

Bayesian Analysis of Formula One Race Results: Disentangling Driver Skill and Constructor Advantage

Erik-Jan van Kesteren & Tom Bergkamp

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Abstract

Successful performance in Formula One is determined by combination of both the driver’s skill and race-car constructor advantage. This makes key performance questions in the sport difficult to answer. For example, who is the best Formula One driver, which is the best constructor, and what is their relative contribution to success? In this paper, we answer these questions based on data from the hybrid era in Formula One (2014 — 2021 seasons). We present a novel Bayesian multilevel Beta regression method to model individual race success as the proportion of outperformed competitors. We show that our modelling approach describes our data well, which allows for precise inferences about driver skill and constructor advantage. We conclude that Hamilton and Verstappen are the best drivers in the hybrid era (both in wet and dry weather), the top-three teams (Mercedes, Ferrari, and Red Bull) clearly outperform other constructors, and around 86% of the variance in race results is explained by the constructor. We argue that this modeling approach may prove useful for sports beyond Formula One, as it creates performance ratings for independent components contributing to success.

1 Introduction

In most competitive sports with a large individual component (e.g., chess, athletics, swimming, or tennis) success is determined primarily by the relative skill (i.e., ability or talent) of the contestants. Official competitions in such sports naturally result in official rankings representing this skill level on an individual basis. Unlike these skill-based sports, competitive motor racing has an additional key factor contributing to success: the material, i.e., the race car or motor bike. In Formula One in particular, the influence of the car on the results is considered to be substantial (Budzinski and Feddersen, 2020). In contrast to “spec” series, where cars have the same specifications and are built by the same constructor, Formula One cars are each built from the ground up by different constructors with differing levels of technological and financial resources. These resource gaps can lead to large differences in performance between cars, despite rules imposed to counteract such performance differences. Arguably, the presence of these large differences in constructor advantage have led to a single constructor (Mercedes) dominating the sport in the “hybrid era”, from 2014 to 2020.

The dependence on materials reduces the relative influence of driver skill on success — a race win is an entangled combination of both driver and constructor performance. Therefore, ranking drivers in terms of skill level by simply using competition race results is complex. Because of this problem, perennial questions such as “who is the best Formula One driver?”, “which constructor is the best?” and “is the driver or the constructor-team more important to success?” are difficult to answer scientifically. These questions are persistent in the sport; the 2016 world champion Niko Rosberg famously posed that 80% of success in Formula One can be attributed to the car and 20% to the driver (Bol, 2020). In this article, we argue that it is now possible to answer these questions due the widespread availability of race result

data and the accessibility of advanced statistical methodology. We propose a novel Bayesian multilevel Beta regression model to answer three interrelated questions for the hybrid era in Formula One: (a) what is the relative influence of the driver and constructor on race results, (b) how do drivers rank in terms of skill level, and (c) how do constructors rank in terms of race car advantage?

Several attempts have been made to disentangle driver and constructor performance. Eichenberger and Stadelmann (2009) used linear regression with dummy variables and several covariates to estimate driver and constructor-year effects on race finishing position in the 1950-to-2006 Formula One seasons. The driver-specific effects on race outcomes were then used to compute a ranking of drivers' skill level. Additionally, the authors found that predictors for weather and circuit type (street circuit vs. permanent circuit) were relevant additions to the model. One shortcoming of this study is the choice of outcome variable: because the number of contestants per race changed over seasons (and sometimes even within seasons), the interpretation of "finishing position" changed as well. This shortcoming was addressed by Phillips (2014), who analysed Formula One race data from 1950-to-2013. Here, the effects of driver performance, constructor performance and season difficulty were estimated using an adjusted "points scored" outcome variable, based on the official points scoring system used in the Formula One between 1991 and 2002: 10 points for first place, 6 for second, down to 1 point for sixth place. With this approach, the authors ranked drivers in terms of skill and concluded that Juan Manuel Fangio was the best driver of all time.

As Phillips (2014) noted, there are two reasons that such models can differentiate driver from constructor effects: (a) throughout the history of Formula One, constructors have generally had two cars enter a race. Barring minor differences in individual races, these cars have the same performance, which allows for direct comparisons of a driver's skill level against their teammates. Furthermore, (b) drivers generally switch/move to different constructor-teams throughout their career, meaning their teammates also change. This allows for simultaneous estimation of driver and constructor effects based on race results.

A disadvantage of the aforementioned studies is that they used models with many dummy variables (fixed effects) instead a multilevel (random effects) model. A multilevel approach is beneficial, as it makes the models tractable with fewer observations per driver and improves predictions for nested data (Gelman, 2006). A similar argument was made by Bell et al. (2016), who used the same "points scored" outcome variable as Phillips (2014), but used a multilevel (random-coefficients) linear model to determine the driver and constructor-year effects. Using this approach, the authors were the first to directly estimate a parameter for comparing driver skill and constructor advantage: they concluded that the constructor accounts for 86% of the variance in points scored, and the driver for 14%.

While the approach presented by Bell et al. (2016) provides an answer to the research questions posed above (pre-2014), we here present several improvements which enable a more accurate insight into the current state of Formula One. First and foremost, we argue that the "points scored" variable leads to information loss: any result below sixth place leads to zero points, making it impossible to differentiate constructors and drivers who consistently finish below this threshold. Our approach, explained in section 3, is to compute a "proportion of outperformed competitors": 1 for finishing in first place, 0.5 for the middle, 0 for last place. Second, we focus on the current hybrid era in Formula One, from 2014 – 2021, using data from a publicly available resource (Newell, 2021). This focus enables us to closely inspect changes across these most recent seasons with comparable regulations. Third, we apply a Bayesian workflow for model development (Gelman et al., 2013), visualisation (Gabry et al., 2019), and comparison (Vehtari et al., 2017), which makes the parameters interpretable and clarifies the model's implications. Using our approach, the parameters for driver skill and constructor advantage in our model are directly interpretable as log-odds ratios of beating competitors, similar to Elo ratings in chess (e.g., Van Der Maas and Wagenmakers, 2005). These parameters (and their uncertainty intervals), can then be used to create rankings which provide clear insight into the relative performance of drivers and constructors in

Formula One.

The paper is structured as follows. First, in Section 2 we describe the data source and processing steps we performed to obtain the predictors and the outcome of interest. Then, in Section 3 we introduce the proposed Bayesian multilevel Beta regression model and its interpretation. In the same section, we also perform model selection and model checking to validate the estimation procedure. In Section 4 we perform inference for the 2014-to-2021 Formula One seasons to answer the research questions surrounding driver skill and constructor advantage. This includes a driver ranking for the 2021 season and investigation of a counterfactual statement based on the estimated model: would Hamilton in an Alfa Romeo beat Räikkönen in a Mercedes? Last, in Section 5 we place our contributions in context, discuss its implications, and provide suggestions for future work. All analysis scripts and pre-processed data are openly available in the supplementary material at <https://doi.org/10.5281/zenodo.6358648> (van Kesteren and Bergkamp, 2022).

