$$p\left(\mathbf{y}_i, \boldsymbol{\beta}_i, \sigma^2, \boldsymbol{\mu}, \Sigma; \Theta\right) = p\left(\mathbf{y}_i \mid \boldsymbol{\beta}_i, \sigma^2\right) p\left(\boldsymbol{\beta}_i \mid \boldsymbol{\mu}, \Sigma\right) p\left(\sigma^2\right) p\left(\boldsymbol{\mu}, \Sigma; \Theta\right)$$

$$p\left(\boldsymbol{\beta}_i \mid \mathbf{y}_i, \sigma^2, \boldsymbol{\mu}, \Sigma; \Theta\right) \propto p\left(\mathbf{y}_i \mid \boldsymbol{\beta}_i, \sigma^2\right) p\left(\boldsymbol{\beta}_i \mid \boldsymbol{\mu}, \Sigma\right)$$

$$= \exp\left(-\frac{1}{2\sigma^2} \sum_{t=1}^{n_i} \left(y_{it} - x_{it}^\top \boldsymbol{\beta}_i\right)^\top \left(y_{it} - x_{it}^\top \boldsymbol{\beta}_i\right)\right) \exp\left(-\frac{1}{2}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)^\top \Sigma^{-1}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)\right)$$

$$= \exp\left(-\frac{1}{2\sigma^2}\left(\mathbf{y}_i - X_i \boldsymbol{\beta}_i\right)^\top \left(\mathbf{y}_i - X_i \boldsymbol{\beta}_i\right)\right) \exp\left(-\frac{1}{2}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)^\top \Sigma^{-1}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)\right)$$

$$= \exp\left(-\frac{1}{2}\left(\left(\mathbf{y}_i - X_i \boldsymbol{\beta}_i\right)^\top \left(\sigma^2 I\right)^{-1}\left(\mathbf{y}_i - X_i \boldsymbol{\beta}_i\right) + \left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)^\top \Sigma^{-1}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)\right)\right)$$

$$\propto \exp\left(-\frac{1}{2}\left(-2\sigma^{-2} \mathbf{y}_i^\top X_i \boldsymbol{\beta}_i + \sigma^{-2} \boldsymbol{\beta}_i^\top X_i^\top X_i \boldsymbol{\beta}_i + \boldsymbol{\beta}_i^\top \Sigma^{-1} \boldsymbol{\beta}_i - 2\boldsymbol{\mu}^\top \Sigma^{-1} \boldsymbol{\beta}_i\right)\right)$$

$$= \exp\left(-\frac{1}{2}\left(\boldsymbol{\beta}_i^\top \left(\sigma^{-2} X_i^\top X_i + \Sigma^{-1}\right)\boldsymbol{\beta}_i - 2\left(\boldsymbol{\mu}^\top \Sigma^{-1} + \sigma^{-2} \mathbf{y}_i^\top X_i\right)\boldsymbol{\beta}_i\right)\right)$$

$$\sim \mathcal{N}\left(\left(\sigma^{-2} X_i^\top X_i + \Sigma^{-1}\right)\left(\boldsymbol{\mu}^\top \Sigma^{-1} + \sigma^{-2} \mathbf{y}_i^\top X_i\right), \left(\sigma^{-2} X_i^\top X_i + \Sigma^{-1}\right)^{-1}\right)$$

$$p\left(\sigma^2 \mid \mathbf{y}, \boldsymbol{\beta}_i, \boldsymbol{\mu}, \Sigma; \Theta\right) \propto p\left(\mathbf{y} \mid \boldsymbol{\beta}_i, \sigma^2\right) p\left(\sigma^2\right)$$

$$\propto \sigma^{-2\frac{(n_i+1)}{2}} \exp\left(-\frac{1}{2\sigma^2} \sum_{i=1}^{n_s}\left(\mathbf{y}_i - X_i \boldsymbol{\beta}_i\right)^\top \left(\mathbf{y}_i - X_i \boldsymbol{\beta}_i\right)\right)$$

$$\sim \mathrm{I}\Gamma\left(\frac{n_i}{2}, \frac{1}{2} \sum_{i=1}^{n_s}\left(\mathbf{y}_i - X_i \boldsymbol{\beta}_i\right)^\top \left(\mathbf{y}_i - X_i \boldsymbol{\beta}_i\right)\right)$$

$$p\left(\boldsymbol{\mu}, \Sigma \mid \mathbf{y}, \boldsymbol{\beta}, \sigma^2; \Theta\right) \propto \mathcal{NW}\left(\mathbf{m}, v, C, d\right)$$

$$\mathbf{m} = \frac{v_0}{v_0 + n_s} \mathbf{m}_0 + \frac{n_s}{v_0 + n_s} \bar{\boldsymbol{\beta}}$$

$$v = v_0 + n_s$$

$$d = d_0 + n_s$$

$$C = C_0 + \sum_{i=1}^{n_s}\left(\mathbf{y}_i - X_i \boldsymbol{\beta}_i\right)^\top \left(\mathbf{y}_i - X_i \boldsymbol{\beta}_i\right) + \frac{v_0 n_s}{v_0 + d}\left(\bar{\boldsymbol{\beta}} - \mathbf{m}_0\right)\left(\bar{\boldsymbol{\beta}} - \mathbf{m}_0\right)^\top$$