Variance-Gamma and Monte Carlo

Rohit Arora, Ying Wang

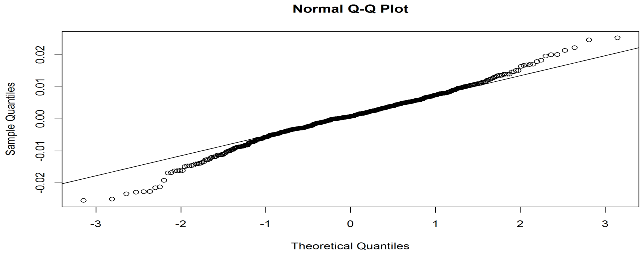
https://github.com/rpackage/VG-MonteCarlo.git

June 6, 2014

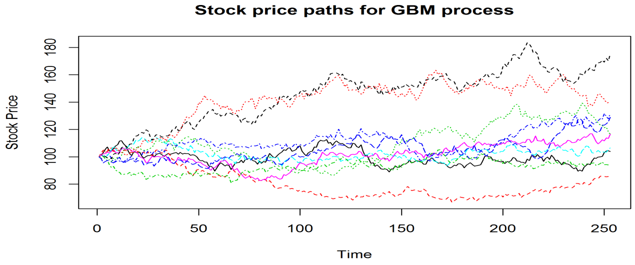
Table of contents

Motivation

- Asymmetric
- Heavy-Tailed

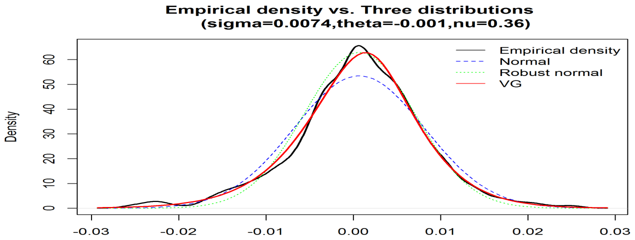


- No Jumps

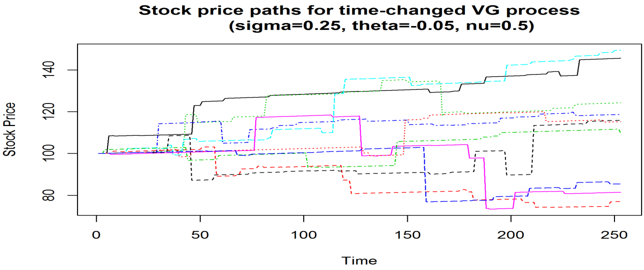


(Why VG?)

- Asymmetric
- Heavy-Tailed



- No Jumps



$VG\left(\theta, \sigma, \nu\right)$

Density

$$f\left(x\right) = \int_0^\infty \frac{1}{\sigma\sqrt{2\pi g}} \exp\left(-\frac{\left(x-\theta g\right)}{2\sigma^2 g}\right) \frac{g^{t/\nu-1}\exp\left(\frac{-g}{\nu}\right)}{\nu^{t/\nu}\Gamma\left(\frac{t}{\nu}\right)} dg$$

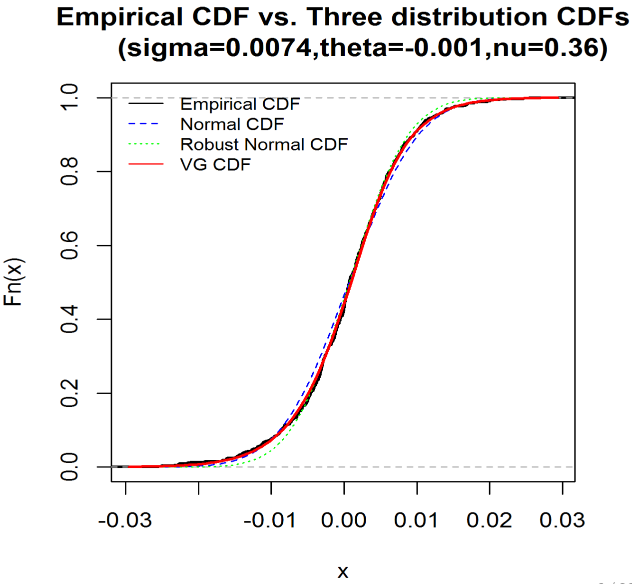
Characteristic function

$$\phi\left(u\right) = \left(\frac{1}{1-\theta\nu u + \left(\sigma^2\nu/2\right)u^2}\right)^{t/\nu}$$