2 Data processing

We collected race results (driver id, constructor id, season (year), race number, finishing position and status) for the 2014-to-2021 Formula One seasons from the dataset behind the Ergast motorsports API (Newell, 2021). This resulted in a dataset of 160 races, with 51 unique drivers and 19 distinct constructors. In addition, we performed a data enrichment step by scraping and parsing further race information from Wikipedia. In this step, two predictors were added to the dataset which were previously found to be relevant in the work by Eichenberger and Stadelmann (2009) and Bell et al. (2016). We constructed a variable indicating whether the race was wet or dry, and we also collected information about the circuit type (street circuit or permanent circuit). As the information was not complete, especially for weather type in the 2018 season, we manually completed this data using publicly available race summaries. In total, in the period of interest, there were 143 dry races and 17 wet races.

For the main analysis, non-finishers were removed from the analysis, removing 590 rows (from the original 3267 rows) in the dataset. Thus, we only examined finished races for each driver: accidents and other reasons for non-finishing or non-starting can be due to any number of reasons, which adds noise and complexity to the outcome of interest. This removal of non-finishes has implications for the interpretation of the model, which we discuss in section 3. Note that there are other ways of handling non-finishing, for example by jointly modelling the finish probability as in Ingram (2021).

After the data preparation, the finishing position was processed into the final outcome of interest: the *proportion of outperformed competitors*. For each driver in each race we computed this proportion, meaning the driver received a 1 if they finished in first place, and 0 if the driver finished last (after excluding the non-finishers). After this, we smoothed the proportion to make the outcome amenable for a Beta regression model, following recommendations from Zeileis et al. (2010). In short, this smoothing procedure pulls the extreme values (0 and 1) a towards 0.5; strongly if there are few competitors and weakly if there are many competitors. A visual display of the proportion of outperformed competitors for three drivers (Räikkönen, Hamilton, and Giovinazzi) in the 2015-to-2020 seasons is shown in Figure 1.

3 Bayesian multilevel Beta regression model

In this section, we describe in detail the process used to model the proportion of drivers beaten as a function of driver skill, yearly driver form, constructor advantage, and yearly constructor form. All models in this paper were estimated using the software package `brms` (Bürkner, 2017) with the default priors for all parameter types (see section 2.1 of Bürkner

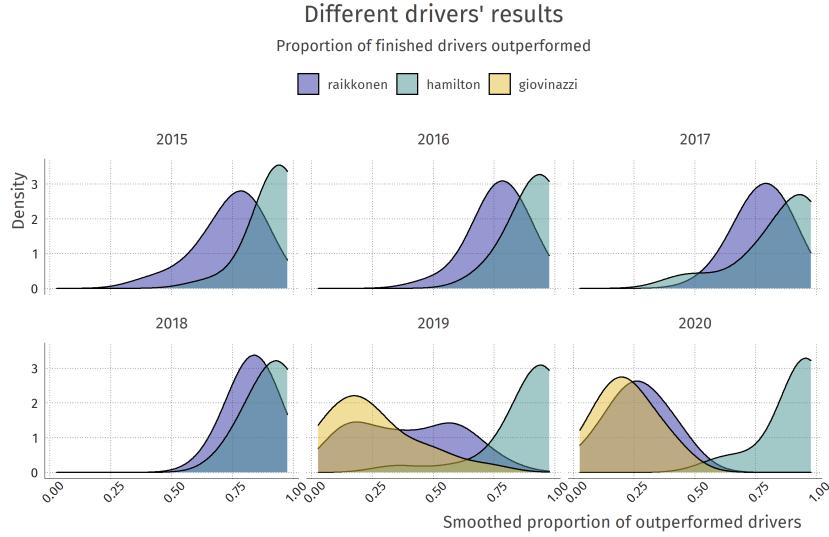


Figure 1: Smoothed finish proportions for Räikkönen, Hamilton, and Giovinazzi for the seasons 2015 – 2020. Räikkönen’s move from Ferrari to Alfa Romeo in 2019 is clearly visible in the proportion of outperformed drivers. At Alfa Romeo, Räikkönen became teammates with Giovinazzi, who he on average outperforms slightly (as indicated by the higher proportion of drivers beaten).

(2017) for more details).

3.1 Basic model specification and parameter interpretation

For driver d and constructor c in season s , we specify the following generative multilevel model for the proportion of drivers beaten y_{dcs} :

$$y_{dcs} \sim \text{Beta}(\mu_{dcs}, \phi) \quad (1)$$

$$\begin{aligned} \text{logit}(\mu_{dcs}) &= \beta_d + \beta_{ds} + \beta_c + \beta_{cs} \\ \beta_d &\sim \mathcal{N}(0, \sigma_d^2) \\ \beta_{ds} &\sim \mathcal{N}(0, \sigma_{ds}^2) \\ \beta_c &\sim \mathcal{N}(0, \sigma_c^2) \\ \beta_{cs} &\sim \mathcal{N}(0, \sigma_{cs}^2) \end{aligned} \quad (2)$$

The Beta distribution on line 1 of the model does not follow the standard α, β parameterization, but rather a Beta regression formulation with a mean parameter μ and a dispersion parameter ϕ (Ferrari and Cribari-Neto, 2004):

$$f(y) = \frac{\Gamma(\phi)}{\Gamma(\mu\phi)\Gamma((1-\mu)\phi)} y^{\mu\phi-1} (1-y)^{(1-\mu)\phi-1} \quad (3)$$

where $\Gamma(\cdot)$ is the Gamma function.

This model form leads to a specific, natural interpretation for the parameters β . The logit link function, in combination with the omission of an overall intercept, ensures that the (hypothetical) average driver at an average team with an average seasonal form will on average have $\mu_{dcs} = 0$, which translates into a probability of 0.5 of beating other drivers. Then, β_d represents the mean driver skill as a log-odds ratio; e.g., if $\beta_d = 0.3$, this means (*ceteris paribus*) that the probability of beating other drivers is $1/(1 + e^{-0.3}) \approx 0.57$. This

parameter represents a deviation from the average driver, so negative values mean worse than average skill, and positive values mean better than average skill. We also include the seasonal driver form parameter β_{ds} , which represents yearly deviations from this long-term average driver skill.

A similar interpretation holds for β_c , which indicates the long-term average constructor advantage. Constructors with positive values on this parameter tend to produce cars which are better than average, and negative values indicate cars which are worse on average. We also include the seasonal constructor form parameter β_{cs} , which represents yearly deviations from this long-term average constructor advantage. For more detailed parameter interpretations and conclusions, see Section 4.

Note that with this model formulation, we implicitly assume no correlation between the random intercepts for driver and constructor; there are no interactions at all between driver skill and constructor advantage. This means that a driver's skill is independent of the constructor advantage, i.e., the driver skill does not change when the driver moves to a different constructor.

