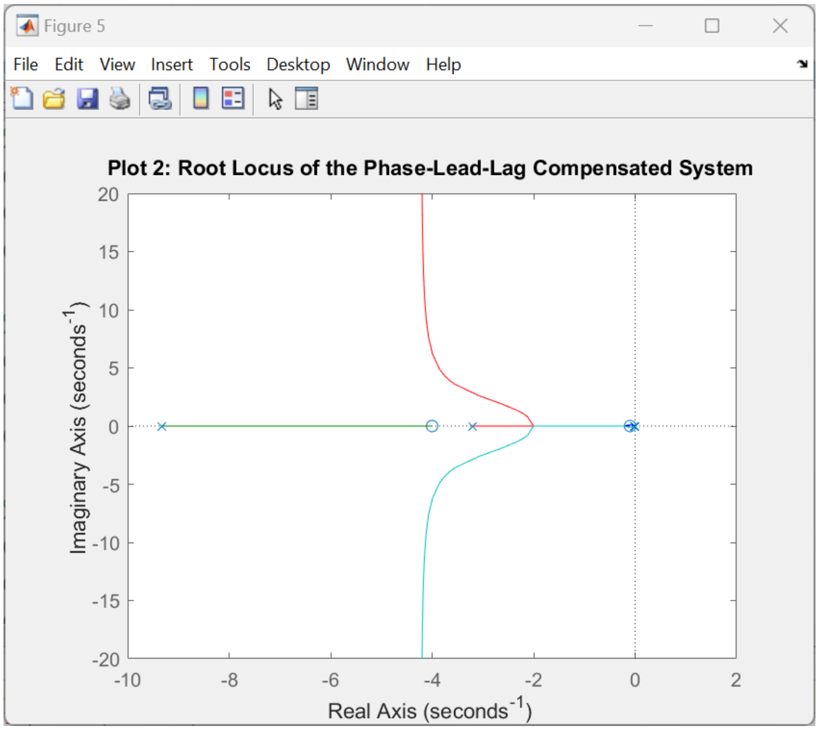
$$P.O = 10\%$$
$$T_s = 1s$$
$$K_{v2} = 69.9 \quad (10 \times K_v)$$

Question 2

To satisfy the requirement of increasing K_v by a factor of 10 while minimizing impact on overshoot and settling time, the phase lag compensator's zero (z_{lag}) and pole (p_{lag}) are placed close to the origin with $\frac{z_{lag}}{p_{lag}} = 10$. Choosing $z_{lag} = 0.1$ and $p_{lag} = 0.01$ ensures the K_v boost while their proximity guarantees negligible net angle contribution to the dominant poles. From the root locus perspective, since z_l and p_{lag} are near $s = 0$, their angles to the dominant poles nearly cancel out ($\theta_z - \theta_p \approx 0$), preserving the transient response shaped by the lead compensator.

Question 3





Question 4

From lab 4, the parameters were:

$$K_c = 2.180$$
$$z = -13.33$$
$$p = -32.60$$
$$K_v = 24.95$$

Question 5

Similar to Question 2, we select the phase lag compensator poles/zeros ($z_l = 0.1$, $p_l = 0.01$) to achieve:

- A $10\times$ boost in K_v (since $\frac{z_l}{p_l} = 10$),
- Minimal impact on transient response (angles from z_l and p_l cancel out near $s = 0$).

The lead-lag compensator is:

$$G_c(s) = G_{\text{lead}}(s) \cdot \frac{s+0.1}{s+0.01}.$$

Question 6

The lead-lag compensator $\overline{G}_c(s)$ and DC motor $G(s)$ combine as:

$$\overline{G}_c(s)G(s) = \underbrace{2.180\frac{(s+0.1)(s+13.33)}{(s+0.01)(s+32.60)}}_{\text{Lead-Lag Compensator}} \cdot \underbrace{\frac{279.3}{s(s+9.98)}}_{\text{DC Motor}},$$

yielding:

Question 7

$$\overline{G}_c(s)G(s) = \frac{(2.180)(279.3)(s+13.33)(s+0.1)}{s(s+9.98)(s+32.60)(s+0.01)}.$$

The closed loop transfer functions are:

$$T_{\text{lead}}(s) = \frac{595.8s + 7942}{s^3 + 42.58s^2 + 921.1s + 7942} \quad \text{(Lead-only)} \tag{1}$$

$$T_{\text{lead-lag}}(s) = \frac{595.8s^2 + 8002s + 794.2}{s^4 + 42.59s^3 + 921.6s^2 + 8005s + 794.2} \quad \text{(Lead-lag)} \tag{2}$$

Table 1: Comparison of System Characteristics

Feature	**Lead-Only**	**Lead-Lag**
Transfer Function Characteristics		
System Order	3rd	4th
Number of Zeros	1	2
Pole-Zero Locations		
Zeros	-13.33	-13.33, -0.1
Poles	$-13.28 \pm 17.87j$, -16.02	$-13.22 \pm 17.84j$, -16.05, -0.1004
Performance Characteristics		
Dominant Poles	$-13.28 \pm 17.87j$	$-13.22 \pm 17.84j$
Additional Dynamics	None	Pole at -0.1004, Zero at -0.1
K_v Improvement	$1\times$	$10\times$

The lead-lag compensator introduces a pole-zero pair near the origin (pole at -0.1004 and zero at -0.1). These additions produce two key effects: First, their ratio $\frac{0.1}{0.1004} \approx 1$ boosts K_v by $10\times$, improving the system's ability to track ramp inputs, reducing steady-state error. Second, the dominant poles governing transient response shift only minimally from $-13.28 \pm 17.87j$ to $-13.22 \pm 17.84j$, preserving the system's overshoot and settling time characteristics. The physical proximity of the new pole and zero ensures their angle contributions effectively cancel out in the root locus plot, leaving the dominant pole locations and consequently the system's speed and stability margins virtually unchanged.

Question 8 The lead and lead-lag systems show nearly identical step responses, with similar overshoot and settling time. The lag compensator's extra pole-zero pair (-0.01 and -0.1) has minimal effect on transient performance.