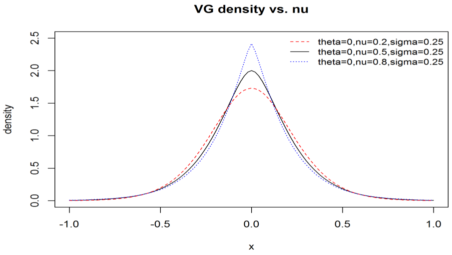
API Functions

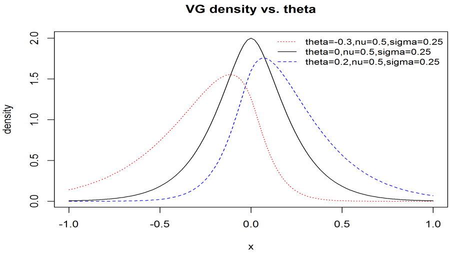
[E. Seneta (2004)]

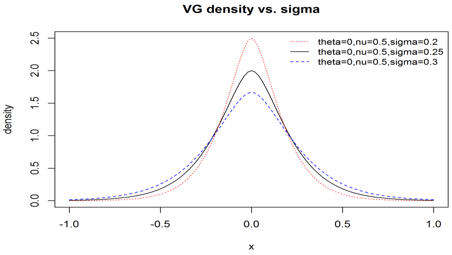
- dVG
- pVG
- qVG
- fit.VG

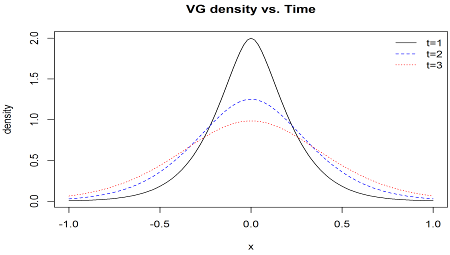


Parameter Sensitivity









$VG\left(\theta, \sigma, \nu, t\right)$

Process

$$S_t = S_0 e^{(r+\omega)t + X_t}$$

$$X_t = \theta G_t + \sigma W_{G_t} \qquad \omega = \frac{1}{\nu} \ln\left(1 - \theta\nu - \sigma^2\nu/2\right)$$

Characteristic function of $\log\left(S_T\right)$

$$\phi\left(u\right) = S_0 e^{(r+\omega)iut} \left(\frac{1}{1 - \theta\nu u + \left(\sigma^2\nu/2\right) u^2}\right)^{t/\nu}$$

Two Forms of VG process

Time Changed Brownian Motion

$$X_t = B^{(\theta,\sigma)}_{\gamma^\nu_t} = \theta\gamma^\nu_t + \sigma W_{\gamma^\nu_t}$$

Difference of Gamma Processes

$$X_t = \gamma_t^{(\mu_+,\nu_+)} - \gamma_t^{(\mu_-,\nu_-)}$$

$$\mu_{\pm} = \frac{1}{2}\left(\sqrt{\theta^2 + 2\frac{\sigma^2}{\nu}} \pm \theta\right) \qquad \nu_{\pm} = \mu_{\pm}^2\nu$$

Simulating VG Paths

1. Generate $\Delta G \sim \Gamma\left(\frac{\Delta t_i}{\nu}, \nu\right),\ Z_i \sim \mathcal{N}(0,1)$ independently
2. $X_{t_i} = X_{t_{i-1}} + \theta \Delta G_i + \sigma \sqrt{\Delta G_i} Z_i$
3. $S_{t_i} = S_0 \exp\left\{(r+\omega)\, t_i + X_{t_i}\right\}$

