$$p\left(\mathbf{y},\boldsymbol{\beta},\sigma^2,\boldsymbol{\mu},\Sigma;\Theta\right)=p\left(\sigma^2\right)p\left(\boldsymbol{\mu},\Sigma;\Theta\right)\prod_{i=1}^{n_s}p\left(\mathbf{y}_i\mid\boldsymbol{\beta}_i,\sigma^2\right)p\left(\boldsymbol{\beta}_i\mid\boldsymbol{\mu},\Sigma\right)$$

$$\begin{aligned}
p\left(\boldsymbol{\beta}_i\mid\mathbf{y}_i,\sigma^2,\boldsymbol{\mu},\Sigma;\Theta\right) &\propto p\left(\mathbf{y}_i\mid\boldsymbol{\beta}_i,\sigma^2\right)p\left(\boldsymbol{\beta}_i\mid\boldsymbol{\mu},\Sigma\right)\\
&=\exp\left(-\frac{1}{2\sigma^2}\sum_{t=1}^{n_i}\left(y_{it}-x_{it}^{\top}\boldsymbol{\beta}_i\right)^{\top}\left(y_{it}-x_{it}^{\top}\boldsymbol{\beta}_i\right)\right)\exp\left(-\frac{1}{2}\left(\boldsymbol{\beta}_i-\boldsymbol{\mu}\right)^{\top}\Sigma^{-1}\left(\boldsymbol{\beta}_i-\boldsymbol{\mu}\right)\right)\\
&=\exp\left(-\frac{1}{2\sigma^2}\left(\mathbf{y}_i-X_i\boldsymbol{\beta}_i\right)^{\top}\left(\mathbf{y}_i-X_i\boldsymbol{\beta}_i\right)\right)\exp\left(-\frac{1}{2}\left(\boldsymbol{\beta}_i-\boldsymbol{\mu}\right)^{\top}\Sigma^{-1}\left(\boldsymbol{\beta}_i-\boldsymbol{\mu}\right)\right)\\
&=\exp\left(-\frac{1}{2}\left(\left(\mathbf{y}_i-X_i\boldsymbol{\beta}_i\right)^{\top}\left(\sigma^2 I\right)^{-1}\left(\mathbf{y}_i-X_i\boldsymbol{\beta}_i\right)+\left(\boldsymbol{\beta}_i-\boldsymbol{\mu}\right)^{\top}\Sigma^{-1}\left(\boldsymbol{\beta}_i-\boldsymbol{\mu}\right)\right)\right)\\
&\propto\exp\left(-\frac{1}{2}\left(-2\sigma^{-2}\mathbf{y}_i^{\top}X_i\boldsymbol{\beta}_i+\sigma^{-2}\boldsymbol{\beta}_i^{\top}X_i^{\top}X_i\boldsymbol{\beta}_i+\boldsymbol{\beta}_i^{\top}\Sigma^{-1}\boldsymbol{\beta}_i-2\boldsymbol{\mu}^{\top}\Sigma^{-1}\boldsymbol{\beta}_i\right)\right)\\
&=\exp\left(-\frac{1}{2}\left(\boldsymbol{\beta}_i^{\top}\left(\sigma^{-2}X_i^{\top}X_i+\Sigma^{-1}\right)\boldsymbol{\beta}_i-2\left(\boldsymbol{\mu}^{\top}\Sigma^{-1}+\sigma^{-2}\mathbf{y}_i^{\top}X_i\right)\boldsymbol{\beta}_i\right)\right)\\
&\sim\mathcal{N}\left(\left(\sigma^{-2}X_i^{\top}X_i+\Sigma^{-1}\right)\left(\boldsymbol{\mu}^{\top}\Sigma^{-1}+\sigma^{-2}\mathbf{y}_i^{\top}X_i\right),\left(\sigma^{-2}X_i^{\top}X_i+\Sigma^{-1}\right)^{-1}\right)
\end{aligned}$$

$$\begin{aligned}
p\left(\sigma^2\mid\mathbf{y},\boldsymbol{\beta},\boldsymbol{\mu},\Sigma;\Theta\right) &\propto p\left(\mathbf{y}\mid\boldsymbol{\beta},\sigma^2\right)p\left(\sigma^2\right)=p\left(\sigma^2\right)\prod_{i=1}^{n_s}p\left(\mathbf{y}_i\mid\boldsymbol{\beta}_i,\sigma^2\right)\\
&\propto\sigma^{-2\frac{(n+1)}{2}}\exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^{n_s}\sum_{t=1}^{n_t}\left(y_{it}-x_{it}^{\top}\boldsymbol{\beta}_i\right)^{\top}\left(y_{it}-x_{it}^{\top}\boldsymbol{\beta}_i\right)\right)\\
&\sim\mathrm{I}\Gamma\left(\frac{n}{2},\frac{1}{2}\sum_{i=1}^{n_s}\sum_{t=1}^{n_t}\left(y_{it}-x_{it}^{\top}\boldsymbol{\beta}_i\right)^{\top}\left(y_{it}-x_{it}^{\top}\boldsymbol{\beta}_i\right)\right)
\end{aligned}$$

$$p\left(\boldsymbol{\mu},\Sigma\mid\mathbf{y},\boldsymbol{\beta},\sigma^2;\Theta\right)\propto\mathcal{NW}\left(\mathbf{m},v,C,d\right)$$

$$\begin{aligned}
\mathbf{m}&=\frac{v_0}{v_0+n_s}\mathbf{m}_0+\frac{n_s}{v_0+n_s}\overline{\boldsymbol{\beta}}\\
v&=v_0+n_s\\
d&=d_0+n_s\\
C&=C_0+\sum_{i=1}^{n_s}\left(\mathbf{y}_i-X_i\boldsymbol{\beta}_i\right)^{\top}\left(\mathbf{y}_i-X_i\boldsymbol{\beta}_i\right)+\frac{v_0 n_s}{v_0+d}\left(\overline{\boldsymbol{\beta}}-\mathbf{m}_0\right)\left(\overline{\boldsymbol{\beta}}-\mathbf{m}_0\right)^{\top}
\end{aligned}$$