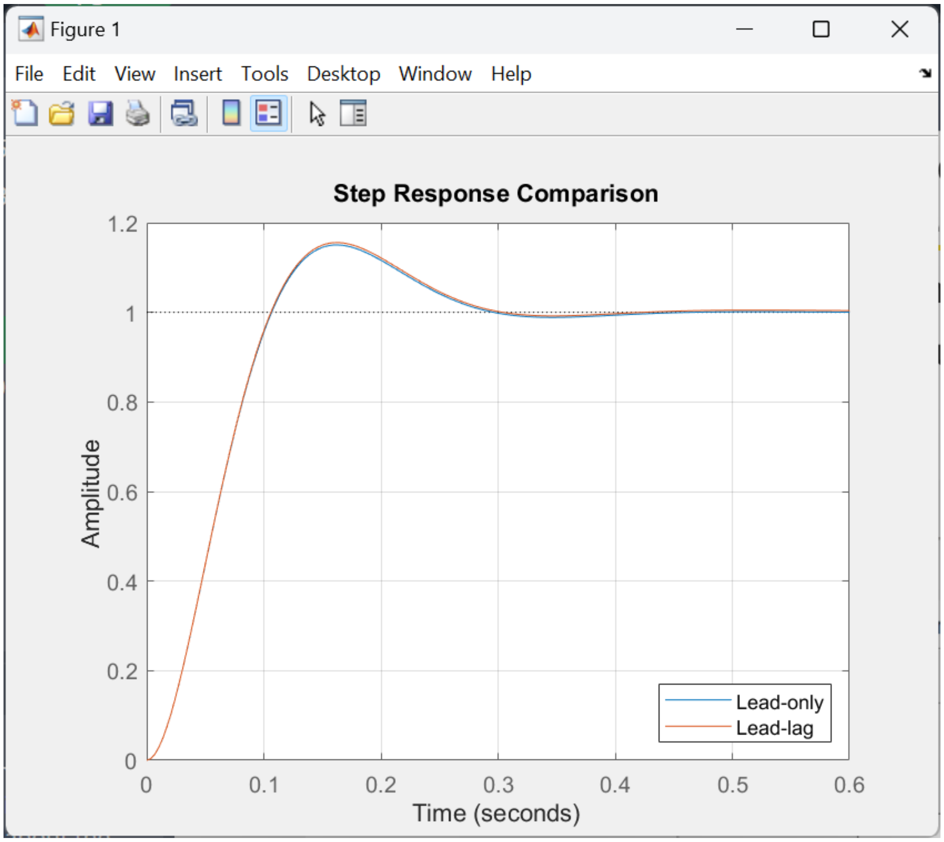


Figure 1
File Edit View Insert Tools Desktop Window Help
Step Response Comparison
Amplitude
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
Lead-only
Lead-lag

