

$$p(\mathbf{y}, \boldsymbol{\gamma}, \sigma^2, \Sigma; \Theta) = p(\sigma^2) p(\Sigma; \Theta) \prod_{i=1}^{n_s} p(\mathbf{y}_i | \boldsymbol{\gamma}_i, \sigma^2) p(\boldsymbol{\gamma}_i | \Sigma)$$

$$\begin{aligned} p(\boldsymbol{\gamma}_i | \mathbf{y}_i, \sigma^2, \Sigma; \Theta) &\propto p(\mathbf{y}_i | \boldsymbol{\gamma}_i, \sigma^2) p(\boldsymbol{\gamma}_i | \Sigma) \\ &= \exp\left(-\frac{1}{2\sigma^2} \sum_{t=1}^{n_i} (y_{it} - z_{it}^\top \boldsymbol{\gamma}_i)^\top (y_{it} - z_{it}^\top \boldsymbol{\gamma}_i)\right) \exp\left(-\frac{1}{2} \boldsymbol{\gamma}_i^\top \Sigma^{-1} \boldsymbol{\gamma}_i\right) \\ &= \exp\left(-\frac{1}{2\sigma^2} (\mathbf{y}_i - \mathbf{Z}_i \boldsymbol{\gamma}_i)^\top (\mathbf{y}_i - \mathbf{Z}_i \boldsymbol{\gamma}_i)\right) \exp\left(-\frac{1}{2} \boldsymbol{\gamma}_i^\top \Sigma^{-1} \boldsymbol{\gamma}_i\right) \\ &= \exp\left(-\frac{1}{2} \left((\mathbf{y}_i - \mathbf{Z}_i \boldsymbol{\gamma}_i)^\top (\sigma^2 \mathbf{I})^{-1} (\mathbf{y}_i - \mathbf{Z}_i \boldsymbol{\gamma}_i) + \boldsymbol{\gamma}_i^\top \Sigma^{-1} \boldsymbol{\gamma}_i \right)\right) \\ &\propto \exp\left(-\frac{1}{2} \left(-2\sigma^{-2} \mathbf{y}_i^\top \mathbf{Z}_i \boldsymbol{\gamma}_i + \sigma^{-2} \boldsymbol{\gamma}_i^\top \mathbf{Z}_i^\top \mathbf{Z}_i \boldsymbol{\gamma}_i + \boldsymbol{\gamma}_i^\top \Sigma^{-1} \boldsymbol{\gamma}_i \right)\right) \\ &= \exp\left(-\frac{1}{2} \left(\boldsymbol{\gamma}_i^\top (\sigma^{-2} \mathbf{Z}_i^\top \mathbf{Z}_i + \Sigma^{-1}) \boldsymbol{\gamma}_i - 2\sigma^{-2} \mathbf{y}_i^\top \mathbf{Z}_i \boldsymbol{\gamma}_i \right)\right) \\ &\sim \mathcal{N}\left(\left(\sigma^{-2} \mathbf{Z}_i^\top \mathbf{Z}_i + \Sigma^{-1}\right)^{-1} \sigma^{-2} \mathbf{y}_i^\top \mathbf{Z}_i, \left(\sigma^{-2} \mathbf{Z}_i^\top \mathbf{Z}_i + \Sigma^{-1}\right)^{-1}\right) \end{aligned}$$

$$\begin{aligned} p(\sigma^2 | \mathbf{y}, \boldsymbol{\gamma}, \Sigma; \Theta) &\propto p(\mathbf{y} | \boldsymbol{\gamma}, \sigma^2) p(\sigma^2) = p(\sigma^2) \prod_{i=1}^{n_s} p(\mathbf{y}_i | \boldsymbol{\gamma}_i, \sigma^2) \\ &\propto \sigma^{-\frac{n(n+1)}{2}} \exp\left(-\frac{1}{2\sigma^2} \sum_{i=1}^{n_s} \sum_{t=1}^{n_i} (y_{it} - z_{it}^\top \boldsymbol{\gamma}_i)^\top (y_{it} - z_{it}^\top \boldsymbol{\gamma}_i)\right) \\ &\sim \Pi\left(\frac{n}{2}, \frac{1}{2} \sum_{i=1}^{n_s} \sum_{t=1}^{n_i} (y_{it} - z_{it}^\top \boldsymbol{\gamma}_i)^\top (y_{it} - z_{it}^\top \boldsymbol{\gamma}_i)\right) \end{aligned}$$

$$p(\Sigma | \mathbf{y}, \boldsymbol{\gamma}, \sigma^2; \Theta) \propto \mathcal{IW}\left(d_0 + n_s, \mathbf{C}_0 + \sum_{i=1}^{n_s} \boldsymbol{\gamma}_i \boldsymbol{\gamma}_i^\top\right)$$