$$p(\theta, w \mid \mathbf{y}) \propto w^{(n+d+1)/2-1} \exp\left(-w(n+\kappa) \frac{\left(\theta - \frac{n\bar{y} + \mu\kappa}{n+\kappa}\right)^2}{2} \right) \exp\left(-\frac{w}{2}\left(\mathbf{y}^\top \mathbf{y} + \kappa\mu^2 + \eta\right)\right)$$

$$p(\theta \mid \mathbf{y}) \propto \int_0^\infty w^{(n+d+1)/2-1} \exp\left(-w(n+\kappa) \frac{\left(\theta - \frac{n\bar{y} + \mu\kappa}{n+\kappa}\right)^2}{2} \right) \exp\left(-\frac{w}{2}\left(\mathbf{y}^\top \mathbf{y} + \kappa\mu^2 + \eta\right)\right)$$

$$\propto \int_0^\infty w^{(n+d+1)/2-1} \exp\left(-\frac{w}{2}\left((n+\kappa)\left(\theta - \frac{n\bar{y} + \mu\kappa}{n+\kappa}\right)^2 + \left(\mathbf{y}^\top \mathbf{y} + \kappa\mu^2 + \eta\right)\right)\right) dw$$

$$\propto \Gamma\left(\frac{n+d+1}{2}\right)\left((n+\kappa)\left(\theta - \frac{n\bar{y} + \mu\kappa}{n+\kappa}\right)^2 + \left(\mathbf{y}^\top \mathbf{y} + \kappa\mu^2 + \eta\right)\right)^{-1}$$

$$\propto \left(\mathbf{y}^\top \mathbf{y} + \kappa\mu^2 + \eta\right)^{-1} \Gamma\left(\frac{n+d+1}{2}\right)\left(\frac{(n+\kappa)(n+d)}{\left(\mathbf{y}^\top \mathbf{y} + \kappa\mu^2 + \eta\right)} \frac{1}{n+d}\left(\theta - \frac{n\bar{y} + \mu\kappa}{n+\kappa}\right)^2 + 1\right)^{-1}$$

$$\propto \left(\mathbf{y}^\top \mathbf{y} + \kappa\mu^2 + \eta\right)^{-1} \Gamma\left(\frac{n+d+1}{2}\right)\left(\frac{1}{n+d}\left(\frac{\theta - \frac{n\bar{y} + \mu\kappa}{n+\kappa}}{\sqrt{\frac{\left(\mathbf{y}^\top \mathbf{y} + \kappa\mu^2 + \eta\right)}{(n+\kappa)(n+d)}}}\right)^2 + 1\right)^{-1}$$