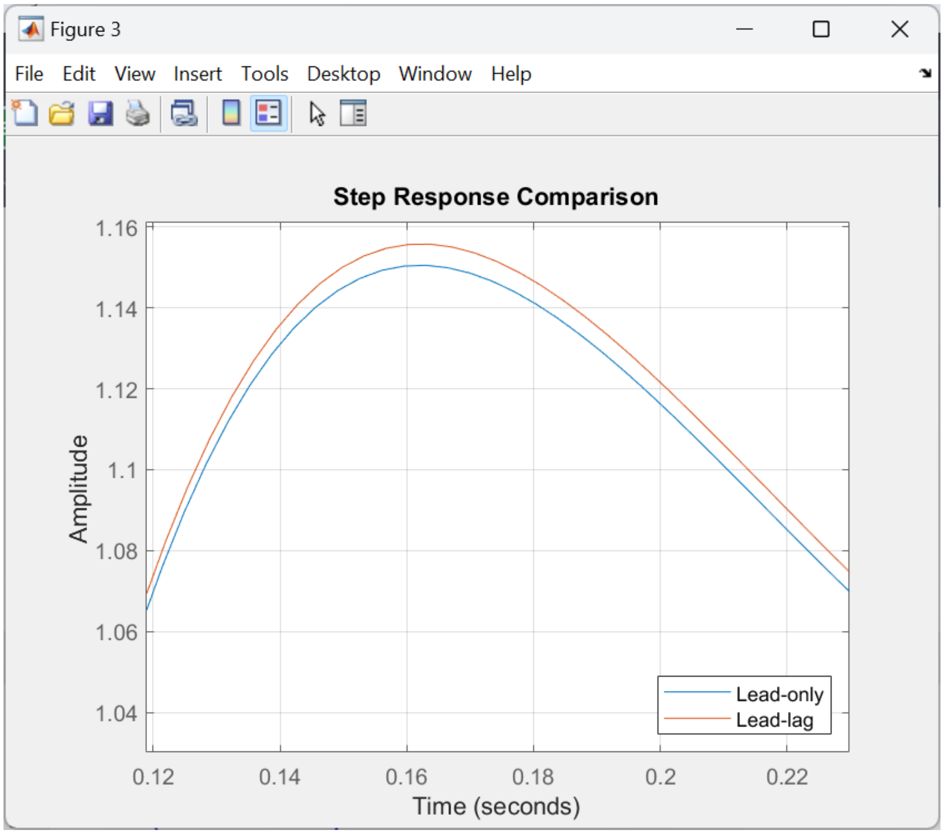


Figure 3
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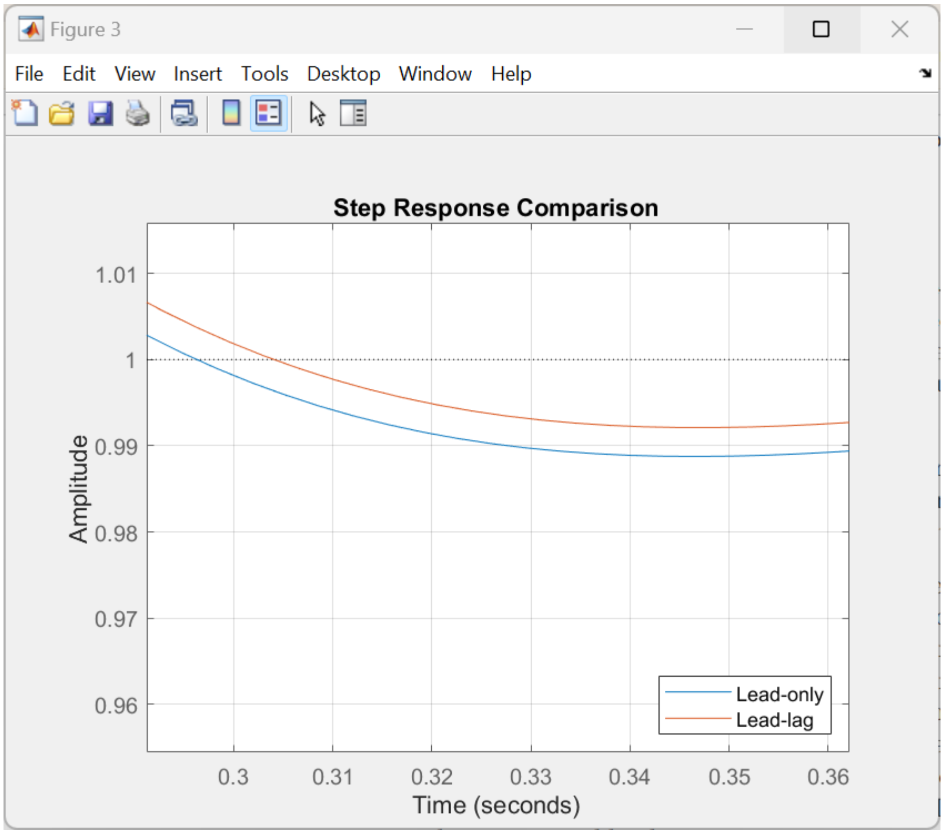
Step Response Comparison
Amplitude
Time (seconds)
1.16
1.14
1.12
1.1
1.08
1.06
1.04
0.12
0.14
0.16
0.18
0.2
0.22
Lead-only
Lead-lag



Question 9 For ramp inputs, the lead-lag system reduces steady-state error by 10× due to its 0.1/0.01 pole-zero ratio. This improves K_v as designed, while maintaining the lead compensator's good transient response.