BenchMarking

Fourier Transform of Call Price [Carr-Madan (1999)]

$$C_T(k) = \frac{e^{-\alpha k}}{2\pi} \int_0^{\infty} e^{-iuk} \Psi_T(u)\, du$$

Fourier Transform of Asian Call Price [Fusai-Meucci (2008)]

$$G_T = \left(\prod_{i=0}^{N} S_{t_i} \right)^{1/(N+1)}$$

$$\phi_{\ln(G_T)} = \exp\left\{ i\omega \left(\ln \left(S_0 + m \frac{\Delta N}{2} \right) \right) + \sum_{k=1}^{N} \Psi_\Delta \left(\omega \frac{N-k+1}{N+1} \right) \right\}$$

Pricing with VG

	Estimated Price	Standard Error
VG Time Changed	7.3624611	0.0502044
VG Diff Gamma	7.43042828	0.05110586
VG FFT	7.402404	NA
GBM	8.05231697	0.04896859
GBM FFT	8.026385	NA

Table: European Call

	Estimated Price	Standard Error
VG Time Changed	12.00348248	0.04563806
VG Diff Gamma	11.95911826	0.04570866
VG FFT	12.03764	NA
GBM	12.73824066	0.04516489
GBM FFT	12.66162	NA

Table: European Put

Pricing with VG (Contd.)

	Estimated Price	Standard Error
VG Time Changed	2.39295990	0.02190057
VG FFT	2.393005	NA
GBM	2.73848509	0.02005997
GBM FFT	2.76867	NA

Table: Asian Call

	Estimated Price	Standard Error
VG Time Changed	10.0600321	0.0318425
VG FFT	10.00268	NA
GBM	10.37265283	0.03154842
GBM FFT	10.38007	NA

Table: Asian Put

Control Variate

Discretely monitored fixed strike Geometric option
Control Variate: Same option but using BS framework

	Estimated Price	**Standard Error**
VG Time Changed	2.395449	8.785859e-05

Table: Asian Call

	Estimated Price	**Standard Error**
VG Time Changed	10.03663	0.0001258223

Table: Asian Put

Bridge Sampling [Avramidis-L′Ecuyer (2006)]

1. $X_0 = 0 \quad \gamma_0 = 0$
2. Generate $\gamma_T \sim \Gamma\left(\frac{T}{\nu}, \nu\right), X_T \sim \mathcal{N}\left(\theta\gamma_T, \sigma^2\gamma_T\right)$
3. Loop from k = 1 to M: $n \leftarrow 2^{M-k}$

   Loop from j =1 to 2^{k-1}

   $Y_i \sim \beta\left(\frac{t_i - t_{i-n}}{\nu}, \frac{t_{i+n} - t_i}{\nu}\right)$

   $\gamma_{t_i} \leftarrow \gamma_{t_{i-n}} + Y_i\left(\gamma_{t_{i+n}} - \gamma_{t_{i-n}}\right)$

   $Z_i \sim \mathcal{N}\left(0, \left(\gamma_{t_{i+n}} - \gamma_{t_i}\right)\sigma^2 Y_i\right)$

   $X_{t_i} \leftarrow Y X_{t_{i+n}\, i} + \left(1 - Y_i\right) X_{t_{i-n}} + Z_i$

What happens when T is small or ν is large?

Importance Sampling

Exponential Twisting [Radon-Nikodym derivative]

$$\frac{dP}{dP'} = \exp\left(-t\int_{-\infty}^{+\infty}\left(k\left(x\right)-k'\left(x\right)\right)dx\right)\cdot\varphi^{+}\left(\tilde{\gamma}_t^{+}\right)\cdot\varphi^{-}\left(-\tilde{\gamma}_t^{-}\right)$$

$$\varphi^{\pm}\left(x\right) = \exp\left(2\cdot\left(\frac{\mu'_{\mp}}{\sigma'^2}-\frac{\mu_{\mp}}{\sigma^2}\right)\cdot|x|\right)$$

σ	ν	θ	σ'	ν'	θ'
0.25	-0.05	0.5	0.35	-0.1	0.5

	Estimated Price	**Standard Error**
VG Diff Gamma	0.025807280	0.001583751
VG Importance Sampling	0.0237484936	0.0005620372