3.2 Extending the basic model

Previous work has shown that several predictors may change the race results (Bell et al., 2016). The first extension we make reflects the knowledge that wet races are different from dry races. Wet races require a specific set of skills, which rely less on the car and more on the driver. Like Bell et al. (2016), we represent this knowledge by splitting the driver average skill parameter into a random intercept parameter γ_{0d} and a random slope parameter γ_{1d} as in Equation 4:

$$\beta_d = \gamma_{0d} + \gamma_{1d} \cdot \text{wet_race} \quad (4)$$

where `wet_race` is an indicator (dummy) variable with a 1 if the race was wet and 0 if the race was dry (see Section 2 for details). The driver average skill in dry races is then γ_{0d} , and in wet races it is $\gamma_{0d} + \gamma_{1d}$.

The second extension we make reflects the knowledge that different constructors have different car philosophies. Theoretically, high-downforce concept cars (e.g., Red Bull cars in the hybrid era) are relatively better suited to narrow, curvy street circuits such as the famous Monaco circuit, but this advantage disappears on fast, permanent circuits such as Monza (Italy). Therefore, we add a random slope to the constructor advantage parameter, splitting it up as in Equation 5:

$$\beta_d = \gamma_{0c} + \gamma_{1c} \cdot \text{permanent_circuit} \quad (5)$$

where `permanent_circuit` is an indicator (dummy) variable with a 1 if the race was on a permanent circuit and 0 if the race was on a street circuit (see Section 2 for details).

With these two extensions, four models are possible: (a) the basic model, (b) a weather model, (c) a circuit type model, and (d) a weather and circuit type model. In the next subsection, we compare these models to select our final model, on which we perform inference.

3.3 Model selection

We used efficient leave-one-out cross-validation (LOO, Vehtari et al., 2017) to compare the four possible models. In short, the LOO implementation in `brms` computes the expected log posterior density (ELPD) for each model, which is an alternative to the standard information criteria in Bayesian model comparison such as the Bayes Factor (marginal density) or DIC. For the four tested models, the results are shown in Table 1.

The basic model is shown to be the best model in terms of ELPD. Although the differences are small, this indicates that in terms of out-of-sample predictive performance, the most parsimonious model is the best. In the following sections, we use the basic model for model assessment, inference, and prediction.

	$ELPD$	SE_{ELPD}	Δ	SE_Δ
Basic	1444.12	46.62		
Circuit	1443.08	46.59	-1.03	0.51
Weather	1442.96	46.60	-1.16	1.33
Circuit + Weather	1442.22	46.59	-1.90	1.41

Table 1: Model comparison results showing the expected log posterior density (ELPD), its standard error, the difference between each model and the best model, and the standard error of this difference. Note: a higher ELPD indicates a better fit.

3.4 Model assessment

Posterior samples were obtained via Hamiltonian Monte Carlo sampling with four chains of 2500 samples each after 1000 burn-in iterations. The effective sample size for all parameters was higher than 2500, and their R-hat value was smaller than 1.01, indicating adequate convergence. Trace plots for the main parameters of the weather model are shown in Appendix A.

Posterior predictive checks (PPCs) are a vital part of the Bayesian workflow (Gabry et al., 2019). In a visual PPC, simulated data \tilde{y} from the posterior predictive distribution is compared to the observed data y . If \tilde{y} approximates y well, then the model captures the outcome well. For our model, we performed two PPCs: one for the 2015 season (early hybrid era) and one for the 2019 season (late hybrid era). The results are shown in Figures 2 and 3.

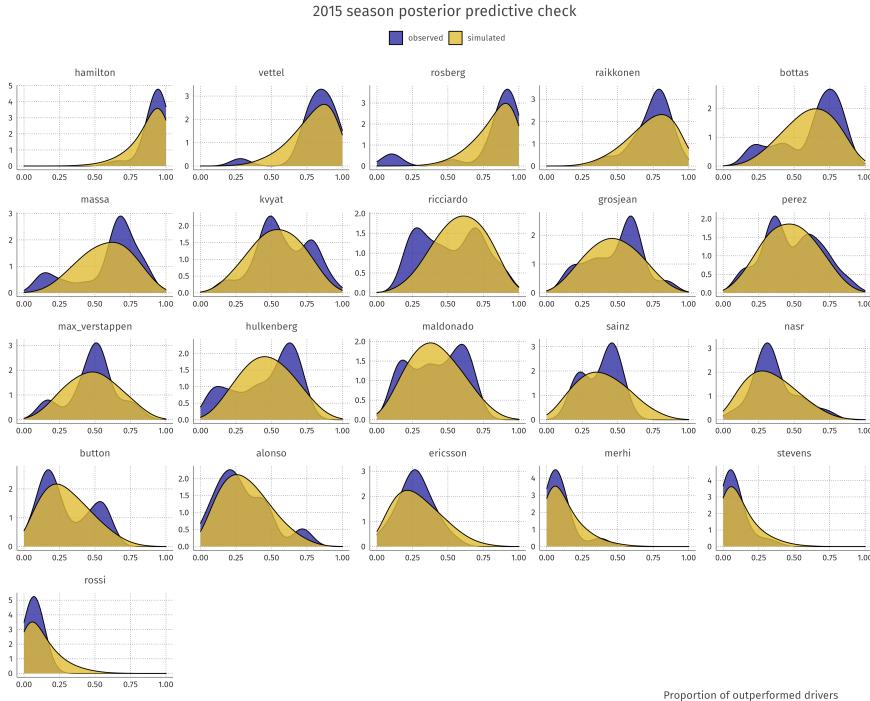


Figure 2: Posterior predictive check for driver finishing proportions in the 2015 season.

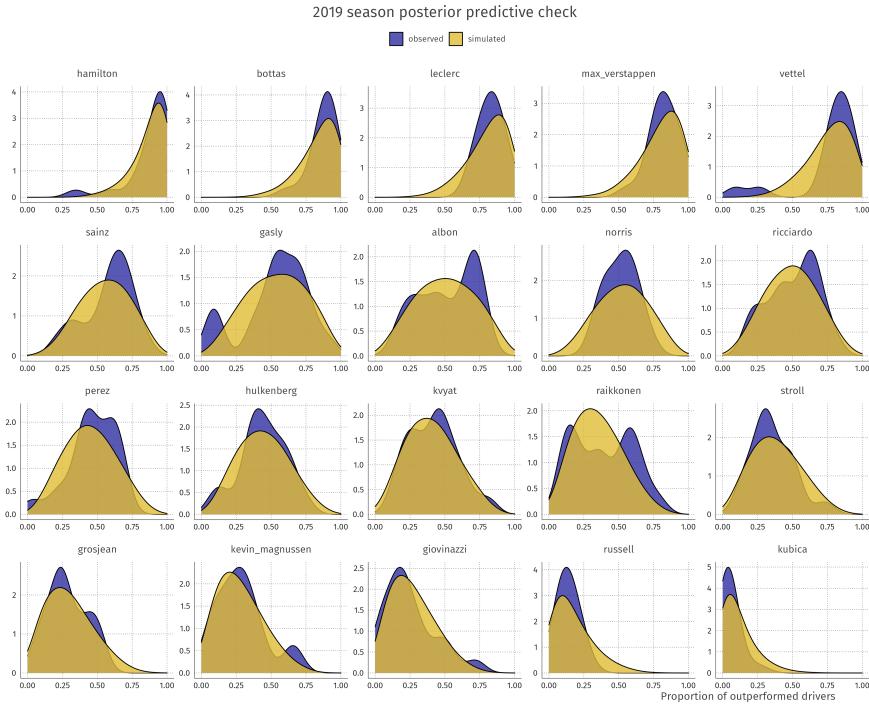


Figure 3: Posterior predictive check for driver finishing proportions in the 2019 season.