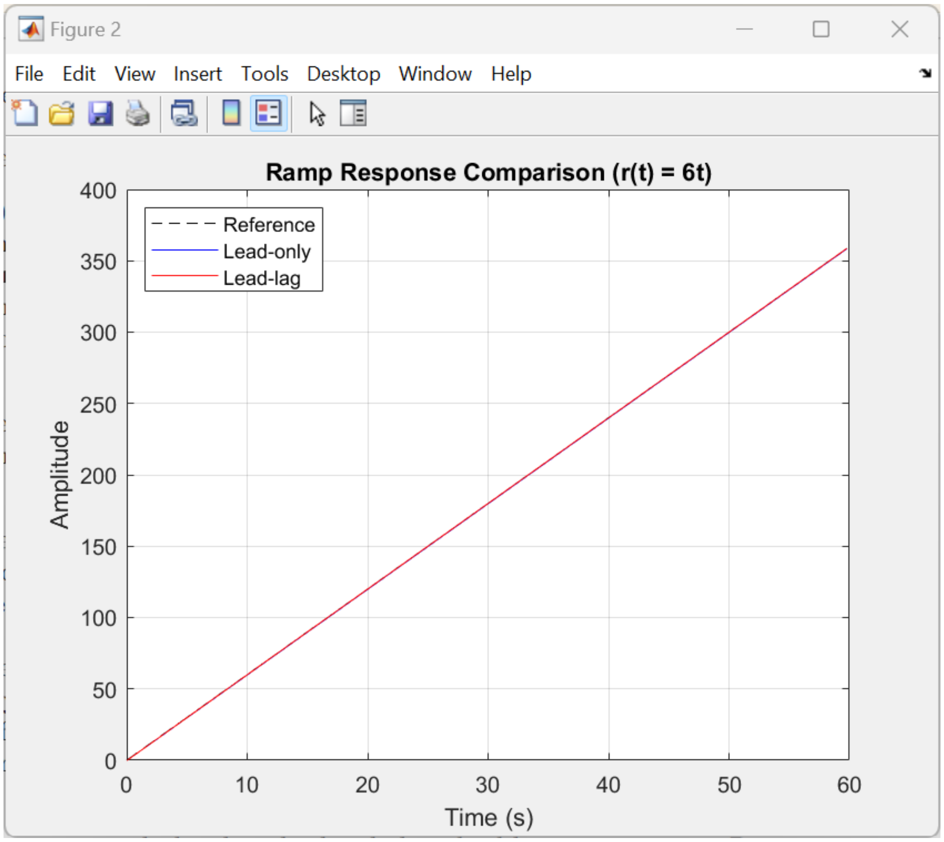


Figure 2
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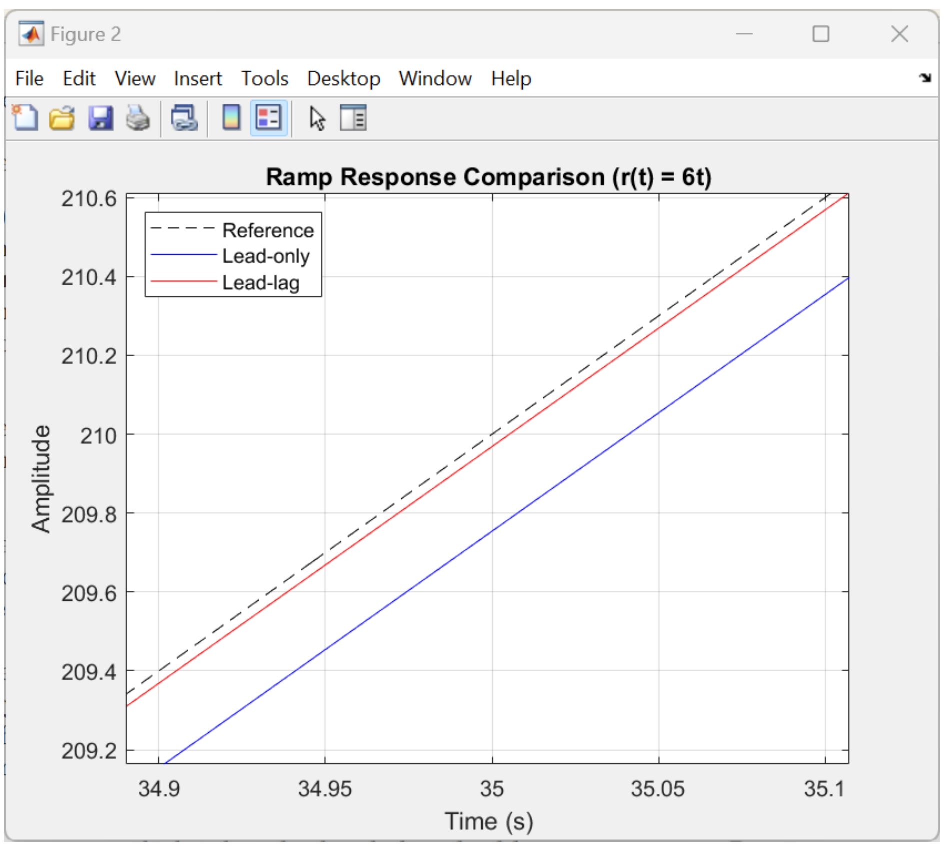
Ramp Response Comparison (r(t) = 6t)
Reference
Lead-only
Lead-lag
Amplitude
Time (s)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Ramp Response Comparison (r(t) = 6t)
Reference
Lead-only
Lead-lag
Amplitude
Time (s)