$$p(\beta \mid \mathbf{y}) = \frac{p(\beta, \mathbf{y})}{p(\mathbf{y})} = \frac{\int_0^\infty p(\beta, w, \mathbf{y})\,dw}{p(\mathbf{y})} = \frac{\int_0^\infty p(\beta \mid w, \mathbf{y})\,p(w, \mathbf{y})\,dw}{p(\mathbf{y})}$$

$$= \frac{\int_0^\infty p(\beta \mid w, \mathbf{y})\,p(w \mid \mathbf{y})\,p(\mathbf{y})\,dw}{p(\mathbf{y})} = \int_0^\infty p(\beta \mid w, \mathbf{y})\,p(w \mid \mathbf{y})\,dw$$

$$\begin{aligned}
p(\beta \mid \mathbf{y}) &= \int_0^\infty p(\beta \mid w, \mathbf{y})\,p(w \mid y)\,dw \\
&\propto \int_0^\infty w^{d^*/2-1} \exp\left(-\frac{w}{2}(\boldsymbol{\beta} - \boldsymbol{\mu}^*)^\top \kappa^* (\boldsymbol{\beta} - \boldsymbol{\mu}^*)\right) \exp\left(-w\frac{\eta^*}{2}\right) dw \\
&\propto \int_0^\infty w^{d^*/2-1} \exp\left(-\frac{w}{2}\left((\boldsymbol{\beta} - \boldsymbol{\mu}^*)^\top \kappa^* (\boldsymbol{\beta} - \boldsymbol{\mu}^*) + \eta^*\right)\right) dw \\
&\propto \Gamma\left(\frac{d^*}{2}\right)\left((\boldsymbol{\beta} - \boldsymbol{\mu}^*)^\top \kappa^* (\boldsymbol{\beta} - \boldsymbol{\mu}^*) + \eta^*\right)^{-d^*/2} \\
&\propto (\eta^*)^{-d^*/2}\,\Gamma\left(\frac{d^*}{2}\right)\left(\frac{1}{\nu}\frac{(\boldsymbol{\beta} - \boldsymbol{\mu}^*)^\top \kappa^* (\boldsymbol{\beta} - \boldsymbol{\mu}^*)}{\eta^*/\nu} + 1\right)^{-(\nu+p)/2} \qquad (d^* = n + d + p = \nu + p)
\end{aligned}$$

$$\boldsymbol{\mu} = \boldsymbol{\mu}^*, \boldsymbol{\Sigma} = \left(\frac{\nu}{\eta^*}\kappa^*\right)^{-1}, \nu = n + d$$

$$\beta \mid \mathbf{y} \sim \mathbf{t}_\nu(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$