The plots show satisfactory recovery of individual performances per driver-constructor-season combination. One noteworthy aspect visible in these plots is that for consistently high finishers (mainly Hamilton) and low finishers (e.g., Rossi in 2015), the posterior predictive distribution is perhaps slightly overdispersed (ϕ is too high), as the observations show less variation than the simulations. Instead, for midfield drivers (e.g., Ricciardo in 2015, Räikkönen in 2019) the Beta distribution is slightly underdispersed (ϕ is too low), as the observations show more variation than the simulations. This does not seem to bias the mean estimates much, especially in the midfield where the dispersion problems are stronger. Appendix B shows the posterior predictive checks for the same seasons on the original outcome scale (finishing position). The conclusion from these checks is the same. In general, we conclude that the model fits the observed data well. In the next section, we use the model to perform parameter inference.

4 Results

In this section, we perform inference with the model that resulted from the modeling procedure described in Section 3. To narrow down our inference efforts, we focus on a subset of the drivers and teams competing in the 2021 season. The results for all drivers, teams, and seasons are available in the supplementary material (van Kesteren and Bergkamp, 2022).

4.1 Driver skill

After estimation of the preferred model, it is possible answer the question of which driver is the most skilled, while taking into account constructor advantage, constructor form, as well as all parameter uncertainties. In order to produce a ranking, we obtained the posterior means and 89% credible intervals (see McElreath, 2018) of $\beta_d + \beta_{ds}$ for the season 2021. These summaries are shown in Figure 4.

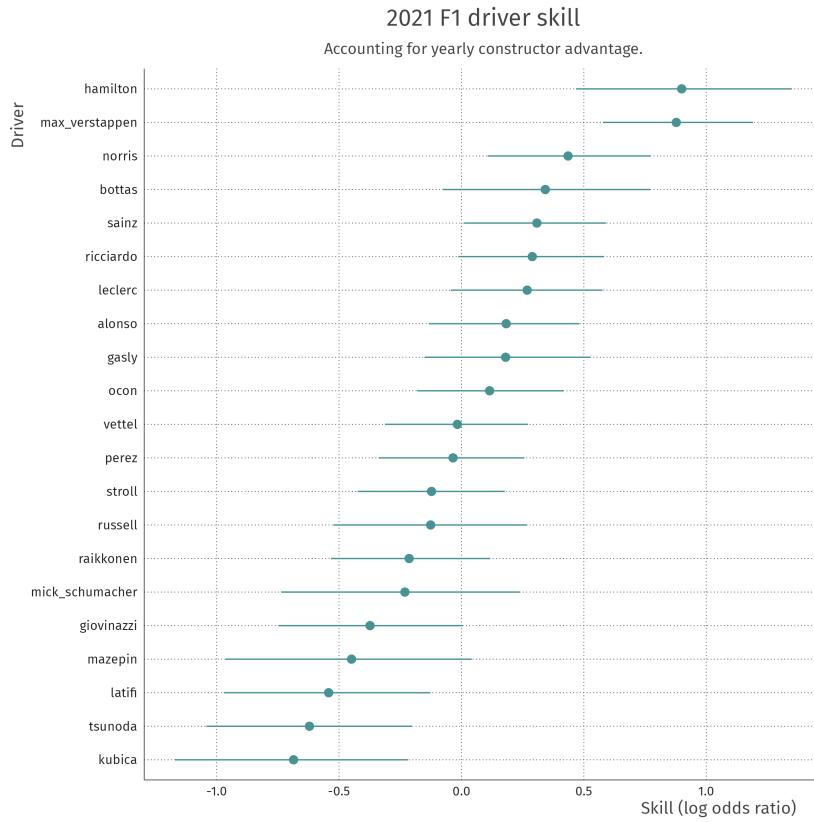


Figure 4: Driver skill in 2021 on the log odds ratio scale, accounting for yearly driver skill and constructor form. Error bars indicate 89% credible interval.

In Figure 4 it is apparent that of the 2021 drivers, Hamilton and Verstappen are ranked as the most skilled driver. Note that this ranking comes directly from the model, which is estimated only on the hybrid-era data. Earlier performances by drivers such as Vettel (four-time world champion in the period 2010–2013) and Räikkönen (world champion in 2007) have not been taken into account, explaining their lower position on the ranking.

Because the model contains a yearly form parameter, we can also visualize the latent skill trajectories of several drivers throughout the hybrid era, with their credible intervals. The result of this visualization for 12 drivers of the 2021 season is shown in Figure 5.

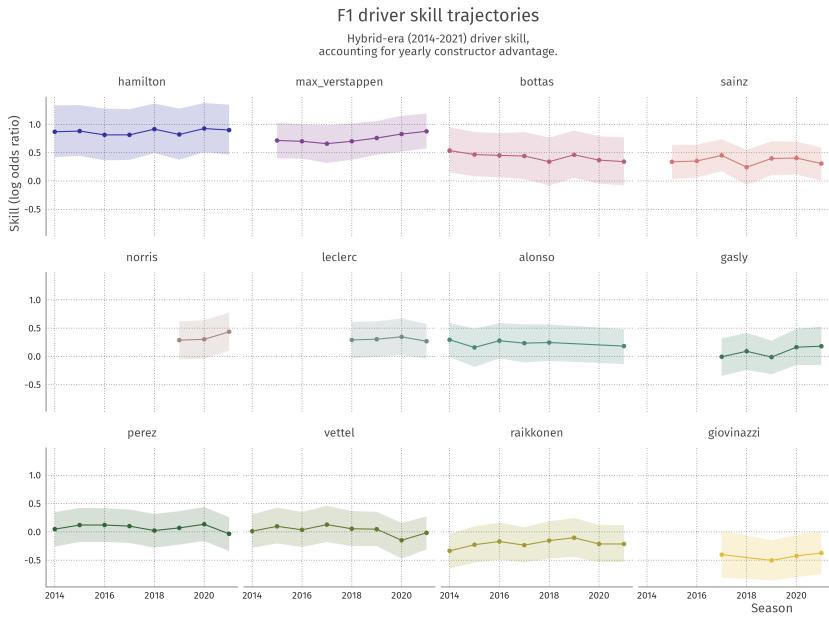


Figure 5: Driver skill trajectories for 12 drivers in the hybrid era on the log odds ratio scale. Ribbons indicate 89% credible interval.