Table: European Call

Greeks

Pathwise Estimation

$$\frac{dY(\chi)}{d\chi} = \frac{dE\left[e^{-rT}payoff\left(S_T\right)\right]}{d\chi} = E\left[\frac{d\left(e^{-rT}payoff\left(S_T\right)\right)}{d\chi}\right]$$

European Calls

$$\Delta = \frac{de^{-rT}(S_T - K)^+}{dS_0} = e^{-rT}\mathbf{1}_{\{S_T > K\}}\frac{S_T}{S_0}$$

$$\rho = \frac{de^{-rT}(S_T - K)^+}{dr} = e^{-rT}\mathbf{1}_{\{S_T > K\}}TK$$

$$\theta = \frac{de^{-rT}(S_T - K)^+}{d\theta} = e^{-rT}\mathbf{1}_{\{S_T > K\}}S_T\left(T\frac{d\omega}{d\theta} + \gamma_T^{(\nu)}\right)$$

$$\kappa = \frac{de^{-rT}(S_T - K)^+}{d\sigma} = e^{-rT}\mathbf{1}_{\{S_T > K\}}S_T\left(T\frac{d\omega}{d\sigma} + W_{\gamma_T^{(\nu)}}\right)$$

Greeks Results

	Estimated Value	Standard Error
Finite Difference	0.47934777	0.03566071
Pathwise	0.465494395	0.001945311
Fourier	0.4653853	NA

Table: Delta European Call

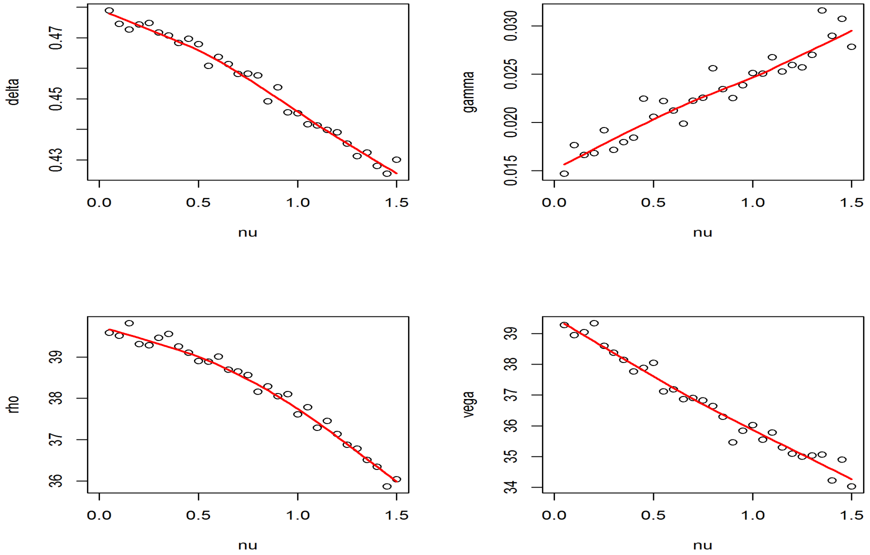
	Estimated Value	Standard Error
Finite Difference	0.09000361	0.12400942
Pathwise	0.018325135	0.001374918
Fourier	0.01978011	NA

Table: Gamma European Call

	Estimated Value	Standard Error
Finite Difference	39.6033748	0.3656007
Pathwise	38.9640694	0.1599638
Fourier	39.13612	NA

