

$$p(\theta, w | \mathbf{y}) \propto p(\mathbf{y} | \theta, w) p(\theta | w) p(w)$$

$$\mathbf{y} | \theta, w \sim \mathcal{N}(\theta, w^{-1}), \theta | w \sim \mathcal{N}(\mu, (w\kappa)^{-1}), w \sim \text{InvGamma}\left(\frac{d}{2}, \frac{\eta}{2}\right)$$

$$\propto w^{n/2} \exp\left(-\frac{1}{2} \sum_{i=1}^n \left(\frac{y_i - \theta}{\sqrt{w^{-1}}}\right)^2\right) w^{1/2} \exp\left(-\frac{1}{2} \left(\frac{\theta - \mu}{\sqrt{(w\kappa)^{-1}}}\right)^2\right) w^{d/2-1} \exp\left(-w \frac{\eta}{2}\right)$$

$$\propto w^{(n+d+1)/2-1} \exp\left(-\frac{w}{2} \left((\mathbf{y} - \theta \mathbf{1})^\top (\mathbf{y} - \theta \mathbf{1}) + \kappa (\theta - \mu)^2\right)\right) \exp\left(-w \frac{\eta}{2}\right)$$

$$\propto w^{(n+d+1)/2-1} \exp\left(-\frac{w}{2} \left((\mathbf{y}^\top \mathbf{y} - 2\theta n\bar{y} + n\theta^2) + \kappa (\theta^2 - 2\mu\theta + \mu^2)\right)\right) \exp\left(-w \frac{\eta}{2}\right)$$

$$\propto w^{(n+d+1)/2-1} \exp\left(-\frac{w}{2} \left((n + \kappa)\theta^2 - 2\theta(n\bar{y} + \mu\kappa)\right)\right) \exp\left(-\frac{w}{2} (\mathbf{y}^\top \mathbf{y} + \kappa\mu^2 + \eta)\right)$$

$$\propto w^{(n+d+1)/2-1} \exp\left(-(n + \kappa) \frac{w}{2} \left(\theta^2 - 2\theta \left(\frac{n\bar{y} + \mu\kappa}{n + \kappa}\right) + \left(\frac{n\bar{y} + \mu\kappa}{n + \kappa}\right)^2\right)\right) \exp\left(-\frac{w}{2} \left(\mathbf{y}^\top \mathbf{y} + \kappa\mu^2 + \eta - \frac{(n\bar{y} + \mu\kappa)^2}{n + \kappa}\right)\right)$$

$$\propto w^{(n+d+1)/2-1} \exp\left(-w(n + \kappa) \frac{\left(\theta - \frac{n\bar{y} + \mu\kappa}{n + \kappa}\right)^2}{2}\right) \exp\left(-\frac{w}{2} \left(\mathbf{y}^\top \mathbf{y} + \kappa\mu^2 + \eta - \frac{(n\bar{y} + \mu\kappa)^2}{n + \kappa}\right)\right)$$

$$d^* = n + d, \eta^* = \mathbf{y}^\top \mathbf{y} + \kappa\mu^2 + \eta - \frac{(n\bar{y} + \mu\kappa)^2}{n + \kappa}, \kappa^* = (n + \kappa), \mu^* = \bar{y} \frac{n}{n + \kappa} + \frac{\kappa}{n + \kappa} \mu$$