The figure shows that there are slight changes in skill across seasons. Drivers such as Verstappen, Norris, and Gasly tend to improve over years, on average, whereas Bottas displays a slight decline. Notably, Hamilton is consistently at the top, Perez and Sainz are consistently slightly above average, and Räikkönen is consistently slightly below average in terms of skill and performance.

4.2 Constructor advantage

For the constructors competing in 2021, we here investigate how much of an advantage their car yields. We do this by computing the posterior means and 89% credible intervals of β_c from the model. The result is shown in Figure 6.

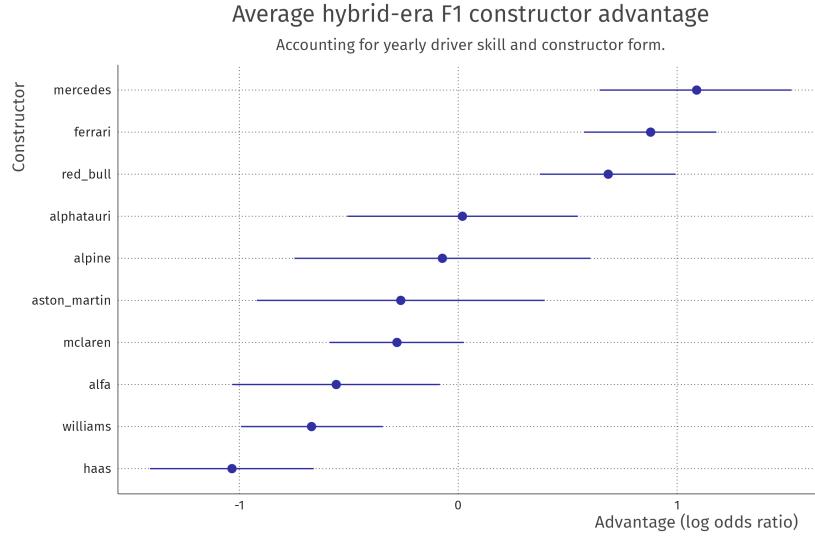


Figure 6: Long-run average constructor advantage on the log odds scale. Error bars indicate 89% credible interval.

One feature that becomes clear from this constructor advantage plot is the relative advantage of the *big three* teams: Mercedes, Ferrari, and Red Bull have the largest budget and the most resources to spend on developing their car, which has resulted in these three teams excelling in the hybrid era. Another interesting feature in this plot is the large uncertainty around the teams that have competed in only a few seasons. For example, Alpine and Aston Martin were new teams in 2021, and therefore have not had a chance compete in many races, resulting in uncertainty around where it is placed in this constructor ranking. Again, these parameters need to be interpreted with care: they represent the average constructor advantage over the entire hybrid era.

The last random intercept component is the constructor-year effects β_{cs} . These represent yearly constructor form, as a deviation from their long-term average advantage. In Figure 7, the yearly constructor advantage trajectories for a selection of teams is shown from 2014 to 2021.

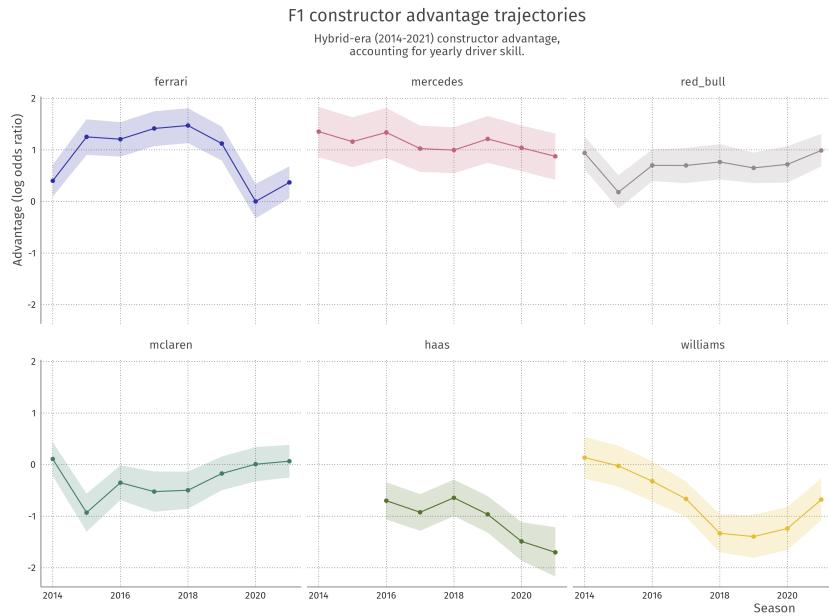


Figure 7: Yearly constructor advantage (summing constructor and constructor-year random effects) for Ferrari, Mercedes, Red Bull, McLaren, Haas, and Williams in the period 2014 – 2020. Ribbons indicate 89% credible interval.

One of the most striking features from this graph is Ferrari’s drop in form from 2019 to 2020. There is a good explanation for this: after the 2019 season, there were allegations that Ferrari’s engine in the previous year did not match league regulations. Ferrari subsequently reached a settlement with the Formula One Management (Formula1.com, 2020), which left the team with a relatively weak engine in the 2020 season. This change is aptly reflected in the constructor form parameter for Ferrari, but also to a lesser extent in that of McLaren, which profited from Ferrari’s drop and ended up third in the constructor’s championship that year.

4.3 Season performance

By combining driver skill and constructor advantage, we produce a theoretical ranking using the posterior predictive distributions for the 2021 season (Table 2). Here, we take the mean performance measure (proportion of competitors beaten) and transform it into an expected posterior rank. This posterior rank corresponds closely to the achieved rank in the 2021 season (shown in the last column of Table 2).

4.4 Relative contributions of drivers and constructors

In order to investigate the contributions of drivers and constructors to the race results, we investigate the standard deviations of the random intercepts in the model. By investigating these standard deviations, we can make conclusions about which matters more in terms of race results: the driver or the car? The posteriors for the variation coefficients are shown in Table 3.

