$$\mathbf{y} \mid \boldsymbol{\theta} \sim \mathcal{N}\left(\mathbf{R}\boldsymbol{\theta}, \boldsymbol{\Sigma}\right), \;\; \boldsymbol{\theta} \sim \mathcal{N}\left(\mathbf{m}, \mathbf{V}\right)$$

$$\begin{aligned} p\left(\mathbf{z}\right) &= \frac{1}{\left(2\pi\right)^{d/2}\sqrt{\left|\mathbf{C}\right|}} \exp\left\{-\frac{1}{2}\left(\mathbf{z}-\boldsymbol{\mu}\right)^{\top}\mathbf{C}^{-1}\left(\mathbf{z}-\boldsymbol{\mu}\right)\right\} \\ &= \exp\left(\zeta + \boldsymbol{\mu}^{\top}\mathbf{C}^{-1}\mathbf{z} - \frac{1}{2}\mathbf{z}^{\top}\mathbf{C}^{-1}\mathbf{z}\right) \end{aligned}$$

$$\mathbf{y} \mid \boldsymbol{\theta} \sim \mathcal{N}\left(\mathbf{R}\boldsymbol{\theta}, \boldsymbol{\Sigma}\right) = \exp\left(\zeta_1 - \frac{1}{2}\boldsymbol{\theta}^{\top}\mathbf{R}^{\top}\boldsymbol{\Sigma}^{-1}\mathbf{R}\boldsymbol{\theta} + \boldsymbol{\theta}^{\top}\mathbf{R}^{\top}\boldsymbol{\Sigma}^{-1}\mathbf{y} - \frac{1}{2}\mathbf{y}^{\top}\boldsymbol{\Sigma}^{-1}\mathbf{y}\right)$$

$$\boldsymbol{\theta} \sim \mathcal{N}\left(\mathbf{m}, \mathbf{V}\right) = \exp\left(\zeta_2 + \mathbf{m}^{\top}\mathbf{V}^{-1}\boldsymbol{\theta} - \frac{1}{2}\boldsymbol{\theta}^{\top}\mathbf{V}^{-1}\boldsymbol{\theta}\right)$$

$$\begin{aligned} p\left(\mathbf{y}, \boldsymbol{\theta}\right) &= p\left(\mathbf{y} \mid \boldsymbol{\theta}\right) p\left(\boldsymbol{\theta}\right) \\ &= \exp\left(\zeta_1 - \frac{1}{2}\boldsymbol{\theta}^{\top}\mathbf{R}^{\top}\boldsymbol{\Sigma}^{-1}\mathbf{R}\boldsymbol{\theta} + \boldsymbol{\theta}^{\top}\mathbf{R}^{\top}\boldsymbol{\Sigma}^{-1}\mathbf{y} - \frac{1}{2}\mathbf{y}^{\top}\boldsymbol{\Sigma}^{-1}\mathbf{y}\right)\exp\left(\zeta_2 + \mathbf{m}^{\top}\mathbf{V}^{-1}\boldsymbol{\theta} - \frac{1}{2}\boldsymbol{\theta}^{\top}\mathbf{V}^{-1}\boldsymbol{\theta}\right) \\ &= \exp\left(\left(\zeta_1 + \zeta_2\right) + \left(\boldsymbol{\theta}^{\top}\mathbf{R}^{\top}\boldsymbol{\Sigma}^{-1}\mathbf{y} + \mathbf{m}^{\top}\mathbf{V}^{-1}\boldsymbol{\theta}\right) - \frac{1}{2}\left(\mathbf{y}^{\top}\boldsymbol{\Sigma}^{-1}\mathbf{y} + \boldsymbol{\theta}^{\top}\left(\mathbf{V}^{-1} + \mathbf{R}^{\top}\boldsymbol{\Sigma}^{-1}\mathbf{R}\right)\boldsymbol{\theta}\right)\right) \\ &= \exp\left(\left(\zeta_1 + \zeta_2\right) + \left(\boldsymbol{\theta}^{\top}\left(\mathbf{R}^{\top}\boldsymbol{\Sigma}^{-1} + c\right)\mathbf{y} + \mathbf{m}^{\top}\mathbf{V}^{-1}\boldsymbol{\theta}\right) - \frac{1}{2}\begin{bmatrix}\mathbf{y} & \boldsymbol{\theta}\end{bmatrix}^{\top}\begin{bmatrix}\boldsymbol{\Sigma}^{-1} & c \\ c^{\top} & \mathbf{V}^{-1} + \mathbf{R}^{\top}\boldsymbol{\Sigma}^{-1}\mathbf{R}\end{bmatrix}\begin{bmatrix}\mathbf{y} \\ \boldsymbol{\theta}\end{bmatrix}\right) \\ &= \exp\left(\left(\zeta_1 + \zeta_2\right) + \begin{bmatrix}a & b\end{bmatrix}^{\top}\begin{bmatrix}\boldsymbol{\Sigma}^{-1} & c \\ c^{\top} & \mathbf{V}^{-1} + \mathbf{R}^{\top}\boldsymbol{\Sigma}^{-1}\mathbf{R}\end{bmatrix}\begin{bmatrix}\mathbf{y} \\ \boldsymbol{\theta}\end{bmatrix} - \frac{1}{2}\begin{bmatrix}\mathbf{y} & \boldsymbol{\theta}\end{bmatrix}^{\top}\begin{bmatrix}\boldsymbol{\Sigma}^{-1} & c \\ c^{\top} & \mathbf{V}^{-1} + \mathbf{R}^{\top}\boldsymbol{\Sigma}^{-1}\mathbf{R}\end{bmatrix}\begin{bmatrix}\mathbf{y} \\ \boldsymbol{\theta}\end{bmatrix}\right) \end{aligned}$$

$$\left(\mathbf{y}, \boldsymbol{\theta}\right) \sim \mathcal{N}\left(\begin{bmatrix}a \\ b\end{bmatrix}, \begin{bmatrix}\boldsymbol{\Sigma}^{-1} & c \\ c^{\top} & \mathbf{V}^{-1} + \mathbf{R}^{\top}\boldsymbol{\Sigma}^{-1}\mathbf{R}\end{bmatrix}\right)$$