$$p(\theta, w \mid \mathbf{y}) \propto p(\mathbf{y} \mid \theta, w)\, p(\theta \mid w)\, p(w)$$

$$\mathbf{y} \mid \theta, w \sim \mathcal{N}\left(\theta, w^{-1}\right),\ \theta \mid w \sim \mathcal{N}\left(\mu, (w\kappa)^{-1}\right),\ w \sim \text{InvGamma}\left(\frac{d}{2}, \frac{\eta}{2}\right)$$

$$\propto w^{n/2} \exp\left(-\frac{1}{2}\sum_{i=1}^{n}\left(\frac{y_i - \theta}{\sqrt{w^{-1}}}\right)^2\right) w^{1/2} \exp\left(-\frac{1}{2}\left(\frac{\theta - \mu}{\sqrt{(w\kappa)^{-1}}}\right)^2\right) w^{d/2-1} \exp\left(-w\frac{\eta}{2}\right)$$

$$\propto w^{(n+d+1)/2-1} \exp\left(-\frac{w}{2}\left((\mathbf{y} - \theta\mathbf{1})^\top (\mathbf{y} - \theta\mathbf{1}) + \kappa(\theta - \mu)^2\right)\right) \exp\left(-w\frac{\eta}{2}\right)$$

$$\propto w^{(n+d+1)/2-1} \exp\left(-\frac{w}{2}\left(\left(\mathbf{y}^\top\mathbf{y} - 2\theta n\bar{y} + n\theta^2\right) + \kappa\left(\theta^2 - 2\mu\theta + \mu^2\right)\right)\right) \exp\left(-w\frac{\eta}{2}\right)$$

$$\propto w^{(n+d+1)/2-1} \exp\left(-\frac{w}{2}\left((n+\kappa)\theta^2 - 2\theta\left(n\bar{y} + \mu\kappa\right)\right)\right) \exp\left(-\frac{w}{2}\left(\mathbf{y}^\top\mathbf{y} + \kappa\mu^2 + \eta\right)\right)$$

$$\propto w^{(n+d+1)/2-1} \exp\left(-(n+\kappa)\frac{w}{2}\left(\theta^2 - 2\theta\left(\frac{n\bar{y} + \mu\kappa}{n+\kappa}\right)\right)\right) \exp\left(-\frac{w}{2}\left(\mathbf{y}^\top\mathbf{y} + \kappa\mu^2 + \eta\right)\right)$$

$$\propto w^{(n+d+1)/2-1} \exp\left(-w(n+\kappa)\frac{\left(\theta - \frac{n\bar{y} + \mu\kappa}{n+\kappa}\right)^2}{2}\right) \exp\left(-\frac{w}{2}\left(\mathbf{y}^\top\mathbf{y} + \kappa\mu^2 + \eta\right)\right)$$

$$d^* = n + d,\ \eta^* = \mathbf{y}^\top\mathbf{y} + \kappa\mu^2 + \eta,\ \kappa^* = (n+\kappa),\ \mu^* = \bar{y}\frac{n}{n+\kappa} + \frac{\kappa}{n+\kappa}\mu$$