The standard deviation of the constructor is larger than that for the driver. This means that on average, the constructor has a larger impact on race results than the driver. Rephrasing this, on average the correlation in the outcome is stronger for two different drivers driving for the same team than for the same driver driving for different teams. This interpretation is also exemplified by the race results shown in Figure 1: Räikkönen driving for Alfa Romeo

Driver	Constructor	Performance [89% CI]	Rank	WDC Rank
Verstappen	Red Bull	0.865 [0.602 - 0.995]	1	1
Hamilton	Mercedes	0.852 [0.581 - 0.993]	2	2
Bottas	Mercedes	0.770 [0.469 - 0.970]	3	3
Perez	Red Bull	0.719 [0.401 - 0.947]	4	4
Sainz	Ferrari	0.664 [0.344 - 0.917]	5	5
Leclerc	Ferrari	0.651 [0.329 - 0.912]	6	<u>7</u>
Norris	McLaren	0.620 [0.297 - 0.892]	7	<u>6</u>
Gasly	Alpha Tauri	0.585 [0.270 - 0.867]	8	<u>9</u>
Ricciardo	McLaren	0.584 [0.269 - 0.873]	9	<u>8</u>
Alonso	Alpine	0.523 [0.214 - 0.823]	10	10
Ocon	Alpine	0.507 [0.207 - 0.813]	11	11
Vettel	Aston Martin	0.417 [0.133 - 0.735]	12	12
Stroll	Aston Martin	0.392 [0.111 - 0.712]	13	13
Tsunoda	Alpha Tauri	0.392 [0.112 - 0.708]	14	14
Räikkönen	Alfa Romeo	0.324 [0.077 - 0.644]	15	<u>16</u>
Russell	Williams	0.307 [0.067 - 0.622]	16	<u>15</u>
Giovinazzi	Alfa Romeo	0.291 [0.059 - 0.604]	17	<u>18</u>
Latifi	Williams	0.229 [0.029 - 0.531]	18	<u>17</u>
Schumacher	Haas	0.130 [0.004 - 0.392]	19	19
Mazepin	Haas	0.107 [0.002 - 0.350]	20	20

Table 2: For drivers in the 2021 season, the posterior expectations and 89% credible interval for performance in terms of proportion of opponents beaten per race, the posterior rank resulting from this, and the realized 2021 world driver’s championship (WDC) rank. Where the WDC rank does not equal the posterior predictive rank, the WDC rank has been underlined.

Component	Symbol	Estimate	Est.Error	Lower	Upper
Constructor advantage	σ_c	0.95	0.19	0.68	1.29
Constructor form	σ_{cs}	0.43	0.05	0.35	0.52
Driver skill	σ_d	0.39	0.08	0.27	0.52
Driver form	σ_{ds}	0.14	0.04	0.07	0.21

Table 3: Standard deviations (σ) for the random effects in the model. Lower and upper represent bounds of the 89% credible intervals.

(2020) looks more like his teammate (Giovinazzi) than like Räikkönen driving for Ferrari (2018).

Quantifying the relative importance of long-term constructor advantage compared to driver skill is also possible directly from the numerical summaries. The posterior estimates for the variances are as follows: $\sigma_c^2 \approx 0.89$, $\sigma_{ds}^2 \approx 0.19$, $\sigma_d^2 \approx 0.15$, and $\sigma_{cs}^2 \approx 0.02$. Following the methodology of Bell et al. (2016, §4.2), this means that constructor effects account for around 86% of the variance in the model (89% CI [0.857, 0.886]), which is very similar to the aforementioned authors who also reported 86%.

4.5 Counterfactual inference

Using samples from the posterior distributions of the parameters, we can answer some counterfactual questions about the drivers in the model. The main approach for this is by comparing the predicted outcome (proportion of competitors beaten) for different configurations of the predictors. An example question would be: “According to the model, would Hamilton be expected to beat Räikkönen in a race in 2021 if Hamilton drove for Alfa Romeo

and Räikkönen for Mercedes?”.

We can answer this question by computing the posterior of the difference δ of the predicted proportion of competitors beaten \tilde{y} :

$$\tilde{y}_{ham:alfa:2021} = \text{logit}^{-1}(\beta_{ham} + \beta_{ham:2021} + \beta_{alfa} + \beta_{alfa:2021}) \quad (6)$$

$$\tilde{y}_{rai:merc:2021} = \text{logit}^{-1}(\beta_{rai} + \beta_{rai:2021} + \beta_{merc} + \beta_{merc:2021}) \quad (7)$$

$$\delta = \tilde{y}_{ham:alfa:2021} - \tilde{y}_{rai:merc:2021} \quad (8)$$

The posterior of δ is shown in Figure 8. It is shown that Räikkönen is expected to beat Hamilton in this scenario ($E[\delta]$ is negative), with a clear degree of uncertainty. Note that this counterfactual prediction is a way to summarise the model, meaning the same assumptions that accompany the model also accompany the counterfactual predictions. For example, for these predictions the data from before 2014 is irrelevant, and driver talent is independent of the constructor advantage.

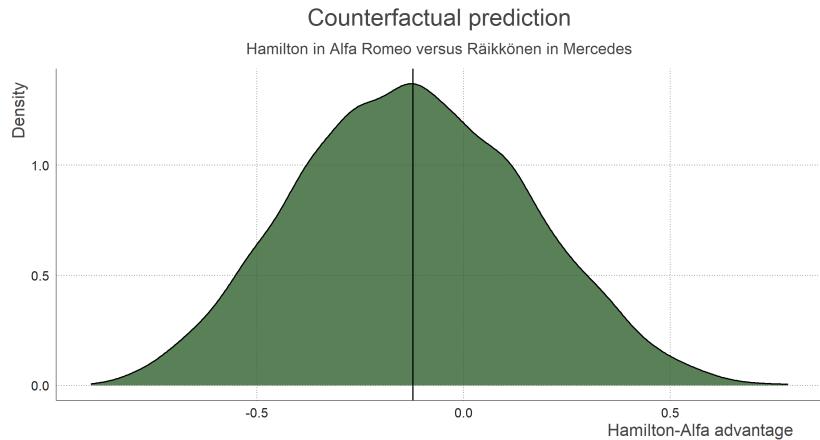


Figure 8: Posterior distribution of the advantage of Hamilton in an Alfa Romeo over Räikkönen in a Mercedes in a wet race during the 2020 season. The expected value of the distribution (vertical line) is -0.113, meaning that Räikkönen is expected to beat Hamilton in this scenario.

5 Discussion

In this paper, we used Bayesian multilevel Beta regression to model the proportion of drivers beaten in Formula 1 races of the hybrid era (2014 – 2021). After model development, comparison with leave-one-out cross-validation, and validation with posterior predictive checks, we have made inferences about the drivers and constructors competing in the 2021 season. In terms of the drivers, Hamilton and Verstappen are a step above the rest and closely matched in skill. In terms of the constructors, the top three teams (Mercedes, Ferrari, and Red Bull) clearly outperform the rest on average in the hybrid era. Additionally, the model accurately represents changes in constructors’ seasonal form, for example reproducing a drop in performance by Ferrari in 2020. Comparing driver contributions to team contributions, we have concluded that the car is more important than the driver when it comes to race results. Using our model and posterior sampling, it is possible to obtain answers of counterfactual questions as posterior distributions, which we have shown in the last part of Section 4.

While generally the driver ranking aligns with public opinion, one outlier in this data is Giovinazzi: while not generally regarded as the best driver, he is not regarded as the

worst either. His low ranking can be explained by the fact that his only teammate has been Räikkönen (who has outperformed him) and Räikkönen’s only teammate in this data before that has been Vettel (who, in turn, has outperformed him). Again, this ranking should be carefully interpreted as a summary of the individual talent *shown in the race results* in the period 2014 – 2020. Additionally, the difference between dry and wet-weather performance aligns well with public opinion. A notable example is Bottas, who is considered a good driver but not in wet weather, and this is reflected in the model parameters.

