$$p(\theta, w \mid \mathbf{y}) \propto p(\mathbf{y} \mid \theta, w)\, p(\theta \mid w)\, p(w)$$

$$\mathbf{y} \mid \theta, w \sim \mathcal{N}\left(\theta, \sigma^2\right),\ \theta \mid w \sim \mathcal{N}\left(\mu, (w\kappa)^{-1}\right),\ w \sim \text{InvGamma}\left(\frac{d}{2}, \frac{\eta}{2}\right)$$

$$\propto \exp\left(-\frac{w}{2}\sum_{i=1}^{n}\left(y_i - \theta\right)^2\right)\exp\left(-\frac{1}{2}\left(\frac{\theta-\mu}{\sqrt{(w\kappa)^{-1}}}\right)^2\right) w^{(d+1)/2-1}\exp\left(-w\frac{\eta}{2}\right)$$

$$\propto w^{(d+1)/2-1}\exp\left(-\frac{w}{2}\left(\left(\mathbf{y}-\theta\mathbf{1}\right)^{\top}\left(\mathbf{y}-\theta\mathbf{1}\right)+\kappa\left(\theta-\mu\right)^2\right)\right)\exp\left(-w\frac{\eta}{2}\right)$$

$$\propto w^{(d+1)/2-1}\exp\left(-\frac{w}{2}\left(\left(\mathbf{y}^{\top}\mathbf{y}-2\theta n\bar{y}+n\theta^2\right)+\kappa\left(\theta^2-2\mu\theta+\mu^2\right)\right)\right)\exp\left(-w\frac{\eta}{2}\right)$$

$$\propto w^{(d+1)/2-1}\exp\left(-\frac{w}{2}\left(\left(n+\kappa\right)\theta^2-2\theta\left(n\bar{y}+\mu\kappa\right)\right)\right)\exp\left(-w\frac{\eta}{2}\right)$$

$$\propto w^{(d+1)/2-1}\exp\left(-\left(n+\kappa\right)\frac{w}{2}\left(\theta^2-2\theta\left(\frac{n\bar{y}+\mu\kappa}{n+\kappa}\right)\right)\right)\exp\left(-w\frac{\eta}{2}\right)$$

$$\propto w^{(d+1)/2-1}\exp\left(-w\left(n+\kappa\right)\frac{\left(\theta-\frac{n\bar{y}+\mu\kappa}{n+\kappa}\right)^2}{2}\right)\exp\left(-w\frac{\eta}{2}\right)$$

$$d^* = d,\ \eta^* = \eta,\ \kappa^* = \left(n+\kappa\right),\ \mu^* = \bar{y}\frac{n}{n+\kappa}+\frac{\kappa}{n+\kappa}\mu$$