$$p(\theta, w \mid \mathbf{y}) \propto w^{(n+d+1)/2-1} \exp\left(-w(n+\kappa) \frac{\left(\theta - \frac{n\bar{y} + \mu\kappa}{n+\kappa}\right)^2}{2} \right) \exp\left(-\frac{w}{2}\left(\mathbf{y}^\top \mathbf{y} + \kappa\mu^2 + \eta\right)\right)$$

$$p(w \mid \mathbf{y}) \propto \int_{-\infty}^{\infty} w^{(n+d+1)/2-1} \exp\left(-w(n+\kappa) \frac{\left(\theta - \frac{n\bar{y} + \mu\kappa}{n+\kappa}\right)^2}{2} \right) \exp\left(-\frac{w}{2}\left(\mathbf{y}^\top \mathbf{y} + \kappa\mu^2 + \eta\right)\right) d\theta$$

$$\propto w^{(n+d+1)/2-1} \exp\left(-\frac{w}{2}\left(\mathbf{y}^\top \mathbf{y} + \kappa\mu^2 + \eta\right)\right) \int_{-\infty}^{\infty} \exp\left(-w(n+\kappa) \frac{\left(\theta - \frac{n\bar{y} + \mu\kappa}{n+\kappa}\right)^2}{2} \right) d\theta$$

$$\propto w^{(n+d+1)/2-1} \exp\left(-\frac{w}{2}\left(\mathbf{y}^\top \mathbf{y} + \kappa\mu^2 + \eta\right)\right) \int_{-\infty}^{\infty} \exp\left(-\frac{1}{2}\left(\frac{\theta - \frac{n\bar{y} + \mu\kappa}{n+\kappa}}{\sqrt{\left(w(n+\kappa)\right)^{-1}}} \right)^2 \right) d\theta$$

$$\propto w^{(n+d+1)/2-1} \exp\left(-\frac{w}{2}\left(\mathbf{y}^\top \mathbf{y} + \kappa\mu^2 + \eta\right)\right) \left(w(n+\kappa)\right)^{-1}$$

$$\propto w^{(n+d+1)/2-2} \exp\left(-\frac{w}{2}\left(\mathbf{y}^\top \mathbf{y} + \kappa\mu^2 + \eta\right)\right)$$

$$p(w \mid \mathbf{y}) \sim \Gamma\left(\frac{n+d-2}{2}, \frac{\mathbf{y}^\top \mathbf{y} + \kappa\mu^2 + \eta}{2} \right)$$