In terms of parameter interpretation, there is an interesting parallel between this model and Elo ratings (Elo, 1978) in chess. Both the Elo rating and our driver talent parameters can be transformed using an inverse logit to compare the relative strength of the competitors, and thus how likely it is that one competitor wins. The larger the difference between the ratings, the more certain it is that the competitor with the higher rating wins. We have shown such a comparison in a counterfactual situation in Section 4. Note that this Bayesian hierarchical approach has been applied before to different sports (e.g., in tennis; Ingram, 2019). However, in our model not only the athletes get ratings, but also the constructors, so comparisons can be made at this level as well. This approach could be used in other sports where multiple independent components contribute to competition results.

While this model does well at describing past data, for example closely reproducing the ranking of the 2021 season, it is probably not suitable for prediction. It uses very limited information (only the driver, constructor, and year) and for each year a specific effect needs to be estimated. Before a season starts, there is no data on that season, meaning that these year-effects are unavailable (even though they are important components of this model). For forecasting, approaches such as that of Henderson et al. (2018) may be more suitable relative to the baseline of bookmaker odds.

There are several areas where this model may be improved. One area is in team continuity: teams can officially change their name, when behind the scenes it is the same team, with the same long-run performance. For example, Alpha Tauri is a re-branding of the Italian Toro Rosso team, but it enters our dataset as a completely new team, with understandably large credible intervals around performance in 2020. By not accounting for team continuity across different team names, we had in total 17 different constructors in the dataset. On the other hand, team name changes often do go hand-in-hand with some structural changes, and it is hard to determine the extent to which this happens: where to draw the line between a “re-brand” and a new team?

While our model accurately represents driver and constructor performances in the seasons 2014 – 2020, the data range could be expanded in order to provide results that better reflect the careers of certain drivers. As explained in the discussion of the driver talent parameters in Section 4, Giovinazzi is considered the worst driver by the model but only because he has not had the chance to “prove himself” against anyone but Räikkönen, who is at the end of his career. Because the biggest part of Räikkönen’s career is missing from the data, his 2007 world championship has not been appropriately taken into account. These errors propagate, (e.g., his teammate will only profit very little by beating him), leading to surprising results in the rankings.

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Appendices

A Traceplot

Figure 9 shows the traceplot for the main parameters of the model that includes the weather random effect. From top to bottom, the standard deviations for the three random intercepts γ_{0d} , β_c , and β_{cs} , the standard deviation for the weather random slope γ_{1d} , the correlation ρ_d between the random intercept and random slope for the driver, and the dispersion ϕ of the Beta distribution used to model the proportion of drivers beaten.

All traceplots for the four different chains overlap as expected from an appropriately converged model.

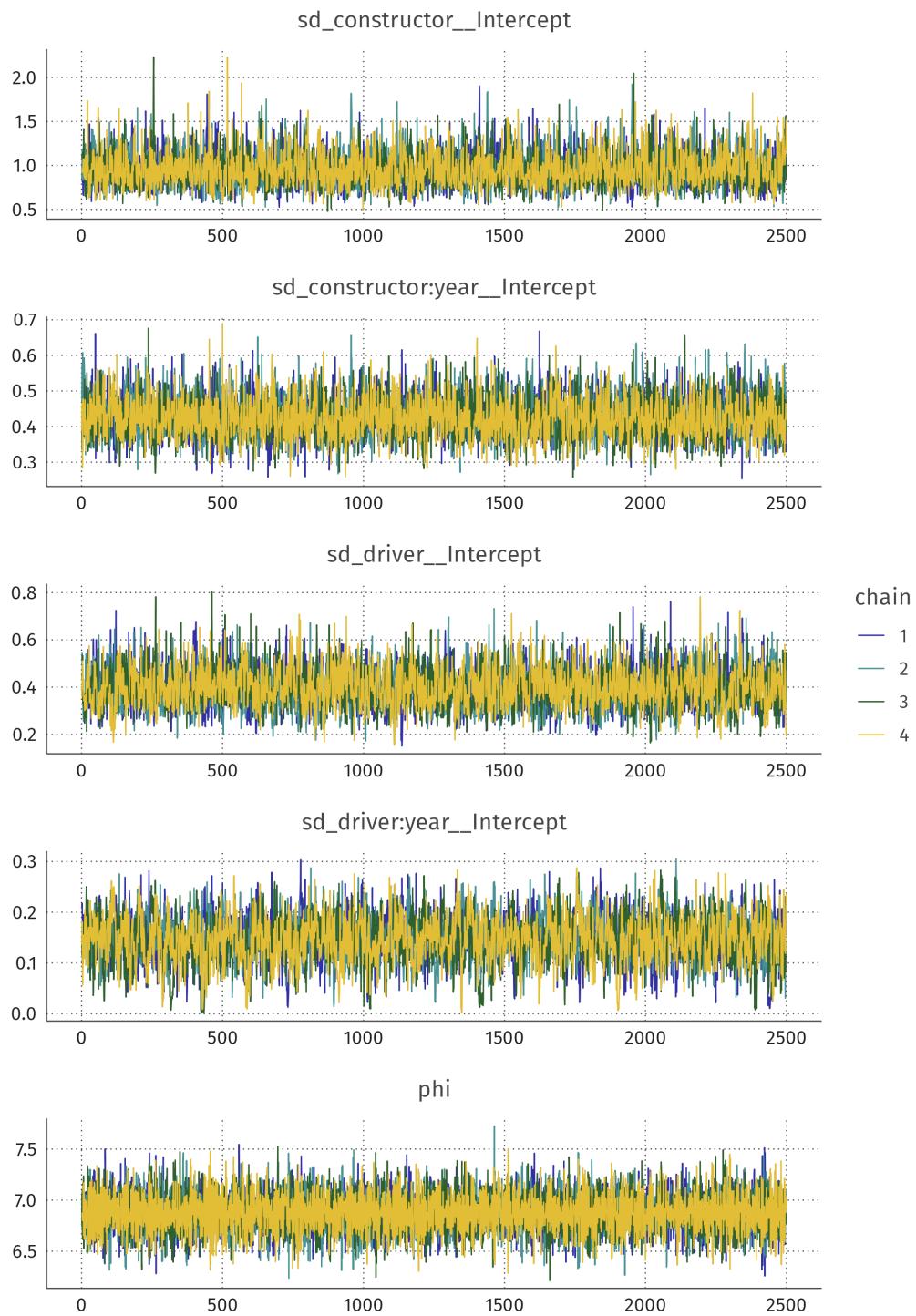


Figure 9: Monte carlo markov chain visualisation for the main model parameters, indicating satisfactory convergence for all shown parameters.