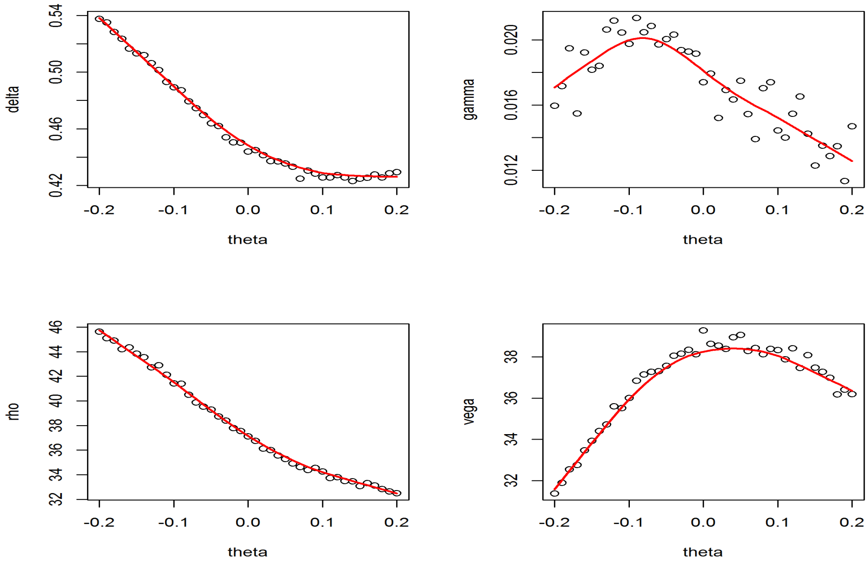
Table: Rho European Call

Greeks Vs ν

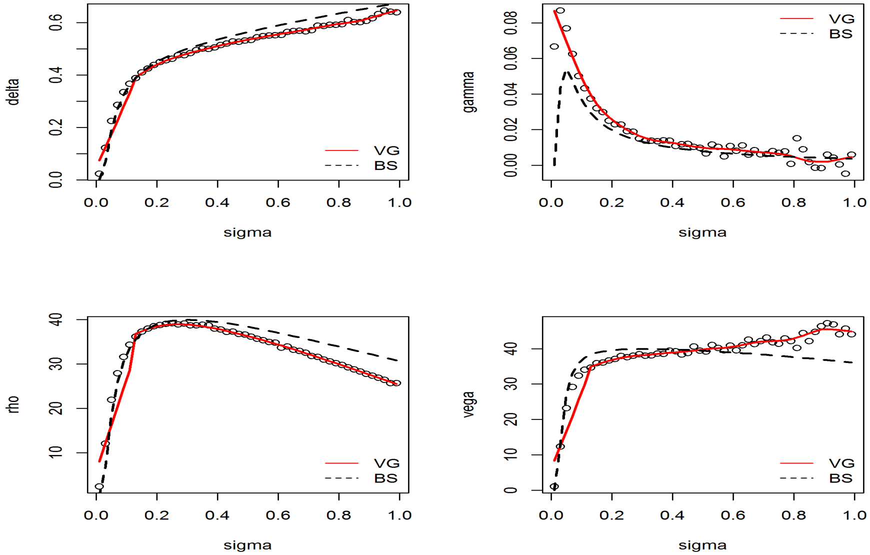


delta
0.43
0.45
0.47
0.0
0.5
1.0
1.5
nu
gamma
0.015
0.020
0.025
0.030
0.0
0.5
1.0
1.5
nu
rho
36
37
38
39
0.0
0.5
1.0
1.5
nu
vega
34
35
36
37
38
39
0.0
0.5
1.0
1.5
nu

Greeks Vs θ



Greeks Vs σ



References

1. Madan, Dilip B.; Seneta, Eugene (1990). "The Variance Gamma (V.G.) Model for Share Market Returns". Journal of Business 63 (4): 511524
2. Fu, MichaelC. "Variance-Gamma and Monte Carlo." In Advances in Mathematical Finance, edited by MichaelC Fu, RobertA Jarrow, Ju-YiJ Yen and RobertJ Elliott, 21-34: Birkhuser Boston, 2007.
3. Dilip Madan, Peter Carr, Eric Chang (1998). "The Variance Gamma Process and Option Pricing". European FinanceReview 2: 79105
4. Carr, P., and Madan, D. (1999). "Option valuation using the fast Fourier transform", The Journal of Computational Finance 2(4), 6173
5. Fusai G, Meucci, A. (2008), Pricing Discretely Monitored Asian Options under Levy Processes, Journal of Banking and Finance, 32(10), p.2076-2088
6. E. Seneta (2004). Fitting the variance-gamma model to financial data, Stochastic methods and their applications, J. Appl. Probab. 41A, 177187.

Thank you for your attention
Any Questions