B Posterior predictive check on the rank scale

In order to simulate data on the rank scale, the transformed outcome variable (proportion of drivers beaten) needs to be back-transformed into a rank. We use the following procedure

1. For each driver-team-year under consideration, obtain a posterior sample of the "proportion of drivers beaten"
2. Consider these proportions as expectations for the relative finishing positions: transform these proportions into ranks

The result is a matrix filled with integers indicating finish positions. Each column represents a different driver(-team-year), and each row represents a simulated "race". Then, in order to take into account the varying number of contestants per race in the data and the probability distribution of non-finishes, one more post-processing step needs to be performed: due to accidents and other errors, it is much less likely that a driver finishes 19th than 13th. In order to include this knowledge in the PPC, we resample the simulated finishing position in each column (i.e., for each driver) using probabilities based on the empirical finishing distribution shown in Figure 10. After this resampling, we have transformed the simulated number of drivers beaten \tilde{y} into a simulated finishing position \tilde{y}^* , assuming that the same empirical finishing distribution holds for all drivers.

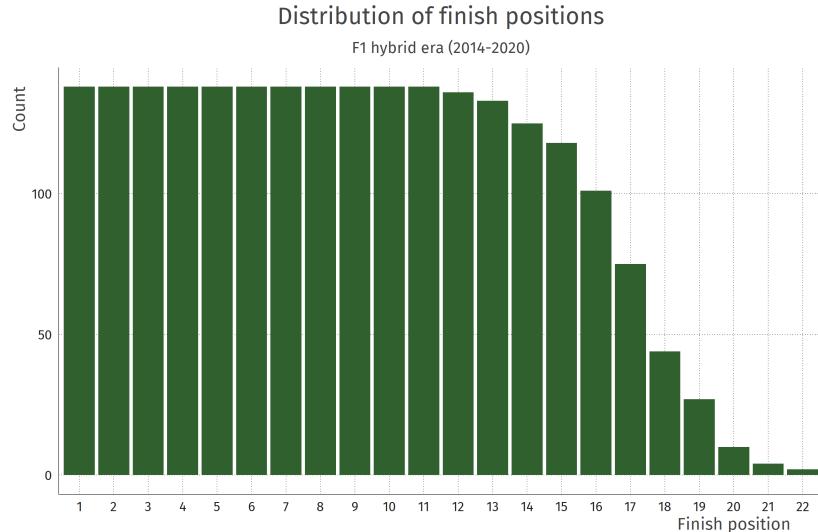


Figure 10: Finish positions in the data, after removing the non-finishes due to accidents and other problems.

Visual posterior predictive checks for the seasons 2015 and 2019 are shown in Figures 11 and 12. For each of these figures, we simulated a single "season" of 23 races. Similarly to the proportion-based PPCs shown in Figures 2 and 3, this rank-based PPC shows adequate representation of the observed data by the model, indicating good fit.

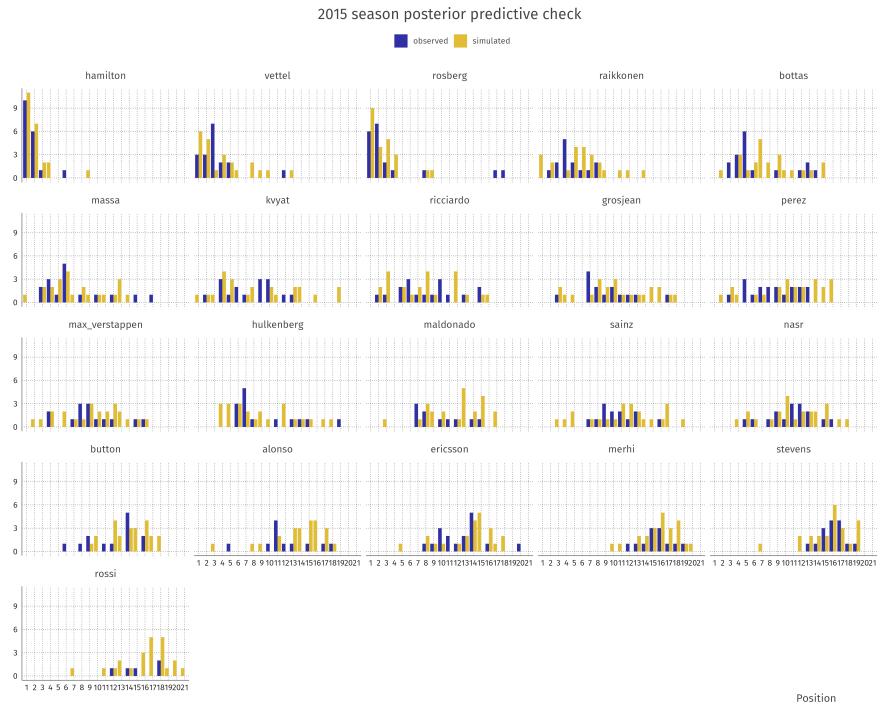


Figure 11: Posterior predictive check for driver finish position in the 2015 season.

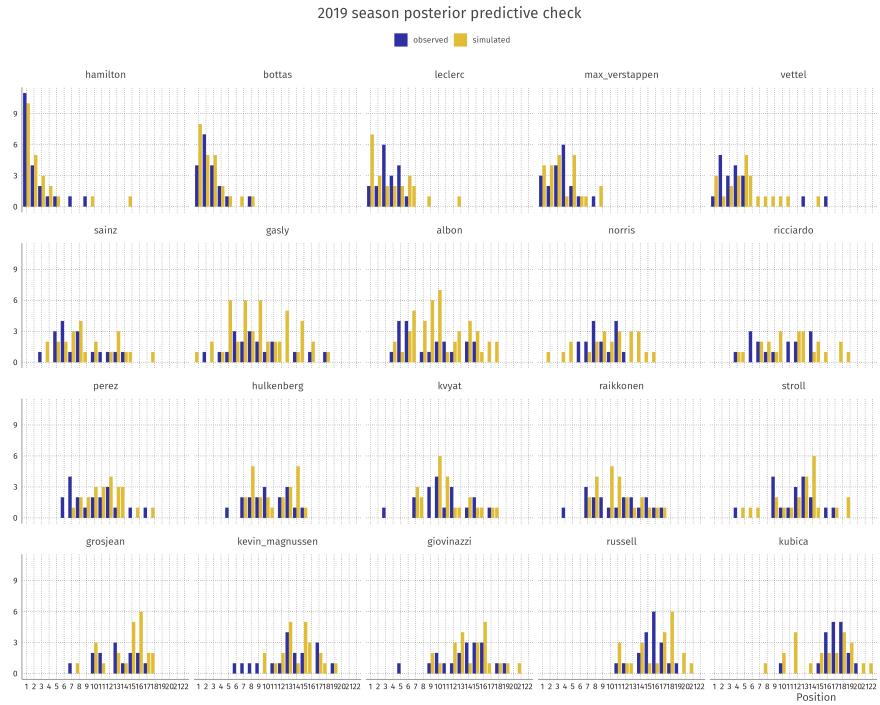


Figure 12: Posterior predictive check for driver finish position in the 2019 season.